Representation of Symbolic 2-Plithogenic Matrices by Symbolic 2-Plithogenic Linear Transformations

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Abstract

The main purpose of this article is to study about the representation symbolic 2-plithogenic matrices by linear transformations between symbolic 2-plithogenic vector spaces, where it proves that every symbolic 2-plithogenic matrix can be represented uniquely by a linear transformation between symbolic 2-plithogenic vector spaces. Also, this work proves that any linear function between symbolic 2-plithogenic vector spaces must be an AH-linear transformation.

Keywords: Symbolic 2-plithogenic vector space; symbolic 2-plithogenic matrice; linar transformation.

1 Introduction

The concept of refined neutrosophic structure was studied by many authors in.[1–3,8] Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation.[20]

In,[14] the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings are studied. In,[9] some more algebraic properties of symbolic 2-plithogenic rings are studied. Further, Taffach[21,22] studied the concepts of symbolic 2-plithogenic vector spaces and modules.

Recently, in,[7] the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepci et.al,[15] studied the the symbolic 2-plithogenic number theory and integers.

Moreover, Mohammad Abobala[4] presented the representation of neutrosophic matrices defined over a neutrosophic field by neutrosophic linear transformations between neutrosophic vector spaces, where he proved that every neutrosophic matrix can be represented uniquely by a neutrosophic linear transformation. As the continuation in[5] Mohammad Abobala et.al studied the neutrosophic linear transformations and their relationship with neutrosophic matrices to the case of refined neutrosophic matrices and refined neutrosophic linear transformations.

Motivated by these works, in this article the symbolic 2-plithogenic linear transformations and their relationship with symbolic 2-plithogenic matrices are studied. Also, many examples are illustrated to clarify the validity of the work.

All symbolic 2-plithogenic matrices are defined over the symbolic 2-plithogenic field of real numbers $2-SP_R$.

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2 Preliminaries

Definition 2.1. [14] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2-SP_R = \{a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{max(1,2)} = P_2\}$$

Smarandache has defined algebraic operations on $2-SP_R$ as follows:
Addition:
$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$

Multiplication:
$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = (a_0b_0) + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2$.

It is clear that $2-SP_R$ is a ring. If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Definition 2.2. [7] A symbolic 2-plithogenic real square matrix is a matrix with symbolic 2-plithogenic real entries.

Theorem 2.3. [7] *Let $S = S_0 + S_1P_1 + S_2P_2$ be a symbolic 2-plithogenic real square matrix, then*

1. *S is invertible if and only if $S_0, S_0 + S_1, S_0 + S_1 + S_2$ are invertible.*
2. *If S is invertible then*

$$S^{-1} = S_0^{-1} + [(S_0 + S_1)^{-1} - S_0^{-1}]P_1 + [(S_0 + S_1 + S_2)^{-1} - (S_0 + S_1)^{-1}]$$

3. *$S^m = S_0^m + [(S_0 + S_1)^m - S_0^m]P_1 + [(S_0 + S_1 + S_2)^m - (S_0 + S_1)^m]$ for $m \in N$.*

Definition 2.4. [21] Let V be a vector space over the field F, let $2-SP_F$ be the corresponding symbolic 2-plithogenic field. $2-SP_F = \{x + yP_1 + zP_2; x, y, z \in F, P_i^2 = P_i, P_1 \times P_2 = P_2, P_2 \times P_1 = P_2\}$. We define the symbolic 2-plithogenic vector space as follows:

$$2-SP_V = V + VP_1 + VP_2 = \{a + bP_1 + cP_2; a, b, c \in V\}.$$

Operations on $2-SP_V$ can be defined as follows: Addition:
$[x_0 + x_1P_1 + x_2P_2] + [y_0 + y_1P_1 + y_2P_2] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2$

Multiplication:
$[a + bP_1 + cP_2].[x_0 + x_1P_1 + x_2P_2] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2$.
where $x_i, y_i \in V, a, b, c \in F$.

Definition 2.5. [21] Let V, W be two vector spaces over the field F. Let $2-SP_V$, $2-SP_W$ be the corresponding symbolic 2-plithogenic vector spaces over $2-SP_F$.
Let $L_0, L_1, L_2 : V \to W$ be three linear transformations, we define the AH-linear transformations as follows:
$L : 2-SP_V \to 2-LP_W, L = L_0 + L_1P_1 + L_2P_2; L(x + yP_1 + zP_2) = L_0(x) + L_1(y)P_1 + L_2(z)P_2$.
If $L_0 = L_1 = L_2$, then L is called AHS-Linear transformation.

