Common Fixed Point Theorems For Weakly Compatible Mappings In Complex Valued $\mathcal{M}-$ Fuzzy Metric Spaces

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Abstract

In this paper, we introduce the notion of complex valued $\mathcal{M}$- fuzzy metric spaces. We are proving a common fixed point theorem for weakly compatible mappings satisfying common E.A. Like property in complex valued $\mathcal{M}$- fuzzy metric space. Our results improve and extend the results of Singh et al.[12]

Keywords: Metric Space; Complex Valued; Common E.A. Like Property; Compatible Maps; Weakly Compatible Maps.

1 introduction

In 1965, Zadeh[14] introduced the concept of fuzzy set. Following the concept of fuzzy sets Kramosil and Michalek[9] introduced the concept of fuzzy metric space in 1975. George and Veeramani[4] modified the notion of fuzzy metric spaces with the help of continuous t-norm, which shows a new way for further development of analysis in such spaces. It has been seen that the study of Kramosil and Michalek[9] of fuzzy metric space covered almost all the points in the way in developing this theory in the field of fixed point theorem, in particular for the study of contractive type maps. They have also shown that every metric induces a fuzzy metric. In 2006, Sedghi and Shobe[12] defined $\mathcal{M}-$ fuzzy metric spaces and proved a common fixed point theorem for four weakly compatible mappings in this space. Fuzzy complex numbers and fuzzy complex analysis were first introduced by Buckley.[2] Acknowledging the Buckley's work some authors continued research in fuzzy complex numbers. In this series Ramot et al.[10] extended fuzzy sets to complex fuzzy sets. According to Ramot et al.,[10] the complex fuzzy set is characterized by a membership function, whose range is not limited to [0, 1] but extended to the unit circle in the complex plane. Membership in a complex fuzzy set remains "as fuzzy" as membership in a traditional fuzzy set.

In this paper, we introduce the notion of complex valued $\mathcal{M}$- fuzzy metric spaces. We are proving a common fixed point theorem for weakly compatible mappings satisfying common E.A. Like property in complex valued $\mathcal{M}$- fuzzy metric space.

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2 Preliminaries

Definition 2.1. A binary operation $* : [0,1]e^{i\theta} \times [0,1]e^{i\theta} \to [0,1]e^{i\theta}$, where $[0, \frac{\pi}{2}]$ is fixed, is called a complex valued continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * e^{i\theta} = a$ *, for all* $a \in [0,1]\ e^{i\theta}$ *(existence of identity element* $e^{i\theta} = 1.\ e^{i\theta}$),
4. $a * b \preccurlyeq c * d$, *whenever* a $\preccurlyeq c$ *and* $b \preccurlyeq d$, *for all a, b, c, d* $\in [0,1]e^{i\theta}$.

Example 2.2. (i) $a * b = \min\{a, b\}$ *for all* $a, b \in [0,1]e^{i\theta}$.
(ii) $a * b = \max\{a + b - e^{i\theta}, 0\}$ *for all* $a, b \in [0,1]e^{i\theta}$ *and fixed* $\theta \in [0, \frac{\pi}{2}]$.

Definition 2.3. A 3-tuple $(X, \mathcal{M}, *)$ is called complex valued $\mathcal{M}-$ fuzzy metric space if X is an arbitrary non empty set, $*$ is a complex valued continuous t-norm and $\mathcal{M} : X^3 \times (0, \infty) \to [0,1]e^{i\theta}$ is a complex valued fuzzy set on, satisfying the following conditions: for each $x, y, z, a \in X$ *and* $t, s \succ 0$.

1. $\mathcal{M}(x, y, z, t) \succ 0$,
2. $\mathcal{M}(x, y, z, t) = e^{i\theta}$ *, if and only if* $x = y = z$,
3. $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, *where p is a permutation function,*
4. $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \preccurlyeq \mathcal{M}(x, y, z, t + s)$,
5. $\mathcal{M}(x, y, z, .) : [0, \infty) \to [0,1]e^{i\theta}$ *is continuous,*
6. $\lim_{t\to\infty} \mathcal{M}(x, y, z, t) = e^{i\theta}$ *, for all* $x, y, z \in X$.

Remark 2.4. If $\theta = 0$, then Complex fuzzy metric space becomes ordinary fuzzy metric space in sense of George and Veeramani.

