On Schur Complement in k-Kernel Symmetric Neutrosophic and Intuitionistic Fuzzy Matrices

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Abstract

The present study provides the necessary and sufficient criteria for the k-Kernel symmetry (KS) of a Schur complement (SC) in a k-KS Neutrosophic Fuzzy matrices (NFM) and Intuitionistic Fuzzy Matrices (IFM). Equivalent characterizations of KS and k-KS NFM and IFM are presented in this work. We provide a few fundamental examples about KS NFM and IFM. It is demonstrated that while k-symmetric implies k-KS, but the converse need not be true. A few fundamental characteristics of k-KS IFM and NFM are obtained.

Keywords: NFM; IFM; Schur Complement, KS; k-KS.

1. Introduction

Consider (p^T, p^F) to be IFM. If (p^T, p^F) is a part of (IFM)n is known as k- KS IFM if N((p^T, p^F)) is more significant than N(K$(p^T, p^F)^T$K). Matrices are essential in a variety of areas of research in engineering and science. The conventional matrix theory must address issues with a wide range of uncertainty. Zadeh [1] introduced the fuzzy set (FS) concept by using membership numbers. A fuzzy set designed intuitively by Atanassov [2] is a good choice for providing values for an element in membership and non-membership grades.

Let IFM_{mn} indicates the set of every m×n IFM over the nuetrosophic fuzzy algebra (NF)n In short IFM_{nn} is indicated as NFM_n. We denote a solution Z of the equation $(p^T, p^F)Z(p^T, p^F) = (p^T, p^F)$ by $(p^T, p^F)^-$.For a complex matrix P subdivided in the form $P = \begin{bmatrix} (p^T, p^F) & (q^T, q^F) \\ (r^T, r^F) & (s^T, s^F) \end{bmatrix}$ a Schur complement of (p^T, p^F) in P denoted by (s^T, s^F)-$(r^T, r^F)(p^T, p^F)^-(q^T, q^F)$. This is known as the complex matrix generalized Schur complement. If $(p^T, p^F)Z(p^T, p^F) = (p^T, p^F)$ has a solution, (p^T, p^F) is considered regular. A solution Z to the equation $(p^T, p^F)Z(p^T, p^F) = (p^T, p^F)$ is denoted by $(p^T, p^F)^-$ and is referred to as a generalized inverse, or generalized inverse of (p^T, p^F).

A IFM (p^T, p^F) is range symmetric if R(p^T, p^F) = R$(p^T, p^F)^T$ and KS if N(p^T, p^F) = N$(p^T, p^F)^T$. For NFM $(p^T, p^F) \in$ IFMn, (p^T, p^F) is range symmetric, that is, R(p^T, p^F) = R$(p^T, p^F)^T$ implies N(p^T, p^F) = N$(p^T, p^F)^T$ but converse needs not be true.

Kim and and Roush [3] have characterized Generalized fuzzy matrices. Meenakshi [4] has discussed on Fuzzy Matrix: Theory and Applications. Hill and Waters [5] have characterized On κ-real and κ-Hermitian matrices. Baskett and Katz [6] have studied "Theorems on products of EPr matrices. Meenakshi andn Krishnamoorthy [7] have focused On κ-EP matrices. Meenakshi and Krishnamoorthy [8] have studied On SC in k-EP matrices.

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The structure of the article is as follows. In subdivision 2, We present some elementary definitions and findings. In subdivision 3 we provided k-KS IFM and NFM. In subdivision 4 we introduced SC in k-KS IFM and NFM.

1.1 Research Gap:

Meenakshi and Jayasree presented the concept of On Schur Complement in k-KS Matrices and k-KSM. We have applied the Schur Complement in k-KSM and k-KSM principles to NFM and IFM in this context. We have examined some of the results and extended both concepts to NFM and IFMs. We first present equivalent characterizations for Schur Complement in k-KSIFM and a k-KSIFM. We then derive the equivalent conditions that NFM and IFMs must meet to show KS and we explore the relationship between KS and k-KS.

1.2 Notation

let $(p^T, p^F)^T$ transpose of (p^T, p^F),

$R((p^T, p^F))$ Row space of (p^T, p^F)

$N((p^T, p^F))$ Null space of (p^T, p^F)

$(p^T, p^F)^+$ Moore-Penrose inverse of (p^T, p^F)

$\rho(p^T, p^F)$ Rank of (p^T, p^F)

$C((p^T, p^F))$ column space of (p^T, p^F)

IFM =Intuitionistic fuzzy matrices

NFM =Neutrosophic fuzzy matrices

KSIFM=Kernel symmetric Intuitionistic fuzzy matrices.

2. Definitions and Theorems

Definition 2.1 A Intuitionistic fuzzy matrix $(p^T, p^F) \in (IFM)_n$ is said to be k-KS if $N((p^T, p^F)) = N(K(p^T, p^F)^T K)$.

Definition 2.2 For $(p^T, p^F) \in (IFM)_n$ is KS if $N(p^T, p^F) = N((p^T, p^F)^T)$ where $N((p^T, p^F))$ $= \{x/x(p^T, p^F) = (0,1)$ and $x \in IFM_{1n}\}$.