Theorem 2.6. [21] *Let $T = \{t_1, t_2, \cdots, t_n\}$ be a basis of the vector space V over the field F, then the set:*

$$T_P = \{t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \le i, j, k \le n\}$$

is a basis of $2-SP_V$.

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Definition 2.7. [5] Let V, W be two vector spaces over R, and consider any three linear transformations $g, h, q : V \to W$. We define the corresponding full AH-refined linear transformation as follows:

$$f : V(I_1, I_2) \to W(I_1, I_2)$$
$$f(x + yI_1 + zI_2) = g(x) + I_1[(g + h + q)(x + y + z) - (g + q)(x + z)] + I_2[(g + q)(x + z) - g(x)].$$

We denote it by $f = g + hI_1 + qI_2$.

Theorem 2.8. [5] *Let $f = g + hI_1 + qI_2 : V(I_1, I_2) \to W(I_1, I_2)$ be any full AH-linear transformation, then f is linear by classical meaning.*

Definition 2.9. [5] Let $M = A + BI_1 + CI_2$ be a matrix over $R(I_1, I_2)$. We say that M is the matrix of $f = g + hI_1 + qI_2$ if and only if $f(x + yI_1 + zI_2) = M \cdot (x + yI_1 + zI_2)$

Theorem 2.10. [5] *Let $f = g + hI_1 + qI_2 : V(I_1, I_2) \to W(I_1, I_2)$ be any full AH-linear transformation, then $M = A + BI_1 + CI_2$ is the matrix of f if and only if A is the matrix of g, B is the matrix of h, C is the matrix of q.*

3 Matrix Representation of Symbolic 2-Plithogenic Linear Transformations

We begin this section with the following definition.

Definition 3.1. Let V, W be two vector spaces over R and let g, h, q be any three linear transformations from V to W. We define the corresponding full AH-linear transformation $f : 2 - SP_V \to SP_W$ as follows:

$$f(x + yP_1 + zP_2) = g(x) + [(g + h)(x + y) - g(x)]P_1 + [(g + h + q)(x + y + z) - (g + h)(x + y)]P_2.$$

We denote it by $f = g + hP_1 + qP_2$

Theorem 3.2. *Let $f = g + hP_1 + qP_2 : 2 - SP_V \to SP_W$ be a full AH-linear transformations, then f is linear by classical meaning.*

Proof. For all $X = x + yP_1 + zP_2, Y = x' + y'P_1 + z'P_2 \in 2 - SP_V$, we have:

$$\begin{aligned}
f(X + Y) &= f[(x + x') + (y + y')P_1 + (z + z')P_2] \\
&= g(x + x') + [(g + h)(x + x' + y + y') - g(x + x')]P_1 \\
&\quad +[(g + h + q)(x + x' + y + y' + z + z') - (g + h)(x + x' + y + y')]P_2 \\
&= g(x) + g(x') + [(g + h)(x + y) + (g + h)(x' + y') - g(x) - g(x')]P_1 \\
&\quad +[(g + h + q)(x + y + z) + (g + h + q)(x' + y' + z') - (g + h)(x + y) - (g + h)(x' + y')]P_2 \\
&= [g(x) + [(g + h)(x + y) - g(x)]P_1 + [(g + h + q)(x + y + z) - g + h(x + y)]P_2] \\
&\quad +[g(x') + [(g + h)(x' + y') - g(x')]P_1 + [(g + h + q)(x' + y' + z') - g + h(x' + y')]P_2] \\
&= f(X) + g(X).
\end{aligned}$$

For all $r = r_0 + r_1P_1 + r_2P_2$, we have:

$$r \cdot X = r_0x + [(r_0 + r_1)(x + y) - r_0x]P_1 + [(r_0 + r_1 + r_2)(x + y + z) - (r_0 + r_1)(x + y)]$$

Now,

$$\begin{aligned}
f(r \cdot X) &= g(r_0x) + [(g + h)[(r_0 + r_1)(x + y)] - g(r_0x)]P_1 \\
&\quad +[(g + h + q)[(r_0 + r_1 + r_2)(x + y + z)] - (g + h)[(r_0 + r_1)(x + y)]]P_2 \\
&= r_0g(x) + [(r_0 + r_1)(g + h)(x + y) - r_0g(x)]P_1 + \\
&\quad [(r_0 + r_1 + r_2)(g + h + q)(x + y + z) - (r_0 + r_1)(g + h)(x + y)]P_2 \\
&= (r_0 + r_1P - 1 + r_2P_2)[g(x) + [(g + h)(x + y) - g(x)]P_1 \\
&\quad +[(g + h + q)(x + y + z) - (g + h)(x + y)]P_2] \\
&= r \cdot f(X).
\end{aligned}$$