Definition 2.5. Let $(X, \mathcal{M}, *)$ be a complex valued $\mathcal{M}-$ fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $r \in C,\ 0 \prec r \prec e^{i\theta}$ as

$$B_{\mathcal{M}}(x, r, t) = \{y \in X : \mathcal{M}(x, y, y, t) \succ e^{i\theta} - r\} \text{ where } \theta \in [0, \frac{\pi}{2}].$$

A point $x \in X$ is called an interior point of set $A \subset X$, whenever there exists $r \in C, 0 \prec r \prec e^{i\theta}$ such that $B_{\mathcal{M}}(x, r, t) = \{y \in X : \mathcal{M}(x, y, y, t) \succ e^{i\theta} - r\} \subset A$, where $\theta \in [0, \frac{\pi}{2}]$.
The subset A of X is called open whenever each element of A is an interior point of A.

Theorem 2.6. *Every complex valued* $\mathcal{M}-$ *fuzzy metric space is a Hausdorff space.*

Proof. Let (X, $\mathcal{M}$, $*$) be a complex valued $\mathcal{M}-$ fuzzy metric space and x, y be two distinct point of X. Then $0 \prec \mathcal{M}(x, y, y, t) \prec e^{i\theta}$. Put $\mathcal{M}(x, y, y, t) = e^{i\theta}$ for some $r \in C,\ 0 \prec r \prec e^{i\theta}$. Then for each r with $r \prec r_0 \prec ei\theta$, there exists r_1 such that $r_1 * r_1 \geq r_0$. Now consider the open ball $B_{\mathcal{M}}(x, e^{i\theta} - r_2, \frac{t}{2})$ and $B_{\mathcal{M}}(y, e^{i\theta} - r_2, \frac{t}{2})$. Clearly, we have $B_{\mathcal{M}}(x, e^{i\theta} - r_2, \frac{t}{2}) \cap B_{\mathcal{M}}(y, e^{i\theta} - r_2, \frac{t}{2}) = \varphi$. In fact if there exists $z \in B_{\mathcal{M}}(x, e^{i\theta} - r_2, \frac{t}{2}) \cap B_{\mathcal{M}}(y, e^{i\theta} - r_2, \frac{t}{2})$, then we have

$$\begin{aligned} r = \mathcal{M}(x, y, y, t) = \mathcal{M}(x, x, y, t) &\succcurlyeq \mathcal{M}(x, x, z, \frac{t}{2}) * \mathcal{M}(z, y, y, \frac{t}{2}) \\ &= \mathcal{M}(x, z, z, \frac{t}{2}) * \mathcal{M}y, z, z, \frac{t}{2}) \\ &\succcurlyeq r_1 * r_1 \succcurlyeq r_0 \succ r. \end{aligned}$$

which is a contradiction. Hence $(X, \mathcal{M}, *)$ is a Hausdorff space.

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Example 2.7. Let $X = R$ and $\mathcal{M}(x, y, z, t) = \frac{t}{t+|x-y|+|y-z|+|z-x|}$, for every x, y, z and $t \succ 0$, let A and B be defined as $Ax = 2x + 1, Bx = x + 2$, Consider the sequence $x_n = \frac{1}{n} + 1$, $n = 1, 2, \cdots$. Thus, we have $\lim_{n\to\infty} \mathcal{M}(Ax_n, 3, 3, t) = \lim_{n\to\infty} \mathcal{M}(Bx_n, 3, 3, t) = e^{i\theta}$ and for every $t \succ 0$. Then A and B said to satisfy the property $(E.A.)$.

Definition 2.8. A sequence x_n in a complex valued $\mathcal{M}-$ fuzzy metric spaces $(X, \mathcal{M}, *)$ is a Cauchy sequence if and only in $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t) = e^{i\theta}$, for all $t \succ 0$ and $p \succ 0$ or $\lim_{n\to\infty} |\mathcal{M}(x_{n+p}, x_{n+p}, x_n, t)| = 1$, for all $t \succ 0$ and $p \succ 0$.

Definition 2.9. A Complex valued $\mathcal{M}-$ fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.10. A function is complex valued continuous in complex valued $\mathcal{M}-$ fuzzy metric space if and only if whenever $x_n \to x, y_n \to y$ and $z_n \to z$, then $\lim_{n\to\infty} \mathcal{M}(x_n, y_n, z_n, t) = \mathcal{M}(x, y, z, t)$ for all $t \succ 0$.

Definition 2.11. Let A and B be mappings from complex valued $\mathcal{M}-$ fuzzy metric space $(X, \mathcal{M}, *)$ into itself. The mappings A and B are said to be weakly compatible if they commute at their coincidence points, i.e. $Ax = Bx \Longrightarrow ABx = BAx$.

