Partial orderings, Characterizations and Generalization of k-idempotent Neutrosophic fuzzy matrices

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Abstract

In this article, First, we study the different orderings for k-idempotent Neutrosophic fuzzy matrices (NFM). With this idea, we also discover some properties for the k- Neutrosophic fuzzy matrices and demonstrate the connection between the generalized inverse and different orderings. We also go through some properties for the T-ordering, T- reverse ordering, minus, and space ordering in k-idempotent Neutrosophic fuzzy matrices using the g-inverses with numerical examples is given. Minus ordering is a partial ordering in the set of all regular fuzzy matrices. We have introduced ordering on k− idempotent fuzzy matrices and developed the theory of fuzzy matrix partial ordering. The minus ordering and k−space ordering are identical for k− idempotent matrices. Next, we introduce and study the concept of k–Idempotent Neutrosophic fuzzy matrix as a generalization of idempotent NFM via permutations. It is shown that a k-idempotent NFM reduces to an idempotent NFM if and only if PK = KP. The Conditions for power symmetric NFM to be k-idempotent are derived and some related results are given.

Keywords: k−idempotent NFM; T−ordering; minus ordering; space ordering and inverses; Idempotent NFM; permutation IFM; k-symmetric NFM.

1. Introduction

Matrices are crucial in many fields of research in science and engineering. The traditional matrix theory is unable to address problems involving numerous kinds of uncertainties. To overcome this situation,

Zadeh [1] first introduced fuzzy sets (FSs) in 1965. These are traditionally defined by their membership value or grade of membership. Assigning membership values to a fuzzy set can sometimes be challenging. Atanassov [2] introduced intuitionistic FSs to solve the problem of assigning non-membership values. Smarandache [3] introduced the concept of neutrosophic sets (NSs) to handle indeterminate information and deal with problems that involve imprecision, uncertainty, and inconsistency. Fuzzy matrices are used to solve certain kinds of issues. Many researchers have since completed numerous works. Only membership values are addressed by fuzzy matrices. These matrices cannot handle values that are not membership.

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Ben Isral and Greville [4] have studied Generalized Inverse Theory and Application. Kim and Roush [5] have discussrd on Generalized FM. Anandhkumar, et.al.,[6] have discussed Pseudo Similarity of NFM. Anandhkumaret. al.[7] have characterizes On various Inverse of NFM. Anandhkumar, et.al. [8] have fuccussed on Reverse Sharp and Left-T Right-T Partial Ordering on NFM. Thomason [9] has studied Convergence of possets of a fuzzy matrix. Partial orderings on fuzzy matrices, the analogue of star orderings on complex matrices, were first introduced by Jian Miao Chen [10].

Mitra, Bhimasankaram and Malik [11] have characterizes Matrix partial orders, shorted operators and applications .Meenakshi [12] has focused on FM – Theory and its applications. Muthugurupackiam and Krishnamohan [13] have introduced Generalization of Idempotent FM. Muthugurupackiam and Krishnamohan [14] have discussed k−idempotent FM. Muthu Guru Packiam and Krishna Mohan [15] have studied Partial orderings on k−idempotent fuzzy matrices. Anandhkumar et.al [16] has discussed Generalized Symmetric Neutrosophic Fuzzy Matrices. Anandhkumar et.al [17] has studied, Reverse Tilde (T) and Minus Partial Ordering on Intuitionistic Fuzzy Matrices.

Table:1 **Review of the Extension of NFM.**

References	**Extension of Neutrosophic Fuzzy Matrices.**	**Year**
[14]	Generalisation of Idempotent fuzzy matrices	2018
[15]	Some inverses on Generalised idempotent fuzzy matrices,	2019
[16]	Partial orderings on k−idempotent fuzzy matrices	2019
Proposed	Partial orderings, Characterizations and Generalization of k-idempotent Neutrosophic fuzzy matrices	2023

According to a survey of the literature, no research has been done on Partial orderings, Characterizations and Generalization of k-idempotent Neutrosophic fuzzy matrices and marge this gap.

