New type of Diophantine neutrosophic aggregation operators and its extension

M. Palanikumar[1]**, K. Arulmozhi**[2]**, K. Sundareswari**[3]**, Aiyared Iampan**[4,*]
[1]Department of Mathematics, Saveetha School of Engineering, Saveetha University, Saveetha Institute of Medical and Technical Sciences, Chennai-602105, India

[2]Department of Mathematics, Bharath Institute of Higher Education and Research, Tamil Nadu, Chennai-600073, India

[3]Department of Mathematics, Al-Ameen Engineering College, Erode.
[4]Fuzzy Algebras and Decision-Making Problems Research Unit, School of Science, University of Phayao, 19 Moo 2, Tambon Mae Ka, Amphur Mueang, Phayao 56000, Thailand.

Emails:palanimaths86@gmail.com; arulmozhiems@gmail.com; k.sundareswari@gmail.com; aiyared.ia@up.ac.th

Abstract

We introduce the concept of new type of Diophantine neutrosophic set. A Diophatine neutrosophic set is the new type of a neutrosophic set and Diophatine fuzzy set (DioFS). We discuss Diophantine neutrosophic weighted averaging (DioNWA), Diophantine neutrosophic weighted geometric (DioNWG), generalized Diophantine neutrosophic weighted averaging(GDioNWA), generalized Diophantine neutrosophic weighted geometric (GDioNWG). In this article, we define the Euclidean distance (ED), Hamming distance (HD) and operator laws. By analyzing new type of Diophantine neutrosophic set through algebraic operations, we discuss its properties.

Keywords: DioNWA; DioNWG; GDioNWA; GDioNWG.

1 Introduction

To deal with the ambiguities, many uncertain theories including fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS)[3] and spherical FS (SFS).[4] A FS is a set of elements with membership grade (MD) in the given set values from zero to one; Later, Atanassov proposed the concept of an IFS that is divided into categories using non-membership grade (NMD), which cannot exceed one.[2] In cases where the combined MD and NMD grades are greater than one, a single problem may be conveyed to the decision making (DM). The square sum of the MD and NMD of an IFS with a value less than one is classified as PFS, which can be characterized by Yager.[3] The picture FS concept was developed by Cuong et al.[5] and its three pointers are positive MD, neutral MD, and negative MD. Furthermore, it offers more benefits than PFS and IFS. Liu et al.[6] examined the concept of certain types of picture FS with AOs, which is a generalization of picture FS. Generalized PFS with AO and applications were proposed by Liu et al.[7] The features of AOs[8] using PFS and interval value. Liu et al.[6] introduced a brand new picture FS based on AO. Rarely, the sum of the values for the positive, neutral, and negative MDs exceed one in the DM approach challenge. Ashraf et al.[4] present the concept of SFS, where the square sum of the positive, neutral, and negative grades does not exceed one. Fatmaa et al.[9] investigated the notion of SFS using TOPSIS models.

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Peng et al.[10] explore NS with MADM using MABAC and TOPSIS approaches. A generalization of PFS using TOPSIS was presented by Lhang et al.[11] A practical application of MADM is shown by Hwang et al.[12] Linear Diophantine FS (LDFS) with reference parameters were discussed by Riaz et al.[13] In contrast to other methods, the LDFS is more flexible and effective because it uses reference parameters. LDFS also classifies the data in MADM problems based on changes to the physical meaning of reference parameters. Recently, Palanikumar et al. discussed many algebraic structures and aggregation operators with applications[14]-.[15] Many researchers[16]-[17] discussed the concept of Pythagorean with its extension based on DM. Bromi et al.[18]-[19] discussed neutrosophic sets and its extension with applications. This article aims to provide a more comprehensive definition of new type of DioNNS information. We use AOs to obtain new type of DioNNS data. Then, we will use this ranking to solve decisions based on these operators. Following are the six sections of the paper. Section 2 provides concise explanations of PFS and NS concepts. The concept of new type of Diophantine neutrosophic number (new type of DioNN) addressed in Section 3. Section 4 discussed the different distance measure between new type of DioNNs. Section 5 discussed MADM using some AOs for new type of DioNN. The conclusion can be found in Section 6. As a result, this work makes the following main contributions: It introduces ED and HD measuring using new type of DioNSs. The new type of DioNN with aggregation operators.

2 Background

For our further learning, it is vital that we review a few of the important definitions in this section.

Definition 2.1. [8] Let $\mathcal{X}$ be an universal. The PIVFS $L = \Big\{\tau, \Big\langle \widetilde{\Psi_L^{\mathcal{T}}(\tau)}, \widetilde{\Psi_L^{\mathcal{F}}(\tau)} \Big\rangle \Big| \tau \in \mathcal{X}\Big\}$, where $\widetilde{\Psi_L^{T}} : \mathcal{X} \to Int([0,1])$ and $\widetilde{\Psi_L^{F}} : \mathcal{X} \to Int([0,1])$ denote the MD and NMD of $\tau \in \mathcal{X}$ to the set L, respectively, and $0 \leq (\Psi_L^{\mathcal{T}}(\tau))^2 + (\Psi_L^{\mathcal{F}}(\tau))^2 \leq 1$. For convenience, $L = \Big\langle \Big[\Psi_L^{\mathcal{T}}, \Psi_L^{\mathcal{T}}\Big], \Big[\Psi_L^{\mathcal{F}}, \Psi_L^{\mathcal{F}}\Big] \Big\rangle$ is called a Pythagorean interval-valued fuzzy number (PyIVFN).

Definition 2.2. The NS $L = \Big\{x, \big\langle \Psi_L^{\mathcal{T}}(\tau), \Psi_L^{\mathcal{I}}(\tau), \Psi_L^{\mathcal{F}}(\tau) \big\rangle \big| \tau \in \mathcal{X}\Big\}$, where $\Psi_L^{\mathcal{T}} : \mathcal{X} \to [0,1]$, $\Psi_L^{\mathcal{I}} : \mathcal{X} \to [0,1]$ and $\Psi_L^{\mathcal{F}} : \mathcal{X} \to [0,1]$ is denote the positive MD, neutral MD and negative MD of $\tau \in \mathcal{X}$, respectively and $0 \leq (\Psi_L^{\mathcal{T}}(\tau)) + (\Psi_L^{\mathcal{I}}(\tau)) + (\Psi_L^{\mathcal{F}}(\tau)) \leq 2$. For $M = \big\langle \Psi_L^{\mathcal{T}}, \Psi_L^{\mathcal{I}}, \Psi_L^{\mathcal{F}} \big\rangle$ is called a neutrosophic number (NN).

Definition 2.3. The Pythagorean NS $L = \Big\{\tau, \big\langle \Psi_L^{\mathcal{T}}(\tau), \Psi_L^{\mathcal{I}}(\tau), \Psi_L^{\mathcal{F}}(\tau) \big\rangle \big| \tau \in \mathcal{X}\Big\}$, where $\Psi_L^{\mathcal{T}} : \mathcal{X} \to [0,1]$, $\Psi_L^{\mathcal{I}} : \mathcal{X} \to [0,1]$ and $\Psi_L^{\mathcal{F}} : \mathcal{X} \to [0,1]$ is denote the positive MD, neutral MD and negative MD of $\tau \in \mathcal{X}$, respectively and $0 \leq (\Psi_L^{\mathcal{T}}(\tau))^2 + (\Psi_L^{\mathcal{I}}(\tau))^2 + (\Psi_L^{\mathcal{F}}(\tau))^2 \leq 2$. For $M = \big\langle \Psi_L^{\mathcal{T}}, \Psi_L^{\mathcal{I}}, \Psi_L^{\mathcal{F}} \big\rangle$ is called a Pythagorean neutrosophic number (PyNN).