Definition 2.3 Suppose p and q are two NFM elements $p = < p_{ij\alpha}, p_{ij\beta}, p_{ij\gamma} >, q = < q_{ij\alpha}, q_{ij\beta}, q_{ij\gamma} >,$ are component wise addition and multiplication are described as,

$$p + q = < \max\{p_{ij\alpha}, q_{ij\alpha}\}, \max\{p_{ij\beta}, q_{ij\beta}\}, \min\{p_{ij\gamma}, q_{ij\gamma}\} >$$

and $p.q = < \min\{p_{ij\alpha}, q_{ij\alpha}\}, \min\{1 - p_{ij\beta}, 1 - q_{ij\beta}\}, \max\{p_{ij\gamma}, q_{ij\gamma}\} >$

Definition2.4 (Transpose) The transpose P^T of an NFM $P = [p_{ij}]_{m\times n}$ is defined as $P^T = [p_{ji}]_{n\times m}$ where $p_{ji} = < p_{ji\alpha}, p_{ji\beta}, p_{ji\gamma} >$.

Definition 2.5 (IFPM) If each row and each column contains accurately one <1,1,0> and all other entries are <0,0,1> in a square NFM, it is known as intuitionistic Fuzzy permutation matrix.

Theorem 2.1 For A ∈ Fn, the following conditions are equivalent:

(i) A is KS,

(ii) PAP^T is KS for some PM P,

(iii) There exists a PM such that $PAP = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}$ with det D > 0

3. k-Kernel Symmetric NFM

Remark 3.1. In particular, when κ(i)= i for each i 1 to n, the associated permutation matrix K reduces to the

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identity matrix and Definition 3.1 reduces to N((p^T, p^I, p^F))= N((p^T, p^I, p^F))^T , that is, (p^T, p^I, p^F) is KS. If (p^T, p^I, p^F) is symmetric, then (p^T, p^I, p^F) is k-KS for all transpositions k in S_n.

Further, (p^T, p^I, p^F) is k-Symmetric implies it is k-KS, for (p^T, p^I, p^F) = K (p^T, p^I, p^F) ^{T}K automatically implies N((p^T, p^I, p^F))= N(K (p^T, p^I, p^F) ^{T}K). The opposite need not be true, though. This is demonstrated in the example that follows.

Example: 12 Let us Consider NFM

$$(p^T, p^I, p^F) = \begin{bmatrix} (0,0,0.5) & (0,0,0.4) & (0.3,0.4,0.5) \\ (0.5,0.4,0.6) & (0.1,0.4,0.6) & (0,0,0.4) \\ (0.4,0.5,0.3) & (0.3,0.4,0.5) & (0,0,0.3) \end{bmatrix}, K = \begin{bmatrix} (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (1,1,0) & (0,0,1) \\ (1,1,0) & (0,0,1) & (0,0,1) \end{bmatrix}$$

$$K(p^T, p^I, p^F)^T K = \begin{bmatrix} (0,0,0.3) & (0,0,0.4) & (0.3,0.4,0.5) \\ (0.3,0,0.5) & (0.1,0,0.6) & (0,0.4,0.4) \\ (0.4,0,0.3) & (0.5,0,0.6) & (0,0.4,0.5) \end{bmatrix}$$

Therefore, (p^T, p^I, p^F) $\neq$ K (p^T, p^I, p^F) ^{T} K

But, N ((p^T, p^I, p^F)) = N(K (p^T, p^I, p^F) ^{T} K) = $(0,0,1)$

Therefore, (p^T, p^I, p^F) is not k-symmetric. For this (p^T, p^I, p^F) , since (p^T, p^I, p^F) has no zero rows and no zero columns. NK (p^T, p^I, p^F) ^{T}K = $(0,0,1)$. Hence (p^T, p^I, p^F) is k-KS, but (p^T, p^I, p^F) is not k-symmetric.

Remark 3.2. In particular, when κ(i)= i for each i 1 to n, the associated permutation matrix K reduces to the identity matrix and Definition 3.1 reduces to N((p^T, p^F))= N((p^T, p^F))^T , that is, (p^T, p^F) is KS. If (p^T, p^F) is symmetric, then (p^T, p^F) is k-KS for all transpositions k in S_n.

Further, (p^T, p^F) is k-Symmetric implies it is k-KS, for (p^T, p^F) = K (p^T, p^F) ^{T}K automatically implies N((p^T, p^F))= N(K (p^T, p^F) ^{T}K). The opposite need not be true, though. This is demonstrated in the example that follows.

Example: 12 Let us Consider NFM

$$(p^T, p^F) = \begin{bmatrix} (0,0.5) & (0,0.4) & (0.3,0.5) \\ (0.5,0.6) & (0.1,0.6) & (0,0.4) \\ (0.4,0.3) & (0.3,0.5) & (0,0.3) \end{bmatrix}, K = \begin{bmatrix} (0,1) & (0,1) & (1,0) \\ (0,1) & (1,0) & (0,1) \\ (1,0) & (0,1) & (0,1) \end{bmatrix}$$

$$K(p^T, p^F)^T K = \begin{bmatrix} (0,0.3) & (0,0.4) & (0.3,0.5) \\ (0.3,0.5) & (0.1,0.6) & (0,0.4) \\ (0.4,0.3) & (0.5,0.6) & (0,0.5) \end{bmatrix}$$

Therefore, (p^T, p^F) $\neq$ K (p^T, p^F) ^{T} K

But, N ((p^T, p^F)) = N(K (p^T, p^F) ^{T} K) = $(0,1)$

Therefore, (p^T, p^F) is not k-symmetric. For this (p^T, p^F) , since (p^T, p^F) has no zero rows and no zero columns. NK (p^T, p^F) ^{T}K = $(0,1)$. Hence (p^T, p^F) is k-KS, but (p^T, p^F) is not k-symmetric.