Definition 2.12. Suppose A and S be two maps from a complex valued $\mathcal{M}-$ fuzzy metric space $(X, \mathcal{M}, *)$ into itself. Then they are said to be semi-compatible if $\lim_{n\to\infty} ASx_n = Sx$ whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$.

Lemma 2.13. *Let $(X, \mathcal{M}, *)$ be a complex valued $\mathcal{M}-$ fuzzy metric space. Then $\mathcal{M}(x, y, z, t)$ is non-decreasing with respect to t, for all $x, y, z \in X$.*

Proof. By Definition 2.2. for each $x, y, z, a \in X$ and $t, s \succ 0$ we have

$\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \preccurlyeq \mathcal{M}(x, y, z, t + s)$. If we get $a = z$, we get

$\mathcal{M}(x, y, z, t) * \mathcal{M}(z, z, z, s) \preccurlyeq \mathcal{M}(x, y, z, t+s)$ that is $\mathcal{M}(x, y, z, t+s) \succcurlyeq \mathcal{M}(x, y, z, t)$

for each $x, y, z, a \in X$ and $t, s \succ 0$ by definition of $(X, \mathcal{M}, *)$.

Lemma 2.14. *Let $\{x_n\}$ be a sequence in a complex valued $\mathcal{M}-$ fuzzy metric space $(X, \mathcal{M}, *)$ with $\lim_{t\to\infty} \mathcal{M}(x, y, z, t) = e^{i\theta}$, $\mathcal{M}(x, y, z, ht) \succcurlyeq \mathcal{M}(x, y, z, t)$ for all $x, y, z \in X$, $0 \prec h \prec 1, t \in (0, \infty)$ then $x = y = z$.*

3 Main Results

Theorem 3.1. *Let $(X, \mathcal{M}, *)$ be a complex valued complete $\mathcal{M}-$ fuzzy metric space where $*$ is a complex valued continuous $t-$ norm and satisfies $t * t \succcurlyeq t$ for all $t \in [0, 1]$. Let A, B, S and T be self-mappings of a complex valued $\mathcal{M}-$ fuzzy metric space satisfying the following conditions:*
(3.1) For all $x, y, z \in X, t \succ 0$ and $h \succ 1$.

$$\mathcal{M}(Ax, By, Bz, ht) \succcurlyeq \min\Big\{\mathcal{M}(Sx, Ax, Ay, t), \mathcal{M}(Ty, By, Bz, t), \frac{r\mathcal{M}(Sx, By, Bz, t) + s\mathcal{M}(Sx, Ty, Tz, t)}{r\mathcal{M}(By, Ty, Bz, t) + s}\Big\},$$

where $r, s \succcurlyeq 0$ with r and s cannot be simultaneously 0.
(3.2) Pairs (A, S) and (B, T) satisfy common $E.A.$ Like property.
(3.3) Pairs (A, S) and (B, T) are weakly compatible.
Then A, B, S and T have a unique common fixed point in X.

Proof. Since (A, S) and (B, T) satisfy common $E.A.$ Like property, therefore there exist two sequences $\{x_n\}$ and $\{y_n\} \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = w$, where $w \in S(X) \cap T(X)$

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or $w \in A(X) \cap B(X)$. Suppose $z \in S(X) \cap T(X)$. Now, we have $\lim_{n\to\infty} Ax_n = w \in S(X)$ then $w = Su$, for some $u \in X$. Now, we claim that $Au = Su$.

Form (3.1) we have,

$$\mathcal{M}(Au, By_n, By_n, ht) \succcurlyeq \min\Big\{\mathcal{M}(Su, Au, Ay_n, t), \mathcal{M}(Ty_n, By_n, By_n, t), \frac{r\mathcal{M}(Su, By_n, By_n, t) + s\mathcal{M}(Su, Ty_n, Ty_n, t)}{r\mathcal{M}(By_n, Ty_n, By_n, t) + s}\Big\}.$$

Taking limit n $\to \infty$, we get

$$\mathcal{M}(Au, By_n, By_n, ht) \succcurlyeq \min\Big\{\mathcal{M}(w, Au, w, t), \mathcal{M}(w, w, w, t), \frac{r\mathcal{M}(w, w, w, t) + s\mathcal{M}(w, w, w, t)}{r\mathcal{M}(w, w, w, t) + s}\Big\}$$

$$\mathcal{M}(Au, w, w, ht) \succcurlyeq \min\left\{\mathcal{M}(w, Au, w, t), e^{i\theta}, e^{i\theta}\right\}$$

$$\mathcal{M}(Au, w, w, ht) \succcurlyeq \mathcal{M}(w, Au, w, t)$$

$$\mathcal{M}(Au, w, w, ht) \succcurlyeq \mathcal{M}(Au, w, w, t).$$

Lemma 2.2. implies that $Au = w = Su$.