3 Operations for new type of DioNNN

We discuss the concept of Diophantine neutrosophic number (new type of DioNN). As a result, the Diophantine neutrosophic number and its operations were defined.

Definition 3.1. The Pythagorean Diophantine neutrosophic set (PyDioNS) $L = \Big\{\tau, \Big\langle \Big(\Psi_L^{\mathcal{T}}(\tau), \Psi_L^{\mathcal{I}}(\tau), \Psi_L^{\mathcal{F}}(\tau)\Big), (\varsigma(\tau)) \Big\rangle \Big| \tau \in \mathcal{X}\Big\}$, where $\Psi_L^{T} : \mathcal{X} \to [0,1]$, $\Psi_L^{I} : \mathcal{X} \to [0,1]$ and $\Psi_L^{F} : \mathcal{X} \to [0,1]$ denote the PMD, neutral MD and NMD of $x \in \mathcal{X}$ to L, respectively and $0 \leq (\varsigma(\tau)\Psi_L^{\mathcal{T}}(\tau))^2 + (\varsigma(\tau)\Psi_L^{\mathcal{I}}(\tau))^2 + (\varsigma(\tau)\Psi_L^{\mathcal{F}}(\tau))^2 \leq 2$, where, $\varsigma \in [0,1]$. For convenience, $L = \Big\langle \Big(\Psi_L^{\mathcal{T}}, \Psi_L^{\mathcal{I}}, \Psi_L^{\mathcal{F}}\Big), \varsigma \Big\rangle$ is represent a PyDioNN.

Definition 3.2. The new type of DioNS $L = \Big\{\tau, \Big\langle \Big(\Psi_L^{\mathcal{T}}(\tau), \Psi_L^{\mathcal{I}}(\tau), \Psi_L^{\mathcal{F}}(\tau)\Big), (\varsigma(\tau)) \Big\rangle \Big| x \in \mathcal{X}\Big\}$, where $\Psi_L^{T} : \mathcal{X} \to [0,1]$, $\Psi_L^{I} : \mathcal{X} \to [0,1]$ and $\Psi_L^{F} : \mathcal{X} \to [0,1]$ denote the PMD, neutral MD and NMD of $x \in \mathcal{X}$ to L, respectively and $0 \leq (\varsigma(\tau)\Psi_L^{\mathcal{T}}(\tau))^{\aleph} + (\varsigma(\tau)\Psi_L^{\mathcal{I}}(\tau))^{\aleph} + (\varsigma(\tau)\Psi_L^{\mathcal{F}}(\tau))^{\aleph} \leq 2$, where, $\varsigma \in [0,1]$ and $\aleph \geq 1$. For convenience, $L = \Big\langle \Big(\Psi_L^{\mathcal{T}}, \Psi_L^{\mathcal{I}}, \Psi_L^{\mathcal{F}}\Big), \varsigma) \Big\rangle$ is represent a new type of DioNN.

Definition 3.3. Let $L = \langle(\Psi^{\mathcal{T}}, \Psi^{\mathcal{I}}, \Psi^{\mathcal{F}}), \varsigma\rangle$, $L_1 = \langle(\Psi_1^{\mathcal{T}}, \Psi_1^{\mathcal{I}}, \Psi_1^{\mathcal{F}}), \varsigma\rangle$ and $L_2 = \langle(\Psi_2^{\mathcal{T}}, \Psi_2^{\mathcal{I}}, \Psi_2^{\mathcal{F}}), \varsigma\rangle$ be any three new type of DioNNs, and $\aleph > 0$. Then

1. $L_1 \uplus L_2 = \begin{bmatrix} \sqrt[\aleph]{(\varsigma\Psi_1^{\mathcal{T}})^{\aleph} + (\varsigma\Psi_2^{\mathcal{T}})^{\aleph} - (\varsigma\Psi_1^{\mathcal{T}})^{\aleph} \cdot (\varsigma\Psi_2^{\mathcal{T}})^{\aleph}}, \\ \sqrt[\aleph]{(\varsigma\Psi_1^{\mathcal{I}})^{\aleph} + (\varsigma\Psi_2^{\mathcal{I}})^{\aleph} - (\varsigma\Psi_1^{\mathcal{I}})^{\aleph} \cdot (\varsigma\Psi_2^{\mathcal{I}})^{\aleph}}, \\ \varsigma\Psi_1^{\mathcal{F}} \cdot \varsigma\Psi_2^{\mathcal{F}} \end{bmatrix}$,

2. $L_1 \oslash L_2 = \begin{bmatrix} \varsigma\Psi_1^{\mathcal{T}} \cdot \varsigma\Psi_2^{\mathcal{T}}, \\ \sqrt[\aleph]{(\varsigma\Psi_1^{\mathcal{I}})^{\aleph} + (\varsigma\Psi_2^{\mathcal{I}})^{\aleph} - (\varsigma\Psi_1^{\mathcal{I}})^{\aleph} \cdot (\varsigma\Psi_2^{\mathcal{I}})^{\aleph}}, \\ \sqrt[\aleph]{(\varsigma\Psi_1^{\mathcal{F}})^{\aleph} + (\varsigma\Psi_2^{\mathcal{F}})^{\aleph} - (\varsigma\Psi_1^{\mathcal{F}})^{\aleph} \cdot (\varsigma\Psi_2^{\mathcal{F}})^{\aleph}} \end{bmatrix}$

3. $\aleph \cdot L = \left[\sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{T}})^{\aleph}\right)^{\aleph}}, \sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{I}})^{\aleph}\right)^{\aleph}}, (\varsigma\Psi^{\mathcal{F}})^{\aleph}\right]$,

4. $L^{\aleph} = \left[(\varsigma\Psi^{\mathcal{T}})^{\aleph}, \sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{I}})^{\aleph}\right)^{\aleph}}, \sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right]$.

4 Find new type of DioNN distance measure

We introduce ED and HD measures for new type of DioNNs and study mathematical properties.