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Lemma 3.1. For $(p^T, p^I, p^F) \in$ NFMn, the following conditions are equivalent:

(i) (p_T, p_I, p_F) is k-KS

(ii) K(p_T, p_I, p_F) is KS

(iii) (p_T, p_I, p_F)K is KS

(iv) $N(p_T, p_I, p_F)^T = N(K(p_T, p_I, p_F))$

(v) $N(p_T, p_I, p_F) = N((p_T, p_I, p_F)K)^T$

Lemma 3.2. Let $(p^T, p^I, p^F) \in$ NFMn, any two of the ensuing statements then suggest the other one,

(i) (p^T, p^I, p^F) is KS

(ii) (p^T, p^I, p^F) is k-KS

(iii) $N(p^T, p^I, p^F)^T = N((p^T, p^I, p^F)K)^T$

Proof. However, (i)and (ii) ⇒ (iii) (p^T, p^I, p^F) is k-KS

(p^T, p^I, p^F) is k-KS $\Rightarrow N(p^T, p^I, p^F) = N(K(p^T, p^I, p^F)^T K)$

$\Rightarrow N(p^T, p^I, p^F) = N(K(p^T, p^I, p^F)^T)$

Hence (i) and (ii) $\Rightarrow N(p^T, p^I, p^F)^T = N(p^T, p^I, p^F) = N((p^T, p^I, p^F)K)^T$

Thus (iii) holds.

Also (i) and (iii) implies (ii)

(p^T, p^I, p^F) is KS $\Rightarrow N(p^T, p^I, p^F) = N(p^T, p^I, p^F)^T$

Hence (i) and (iii) $\Rightarrow N(p^T, p^I, p^F) = N((p^T, p^I, p^F)K)^T$

$\Rightarrow N\left((p^T, p^I, p^F)K\right) = N((p^T, p^I, p^F)K)^T$

$\Rightarrow (p^T, p^I, p^F)$K is KS

$\Rightarrow (p^T, p^I, p^F)$ is k-KS .

Thus (ii) holds.

However (ii) and (iii) implies (i)

(p^T, p^I, p^F) is k-KS $\Rightarrow N(p^T, p^I, p^F) = N(K(p^T, p^I, p^F)^T K)$

$\Rightarrow N(p^T, p^I, p^F) = N((p^T, p^I, p^F)K)^T$

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Hence (ii) and (iii) $\Rightarrow N(p^T, p^I, p^F) = N(p^T, p^I, p^F)^T$

Lemma 3.2. For $(p^T, p^F) \in$ IFMn, (p^T, p^F) exists iff K(p^T, p^F) exists.

Proof. For $(p^T, p^F) \in$ Fmn if (p^T, p^F) exists then $(p^T, p^F) = (p^T, p^F)^T$ which implies $(p^T, p^F)^T$ is a generalized inverse of (p^T, p^F).

Conversely if $(p^T, p^F)^T$ is a generalized inverse of (p^T, p^F),

then $(p^T, p^F)(p^T, p^F)^T(p^T, p^F) = (p^T, p^F)$

$\Rightarrow (p^T, p^F)^T(p^T, p^F)(p^T, p^F)^T = (p^T, p^F)^T$.

Hence $(p^T, p^F)^T$ is a 2 inverse of (p^T, p^F).

Both $(p^T, p^F)(p^T, p^F)^T$ and $(p^T, p^F)^T(p^T, p^F)$ are symmetric.

Hence $(p^T, p^F)^T (p^T, p^F)$:

(p^T, p^F) exists $\Leftrightarrow (p^T, p^F)(p^T, p^F)^T(p^T, p^F) = (p^T, p^F)$

$\Leftrightarrow K(p^T, p^F)(p^T, p^F)^T(p^T, p^F) = K(p^T, p^F)$

$\Leftrightarrow K(p^T, p^F)(K(p^T, p^F))^T K(p^T, p^F) = K(p^T, p^F)$

$\Leftrightarrow (K(p^T, p^F))^T \in K(p^T, p^F)\{1\}$

$\Leftrightarrow (K(p^T, p^F))^+$, exists

Theorem 3.1. For $(p^T, p^F) \in$ IFMn, the following conditions are equivalent:

(vi) (p^T, p^F) is k-KS
(vii) $K(p_T, p_I, p_F)$ is KS
(viii) $(p^T, p^F)K$ is KS
(ix) $N(p^T, p^F)^T = N(K(p^T, p^F))$
(x) $N(p^T, p^F) = N((p^T, p^F)K)^T$

Theorem 3.2. Let $(p^T, p^F) \in$ IFMn, any two of the ensuing statements then suggest the other one,

$(i)(p^T, p^F)$ is KS
(ii) (p^T, p^F) is k-KS
$(iii) N(p^T, p^F)^T = N((p^T, p^F)K)^T$

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Proof. However, (i)and (ii) ⇒ (iii) (p^T, p^F) is k-KS

(p^T, p^F) is k-KS $\Rightarrow N(p^T, p^F) = N(K(p^T, p^F)^T K)$

$\Rightarrow N(p^T, p^F) = N(K(p^T, p^F)^T)$

Hence (i) and (ii) $\Rightarrow N(p^T, p^F)^T = N(p^T, p^F) = N((p^T, p^F)K)^T$

Thus (iii) holds.