1.1 Research Gap:

As was said in the introduction section above, Muthugurupackiam and Krishnamohan [14] introduced the concept of Generalization of Idempotent fuzzy matrices and Partial orderings on k−idempotent fuzzy matrices. Both these concepts play a significant role in hybrid fuzzy structure, and we use the same in Neutrosophic fuzzy matrices and study some of the results in detail.

With this idea, we also discover some properties for the k- Neutrosophic fuzzy matrices and demonstrate the connection between the generalized inverse and partial orderings. Relations between power symmetric NFM ($A^T=A^n$) and k-idempotent NFM are investigated. It is proved that a k-idempotent NFM reduces to an idempotent NFM when it commutes with the associated permutation NFM (i.e.,) PK = KP.

1.2 Notations

NFM = Neutrosophic fuzzy matrices

P ≤T Q = T-ordering

$P \leq Q =$ Minus ordering

$P \leq_k Q =$ Space ordering

PT = Transpose of P

2.T−ORDERING ON K−IDEMPOTENT NFM

Definition:2.1 For P,Q $\in (NFM)_{m\times n}$ the T−ordering $P \leq^T Q$ is defined as $P \leq^T Q \Leftrightarrow P^tP = P^tQ$ and $PP^t = QP^t$.

Definition:2.2 For P,Q $\in (NFM)_{m\times n}$ the T− Reverse ordering $P \overset{T}{\geq} Q$ is defined as $P \overset{T}{\geq} Q \Leftrightarrow Q^tQ = Q^tP$ and $QQ^t = PQ^t$.

Theorem 2.1 Let P,Q $\in (NFM)_{m\times n}$ are k−idempotent NFM, then $P \leq^T Q$ iff $P^2 \leq^T Q^2$

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Proof. Let $P \leq^T Q$, then
(i) $P^tP = P^tQ$ (ii) $PP^t = QP^t$
Multiplying both sides by K,
$KP^tPK = KP^tQK$
$KP^t KKPK = KP^t KKQK$
$(P^t)^2(P)^2 = (P^t)^2 (Q)^2$ .... (1)
From (ii), $KPP^tK = KQP^tK$
$KPKKP^tK = KQKKP^t K$
$(P)^2 (P^t)^2 = (Q)^2 (P^t)^2$ ………………... (2)
From (1) and (2), we have $P^2 \leq^T Q^2$
Conversely, if we assume that $P^2 \leq^T Q^2$
$KP^2 \leq^T KQ^2$
$KP^2K \leq^T KQ^2K$
$P \leq^T Q$.

Theorem 2.2. Let us consider P,Q $\in (NFM)_{m\times n}$ and K is said to be associated permutation NFM of k, then $P \leq^T Q \Leftrightarrow KP \leq^T KQ \Leftrightarrow PK \leq^T QK$

Proof: $P \leq^T Q \Leftrightarrow P^tP = P^tQ$ and $PP^t = QP^t$
$\Leftrightarrow P^tKKP = P^tKKQ$ and $KPP^tK = KQP^tK$
$\Leftrightarrow (KP)^tKP = (KP)^t KP$ and $KP(KP)^t = KQ (KP)^t$
$\Leftrightarrow KP \leq^T KQ$
Similarly, $P \leq^T Q \Leftrightarrow P^tP = P^tQ$ and $PP^t = QP^t$
$\Leftrightarrow KP^tPK = KP^tQK$ and $PKKP^t = QKKP^t$
$\Leftrightarrow (KP)^tPK = (KP)^tQK$ and $PK(PK)^t = QK(PK)^t$
$\Leftrightarrow PK \leq^T QK$.