Definition 4.1. For any two new type of DioNNs $L_1 = \langle(\Psi_1^{\mathcal{T}}, \Psi_1^{\mathcal{I}}, \Psi_1^{\mathcal{F}}), \varsigma\rangle$ and $L_2 = \langle(\Psi_2^{\mathcal{T}}, \Psi_2^{\mathcal{I}}, \Psi_2^{\mathcal{F}}), \varsigma\rangle$. Then

$$\mathcal{D}_E(L_1, L_2) = \frac{1}{2}\sqrt{\begin{bmatrix} \frac{1+(\varsigma\Psi_1^{\mathcal{T}})^2-(\varsigma\Psi_1^{\mathcal{I}})^2-(\varsigma\Psi_1^{\mathcal{F}})^2}{2} \\ -\frac{1+(\varsigma\Psi_2^{\mathcal{T}})^2-(\varsigma\Psi_2^{\mathcal{I}})^2-(\varsigma\Psi_2^{\mathcal{F}})^2}{2} \end{bmatrix}^2 + \frac{1}{2}\begin{bmatrix} \frac{1+(\varsigma\Psi_1^{\mathcal{T}})^2-(\varsigma\Psi_1^{\mathcal{I}})^2-(\varsigma\Psi_1^{\mathcal{F}})^2}{2} \\ -\frac{1+(\varsigma\Psi_2^{\mathcal{T}})^2-(\varsigma\Psi_2^{\mathcal{I}})^2-(\varsigma\Psi_2^{\mathcal{F}})^2}{2} \end{bmatrix}^2}$$

where $\mathcal{D}_E(L_1, L_2)$ is called the ED between L_1 and L_2.

$$\mathcal{D}_H(L_1, L_2) = \frac{1}{2}\begin{bmatrix} \left| \begin{array}{l} \frac{1+(\varsigma\Psi_1^{\mathcal{T}})^2-(\varsigma\Psi_1^{\mathcal{I}})^2-(\varsigma\Psi_1^{\mathcal{F}})^2}{2} \\ -\frac{1+(\varsigma\Psi_2^{\mathcal{T}})^2-(\varsigma\Psi_2^{\mathcal{I}})^2-(\varsigma\Psi_2^{\mathcal{F}})^2}{2} \end{array} \right| \\ +\frac{1}{2}\left| \begin{array}{l} \frac{1+(\varsigma\Psi_1^{\mathcal{T}})^2-(\varsigma\Psi_1^{\mathcal{I}})^2-(\varsigma\Psi_1^{\mathcal{F}})^2}{2} \\ -\frac{1+(\varsigma\Psi_2^{\mathcal{T}})^2-(\varsigma\Psi_2^{\mathcal{I}})^2-(\varsigma\Psi_2^{\mathcal{F}})^2}{2} \end{array} \right| \end{bmatrix}$$

where $\mathcal{D}_H(L_1, L_2)$ is called the HD between L_1 and L_2.

Theorem 4.2. *If any three new type of DioNNs* $L_1 = \langle(\Psi_1^{\mathcal{T}}, \Psi_1^{\mathcal{I}}, \Psi_1^{\mathcal{F}}), \varsigma\rangle$ *and* $L_2 = \langle(\Psi_2^{\mathcal{T}}, \Psi_2^{\mathcal{I}}, \Psi_2^{\mathcal{F}}), \varsigma\rangle$ *and* $L_3 = \langle(\Psi_3^{\mathcal{T}}, \Psi_3^{\mathcal{I}}, \Psi_3^{\mathcal{F}}), \varsigma\rangle$, *then* $\mathcal{D}_E(L_1, L_2)$ *satisfies the following valid statements.*

1. $\mathcal{D}_E(L_1, L_2) = 0$ *if and only if* $L_1 = L_2$.
2. $\mathcal{D}_E(L_1, L_2) = \mathcal{D}_E(L_2, L_1)$.
3. $\mathcal{D}_E(L_1, L_3) \leq \mathcal{D}_E(L_1, L_2) + \mathcal{D}_E(L_2, L_3)$.

Proof. It is straightforward to prove (1) and (2). Now, $\Big(\mathcal{D}_E(L_1, L_2) + \mathcal{D}_E(L_2, L_3)\Big)^2$ implies

$$\begin{aligned} &\frac{1}{4}\Big((O-P)^2 + \frac{1}{2}(O-P)^2\Big) + \frac{1}{4}\Big((P-Q)^2 + \frac{1}{2}(P-Q)^2\Big) \\ &+\frac{1}{2}\left(\sqrt{(O-P)^2 + \frac{1}{2}(O-P)^2} \times \sqrt{(P-Q)^2 + \frac{1}{2}(P-Q)^2}\right), \end{aligned}$$

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where

$$O = \frac{1 + (\varsigma\Psi_1^{\mathcal{T}})^2 - (\varsigma\Psi_1^{\mathcal{I}})^2 - (\varsigma\Psi_1^{\mathcal{F}})^2}{2},$$
$$P = \frac{1 + (\varsigma\Psi_2^{\mathcal{T}})^2 - (\varsigma\Psi_2^{\mathcal{I}})^2 - (\varsigma\Psi_2^{\mathcal{F}})^2}{2},$$
$$Q = \frac{1 + (\varsigma\Psi_3^{\mathcal{T}})^2 - (\varsigma\Psi_3^{\mathcal{I}})^2 - (\varsigma\Psi_3^{\mathcal{F}})^2}{2}.$$

Hence, $\Big(\mathcal{D}_E(L_1, L_2) + \mathcal{D}_E(L_2, L_3)\Big)^2$

$$\begin{aligned}
&\geq \frac{1}{4}\Big((O-P)^2 + \frac{1}{2}(O-P)^2\Big) + \frac{1}{4}\Big((P-Q)^2 + \frac{1}{2}(P-Q)^2\Big) \\
&\quad + \frac{1}{2}\Big((O-P) \times (P-Q) + \frac{1}{2}(O-P) \times (P-Q)\Big) \\
&= \frac{1}{4}\Big((O-P)^2 + (P-Q)^2 + 2(O-P) \times (P-Q)\Big) \\
&\quad + \frac{1}{4}\Big(\frac{1}{2}(O-P)^2 + \frac{1}{2}(P-Q)^2 + (O-P) \times (P-Q)\Big) \\
&= \frac{1}{4}(O-P+P-Q)^2 + \frac{1}{8}(O-P+P-Q)^2 \\
&= \frac{1}{4}\Big[(O-Q)^2 + \frac{1}{2}(O-Q)^2\Big] \\
&= \mathcal{D}_E(L_1, L_3)^2.
\end{aligned}$$

Corollary 4.3. *Let* $L_1 = \langle(\Psi_1^{\mathcal{T}}, \Psi_1^{\mathcal{I}}, \Psi_1^{\mathcal{F}}), \varsigma\rangle$ *and* $L_2 = \langle(\Psi_2^{\mathcal{T}}, \Psi_2^{\mathcal{I}}, \Psi_2^{\mathcal{F}}), \varsigma\rangle$ *and* $L_3 = \langle(\Psi_3^{\mathcal{T}}, \Psi_3^{\mathcal{I}}, \Psi_3^{\mathcal{F}}), \varsigma\rangle$ *be the any three new type of DioNNs. Then*

1. $\mathcal{D}_H(L_1, L_2) = 0$ *if and only if* $L_1 = L_2$.
2. $\mathcal{D}_H(L_1, L_2) = \mathcal{D}_H(L_2, L_1)$.
3. $\mathcal{D}_H(L_1, L_3) \leq \mathcal{D}_H(L_1, L_2) + \mathcal{D}_H(L_2, L_3)$.

Proof. As a result of Theorem 4.2, the proof follows.

5 AOs based on new type of DioNN

Here we describe the AOs using DioNWA, DioNWG, GDioNWA, and GDioNWG.

5.1 DioNWA

Definition 5.1. Let $L_i = \langle(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}), \varsigma\rangle$ be the new type of DioNNs, $W = (\omega_1, \omega_2, ..., \omega_n)$ be the weight of L_i, $\omega_i \geq 0$ and $\uplus_{i=1}^{n}\omega_i = 1$. Then DioNWA $(L_1, L_2, ..., L_n) = \uplus_{i=1}^{n}\omega_i L_i$.