Also (i) and (iii) implies (ii)

(p^T, p^F) is KS $\Rightarrow N(p^T, p^F) = N(p^T, p^F)^T$

Hence (i) and (iii) $\Rightarrow N(p^T, p^F) = N((p^T, p^F)K)^T$

$\Rightarrow N\left((p^T, p^F)K\right) = N((p^T, p^F)K)^T$

$\Rightarrow (p^T, p^F)$ K is KS

$\Rightarrow (p^T, p^F)$ is k-KS .

Thus (ii) holds.

However (ii) and (iii) implies (i)

(p^T, p^F) is k-KS $\Rightarrow N(p^T, p^F) = N(K(p^T, p^F)^T K)$

$\Rightarrow N(p^T, p^F) = N((p^T, p^F)K)^T$

Hence (ii) and (iii) $\Rightarrow N(p^T, p^F) = N(p^T, p^F)^T$

Theorem 3.3. Let $(q^T, q^F) = \begin{bmatrix} (s^T, s^F) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{bmatrix}$ where (s^T, s^F) is r × r The following similar requirements apply if the matrix is a Intuitionistic fuzzy matrix with no zero rows or zero columns:

(i) (q^T, q^F) is k-KS

(ii) $N(q^T, q^F) = N\left((q^T, q^F)K\right)^T$

(iii) $K = \begin{bmatrix} K_1 & (0,1) \\ (0,1) & K_2 \end{bmatrix}$ where K_1 and K_2 are PM of order r and n-r, respectively,

(iv) $k = k_1k_2$ where k_1 is the product of disjoint transpositions on $S_n = \{1, 2,...,n\}$ leaving (r+1,r+2,…,n) fixed and k_2 is the product of disjoint transposition leaving (1, 2,...,r) fixed.

Proof: Since (s^T, s^F) has no zero rows and no zero columns $N(s^T, s^F) = N(s^T, s^F)^T = (0,1)$. Therefore

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$N(q^T,q^F)=N(q^T,q^F)^T \neq (0,1)$. and (q^T,q^F) is Kernel symmetric.

Now we will prove the equivalence of (i),(ii) and (iii) . (q^T,q^F) is k-Kernel symmetric $\Leftrightarrow$ $N(q^T,q^F)=N\left((q^T,q^F)K\right)^T$ follows from By Lemma 3.6

Choose z =[0 y] with all element of $y \neq (0,1)$ and subdivided in conformity with that of $(q^T,q^F)=\begin{bmatrix}(s^T,s^F) & (0,1)\\ (0,1) & (0,1)\end{bmatrix}$

Clearly $z \in N(q^T,q^F)=N(q^T,q^F)^T=N\left((q^T,q^F)K\right)^T$.

Let us subdivided K as $K=\begin{bmatrix}K_1 & K_3\\ K^T{}_3 & K_2\end{bmatrix}$

Then

$$K=\begin{bmatrix}K_1 & K_3\\ K^T{}_3 & K_2\end{bmatrix}\begin{bmatrix}(D^T,D^F)^T & (0,1)\\ (0,1) & (0,1)\end{bmatrix}=\begin{bmatrix}K_1(D^T,D^F)^T & (0,1)\\ K^T{}_3(D^T,D^F)^T & (0,1)\end{bmatrix}$$

$$z=[0\ y]\in N(q^T,q^F)=N\left(K(q^T,q^F)^T\right)$$

$$\Rightarrow [0\ y]\begin{bmatrix}K_1(s^T,s^F)^T & (0,1)\\ K^T{}_3(s^T,s^F)^T & (0,1)\end{bmatrix}=(0,1)$$

$$\Rightarrow yK^T{}_3(s^T,s^F)^T=(0,1)$$

Since $N(s^T,s^F)=(0,1)$, it follows that $yK^T{}_3=(0,1)$.

Since all element of y $\neq (0,1)$ under max-min arrangement $yK^T{}_3(s^T,s^F)^T=(0,1)$ this implies $K^T{}_3$ $=(0,1) \Rightarrow K_3=(0,1)$.

Therefore $K=\begin{bmatrix}K_1 & (0,1)\\ (0,1) & K_2\end{bmatrix}$

Thus, (iii) holds, conversely, if (iii) holds, then

$$K(q^T,q^F)=\begin{bmatrix}K_1(s^T,s^F) & (0,1)\\ (0,1) & K_2\end{bmatrix}, N\left(K(q^T,q^F)^T\right)=N\left((q^T,q^F)\right)$$

Thus (i)⇔(ii)⇔(iii) holds.

However, (iii) ⇔(iv) ,

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(iii) and (iv) is clear from the definition of k.

Theorem 3.4. For $(p^T, p^F) \in$ IFMn and k = k_1k_2 (where k_1k_2 as defined in Lemma 3.7). Then the following are equivalent:

(i) (p^T, p^F) is k-KS of rank r,

(ii) (p^T, p^F) is k-similar to a diagonal block matrix $\begin{bmatrix} (s^T, s^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix}$ with det $(s^T, s^F) > (0,1)$,

(iii) $(p^T, p^F) = KGLG^T$ and $L \in F_r$ with det L > $(0,1)$ and $G^TG = I_r$.