Theorem2.3: Let us consider P,Q $\in (NFM)_{m\times n}$ are k−idempotent NFM , then $P \overset{T}{\geq} Q$ if and only if $P^2 \overset{T}{\geq} Q^2$

Proof. Let $P \overset{T}{\geq} Q$ then
(i) $Q^tQ = Q^tP$ (ii) $QQ^t = PQ^t$
Multiplying both sides by K,
$K Q^tQ K = K Q^tPK$
$KQ^t KKQK = KQ^t KKPK$
$(Q^t)^2(Q)^2 = (Q^t)^2(P)^2$ ………………….(1)
From (ii), $KQQ^t K = KPQ^tK$
$KQKKQ^tK = KPKKQ^tK$
$(Q)^2 (Q^t)^2=(P)^2 (Q^t)^2$ …………………(2)
From (1) and (2), we have $P^2 \overset{T}{\geq} Q^2$

Conversely, if we assume that $P^2 \overset{T}{\geq} Q^2$

$KP^2 \overset{T}{\geq} KQ^2$

$KP^2K \overset{T}{\geq} KQ^2K$

$P \overset{T}{\geq} Q$.

Theorem: 2.4 If K is the permutation NFM of k, then for P, Q $\in (NFM)_{m\times n}$ $P \overset{T}{\geq} Q \Leftrightarrow KP \overset{T}{\geq} KQ \Leftrightarrow PK \overset{T}{\geq} QK$

Proof: Let $P \overset{T}{\geq} Q \Leftrightarrow Q^tQ = Q^tP$ and $QQ^t = PQ^t$
$\Leftrightarrow Q^tKKQ = Q^tKKP$ and $KQQ^tK = KPQ^tK$
$\Leftrightarrow (KQ)^tKQ = (KQ)^t KP$ and $KQ(KQ)^t = KP(KQ)^t$

$KP \overset{T}{\geq} KQ$

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Similarly, $P \overset{T}{\geq} Q \Leftrightarrow Q^tQ = Q^tP$ and $QQ^t = PQ^t$

$\Leftrightarrow KQ^tQK = KQ^tPK$ and $QKKQ^t = PKKQ^t$

$\Leftrightarrow (KB)^tBK = (KB)^tAK$ and $BK(BK)^t = AK(BK)^t$

$PK \overset{T}{\geq} QK$.

Remark 2.1 If Pseudo-inverse of P and Q exists and $P \leq^T Q$, then we have $P = PP_k^{+}Q = QP_k^{+}P = QP_k^{+}Q$

Theorem 2.5 If $P \leq^T Q$ and Q is k−idempotent NFM, then P is also k−idempotent NFM.

Proof. $KP^2K = KPPK$

$= K\{PP^+_k Q\}\{QP^+_k P\}K$

$= PP^+_k KP^2KPP^+_k$

$= \{PP^+_k Q\}P^+_k P$

$= PP^+_k P = P$

Hence P is k−idempotent.

3. MINUS ORDERING ON K−IDEMPOTENT NFM

Definition 3.1. If $P, Q \in (NFM)_{m\times n}$ the minus ordering denoted as $\leq$ is defined as $P \leq Q \Leftrightarrow P_k^{-}P = P_k^{-}Q$ and $PP_k^{-} = QP_k^{-}$ for some $P_K^{-} \in P\{1\}$

Theorem 3.1. If $P \leq Q$ With Q as k−idempotent NFM, show that $KQK \in P\{1\}$

Example 3.1 Let us consider NFM $P = \begin{bmatrix} <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix}$,

$$Q = \begin{bmatrix} <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \end{bmatrix}$$

$$QP = \begin{bmatrix} <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix}$$

$$= \begin{bmatrix} <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \end{bmatrix} = Q$$

$$PQ = \begin{bmatrix} <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix} \begin{bmatrix} <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \end{bmatrix}$$

$$= \begin{bmatrix} <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \\ <1,1,0> & <1,1,0> & <1,1,0> \end{bmatrix} = Q$$

Now, Q = QP = PQ

Hence, $P \leq Q$ with respect to $Q = Q_k^{-} \in P\{1\}$.