□

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Definition 3.3. Let $M = M_0 + M_1P_1 + M_2P_2$ be a symbolic 2-plithogenic matrix. We say that M is the matrix representation of the full AH-linear transformation $f = g+hP_1+qP_2$ if and only if $f(x+yP_1+zP_2) = M \cdot (x + yP_1 + zP_2)$.

Theorem 3.4. *If $f = g + hP_1 + qP_2 : 2 - SP_V \to 2 - SP_W$ is a full AH-linear transformation, then $M = M_0 + M_1P_1 + M_2P_2$ is the matrix representation of $f = g + hP_1 + qP_2$ if and only if M_0 is the matrix representation g, M_1 is the matrix representation of h and M_2 is the matrix representation of q.*

Proof. Assume that $M = M_0 + M_1P_1 + M_2P_2$ is the matrix representation of $f = g + hP_1 + qP_2$. We have to prove that M_0 is the matrix representation g, M_1 is the matrix representation of h and M_2 is the matrix representation of q.

Since M is the matrix representation of f, we have $f(x + yP_1 + zP_2) = M \cdot (x + yP_1 + zP_2)$. This implies that, $g(x) + [(g + h)(x + y) - g(x)]P_1 + [(g + h + q)(x + y + z) - (g + h)(x + y)]P_2 = M_0x+[(M_0+M_1)(x+y)-M_0x]P_1+[(M_0+M_1+M_2)(x+y+z)-(M_0+M_1)(x+y)]P_2$. Hence, we have

$$g(x) = M_0 \cdot x, (g+h)(x+y) = (M_0+M_1)(x+y), (g+h+q)(x+y+z) = (M_0+M_1+M_2)(x+y+z).$$

This implies that, M_0 is the matrix representation of g, $M_0 + M_1$ is the matrix representation of $g + h$ and $M_0 + M_1 + M_2$ is the matrix representation of $g + h + q$.
Hence, M_0 is the matrix representation of g, M_1 is the matrix representation of h, and M_2 is the matrix representation of q.

Conversely, assume that M_0 is the matrix representation of g, M_1 is the matrix representation of h, and M_2 is the matrix representation of q. We have to show that, $M = M_0 + M_1P_1 + m_2P_2$ is the matrix representation of $f = g + hP_1 + qP_2$.
For all $x + yP_1 + zP_2 \in 2 - SP_V$, we have

$$\begin{aligned} M(x + yP_1 + zP_2) &= [M_0 + M_1P_1 + M_2P_2](x + yP_1 + zP_2) \\ &= M_0x + [(M_0 + M_1)(x + y) - M_0x]P_1 \\ &\qquad +[(M_0 + M_1 + M_2)(x + y + z) - (M_0 + M_1)(x + y)]P_2 \\ &= g(x) + [(g + h)(x + y) - g(x)]P_1 + [(g + h + q)(x + y + z) - (g + h)(x + y)]P_2 \\ &= f(x + yP_1 + zP_2). \end{aligned}$$

Hence M is the matrix representation of f. □

Theorem 3.5. *Let $f : 2-SP_V \to 2-SP_W$ be a linear function, then f must be full AH-linear transformation.*

Proof. We have to prove that there exists three classical linear transformations $g, h, q : V \to W$ with the property that $f = g + hP_1 + qP_2$. We know that the image of a basis $v_{i,j,k}$ under a linear transfomation must be a a basis, thus we have:

$$f(v_{i,j,k}) = \{v_i + (v_j - v_i)P_1 + (v_k - v_j)P_2\}) = \{w_i + (w_j - w_i)P_1 + (w_k - w_j)P_2\}$$

is a basis of $2 - SP_W$, where $w_i, w_j, w_k \in W$.
Consider the functions:

$$\begin{gathered} g : V \to W; g(v_i) = w_i \\ h : V \to W; h'(v_j) = w_j \\ q : V \to W; q'(v_k) = w_k \end{gathered}$$

$f(v_{i,j,k}) = g(v_1) + (h'(v_j) - g(v_i))P_1 + (q'(v_k) - h'(v_j))P_2$.
By taking $h = h' - g$ and $q = q' - g - h$, we get

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$f(v_{i,j,k}) = g(v_1) + [(g+h)(v_j) - g(v_i)]P_1 + [(g+h+q)(v_k) - (g+h)(v_j)]P_2$, thus $f = g + hP_1 + qP_2$.