Proof. (i) ⇔(ii) (p^T, p^F) is k-KS ⇔ KA is KS

$$\Leftrightarrow PK(p^T, p^F)P^T = \begin{bmatrix} (E^T, E^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} \text{ with det } (E^T, E^F) > (0,1),$$

for some PM P By Theorem 2.3

$$\Leftrightarrow (p^T, p^F) = KP^T \begin{bmatrix} (E^T, E^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} P$$

$$\Leftrightarrow (p^T, p^F) = \left(KP^TK\right) K \begin{bmatrix} (E^T, E^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} P$$

$$\Leftrightarrow (p^T, p^F) = \left(KP^TK\right) \begin{bmatrix} K_1 & (0,1) \\ (0,1) & K_2 \end{bmatrix} \begin{bmatrix} (E^T, E^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} P$$

$$\Leftrightarrow (p^T, p^F) = \left(KP^TK\right) \begin{bmatrix} K_1(E^T, E^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} P$$

$$\Leftrightarrow (p^T, p^F) = \left(KP^TK\right) \begin{bmatrix} (s^T, s^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} P$$

Thus (p^T, p^F) is k-similar to a diagonal block matrix $\begin{bmatrix} (s^T, s^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix}$, where $(D^T, D^F) = K_1$ (E^T, E^F) and det $(D^T, D^F) > 0$.

However, (ii) ⇔ (iii)

$$\Leftrightarrow (p^T, p^F) = \left(KP^TK\right) \begin{bmatrix} K_1(E^T, E^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix} P$$

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$$\Leftrightarrow (p^T, p^F) = K\begin{bmatrix} P_1^T & P_3^T \\ P_2^T & P_4^T \end{bmatrix}\begin{bmatrix} K_1 & (0,1) \\ (0,1) & K_2 \end{bmatrix}\begin{bmatrix} (s^T, s^F) & (0,1) \\ (0,1) & (0,1) \end{bmatrix}\begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$$

$$\Leftrightarrow (p^T, p^F) = K\begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix} K_1(s^T, s^F)\begin{bmatrix} P_1 & P_2 \end{bmatrix}$$

$$\Leftrightarrow (p^T, p^F) = KGLG^T \;, G = \begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix}, G^T = \begin{bmatrix} P_1 & P_2 \end{bmatrix}, L = K_1(s^T, s^F) \in (IFM)_r$$

$$G^T G = \begin{bmatrix} P_1 & P_2 \end{bmatrix}\begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix} = P_1 P_1^T + P_2 P_2^T = I_r, L \in (IFM)_r$$

Lemma 3.2

For $A, B \in F_n$ and P be a PM $N(A) = N(B) \Leftrightarrow N(PAP^T) = N(PBP^T)$.

4. Schur Complement (SC) in k-Kernel Intuitionistic symmetric Matrices

Theorem 4.1

Let P be a IFM of the form $P = \begin{bmatrix} (p^T, p^F) & (q^T, q^F) \\ (r^T, r^F) & (s^T, s^F) \end{bmatrix}$ with $N((p^T, p^F)) \subseteq N((q^T, q^F))$ and $N(P/(p^T, p^F)) \subseteq N((r^T, r^F))$, then the subsequent statement are equivalent.

(i) (p^T, p^F) is k – KSIFM with $k = k_1 k_2$.

(ii) (p^T, p^F) is k -KS, $P/(p^T, p^F)$ is k -KS,

$N((p^T, p^F)^T) \subseteq N((r^T, r^F)^T)$ and $N((P/(p^T, p^F))^T) \subseteq N((q^T, q^F)^T)$

(iii)Both the matrices and $\begin{bmatrix} (p^T, p^F) & (q^T, q^F) \\ (0,1) & P/(p^T, p^F) \end{bmatrix}$ are k- kernel symmetric.

Proof:(i)⇒ (ii):

To prove (p^T, p^F) is k is k-KS, $P/(p^T, p^F)$ is k-KS

Let $y_1 \in N((p^T, p^F))$ and $y_2 \in N(P/(p^T, p^F))$.Hence $y_1(p^T, p^F) = (0,1)$ and $y_2(P/(p^T, p^F)) = (0,1)$

Define y = [y_1 y_2]

we claim that $yP = [y_1\, y_2]\begin{bmatrix} (p^T p^F) & (q^T, q^F) \\ (r^T, r^F) & (s^T, s^F) \end{bmatrix} = (0,1)$

Since $N(P/(p^T, p^F)) \subseteq N(r^T, r^F)$, $y_2(P/(p^T, p^F)) = (0,1)$

Implies , $y_2(r^T, r^I, r^F) = (0,0,1)$

$N((p^T, p^F)) \subseteq N((q^T, q^F))$,

$y_1(p^T, p^F) = 0 \Rightarrow y_1(q^T, q^F) = 0$

Hence, $y_1(p^T, p^F) + y_2(r^T, r^F) = (0,1)$ and $y_1(q^T, q^F) + y_2(s^T, s^F) = (0,1)$.

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Therefore yP $=(0,1)$ i.e) $y \in N(P)$.