Theorem 5.2. *Let* $L_i = \Big\langle \left(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}\right), \varsigma \Big\rangle$ *be the new type of DioNNs. Then* $DioNWA(L_1, L_2, ..., L_n)$

$$= \begin{bmatrix} \sqrt[\aleph]{1 - \odot_{i=1}^{n}\Big(1 - (\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\Big)^{\omega_i}}, \\ \sqrt[\aleph]{1 - \odot_{i=1}^{n}\Big(1 - (\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\Big)^{\omega_i}}, \odot_{i=1}^{n}(\varsigma\Psi_i^{\mathcal{F}})^{\omega_i} \end{bmatrix}.$$

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Proof. If $n = 2$, then DioNWA$(L_1, L_2) = \omega_1 L_1 \uplus \omega_2 L_2$, where

$$\omega_1 L_1 = \left[\begin{array}{c} \sqrt[\aleph]{1-\left(1-(\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\omega_1}}, \\ \sqrt[\aleph]{1-\left(1-(\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\omega_1}}; (\varsigma\Psi_1^{\mathcal{F}})^{\omega_1}, \end{array}\right]$$

$$\omega_2 L_2 = \left[\begin{array}{c} \sqrt[\aleph]{1-\left(1-(\varsigma\Psi_2^{\mathcal{T}})^{\aleph}\right)^{\omega_2}}, \\ \sqrt[\aleph]{1-\left(1-(\varsigma\Psi_2^{\mathcal{I}})^{\aleph}\right)^{\omega_2}}; (\varsigma\Psi_2^{\mathcal{F}})^{\omega_2}, \end{array}\right].$$

Now,

$$\omega_1 L_1 \uplus \omega_2 L_2 \quad = \left[\begin{array}{c} \sqrt[\aleph]{\begin{array}{c} \left(1-\left(1-(\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\omega_1}\right)+\left(1-\left(1-(\varsigma\Psi_2^{\mathcal{T}})^{\aleph}\right)^{\omega_2}\right) \\ -\left(1-\left(1-(\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\omega_1}\right)\cdot\left(1-\left(1-(\varsigma\Psi_2^{\mathcal{T}})^{\aleph}\right)^{\omega_2}\right) \end{array}}, \\ \\ \sqrt[\aleph]{\begin{array}{c} \left(1-\left(1-(\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\omega_1}\right)+\left(1-\left(1-(\varsigma\Psi_2^{\mathcal{I}})^{\aleph}\right)^{\omega_2}\right) \\ -\left(1-\left(1-(\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\omega_1}\right)\cdot\left(1-\left(1-(\varsigma\Psi_2^{\mathcal{I}})^{\aleph}\right)^{\omega_2}\right) \end{array}}, \\ \\ (\varsigma\Psi_1^{\mathcal{F}})^{\omega_1}(\varsigma\Psi_2^{\mathcal{F}})^{\omega_2}, \end{array}\right]$$

$$= \quad \left[\begin{array}{c} \sqrt[\aleph]{1-\left(1-(\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\omega_1}\left(1-(\varsigma\Psi_2^{\mathcal{T}})^{\aleph}\right)^{\omega_2}}, \\ \sqrt[\aleph]{1-\left(1-(\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\omega_1}\left(1-(\varsigma\Psi_2^{\mathcal{I}})^{\aleph}\right)^{\omega_2}}, (\varsigma\Psi_1^{\mathcal{F}})^{\omega_1}\cdot(\varsigma\Psi_2^{\mathcal{F}})^{\omega_2}, \end{array}\right]$$

Hence, $DioNWA(L_1, L_2)$

$$= \left[\begin{array}{c} \sqrt[\aleph]{1-\odot_{i=1}^{2}\left(1-(\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\omega_i}}, \\ \sqrt[\aleph]{1-\odot_{i=1}^{2}\left(1-(\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\omega_i}}; \odot_{i=1}^{2}(\varsigma\Psi_i^{\mathcal{F}})^{\omega_i}, \end{array}\right].$$

It valid for $n \geq 3$,
Thus, $DioNWA(L_1, L_2, ..., L_l)$

$$= \left[\begin{array}{c} \sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-(\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\omega_i}}, \\ \sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-(\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\omega_i}}; \odot_{i=1}^{l}(\varsigma\Psi_i^{\mathcal{F}})^{\omega_i}, \end{array}\right].$$

If $n = l + 1$, then DioNWA $(L_1, L_2, ..., L_l, L_{l+1})$

$$= \left[\begin{array}{c} \sqrt[\aleph]{\begin{array}{c} \uplus_{i=1}^{l}\left(1-\left(1-(\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\omega_i}\right)+\left(1-\left(1-(\varsigma\Psi_{l+1}^{\mathcal{T}})^{\aleph}\right)^{\omega_{l+1}}\right) \\ -\odot_{i=1}^{l}\left(1-\left(1-(\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\omega_i}\right)\cdot\left(1-\left(1-(\varsigma\Psi_{l+1}^{\mathcal{T}})^{\aleph}\right)^{\omega_{l+1}}\right) \end{array}}, \\ \\ \sqrt[\aleph]{\begin{array}{c} \uplus_{i=1}^{l}\left(1-\left(1-(\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\omega_i}\right)+\left(1-\left(1-(\varsigma\Psi_{l+1}^{\mathcal{I}})^{\aleph}\right)^{\omega_{l+1}}\right) \\ -\odot_{i=1}^{l}\left(1-\left(1-(\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\omega_i}\right)\cdot\left(1-\left(1-(\varsigma\Psi_{l+1}^{\mathcal{I}})^{\aleph}\right)^{\omega_{l+1}}\right) \end{array}}, \\ \odot_{i=1}^{l}(\varsigma\Psi_i^{\mathcal{F}})^{\omega_i}\cdot(\varsigma\Psi_{l+1}^{\mathcal{F}})^{\omega_{l+1}}, \end{array}\right]$$

$$= \quad \left[\begin{array}{c} \sqrt[\aleph]{1-\odot_{i=1}^{l+1}\left(1-(\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\omega_i}}, \\ \sqrt[\aleph]{1-\odot_{i=1}^{l+1}\left(1-(\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\omega_i}}, \odot_{i=1}^{l+1}(\varsigma\Psi_i^{\mathcal{F}})^{\omega_i}, \end{array}\right].$$

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Theorem 5.3. *Let* $L_i = \left\langle \left(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}\right), \varsigma \right\rangle$ *be the new type of DioNNs. Then DioNWA*$(L_1, L_2, ..., L_n) = L$ *(idempotency property).*

Proof. Since $\Psi_i^{\mathcal{T}} = \Psi^{\mathcal{T}}$, $\Psi_i^{\mathcal{I}} = \Psi^{\mathcal{I}}$ and $\Psi_i^{\mathcal{F}} = \Psi^{\mathcal{F}}$ and $\uplus_{i=1}^{n}\omega_i = 1$. Now, $DioNWA(L_1, L_2, ..., L_n)$

$$\begin{aligned}
&= \left[\begin{array}{c} \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\omega_i}}, \\ \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\omega_i}}; \odot_{i=1}^{n}(\varsigma\Psi_i^{\mathcal{F}})^{\omega_i}, \end{array}\right] \\
&= \left[\begin{array}{c} \sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{T}})^{\aleph}\right)^{\uplus_{i=1}^{n}\omega_i}}, \\ \sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{I}})^{\aleph}\right)^{\uplus_{i=1}^{n}\omega_i}}; (\varsigma\Psi^{\mathcal{F}})^{\uplus_{i=1}^{n}\omega_i}, \end{array}\right] \\
&= \left[\sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{T}})^{\aleph}\right)}, \sqrt[\aleph]{1 - \left(1 - (\varsigma\Psi^{\mathcal{I}})^{\aleph}\right)},\ (\varsigma\Psi^{\mathcal{F}}),\right] \\
&= L.
\end{aligned}$$