It is clear that g, h_1 and q_1 are linear hence, h and q are also linear. Hence f is a full AH-linear transformation. □

Remark 3.6. Every linear transformation $f : 2 - SP_V \to 2 - SP_W$ can be represented by a unique symbolic 2-plithogenic matrix, which is equal to the corresponding matrix of the corresponding full AH-linear transformation.

In the following Example 3.7, we illustrate the representation of a AH-linear transformation by symbolic 2-plithogenic matrix .

Example 3.7. Consider the linear transformation $f : 2 - SP_{R^2} \to 2 - SP_{R^2}$ defined by

$$\begin{aligned} f[(x,y) + (s,t)P_1 + (m,n)] &= f[(x + sP_1 + mP_2, y + tP_1 + nP_2)] \\ &= (2(x + sP_1 + mP_2), -(y + tP_1 + nP_2)) \\ &= (2x, -y) + (2s, -t)P_1 + (2m, -n)P_2. \end{aligned}$$

Now, we have to covert f into a full AH-linear transformation. By Theorem 3.5, f must be equal to $g + hP_1 + qP_2$, where $g, h, q; R^2 \to R^2$.

Here, we have $g : R^2 \to R^2; g(x,y) = (2x, -y)$ and $(g+h)[(x,y)+(s,t)]-g(x,y) = (2s,-t)$. This implies that $g(x+s, y+t)+h(x+s, y+t)-(2x,-y) = (2s,-t)$, so that $(2x+2s, -y-t)+h(x+s, y+t)-(2x,-y) = (2s,-t)$, hence $h(x+s, y+t) = (0,0)$ for all x, y, s, t. This implies that $h(s,t) = 0$ is a zero map.

Also, $(g+h+q)[(x,y)+(s,t)+(m,n)] - (g+h)[(x,y)+(s,t)] = (2m,-n)$. Since $h = 0$, we have $g(x+s+m, y+t+n) + q(x+s+m, y+t+n) - g(x+s, y+t) = (2m,-n)$ so that $(2x+2s+2m, -y-t-n) + q(x+s+m, y+t+n) - (2x+2s, -y-t) = (2m,-n)$, hence $q(x+s+m, y+t+n) = (0,0)$ for all x, y, s, t, m, n. This implies that q is a zero map.

Therefore, $f = g + 0P_1 + 0P_2$.

Here, the matrix of g is $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$, the matrix of h is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ and the matrix of q is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Thus the symbolic 2-plithogenic matrix of f is $M = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$ by Theorem 3.4.

Now, we verify that M is the symbolic 2-plithogenic matrix of f by the following computing:

$$\begin{aligned} M \cdot (x + sP_1 + mP_2, y + tP_1 + nP_2) &= \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x + sP_1 + mP_2 \\ y + tP_1 + nP_2 \end{pmatrix} \\ &= (2(x + sP_1 + mP_2), -(y + tP_1 + nP_2)) \\ &= f[(x,y) + (s,t)P_1 + (m,n)]. \end{aligned}$$

In the following Example 3.8, we illustrate how a symbolic 2-plithogenic matrix can be turned into a full AH-linear transformation.

Example 3.8. Consider the symbolic 2-plithogenic matrix

$$\begin{pmatrix} 1 & 1 + P_1 \\ 3 - P_2 & 2P_1 \end{pmatrix}$$

Then M can be represented by a unique symbolic 2-plithogenic linear transformation $f : 2 - SP_{R^2} \to 2 - SP_{R^2}$ as follows:

$f(x + sP_1 + mP_2, y + tP_1 + nP_2) = M \cdot (x + sP_1 + mP_2, y + tP_1 + nP_2) =$
$((x+y) + (s+y+2t)P_1 + (m+2n)P_2, 3x + (3s+2y+2t)P_1 + (2m-x-s+2n)P_2)$.

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4 Conclusion

In this paper, we have studied the problem of representing symbolic 2-plithogenic matrices by linear transformations between two symbolic 2-plithogenic vector spaces, where we proved that every symbolic 2-plithogenic matrix can be represented by a unique linear transformation between two symbolic 2-plithogenic vector spaces. Also, we have illustrated many interesting examples to enhance understanding.

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