Remark3.1. For $P, Q \in (NFM)_{m\times n}$, $P \leq Q \Leftrightarrow P = PP_k^{-}Q = QP_k^{-}P = QP_k^{-}Q$

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Theorem 3.2. For $P, Q \in (NFM)_{m\times n}$ with $P \leq Q$, if Q is k−idempotent IFM show that P is also k−idempotent NFM.

Proof. $KP^2K = KPPK$
$= K\{PP^-_k Q\}\{QP^-_k P\}K$
$= PP^-_k KQ^2KPP^-_k$
$= \{PP^-_k Q\}P^-_k P$
$= PP^-_k P = P$
Hence P is k−idempotent.

Remark 3.2. For $P, Q \in (NFM)_{m\times n}$ the subsequent are equivalent:

(i) $P \leq Q$ (ii) $P = PP_k^- Q = QP_k^- P = QP_k^- Q$.

Theorem 3.3. Let P and Q are k−idempotent NFM, then $P \leq Q$ iff $P^2 \leq Q^2$

Proof: Assume that $P \leq Q$ $P = PP_k^- Q = QP_k^- P = QP_k^- Q$
$\Leftrightarrow KPK = KPP^-_k QK = KQP^-_k PK = KQP^-_k QK$
$\Leftrightarrow P^2 = P^2KPP^-_k KQ^2 = Q^2KP^-_k KP^2 = Q^2KP^-_k KQ^2$
$\Leftrightarrow P^2 = P^2 (P^2)^-_k Q^2 = Q^2 (P^2)^-_k P^2 = Q^2 (P^2)^-_k Q^2$
Therefore, $P^2 \leq Q^2$.

Theorem 3.4. If K is the permutation NFM of K, then for $P, Q \in (NFM)_{m\times n}$, $P \leq Q \Leftrightarrow KP \leq KQ$ $\Leftrightarrow PK \leq QK$

Proof. Assuming that

$P \leq Q \Leftrightarrow P^-{}_k P = P^-{}_k Q$ and $PP^-{}_k = QP^-{}_k$

$\Leftrightarrow KP^-{}_k PK = KP^-{}_k QK$ and $KPP^-{}_k K = KQP^-{}_k K$

$\Leftrightarrow (PK)_k^- PK = (PK)_k^- QK$ and

$KP(KP^-)_k = KQ(KP^-)_k$

$\Leftrightarrow PK \leq QK$

Similarly, $P \leq Q \Leftrightarrow P^-{}_k KKP = P^-{}_k KKQ$ and

$PKKP^-{}_k = QKKP^-{}_k$

$\Leftrightarrow (KP)^-{}_k KP = (KP)^-{}_k KQ$ and $KP(KP)^-{}_k = QK(KP)^-{}_k \Leftrightarrow KP \leq KQ$

4.K−SPACE ORDERING ON NFM

In this section, we define space ordering on NFM. We proved that space ordering is a partial ordering on the set of all idempotent NFM in Fn.

Definition 4.1. Let P and Q are k−idempotent NFM, k−space ordering $P \leq_k Q$ is well-defined as

$P \leq_k Q \Leftrightarrow R(P) \subseteq R(Q)$ and $C(P) \subseteq C(Q)$.

Theorem 4.1 Let {Pi}, i ∈ N be a not infinite set of k−idempotent IFM. Then $P = P_1P_2...P_k$ is k−idempotent NFM if $P_iP_j = P_jP_i$

Proof. Consider $P_1P_2 = P_2P_1 = Q^2{}_1$
Similarly, $P_2P_3 = P_3P_2 = Q_2{}^2$ and so on.
Since $P = P_1P_2...P_k$ are k−idempotent,
Therefore $Q^2{}_1$, $Q^2{}_2$, ..., $Q^2_{\frac{m}{2}}$ are also k−idempotent NFM

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Hence $P = P^2_1 + P^2_2 + \ldots + P^2_{\frac{m}{2}}$ is also k−idempotent NFM.