Since the pair (A, S) is weakly compatible, therefore $Aw = ASu = SAu = Sw$.

Again, $\lim_{n\to\infty} By_n = w \in T(X)$ then $w = Tv$ for some $v \in X$.

Now, we claim that $Tv = Bv$, from (3.1) we have,

$$\mathcal{M}(Ax_n, Bv, Bv, ht) \succcurlyeq \min\Big\{\mathcal{M}(Sx_n, Ax_n, Av, t), \mathcal{M}(Tv, Bv, Bv, t), \frac{r\mathcal{M}(Sx_n, Bv, Bv, t) + s\mathcal{M}(Sx_n, Tv, Tv, t)}{r\mathcal{M}(Bv, Tv, Bv, t) + s}\Big\}.$$

Taking limit $n \to \infty$, we get

$$\mathcal{M}(w, Bv, Bv, ht) \succcurlyeq \min \mathcal{M}(w, w, Av, t), \mathcal{M}(w, Bv, Bv, t), \frac{\mathcal{M}(w,Bv,Bv,t)+\mathcal{M}(w,w,w,t)}{\mathcal{M}(Bv,w,Bv,t)+}\Big\},$$

$$\mathcal{M}(w, Bv, Bv, ht) \succcurlyeq \min\{e^{i\theta}, \mathcal{M}(w, Bv, Bv, t), e^{i\theta}\},$$

$$\mathcal{M}(w, Bv, Bv, ht) \succcurlyeq \mathcal{M}(w, Bv, Bv, t),$$

$$\mathcal{M}(Bv, Bv, w, ht) \succcurlyeq \mathcal{M}(Bv, Bv, w, t).$$

Lemma 2.2. implies that $Bv = w = Tv = Av$.

Since the pair (B, T) is weakly compatible, therefore $Tw = TBv = BTv = Bw$.

Now, we show that $Aw = w$, from (3.1) we have,

$$\mathcal{M}(Aw, By_n, By_n, ht) \succcurlyeq \min\Big\{\mathcal{M}(Sw, Aw, Ay_n, t), \mathcal{M}(Ty_n, By_n, By_n, t), \frac{r\mathcal{M}(Sw, By_n, By_n, t) + s\mathcal{M}(Sz, Ty_n, Ty_n, t)}{r\mathcal{M}(By_n, Ty_n, By_n, t) + \ s}\Big\}.$$

Taking limit $n \to \infty$, we get

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$\mathcal{M}(Aw, w, w, ht) \succcurlyeq \min \mathcal{M}(Aw, Aw, Aw, t), \mathcal{M}(w, w, w, t), \frac{r\mathcal{M}(Aw,w,\ w,t)+s\mathcal{M}(Aw,w,w,t)}{r\mathcal{M}(w,w,w,t)+s} \Big\}$

$\mathcal{M}(Aw, w, w, ht) \succcurlyeq \min \Big\{ e^{i\theta}, e^{i\theta}, \frac{r\mathcal{M}(Aw,w,w,t)}{r\mathcal{M}(w,w,w,t)+s} \Big\}$

$\mathcal{M}(Aw, w, w, ht) \succcurlyeq \mathcal{M}(Aw, w, w, t)$.

Lemma 2.2. implies that $Aw = w$.

Now, we show that $Bw = w$, from (3.1) we have,

$$\mathcal{M}(Ax_n, Bw, Bw, ht) \succcurlyeq \min \Big\{ \mathcal{M}(Sx_n, Ax_n, Aw, t), \mathcal{M}(Tw, Bw, Bw, t), \frac{r\mathcal{M}(Sx_n, Bw, Bw, t) + s\mathcal{M}(Sx_n, Tw, Tw, t)}{r\mathcal{M}(By, Tw, Bw, t) + s} \Big\}$$

Taking limit $n \to \infty$, we get

$\mathcal{M}(w, Bw, Bw, ht) \succcurlyeq \min \Big\{ \mathcal{M}(w, w, w, t), \mathcal{M}(Bw, Bw, Bw, t), \frac{r\mathcal{M}(w,Bw,Bw,t)+s\mathcal{M}(w,Bw,Tw,t)}{r\mathcal{M}(Bw,Bw,Bw,t)+s} \Big\}$

$\mathcal{M}(w, Bw, Bw, ht) \succcurlyeq \min \{ e^{i\theta}, e^{i\theta}, \mathcal{M}(w, Bw, Bw, t) \}$,

$\mathcal{M}(w, Bw, Bw, ht) \succcurlyeq \mathcal{M}(w, Bw, Bw, t)$,

$\mathcal{M}(Bw, Bw, w, ht) \succcurlyeq \mathcal{M}(Bw, Bw, w, t)$,

Lemma 2.2. implies that $Bw = w$.