Theorem 5.4. *Let* $L_i = \left\langle \left(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}\right), \varsigma \right\rangle$ *be the new type of DioNNs. Then DioNWA*$(L_1, L_2, ..., L_n)$ *where* $\overleftarrow{\varsigma\Psi^{\mathcal{T}}} = \min\varsigma\Psi_{ij}^{\mathcal{T}}$, $\overrightarrow{\varsigma\Psi^{\mathcal{T}}} = \max\varsigma\Psi_{ij}^{\mathcal{T}}$, $\overleftarrow{\varsigma\Psi^{\mathcal{I}}} = \min\varsigma\Psi_{ij}^{\mathcal{I}}$, $\overrightarrow{\varsigma\Psi^{\mathcal{I}}} = \max\varsigma\Psi_{ij}^{\mathcal{I}}$, $\overleftarrow{\varsigma\Psi^{\mathcal{F}}} = \min\varsigma\Psi_{ij}^{\mathcal{F}}$, $\overrightarrow{\varsigma\Psi^{\mathcal{F}}} = \max\varsigma\Psi_{ij}^{\mathcal{F}}$ *and where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$. *Then,* $\left\langle \overleftarrow{\varsigma\Psi^{\mathcal{T}}}, \overleftarrow{\varsigma\Psi^{\mathcal{I}}}, \overrightarrow{\varsigma\Psi^{\mathcal{F}}} \right\rangle$

$$\begin{aligned}
&\leq DioNWA(L_1, L_2, ..., L_n) \\
&\leq \overrightarrow{\varsigma\Psi^{\mathcal{T}}}, \overrightarrow{\varsigma\Psi^{\mathcal{I}}}, \overleftarrow{\varsigma\Psi^{\mathcal{F}}}]\Big\rangle.
\end{aligned}$$

(Boundedness property).

Proof. Since, $\overleftarrow{\varsigma\Psi^{\mathcal{T}}} = \min\varsigma\Psi_{ij}^{\mathcal{T}}, \overrightarrow{\varsigma\Psi^{\mathcal{T}}} = \max\varsigma\Psi_{ij}^{\mathcal{T}}$ and $\overleftarrow{\varsigma\Psi^{\mathcal{T}}} \leq \varsigma\Psi_{ij}^{\mathcal{T}} \leq \overrightarrow{\varsigma\Psi^{\mathcal{T}}}$. Now, $\overleftarrow{\varsigma\Psi^{\mathcal{T}}} = \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overleftarrow{\varsigma\Psi^{\mathcal{T}}})^{\aleph}\right)^{\omega_i}} \leq \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\varsigma\Psi_{ij}^{\mathcal{T}})^{\aleph}\right)^{\omega_i}} \leq \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overrightarrow{\varsigma\Psi^{\mathcal{T}}})^{\aleph}\right)^{\omega_i}} = \overrightarrow{\varsigma\Psi^{\mathcal{T}}}$.
Since, $\overleftarrow{\varsigma\Psi^{\mathcal{I}}} = \min\varsigma\Psi_{ij}^{\mathcal{I}}, \overrightarrow{\varsigma\Psi^{\mathcal{I}}} = \max\varsigma\Psi_{ij}^{\mathcal{I}}$ and $\overleftarrow{\varsigma\Psi^{\mathcal{I}}} \leq \varsigma\Psi_{ij}^{\mathcal{I}} \leq \overrightarrow{\varsigma\Psi^{\mathcal{I}}}$. Now, $\overleftarrow{\varsigma\Psi^{\mathcal{I}}} = \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overleftarrow{\varsigma\Psi^{\mathcal{I}}})^{\aleph}\right)^{\omega_i}} \leq \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\varsigma\Psi_{ij}^{\mathcal{I}})^{\aleph}\right)^{\omega_i}} \leq \sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overrightarrow{\varsigma\Psi^{\mathcal{I}}})^{\aleph}\right)^{\omega_i}} = \overrightarrow{\varsigma\Psi^{\mathcal{I}}}$.
Since, $\overleftarrow{\varsigma\Psi^{\mathcal{F}}} = \min\varsigma\Psi_{ij}^{\mathcal{F}}, \overrightarrow{\varsigma\Psi^{\mathcal{F}}} = \max\varsigma\Psi_{ij}^{\mathcal{F}}$ and $\overleftarrow{\varsigma\Psi^{\mathcal{F}}} \leq \varsigma\Psi_{ij}^{\mathcal{F}} \leq \overrightarrow{\varsigma\Psi^{\mathcal{F}}}$. We have, $\overleftarrow{\varsigma\Psi^{\mathcal{F}}} = \odot_{i=1}^{n}(\overleftarrow{\varsigma\Psi^{\mathcal{F}}})^{\omega_i} \leq \odot_{i=1}^{n}(\varsigma\Psi_{ij}^{\mathcal{F}})^{\omega_i} \leq \odot_{i=1}^{n}(\overrightarrow{\varsigma\Psi^{\mathcal{F}}})^{\omega_i} = \overrightarrow{\varsigma\Psi^{\mathcal{F}}}$.
Therefore,

$$\begin{aligned}
&\frac{1}{2} \times \left[\begin{array}{c} \dfrac{\left(\sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overleftarrow{\varsigma\Psi^{\mathcal{T}}})^{\aleph}\right)^{\omega_i}}\right)^2}{2} - \dfrac{\left(\sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overleftarrow{\varsigma\Psi^{\mathcal{I}}})^{\aleph}\right)^{\omega_i}}\right)^2}{2} \\ +2 - \dfrac{\left(\odot_{i=1}^{n}(\overrightarrow{\varsigma\Psi^{\mathcal{F}}})^{\omega_i}\right)^2}{2} \end{array}\right] \\
\leq\ &\frac{1}{2} \times \left[\begin{array}{c} \dfrac{\left(\sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\varsigma\Psi_{ij}^{\mathcal{T}})^{\aleph}\right)^{\omega_i}}\right)^2}{2} - \dfrac{\left(\sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\varsigma\Psi_{ij}^{\mathcal{I}})^{\aleph}\right)^{\omega_i}}\right)^2 +}{2} \\ +2 - \dfrac{\left(\odot_{i=1}^{n}(\varsigma\Psi_{ij}^{\mathcal{F}})^{\omega_i}\right)^2}{2} \end{array}\right] \\
\leq\ &\frac{1}{2} \times \left[\begin{array}{c} \dfrac{\left(\sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overrightarrow{\varsigma\Psi^{\mathcal{T}}})^{\aleph}\right)^{\omega_i}}\right)^2}{2} - \dfrac{\left(\sqrt[\aleph]{1 - \odot_{i=1}^{n}\left(1 - (\overrightarrow{\varsigma\Psi^{\mathcal{I}}})^{\aleph}\right)^{\omega_i}}\right)^2 +}{2} \\ +2 - \dfrac{\left(\odot_{i=1}^{n}(\overleftarrow{\varsigma\Psi^{\mathcal{F}}})^{\omega_i}\right)^2}{2} \end{array}\right].
\end{aligned}$$

Hence, $\left\langle \overleftarrow{\varsigma\Psi^{\mathcal{T}}}, \overleftarrow{\varsigma\Psi^{\mathcal{I}}}, \overrightarrow{\varsigma\Psi^{\mathcal{F}}} \right\rangle \leq DioNWA(L_1, L_2, ..., L_n) \leq \langle \overrightarrow{\varsigma\Psi^{\mathcal{T}}}, \overrightarrow{\varsigma\Psi^{\mathcal{I}}}, \overleftarrow{\varsigma\Psi^{\mathcal{F}}} \rangle$.