Theorem 4.2.Let {Pi}, i ∈N be a not infinite set of k−idempotent NFM and if either $R(P_i) \subseteq R(P_j)$ or $C(P_i) \subseteq C(P_j)$, then $P = P_1P_2 \ldots P_k$ is k−idempotent NFM.

Proof. Taking $R(P_i) \subseteq R(P_j)$

$\Rightarrow R(P_1) \subseteq R(P_2), R(P_2) \subseteq R(P_3), \ldots, R(P_{k-1}) \subseteq R(P_k)$

$\Rightarrow R(P_1) \subseteq R(P_2) \subseteq R(P_3) \subseteq \ldots \subseteq R(P_{k-1}) \subseteq R(P_k)$

$\Rightarrow P = P_1$

Hence P is k−idempotent NFM.

Now $C(P_i) \subseteq C(P_j)$

$\Rightarrow C(P_1) \subseteq C(P_2), C(P_2) \subseteq C(P_3), \ldots, C(P_{k-1}) \subseteq C(P_k)$

$\Rightarrow C(P_1) \subseteq C(P_2) \subseteq C(P_3) \subseteq \ldots \subseteq C(P_{k-1}) \subseteq C(P_k)$

$\Rightarrow P = P_k$

Therefore, P is k−idempotent NFM.

Theorem 4.3. Let {Pi}, i ∈ N be a not infinite set of k−idempotent NFM and Pi ≤ Pj , then $P = P_1P_2 \ldots P_k$ is k−idempotent NFM.

Proof. $P_i \leq P_j \Rightarrow P_iP_j \leq P_j$ and $P_i \leq P_jP_i$

$\Rightarrow K(P_iP_j)^2K \leq P_iP_j$ and

$K(P_iP_j)^2K = P_i(P_iP_j)P_j \geq P_iP_j$

$\Rightarrow K(P_iP_j)^2K = P_iP_j$

$\Rightarrow P_iP_j$ is k−idempotent NFM.

Hence $P = P_1P_2 \ldots P_k$ is k−idempotent NFM

5. CHARACTERIZATIONS OF k – IDEMPOTENT NFM

we introduce and study a new characteristic k – idempotent fuzzy matrix in this paper. If a fuzzy matrix P is obtained by k – permuting the elements of P^2 , then it is called k – idempotent. Here k is the fixed product of disjoint transposition in Sn – the symmetric group of order n.

Definition 5.1 For a fixed product of disjoint transposition $K \in S_n$ a NFM $A = (a_{ij})_{nn}$ is called k-idempotent if $KA^2K = A$, where K is the associated permutation NFM of 'k'. The associated permutation NFM K is a matrix with <1,1,0> on main diagonal and <0,0,1> everywhere else.

Definition: 5.2 For a Neutrosophic fuzzy matrices is k- square symmetric Neutrosophic fuzzy matrices if and only if $P^T = KP^2K$ or $P^2 = KP^TK$.

Definition: 5.3 For a Neutrosophic fuzzy matrices is k- square symmetric Neutrosophic fuzzy matrices if and only if $P^T = KP^3K$ or $P^3 = KP^TK$.

Example:5.1 Let us consider NFM

$$K = \begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> \\ <0,0,1> & <1,1,0> & <0,0,1> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix}, \ P = \begin{bmatrix} <0.8,0.2,0.3> & <0.6,0.3,0.4> & <0.4,0.5,0.6> \\ <0.2,0.7,0.3> & <0.4,0.5,0.6> & <0.3,0.6,0.4> \\ <0.1,0.8,0.5> & <0.1,0.8,0.4> & <0.1,0.8,0.3> \end{bmatrix}$$

Then $KP^2K =$

$$\begin{bmatrix} <0.8,0.2,0.3> & <0.6,0.3,0.4> & <0.4,0.5,0.6> \\ <0.2,0.7,0.3> & <0.4,0.5,0.6> & <0.3,0.6,0.4> \\ <0.1,0.8,0.5> & <0.1,0.8,0.4> & <0.1,0.8,0.3> \end{bmatrix} = P$$

$KP^2K = P$ implies that $KPK = P^2$. It is also possible to establish the following relationships, which are advantageous for computational purposes.