Since P is $\tilde{k}$ - KS, $N(P)=N(KP^T K)$

Then, $YKP^T K=(0,1)$

$$[y_1\, y_2]\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}\begin{bmatrix} (p^T,p^F)^T & (q^T,q^F)^T \\ (r^T,r^F)^T & (s^T,s^F)^T \end{bmatrix}\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}=(0,1)$$

$\Rightarrow y_1K(p^T,p^F)^T K + y_2K(q^T,q^F)^T K=(0,1)$

$\Rightarrow y_1K(p^T,p^F)^T K=(0,1)$ and $y_2K(q^T,q^F)^T K=(0,1)$ and

$\Rightarrow y_1K(r^T,r^F)^T K + y_2K(s^T,s^F)^T K=(0,1)$

$\Rightarrow y_1K(r^T,r^F)^T K=(0,1)$ and $y_2K(s^T,s^F)^T K=(0,1)$

Hence $y_1 \in N\left[K(p^T,p^F)K\right], y_2 \in N\left[K(q^T,q^F)^T K\right]$ and $y_2 \in N\left[K(s^T,s^F)K\right]$

Since $y_1 \in N\left[(p^T,p^F)\right]$ and $y_2 \in N\left[P/(p^T,p^F)\right]$ it follows that

$N\left[(p^T,p^F)\right] \subseteq N\left[K(p^T,p^F)K\right], N\left[P/(p^T,p^F)\right] \subseteq N\left[K(q^T,q^F)^T K\right]$ and

$N\left[P/(p^T,p^F)\right] \subseteq N\left[K(s^T,s^F)^T K\right]$ implies $N\left[P/(p^T,p^F)\right] \subseteq N\left[K(P/(p^T,p^F))K\right]$

Likewise, it may be demonstrated that

$N\left[K(p^T,p^F)K\right] \subseteq N\left[(p^T,p^F)\right]$

Thus (p^T,p^F) is k-KS

Since, $y_1 \in N\left[K(r^T,r^F)K\right]$ and (p^T,p^F) is k-KS

$N\left[(p^T,p^F)\right] = N\left[K(p^T,p^F)^T K\right] \subseteq N\left[K(r^T,r^F)^T K\right]$

$N\left[(p^T,p^F)^T\right] \subseteq N\left[(r^T,r^F)^T\right]$

$P/(p^T,p^F) = (s^T,s^I,s^F) - (r^T,r^I,r^F)\,(p^T,p^F)^{-}(q^T,q^F)$.

Implies $N\left[P/(p^T,p^F)\right] \subseteq N\left[K(P/(p^T,p^F))K\right]$

Likewise, it may be demonstrated that

$N\left[P/(p^T,p^F)\right] \supseteq N\left[K(P/(p^T,p^F))K\right]$

Therefore $N\left[P/(p^T,p^F)\right] = N\left[K(P/(p^T,p^F))K\right]$

Hence $P/(p^T,p^F)$ is k-KS

Since $N\left[P/(p^T,p^F)\right] \subseteq N\left[K(P/(q^T,q^F))K\right]$ and $P/(p^T,p^I,p^F)$ is k-KS,

$N\left[P/(q^T,q^F)\right] \supseteq N\left[K(P/(p^T,p^F))K\right]$.

$N\left[P/(p^T,p^F)\right] \subseteq N\left[(q^T,q^F)\right]$.

Thus (i) implies (ii) holds

(ii) implies (iii)

$$P_1=\begin{bmatrix} (p^T,p^F) & (0,1) \\ (r^T,r^F) & P/(p^T,p^F) \end{bmatrix} \text{ and } P_1=\begin{bmatrix} (p^T,p^F) & (q^T,q^F) \\ (0,1) & P/(p^T,p^F) \end{bmatrix}$$

are k-KS.

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Let $y \in N(P_1)$. Partition y conformity with that P_1 as y = [y_1 y_2] then,

$$[y_1\, y_2]\begin{bmatrix}(p^T,p^F) & (0,1)\\ (r^T,r^F) & (s^T,s^F)\end{bmatrix}=(0,1)$$

$$y_1(p^T,p^F)=(0,1), y_2(r^T,r^F)=(0,1), y_2(r^T,r^F)=(0,1)$$

$$\Rightarrow y_2\left[P/(p^T,p^F)\right]=(0,1)$$

Since (p^T,p^F) and $P/(p^T,p^F)$ are k-KS

$$y_1 \in N\left[(p^T,p^F)\right]=N\left[K(p^T,p^F)K\right]$$

$$\Rightarrow y_1K(p^T,p^F)K=(0,1)$$

$$y_2 \in N\left[P/(p^T,p^F)\right]^T=N\left[K\left(P/(p^T,p^F)\right)^T K\right]$$

$$\Rightarrow y_2\left[K\left(P/(p^T,p^F)\right)^T K\right]=(0,1)$$

Since, $N\left[(p^T,p^F)^T\right]\subseteq N\left[(r^T,r^F)^T\right]$

$$N\left[K(p^T,p^F)^TK\right]\subseteq N\left[K(r^T,r^F)^TK\right]$$

$$\Rightarrow y_1K(r^T,r^F)^TK=(0,1)$$

Now by using $y_1K(p^T,p^F)^TK=(0,1), y_1K(r^T,r^F)^TK=(0,1)$

And $y_2K\left[P/(p^T,p^F)\right]^TK=(0,1)$ it can be verified that

$$[y_1\, y_2]\begin{bmatrix}K(p^T,p^F)^TK & K(r^T,r^F)K\\ (0,1) & K(P/(p^T,p^F)^TK\end{bmatrix}=(0,1)$$

Thus $N\left[P_1\right]\subseteq N\left[KP_1^TK\right]$.