Theorem 5.5. *Let* $L_i = \langle(\Psi^{\mathcal{T}}_{t_{ij}}, \Psi^{\mathcal{I}}_{t_{ij}}, \Psi^{\mathcal{F}}_{t_{ij}}), \varsigma\rangle$ *and* $W_i = \langle(\Psi^{\mathcal{T}}_{h_{ij}}, \Psi^{\mathcal{I}}_{h_{ij}}, \Psi^{\mathcal{F}}_{h_{ij}}), \varsigma\rangle$, *be the DioNWAs. For any* i, *if there is* $(\varsigma\Psi^{\mathcal{T}}_{t_{ij}})^2 \leq (\varsigma\Psi^{\mathcal{T}}_{h_{ij}})^2$ *and* $(\varsigma\Psi^{\mathcal{I}}_{t_{ij}})^2 \leq (\varsigma\Psi^{\mathcal{I}}_{h_{ij}})^2$ *and* $(\varsigma\Psi^{\mathcal{F}}_{t_{ij}})^2 \geq (\varsigma\Psi^{\mathcal{F}}_{h_{ij}})^2$ *or* $L_i \leq W_i$. *Prove that* $DioNWA(L_1, L_2, ..., L_n) \leq DioNWA(W_1, W_2, ..., W_n)$, *where* $(i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *(monotonicity property).*

Proof. For any i, $(\varsigma\Psi^{\mathcal{T}}_{t_{ij}})^2 \leq \left(\varsigma\Psi^{\mathcal{T}}_{h_{ij}}\right)^2$.
Therefore, $1 - (\varsigma\Psi^{\mathcal{T}}_{t_i})^2 \geq 1 - (\varsigma\Psi^{\mathcal{T}}_{h_i})^2$.
Hence, $\odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{T}}_{t_i}\right)^2\right)^{\omega_i} \geq \odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{T}}_{h_i}\right)^2\right)^{\omega_i}$
and $\sqrt[\aleph]{1 - \odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{T}}_{t_i}\right)^{\aleph}\right)^{\omega_i}} \leq \sqrt[\aleph]{1 - \odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{T}}_{h_i}\right)^{\aleph}\right)^{\omega_i}}$.
For any i, $(\varsigma\Psi^{\mathcal{I}}_{t_{ij}})^{\aleph} \leq \left(\varsigma\Psi^{\mathcal{I}}_{h_{ij}}\right)^{\aleph}$.
Therefore, $1 - \left(\varsigma\Psi^{\mathcal{I}}_{t_i}\right)^{\aleph} \geq 1 - \left(\varsigma\Psi^{\mathcal{I}}_{h_i}\right)^{\aleph}$.
Hence, $\odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{I}}_{t_i}\right)^{\aleph}\right)^{\omega_i} \geq \odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{I}}_{h_i}\right)^{\aleph}\right)^{\omega_i}$.
This implies that $\sqrt[\aleph]{1 - \odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{I}}_{t_i}\right)^{\aleph}\right)^{\omega_i}} \leq \sqrt[\aleph]{1 - \odot^n_{i=1}\left(1 - \left(\varsigma\Psi^{\mathcal{I}}_{h_i}\right)^{\aleph}\right)^{\omega_i}}$.
For any i, $\left(\varsigma\Psi^{\mathcal{F}}_{t_{ij}}\right)^2 \geq \left(\varsigma\Psi^{\mathcal{F}}_{h_{ij}}\right)^2$.
Therefore, $2 - \frac{\left(\odot^n_{i=1}\varsigma\Psi^{\mathcal{F}}_{t_{ij}}\right)^2}{2} \leq 2 - \frac{\left(\odot^n_{i=1}\varsigma\Psi^{\mathcal{F}}_{h_{ij}}\right)^2}{2}$.

$$
\begin{aligned}
& \frac{1}{2} \times \left[\begin{array}{c} \frac{\left(\sqrt[\aleph]{1-\odot^n_{i=1}\left(1-(\varsigma\Psi^{\mathcal{T}^L}_{ti})^{\aleph}\right)^{\omega_i}}\right)^2}{2} - \frac{\left(\sqrt[\aleph]{1-\odot^n_{i=1}\left(1-(\varsigma\Psi^{\mathcal{I}^L}_{ti})^{\aleph}\right)^{\omega_i}}\right)^2}{2} \\ +2 - \frac{\left(\odot^n_{i=1}(\varsigma\Psi^{\mathcal{F}^L}_{tij})\right)^2}{2} \end{array}\right] \\
\leq \; & \frac{1}{2} \times \left[\begin{array}{c} \frac{\left(\sqrt[\aleph]{1-\odot^n_{i=1}\left(1-(\varsigma\Psi^{\mathcal{T}^L}_{hi})^{\aleph}\right)^{\omega_i}}\right)^2}{2} - \frac{\left(\sqrt[\aleph]{1-\odot^n_{i=1}\left(1-(\varsigma\Psi^{\mathcal{I}^L}_{hi})^{\aleph}\right)^{\omega_i}}\right)^2}{2} \\ +2 - \frac{\left(\odot^n_{i=1}(\varsigma\Psi^{\mathcal{F}^L}_{hij})\right)^2}{2} \end{array}\right].
\end{aligned}
$$

Hence, $DioNWA\,(L_1, L_2, ..., L_n) \leq DioNWA\,(W_1, W_2, ..., W_n)$.

5.2 DioNWG

Definition 5.6. Let $L_i = \left\langle \left(\Psi^{\mathcal{T}}_i, \Psi^{\mathcal{I}}_i, \Psi^{\mathcal{F}}_i\right), \varsigma\right\rangle$ be the new type of DioNNs. Then DioNWG $(L_1, L_2, ..., L_n) = \odot^n_{i=1} L_i^{\omega_i}$.

Theorem 5.7. *Let* $L_i = \left\langle \left(\Psi^{\mathcal{T}}_i, \Psi^{\mathcal{I}}_i, \Psi^{\mathcal{F}}_i\right), \varsigma\right\rangle$ *be the new type of DioNNs. Then DioNWG* $(L_1, L_2, ..., L_n)$

$$
= \left[\begin{array}{c} \odot^n_{i=1}(\varsigma\Psi^{\mathcal{T}}_i)^{\omega_i}; \\ \sqrt[\aleph]{1-\odot^n_{i=1}\left(1-(\varsigma\Psi^{\mathcal{I}}_i)^{\aleph}\right)^{\omega_i}};\ \sqrt[\aleph]{1-\odot^n_{i=1}\left(1-(\varsigma\Psi^{\mathcal{F}}_i)^{\aleph}\right)^{\omega_i}}, \end{array}\right].
$$

Proof. It follows from Theorem 5.2.