$KP = P^2K$ or $KP^2 = PK$

$KP^3 = P^3K$ or $KP^3K = P^3$

$P^3 = (KP)^2$ or $(PK)^2$

$K^2P = PK^2 = P$

$KP^2K = P^2$

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Theorem 5.1 For P is idempotent, k – idempotent NFM P if and only if PK = KP.
Proof: Assume that PK = KP
Pre multiplying by K, we have , KPK = P
Since P is idempotent, $P^2 = P$
Therefore, P is k – idempotent.
The converse is also true by retracing the steps.
Example:5.2 Let us consider NFM

$$K = \begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> \\ <0,0,1> & <1,1,0> & <0,0,1> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix}, \quad P = \begin{bmatrix} <0.8,0.2,0.3> & <0.6,0.3,0.4> & <0.4,0.5,0.6> \\ <0.2,0.7,0.3> & <0.4,0.5,0.6> & <0.3,0.6,0.4> \\ <0.1,0.8,0.5> & <0.1,0.8,0.4> & <0.1,0.8,0.3> \end{bmatrix}$$

Theorem 5.2 For NFM P be a k-idempotent NFM. Then
(a) NFM P^T and P^- are also k-idempotent
(b) NFM P^n is k -idempotent for all positive integers 'n'
(c) NFM P is $P^4 = P$ and $P^3 = I$.
(d) NFM P^3 is idempotent.
(e) NFM KP and PK are tripotent NFM
Proof: (a) Let P be a k-idempotent NFM
$KP^2 K = P$
$(KP^2 K)^T = P^T$
$KP^T P^T K = P^T$
$K(P^T)^2 K = P^T$
Therefore, P^T is k-idempotent
Similarly, P^- is also k-idempotent
(b) $P^n = (KP^2 K)^n$
$= KP^2 K KP^2 K$…n times
$= KP^{2n} K$
$= K(P^n)^2K$

Therefore, P^n is k -idempotent for all positive integers ' n'
(c) $P^4 = P^2P^2$
= KPKKPK
$= KP^2K$
=P
$P^4 = P$ implies $P^3 = I$.
d) $(P^3)^2 = [(KP)^2]^2$
=KPKPKPKP
$= KPP^2PKP$
=KPKP
$= P^2.P = P^3$
(e)$(KP)^3$=KPKPKP
$= KPP^2P$
= KP
The proof is similar for PK
4.k-idempotency of symmetric NFM
In this section, conditions for power symmetric NFM to be k-idempotent are derived and some related results are given.

Theorem 5.3 For NFM P any two of the following imply the other one.
(a) P is k-idempotent
(b) P is k-symmetric
(c) P is square symmetric

Proof: (a) and (b) implies (c)
$KP^2K=P$ and $KP^TK=P$

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$KP^2K= KP^TK$
$P^2 =P^T$
Therefore, P is square symmetric
(b) and (c) implies (a)
Substituting $P^2 =P^T$ and $KP^TK=P$
Therefore, $KP^2 K=P$ Hence P is k-idempotent.
(c) and (a) implies (b):
Substituting $P^2 =P^T$ and $KP^2K=P$
$KP^TK=P$
Therefore, P is k symmetric.

Theorem5.4 Let P be a k-idempotent IFM. If P is cube symmetric then it reduces to an $P=P^T$
Proof: Since is a k -idempotent matrix, we have
$KP^3 K=P^3$
If P is cube symmetric then
$P^3= P^T$ ………………………………(1)
Pre and post multiplying by K , we have $KP^3K = KP^TK$
$P^3 = KP^TK$
$P^T = KP^TK$
$P = KPK$
$P = P^2$
Substituting (1) , we have $P=P^T$

Theorem 5.5 For NFM P be a k-idempotent NFM. Then the following are equivalent.