$$N\left[KP_1^TK\right]\subseteq N\left[P_1\right]$$

Hence Therefore, $N\left[KP_1^TK\right]=N\left[P_1\right]$

Hence P_1 is k-KS.
Similarly, it may be demonstrated that P_2 is k-KS.
Thus (ii)⇒ (iii) holds.
(iii)⇒ (i)
P_1 is k - KS $\Rightarrow N(P_1)=N(KP^TK)$
P_2 is k- KS$\Rightarrow N(P_2)=N(KP^TK)$
To prove, P is k-KS that is $N(P)=N(KP^TK)$.
Let $y\in N(P)\Rightarrow yP=0$.
M as y = [y_1 y_2] then,

$$[y_1\, y_2]\begin{bmatrix}(p^T,p^F) & (q^T,q^F)\\ (r^T,r^F) & (s^T,s^F)\end{bmatrix}=(0,1)$$

$$y_1(p^T,p^F)+y_2(r^T,r^F)=(0,1)$$

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$\Rightarrow y_1(p^T, p^F) = (0,1), y_2(r^T, r^F) = (0,1)$
$y_1(q^T, q^F) + y_2(s^T, s^F) = (0,1)$
$\Rightarrow y_1(q^T, q^F) = (0,1), y_2(s^T, s^F) = (0,1)$

From the definition of $P/(p^T, p^F)$ A = (s^T, s^F) - (r^T, r^F) c $(p^T, p^F)^{-}(q^T, q^F)$

We have, $y_2(s^T, s^F) = (0,1), y_2(r^T, r^F) = (0,1)$

$\Rightarrow y_2\left(P/(p^T, p^F)\right) = (0,1)$

$y_1(p^T, p^F) + y_2(r^T, r^F) = (0,1)$ and $y_2\left(P/(p^T, p^F)\right) = (0,1)$

And $y_2(p^T, p^F) = (0,1)$,

$y_1(q^T, q^F) + y_2(r^T, r^F) = (0,1)$ and $y_2\left(P/(p^T, p^F)\right) = (0,1)$

$y \in N(P_1) \Rightarrow y \in N(KP_1^T K)$

$y \in N(P_2) \Rightarrow y \in N(KP_2^T K)$

Hence, $y \in N(KP^T K)$.

$N(P) \subseteq N(KP^T K)$

Similarly, $N(P) \supseteq N(KP^T K)$

$N(P) = N(KP^T K)$
Therefore, P is k-KSNFM.

Theorem 4.2

Let P be a IFM of the form $P = \begin{bmatrix} (p^T, p^F) & (q^T, q^F) \\ (r^T, r^F) & (s^T, s^F) \end{bmatrix}$ with $N((p^T, p^F)) \subseteq N((r^T, r^F)^T)$ and $N(\left(P/(p^T, p^F)\right)^T \subseteq N((q^T, q^F)^T)$, then the subsequence are equivalent.

(i) P is k – KSIFM with $k = k_1k_2$.

(ii) (p^T, p^F) is k -KS, $P/(p^T, p^F)$ is k -KS, $N(p^T, p^F) \subseteq N(q^T, q^F)$ and $N((P/(p^T, p^F)) \subseteq N$ (r^T, r^I, r^F)

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(iii) Both the matrices $\begin{bmatrix} (p^T,p^F) & (0,1) \\ (r^T,r^F) & P/(p^T,p^F) \end{bmatrix}$ and $\begin{bmatrix} (p^T,p^F) & (q^T,q^F) \\ (0,1) & P/(p^T,p^F) \end{bmatrix}$ are k- kernel symmetric.

Proof: This theorem is directly supported by Theorem (4.1) and the observation that P is k- KS⇔ P^T is k - KS.

Theorem 4.3

Let P be a IFM of the form $P=\begin{bmatrix} (p^T,p^F) & (q^T,q^F) \\ (r^T,r^F) & (s^T,s^F) \end{bmatrix}$ with $N((p^T,p^F)) \subseteq N((r^T,r^F)^T)$ and $N(\left(P/(p^T,p^F)\right) \subseteq N((r^T,r^F))$, then the following are equivalent.

(i) P is k – KSIFM with $k = k_1k_2$.

(ii) (p^T,p^F) is k -KS, $P/(p^T,p^F)$ is k -KS,

(iii) The matrices $\begin{bmatrix} (p^T,p^F) & (0,1) \\ (r^T,r^F) & P/(p^T,p^F) \end{bmatrix}$ is k- KS.

Remark 4.1 It is crucial to consider the condition that is placed on P in Theorems 3.1 and 3.2. The example that follows serves to illustrate this.

Example:4.1 Let us consider a IFM

$$P=\begin{bmatrix} (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) \\ (1,0) & (0,1) & (1,0) & (1,0) \\ (1,0) & (1,0) & (1,0) & (1,0) \end{bmatrix} \text{ and } K=\begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix}$$

$$P=\begin{bmatrix} (p^T,p^F) & (q^T,q^F) \\ (r^T,r^F) & (s^T,s^F) \end{bmatrix}$$

$$(p^T,p^I,p^F)=\begin{bmatrix} (1,0) & (1,0) \\ (1,0) & (1,0) \end{bmatrix}, (q^T,q^F)=\begin{bmatrix} (1,0) & (0,1) \\ (1,0) & (1,0) \end{bmatrix},$$

$$(r^T,r^I,r^F)=\begin{bmatrix} (1,0) & (0,1) \\ (1,0) & (1,0) \end{bmatrix}, (s^T,s^I,s^F)=\begin{bmatrix} (1,0) & (1,0) \\ (1,0) & (1,0) \end{bmatrix}$$

For this P, since P has ho zero rows and no zero columns N(P) = (0,1).