Theorem 5.8. *Let* $L_i = \left\langle \left(\Psi^{\mathcal{T}}_i, \Psi^{\mathcal{I}}_i, \Psi^{\mathcal{F}}_i\right), \varsigma\right\rangle$ *be the new type of DioNNs and all are equal. Then DioNWG*$(L_1, L_2, ..., L_n) = L$.

Proof. It follows from Theorem 5.3.

Remark 5.9. It has other properties, including boundedness and monotonicity, as well as having DioNWG.

Proof. It follows from Theorem 5.4 and Theorem 5.5.

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5.3 Generalized DioNWA (GDioNWA)

Definition 5.10. Let $L_i = \left\langle \left(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}\right), \varsigma \right\rangle$ be the new type of DioNN. Then GDioNWA $(L_1, L_2, ..., L_n) = \left(\uplus_{i=1}^{n} \omega_i L_i^{\aleph}\right)^{1/\aleph}$.

Theorem 5.11. *Let* $L_i = \left\langle \left(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}}\right), \varsigma \right\rangle$ *be the new type of DioNNs. Then GDioNWA* $(L_1, L_2, ..., L_n)$

$$= \begin{bmatrix} \left(\sqrt[\aleph]{1-\odot_{i=1}^{n}\left(1-\left((\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{1/\aleph}, \left(\sqrt[\aleph]{1-\odot_{i=1}^{n}\left(1-\left((\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{1/\aleph}, \\ \sqrt[\aleph]{1-\left(1-\left(\odot_{i=1}^{n}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i}\right)^{\aleph}\right)^{1/\aleph}}, \end{bmatrix}.$$

Proof. We can prove this first by demonstrating that,

$$\uplus_{i=1}^{n}\omega_i L_i^{\aleph} = \begin{bmatrix} \sqrt[\aleph]{1-\odot_{i=1}^{n}\left(1-\left((\varsigma\Psi_i^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}; \\ \sqrt[\aleph]{1-\odot_{i=1}^{n}\left(1-\left((\varsigma\Psi_i^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}; \odot_{i=1}^{n}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i}, \end{bmatrix}.$$

Put $n = 2$, $\omega_1 L_1 \uplus \omega_2 L_2$

$$= \begin{bmatrix} \sqrt[\aleph]{\begin{array}{c} \left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph} + \left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_2^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph}, \\ -\left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph} \cdot \left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_2^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph} \end{array}} \\ , \\ \sqrt[\aleph]{\begin{array}{c} \left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph} + \left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_2^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph}, \\ -\left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph} \cdot \left(\sqrt[\aleph]{1-\left(1-\left((\varsigma\Psi_2^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph} \end{array}} \\ \left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_1^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_1} \cdot \left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_2^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_1}, \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\aleph]{1-\odot_{i=1}^{2}\left(1-\left((\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}; \\ \sqrt[\aleph]{1-\odot_{i=1}^{2}\left(1-\left((\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}; \odot_{i=1}^{2}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i}; \end{bmatrix}.$$

Hence,

$$\uplus_{i=1}^{l}\omega_i L_i^{\aleph} = \begin{bmatrix} \sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-\left((\varsigma\Psi_1^{\mathcal{T}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}; \\ \sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-\left((\varsigma\Psi_1^{\mathcal{I}})^{\aleph}\right)^{\aleph}\right)^{\omega_i}}; \odot_{i=1}^{l}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i}; \end{bmatrix}.$$

If $n = l + 1$, then $\uplus_{i=1}^{l}\omega_i L_i^{\aleph} + \omega_{l+1} L_{l+1}^{\aleph} = \uplus_{i=1}^{l+1}\omega_i L_i^{\aleph}$.

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Now,$\uplus_{i=1}^{l}\omega_i L_i^{\aleph}+\omega_{l+1}L_{l+1}^{\aleph}=\omega_1 L_1^{\aleph}\uplus\omega_2 L_2^{\aleph}\uplus\ldots\uplus\omega_l L_l^{\aleph}\uplus\omega_{l+1}L_{l+1}^{\aleph}$

$$=\left[\begin{array}{c}
\sqrt[\aleph]{\begin{array}{c}\left(\sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-\left(\left(\varsigma\Psi_i^{\mathcal{T}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{\aleph}+\left(\sqrt[\aleph]{1-\left(1-\left(\left(\varsigma\Psi_{l+1}^{\mathcal{T}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph},\\
-\left(\sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-\left(\left(\varsigma\Psi_i^{\mathcal{T}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{\aleph}\cdot\left(\sqrt[\aleph]{1-\left(1-\left(\left(\varsigma\Psi_{l+1}^{\mathcal{T}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph}\end{array}}\\
\sqrt[\aleph]{\begin{array}{c}\left(\sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-\left(\left(\varsigma\Psi_i^{\mathcal{I}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{\aleph}+\left(\sqrt[\aleph]{1-\left(1-\left(\left(\varsigma\Psi_{l+1}^{\mathcal{I}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph},\\
-\left(\sqrt[\aleph]{1-\odot_{i=1}^{l}\left(1-\left(\left(\varsigma\Psi_i^{\mathcal{I}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{\aleph}\cdot\left(\sqrt[\aleph]{1-\left(1-\left(\left(\varsigma\Psi_{l+1}^{\mathcal{I}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_1}}\right)^{\aleph}\end{array}}\\
\odot_{i=1}^{l}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i}\cdot\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_{l+1}^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_1},
\end{array}\right]$$

$$\uplus_{i=1}^{l+1}\omega_i L_i^{\aleph}=\left[\begin{array}{c}
\sqrt[\aleph]{1-\odot_{i=1}^{l+1}\left(1-\left(\left(\varsigma\Psi_1^{\mathcal{T}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}};\ \sqrt[\aleph]{1-\odot_{i=1}^{l+1}\left(1-\left(\left(\varsigma\Psi_1^{\mathcal{I}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}};\\
\odot_{i=1}^{l+1}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i};\odot_{i=1}^{l+1}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i},
\end{array}\right].$$

$$\left(\uplus_{i=1}^{l+1}\omega_i L_i^{\aleph}\right)^{1/\aleph}=\left[\begin{array}{c}
\left(\sqrt[\aleph]{1-\odot_{i=1}^{l+1}\left(1-\left(\left(\varsigma\Psi_i^{\mathcal{T}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{1/\aleph};\\
\left(\sqrt[\aleph]{1-\odot_{i=1}^{l+1}\left(1-\left(\left(\varsigma\Psi_i^{\mathcal{I}}\right)^{\aleph}\right)^{\aleph}\right)^{\omega_i}}\right)^{1/\aleph};\\
\sqrt[\aleph]{1-\left(1-\left(\odot_{i=1}^{l+1}\left(\sqrt[\aleph]{1-\left(1-(\varsigma\Psi_i^{\mathcal{F}})^{\aleph}\right)^{\aleph}}\right)^{\omega_i}\right)^{2}\right)^{1/\aleph}},
\end{array}\right]$$

Remark 5.12. An operator modified from the GDioNWA operator to the DioNWA operator is performed if $\aleph=1$.

Theorem 5.13. *If all* $L_i=\left\langle\left(\Psi_i^{\mathcal{T}},\Psi_i^{\mathcal{I}},\Psi_i^{\mathcal{F}}\right),\varsigma\right\rangle$ *and all are equal. Then GDioNWA*$(L_1,L_2,...,L_n)=L$.