(i) KP is cube symmetric
(ii) KP is symmetric
(iii) P is square symmetric

Proof: (i) implies (ii)
KP is cube symmetric.
$(KP)^3 = (KP)^T$ (By theorem 2.2 (e))
$KP = (KP)^T$

Therefore, KP is symmetric
(ii) implies (iii)
KP is symmetric
$KP=(KP)^T$
$P^2K= P^TK$
$P^2= P^T$
Therefore, P is square symmetric
(iii) Implies (i)
P is square symmetric
$P^2 = P^T$
$(KP)^3 = KP$
$(KP)^3 = P^2K$
$(KP)^3 = P^TK$
$(KP)^3 = (KP)^T$
Therefore, KP is cube symmetric

Theorem 5.6 For NFM P be a k-idempotent NFM. Then the necessary and sufficient condition for the NFM KP to be square symmetric is

(i) P is idempotent.
(ii) $P =P^TK$

Proof; Assume that KP is square symmetric
$(KP)^2 = (KP)^T$
$P^3 = P^TK$……………………………(1)

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Pre and post multiplying(1) by K respectively
we have $KP^3 = KP^TK$ and $P^3K = P^T$
Since $KP^3 = P^3K$, we have $KP^TK = P^T$
$KPK = P$
$P^2 = P$
Hence P is idempotent and substituting this in (1), we have $P = P^TK$
Conversely, assume that A is idempotent with $P = P^TK$
$(KP)^2 = P^3$
$= P$
$= P^TK$
$= (KP)^T$
Therefore, KP is square symmetric

Theorem 5.7 Let P be a k-idempotent NFM. Then the following are equivalent.

(i) KP is k-cube symmetric
(ii) P is symmetric
(iii) P is k-square symmetric
(iv) KP is k-symmetric

Proof: (i) implies (ii)
KP is k-cube symmetric then $K(KP)^3K = (KP)^T$
$KKAK = P^TK$
$P = P^T$
Hence A is symmetric
(ii) implies (iii)
A is symmetric , $P = P^T$
$P^T = KP^2K$
Therefore, P is k-square symmetric
(iii)Implies (iv)
P is k-square symmetric
$P^T = KP^2K$
$KP^2 = P^TK$
$KP^2 = (KP)^T$
Pre and post multiplying by K , we have
$P^2 K = K(KP)^TK$
$KP = K(KP)^TK$

Therefore, KP is k-symmetric
(iv) Implies (i)
KA is k-symmetric, $KP = K(KP)^TK$
But $(KP)^3 = KP$
$(KP)^3 = K(KP)^TK$

Therefore, KP is k-cube symmetric

Theorem 5.8 If P is symmetric and k-square symmetric then P is k-idempotent.
Proof: Assume that $P^T = P$ and $KP^2K = P^T$
Combining the above two relations, we have $KP^2K = P$
Therefore, P is k-idempotent

6. Conclusion:

The concept of k-idempotent NFM is introduced for NFM and exhibited as a generalization of idempotent NFM. We study the different orderings for k-idempotent Neutrosophic fuzzy matrices (NFM). With this idea, we also discover some properties for the k- Neutrosophic fuzzy matrices and demonstrate the connection between the Generalized

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inverse and different orderings. We introduce and study the concept of k-Idempotent Neutrosophic fuzzy matrix as a generalization of Idempotent NFM via permutations. It is shown that a k-idempotent NFM reduces to an idempotent NFM if and only if PK = KP. The Conditions for power symmetric NFM to be k-idempotent are derived and some related results are given. In future, we shall prove some related properties of g-inverse of k-idempotent Neutrosophic Fuzzy Matrices .

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