$N(KP^TK)$ = (0,1).Thus N(P) = $N(KP^TK)$ ⇒ P is k- KS.

$(p^T,p^F)=\begin{bmatrix} (1,0) & (1,0) \\ (1,0) & (1,0) \end{bmatrix}$ is a g-inverse, with respect to $(p^T,p^F)^-$

$P/(p^T,p^F) = (s^T,s^I,s^F) - (r^T,r^F)\,(p^T,p^F)^-(q^T,q^F)$.

$$P/(p^T,p^F) = \begin{bmatrix} (1,0) & (1,0) \\ (1,0) & (1,0) \end{bmatrix} - \begin{bmatrix} (1,0) & (0,1) \\ (1,0) & (1,0) \end{bmatrix}\begin{bmatrix} (1,0) & (1,0) \\ (1,0) & (1,0) \end{bmatrix}\begin{bmatrix} (1,0) & (0,1) \\ (1,0) & (1,0) \end{bmatrix},$$

$$P/(p^T,p^F) = \begin{bmatrix} (1,0) & (0,1) \\ (1,0) & (1,0) \end{bmatrix},$$

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P/(p^T, p^F) is k - KS, since $N(P/(p^T, p^F)) = N(K(P/(p^T, p^F))^T K)$ = (0,1).

(p^T, p^F) is k - KS, since $N(P)$ = $N(KP^T K)$ = (0,1) for all K.
$N((p^T, p^F)) \subseteq N((q^T, q^F))$ and $N((p^T, p^F)^T) \subseteq N((r^T, r^F)^T)$.

Here, N(P/(p^T, p^F)) =(0,1) = $N((P/(p^T, p^F))^T)$,

N((r^T, r^F)) = (0,1) , $N((q^T, q^F)^T)$ = (0,1)
N(P/(p^T, p^F)) contained in N($(r^T, r^F)^T$) and $N((P/(p^T, p^F))^T)$ contained in $N((q^T, q^F)^T)$.
Further

$$P_1 = \begin{bmatrix} (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (0,1) \end{bmatrix} \text{ and } K = \begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix}$$

$$N(P_1) = (0,1).$$

$$KP_1^T K = \begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix} \begin{bmatrix} (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (1,0) \\ (1,0) & (1,0) & (0,1) & (0,1) \end{bmatrix}$$

$$\begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix}$$

$$KP_1^T K = \begin{bmatrix} (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{bmatrix}$$

$$N(KP_1^T K) = \left\{(0,1), (0,1), (0,1), (a^T, c^F) : (a^T, c^F) \in (IFM)\right\}$$

$\Rightarrow P_1$ is not k- Kernel Symmetric.

$$P_2 = \begin{bmatrix} (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (1,0) & (1,0) & (1,0) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \end{bmatrix}$$

P_2 is not k- Kernel Symmetric.

Remark:4.2 For a KSIFM P of the form $P_1 = \begin{bmatrix} (p^T, p^F) & (q^T, q^F) \\ (r^T, r^F) & (s^T, s^F) \end{bmatrix}$ with $k = k_1 k_2$ following are equivalent.

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$N((p^T, p^F)) \subseteq N((q^T, q^F))$ and $N(P/(p^T, p^F)) \subseteq N((r^T, r^F))$,
$N((p^T, p^F)^T) \subseteq N((r^T, r^F)^T)$, and $N((P/(p^T, p^F))^T) \subseteq N((q^T, q^F)^T)$

Example:4.2 Let us consider a IFM

$$P = \begin{bmatrix} (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (1,0) & (1,0) \end{bmatrix} \text{ and } K = \begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix}$$

$$N(P) = \{(0,1),(0,1),(a^T, c^F),(0,1) : (a^T, c^F) \in (IFM)\}$$

$$KP^TK = \begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix} \begin{bmatrix} (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (0,1) & (0,1) \\ (1,0) & (0,1) & (1,0) & (1,0) \end{bmatrix}$$

$$\begin{bmatrix} (0,1) & (1,0) & (0,1) & (0,1) \\ (1,0) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0,1) & (0,1) & (1,0) \\ (0,1) & (0,1) & (1,0) & (0,1) \end{bmatrix}$$

$$KP^TK = \begin{bmatrix} (1,0) & (1,0) & (0,1) & (0,1) \\ (1,0) & (1,0) & (1,0) & (0,1) \\ (1,0) & (0,1) & (1,0) & (0,1) \\ (0,1) & (1,0) & (1,0) & (0,1) \end{bmatrix}$$

$$N(KP^TK) = (0,1)$$

$$N(P) \neq N(KP^TK)$$

Therefore P is not k - KS.

$$P/(p^T, p^F) = \begin{bmatrix} (0,1) & (0,1) \\ (1,0) & (0,1) \end{bmatrix}$$

Here $N(p^T, p^F) \subseteq N(q^T, q^F), N(P/(p^T, p^F)) \subseteq N(r^T, r^F)$

But $N(p^T, p^F)^T$ is not contained $N(r^T, r^F)^T$.

$N(P/(p^T, p^F))^T$ is not contained $N(q^T, q^F)^T$.

5. Conclusion

The theorems explain the properties of k-KS and Schur Complement in *k*-KS IFM and NFM. With the help of relevant examples.

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