Proof. There is a proof based on the Theorem 5.3.

Remark 5.14. In the GDioNWA operator, boundedness and monotonicity are satisfied.

Proof. There is a proof based on the Theorem 5.4 and Theorem 5.5.

5.4 Generalized DioNWG (GDioNWG)

Definition 5.15. Let $L_i=\left\langle\left(\Psi_i^{\mathcal{T}},\Psi_i^{\mathcal{I}},\Psi_i^{\mathcal{F}}\right),\varsigma\right\rangle$ be the new type of DioNNs. Then GDioNWG $(L_1,L_2,...,L_n)=\frac{1}{\aleph}\left(\odot_{i=1}^{n}\left(\aleph L_i\right)^{\omega_i}\right)$.

Theorem 5.16. *Let* $L_i=\left\langle\left(\Psi_i^{\mathcal{T}},\Psi_i^{\mathcal{I}},\Psi_i^{\mathcal{F}}\right),\varsigma\right\rangle$ *be the new type of DioNNs. Then GDioNWG*$(L_1,L_2,...,L_n)$

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$$= \left[\begin{array}{c} \left(\frac{1}{\aleph} \odot_{i=1}^{n} (\aleph)^{\omega_i}, \frac{1}{\aleph} \odot_{i=1}^{n} (\aleph)^{\omega_i} \right); \\ \sqrt[\aleph]{1 - \left(1 - \left(\odot_{i=1}^{n} \left(\sqrt[\aleph]{1 - \left(1 - (\varsigma \Psi_i^{\mathcal{T}})^{\aleph}\right)^{\aleph}} \right)^{\omega_i} \right)^{\aleph} \right)^{1/\aleph}}; \\ \left(\sqrt[\aleph]{1 - \odot_{i=1}^{n} \left(1 - \left((\varsigma \Psi_i^{\mathcal{I}})^{\aleph} \right)^{\aleph} \right)^{\omega_i}} \right)^{1/\aleph}; \left(\sqrt[\aleph]{1 - \odot_{i=1}^{n} \left(1 - \left((\varsigma \Psi_i^{\mathcal{F}})^{\aleph} \right)^{\aleph} \right)^{\omega_i}} \right)^{1/\aleph}, \end{array} \right].$$

Proof. This proof is based on the Theorem 5.11.

Remark 5.17. There is a conversion that takes place when $\aleph = 1$, which converts the GDioNWG into the DioNWG.

Remark 5.18. Boundness and monotonicity properties that are satisfied by GDioNWG operators.

Proof. This proof is based on the Theorem 5.4 and Theorem 5.5

Theorem 5.19. *If all* $L_i = \left\langle \left(\Psi_i^{\mathcal{T}}, \Psi_i^{\mathcal{I}}, \Psi_i^{\mathcal{F}} \right), \varsigma \right\rangle$ *are equal. Then GDioNWG*$(L_1, L_2, ..., L_n) = L$.

6 Conclusion:

In this study, we established ED and HD measures for new type of DioNSs, which also have the advantage of being mathematically simple. The ED and HD are shown to be superior by employing appropriate. For DioNWA, DioNWG, GDioNWA and GDioNWG, we have proposed improved AO rules. In order to create these operators, we have also talked about a few aspects and provided a few algebraic operations. There is no doubt that the research contained in this article, which is still at a very early stage of its development, will provide a major benefit to future researchers in this field. It is a large field that is open to future academics who have an interest in it, therefore the ideas presented here will be beneficial to them in the future.
The following topics will be discussed in more detail:
(1) The cubic FS and IVPFS using AOs are related.
(2) We examine the normal vague set, normal spherical set and normal NS.
(3) The problem can be solved using complex NWA, complex NWG, complex GNWA and complex GNWG.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] L. A. Zadeh, Fuzzy sets, Information and control, 8(3), (1965), 338-353.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems, 20(1), (1986), 87-96.

[3] R. R. Yager, Pythagorean membership grades in multi criteria decision-making, IEEE Trans. Fuzzy Systems, 22, (2014), 958-965.

[4] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani and T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision making problems, Journal of Intelligent and Fuzzy Systems, 36, (2019), 2829-284.

[5] B.C. Cuong and V. Kreinovich, Picture fuzzy sets a new concept for computational intelligence problems, in Proceedings of 2013 Third World Congress on Information and Communication Technologies (WICT 2013), IEEE, (2013), 1-6.

[6] P. Liu, G. Shahzadi, M. Akram, Specific types of picture fuzzy Yager aggregation operators for decision-making, International Journal of Computational Intelligence Systems, 13(1), (2020), 1072-1091.

[7] W.F. Liu, J. Chang, X. He, Generalized Pythagorean fuzzy aggregation operators and applications in decision making, Control Decis. 31, (2016), 2280-2286.

[8] X. Peng, and Y. Yang, Fundamental properties of interval valued Pythagorean fuzzy aggregation operators, International Journal of Intelligent Systems, (2015), 1-44.

[9] K.G. Fatmaa, K. Cengiza, Spherical fuzzy sets and spherical fuzzy TOPSIS method, Journal of Intelligent and Fuzzy Systems, 36(1), (2019), 337-352.

[10] X. D. Peng and J. Dai, Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function, Neural Computing and Applications, 29(10), (2018), 939-954.

[11] X. Lhang and L. Xu, Extension of TOPSIS to multiple criteria decision-making with Diophantine Pythagorean fuzzy sets, International Journal of Intelligent Systems, 29, (2014), 1061-1078.

[12] C. L. Hwang, K. Yoon, Multiple attributes decision-making methods and applications, Springer, Berlin Heidelberg, 1981.

[13] M. Riaz and M. R. Hashmi, Linear Diophantine fuzzy set and its applications towards multi-attribute decision making problems, Journal of Intelligent and Fuzzy Systems, 37(4), (2019), 5417-5439.

[14] M Palanikumar, K Arulmozhi, A Iampan, Multi criteria group decision making based on VIKOR and TOPSIS methods for Fermatean fuzzy soft with aggregation operators, ICIC Express Letters 16 (10), 2022, 1129–1138.

[15] M Palanikumar, K Arulmozhi, MCGDM based on TOPSIS and VIKOR using Pythagorean neutrosophic soft with aggregation operators, Neutrosophic Sets and Systems, 2022, 538-555.

[16] A. Al Quran, A. G. Ahmad, F. Al Sharqi, A. Lutfi, Q-Complex Neutrosophic Set, International Journal of Neutrosophic Science, 20(2), 2023, 08-19.

[17] F. Al Sharqi, M. U. Romdhini, A. Al-Quran, Group decision making based on aggregation operator and score function of Q-neutrosophic soft matrix, Journal of Intelligent and Fuzzy Systems, 45, 2023, 305–321.

[18] Broumi, S., Sundareswaran, R., Shanmugapriya, M., Bakali, A., Talea, M. Theory and Applications of Fermatean Neutrosophic Graphs. Neutrosophic Sets and Systems, 50, (2022), 248-286.

[19] Broumi, S., Mohanaselvi, S., Witczak, T., Talea, M., Bakali, A., Smarandache, F. Complex fermatean neutrosophic graph and application to decision making. Decision Making: Applications in Management and Engineering, 6(1), (2023), 474-501.

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