Hence, $Aw = Sw = Bw = Tw = w$.

Thus w is a common fixed point of A, B, S and T.

To prove uniqueness, we suppose that p and q are two common fixed points of A, B, S and T such that $p \neq q$, then from (3.1) we have,

$\mathcal{M}(Ap, Bq, Bq, ht) \succcurlyeq \min \Big\{ \mathcal{M}(Sp, Ap, Aq, t), \mathcal{M}(Tq, Bq, Bq, t), \frac{r\mathcal{M}(Sp,Bq,Bq,t)+s\mathcal{M}(Sp,Tq,Tq,t)}{r\mathcal{M}(Bq,Tq,Bq,t)+s} \Big\}$

$\mathcal{M}(p, q, q, ht) \succcurlyeq \min \Big\{ \mathcal{M}(p, p, q, t), \mathcal{M}(q, q, q, t), \frac{r\mathcal{M}(p,q,q,t)+s\mathcal{M}(p,q,q,t)}{r\mathcal{M}(q,q,q,t)+s} \Big\}$,

$\mathcal{M}(p, q, q, ht) \succcurlyeq \min \{ e^{i\theta}, e^{i\theta}, \mathcal{M}(p, q, q, t) \}$

$\mathcal{M}(p, q, q, ht) \succcurlyeq \mathcal{M}(p, q, q, t)$. Lemma 2.2. implies that $p = q$.

Corollary 3.2. *Let $(X, \mathcal{M}, *)$ be a complex valued complete $\mathcal{M}-$ fuzzy metric space where $*$ is a complex valued continuous $t-$norm and satisfies $t * t \succcurlyeq t$ for all $t \in [0, 1]$. Let A, B, S and T be self-mappings of a complex valued $\mathcal{M}-$ fuzzy metric space satisfying the following conditions:*
(3.4) For all $x, y, z \in X, t \succ 0$ and $h \succ 1$.

$$\mathcal{M}(Ax, By, Bz, ht) \succcurlyeq \min \Big\{ \mathcal{M}(Sx, Ax, Ay, t), \mathcal{M}(Ty, By, Bz, t), \frac{r\mathcal{M}(Sx, By, Bz, t) + s\mathcal{M}(Sx, Ty, Tz, t)}{r\mathcal{M}(By, Ty, Bz, t) + s} \Big\},$$

where $r, s \succcurlyeq 0$ with r and s cannot be simultaneously 0.
(3.5) Pairs (A, S) and (B, T) satisfy common $E.A.$ Like property.
(3.6) Pairs (A, S) and (B, T) are semi-compatible.
Then A, B, S and T have a unique common fixed point in X.

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Example 3.3. Let $X = \{0\} \cup \{\frac{1}{n} : n \in N\}$ with the metric D defined by

$D(x, y, z) = |x - y| + |y - z| + |z - x|$, for all $x, y, z \in X$.

For all $x, y, z \in X$ and $t \in (0, \infty)$, we define $\mathcal{M}(x, y, z, t) = e^{i\theta} \frac{t}{t+D(x,y,z)}$.

Clearly $(X, \mathcal{M}, *)$ is complex valued complete $\mathcal{M}-$ fuzzy metric space with $t-$norm $*$ is defined as $a * b = \min\{a, b\}$ where $a, b \in [0, 1]e^{i\theta}$, for a fixed $\theta \in [0, \frac{\pi}{2}]$.

Here, $\lim_{t\to\infty} \mathcal{M}(x, y, z, t) = e^{i\theta}$, for all $x, y, z \in X,\ t \in (0, \infty)$.

Define self-maps A, B, S and T as follows.

$Ax = Bx = \frac{x}{6}$ $Sx = Tx = \frac{x}{3}$. Then conditions (3.1) and (3.3) hold good.

Also, the condition (3.1) holds for $h = \frac{2}{5}$. Therefore, by Theorem 3.1. the self-maps A, B, S and T have a unique common fixed point in X. Here 0 is the unique common fixed point.

4 Conclusion

We make common fixed point theorems for two pairs of weakly compatible mappings in a complex-valued $\mathcal{M}-$ fuzzy metric space fulfilling common $E.A.$ property. Utilizing this thought we will make a common fixed point theorems for two two pairs of weakly compatible mappings in a complex valued fuzzy metric spaces.

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