The Properties of the Spectrals of Fuzzy Compact Linear Operators

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Abstract

The objective of this paper is to present a novel approach for the a-fuzzy standardized area and its fundamental characteristics with basic attributes of fuzzy compact and fuzzy bounded linear functions. Also, it presents some of the basic attributes of the spectral of fuzzy compact linear functions in terms of theorems that clearly draw the elementary properties of these functions and the mathematical relationships between them.

Keywords: Fuzzy Standardized Spaces; Compactness; Bounded Operators; Complete Spaces; Linear Operators

1. Introduction

We are embarking on the development of an innovative framework in fuzzy functional analysis, drawing inspiration from the well-established classical theory of practical evaluation. The works of examination within the research unfold as below: In 2011, Kider explained the inaugural definition of a-fuzzy standardized area [1]. Subsequently, in a separate work [2], he presented a completion for this particular fuzzy standardized area. Continuing this trajectory, Kider explains a novel definition of a-fuzzy standardized area in 2012 [3].

In 2017, Kider and Kadhum made significant contributions by unveiling the fuzzy standardized of linear fuzzy functions with bounds [4, 5]. They then went on to substantiate the foundational attributes associated with this specific fuzzy standarized in the realm of fuzzy bounded functions. Further advancing this line of research, Kider and Ali, in 2018, brought forth the definition of a fuzzy standardized algebra [6, 7], offering insights into critical general attributes characterizing this type of algebra.

In 2019, Kider and Gheeab expanded their area of study by introducing the idea of a broad fuzzy standardized area [8, 9]. After this introduction, they carefully proved basic and important characteristics of these special fuzzy standardized areas. Key insights into the characteristics of fuzzy small linear functions based on fuzzy standardized areas were offered in 2019 by Kider and Kadhum [10].

In 2020, Kider presented a novel kind of fuzzy measure that he called the fuzzy soft measure [11]. He supported this form of measurement by proving a number of characteristics in the context of the fuzzy soft measure area. Kider [12] introduced the notion of an a-fuzzy measure in the exact same time and demonstrated many characteristics of a-fuzzy measure areas. In 2021, Kider and M. Gheeab explored attributes concerning the fuzzy bounded function [13]. In 2012, Khudhair and Kider introduced the notion of an a-fuzzy standardized [15-21], followed by a comprehensive discussion on pivotal general characterizing a-fuzzy standardized area.

This paper's structural arrangement is presented as follows: The second section aims to review a-fuzzy standardized areas and their basic characteristics. Section three delves into the exploration of supplementary attributes related to a linear function exhibiting fuzzy smallness. Lastly, the focal point of this paper, section four, encapsulates the principal findings, concentrating on the exploration of significant attributes associated with the adjoin function of fuzzy bounded fuzzy small functions.

2. The a-fuzzy standardized area and its basic attributes

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Embarking on the exploration of a-fuzzy standardized areas and their fundamental characteristics, we encounter distinct observations.

Observation 2.1: [14] If we denote $\odot$ as a t-conorm, specific conditions emerge:

(i) A constant c, where $0 < c < 1$, exists This way a surpasses $b \odot c$ for all $0 < a < 1, 0 < b < 1, with\ a\ exceeding\ b.$

(ii) (ii) Another element $0 < b < 1$ exists, ensuring $b \odot b$ is less than or equal to a, for every a, where $0 < a < 1$.

Observation 2.2: [15] Given a fuzzy set $aR\colon R \to I$ and a cont. t-conorm $\odot$, aR is termed an a-fuzzy absolute measurement on R, subject to the coming conditions for all r, s ∈ R:

(i) $aR(r)$ belongs to the interval (0,1]

(ii) (ii) $aR(r)$ is zero if and only if r equals zero

(iii) (iii) $aR(\sigma r)$ is less than or equal to $aR(\sigma)$ approximately equal to $aR(r)$

(iv) (iv) $aR(r+s)$ equals $aR(r)$ minus $aR(s)$.

We designate $(R, aR, \odot)$ as a-fuzzy measurements area.

Observation 2.3: [15] An a-fuzzy standardized area $(R, aR, \odot)$ attains fuzzy include if there exists r in R such that rk converges to r for every given fuzzy Cauchy sequence (rk) in R.

Observation 2.4: [15] Let $\mathbb{Q}$ signify a cont. t-conorm, Z represent an arbitrary vector area over R, and (R, aR, $\mathbb{Q}$) depict an a-fuzzy measurement area. Assuming $nZ\colon Z \to [0,1]$ is a fuzzy set, nZ is an a-fuzzy standardization on Z, adhering to the coming criteria for all $z, w \in Z\ and\ for\ all\ 0 = \alpha \in R$:

(i) $nZ(z)$ exists in the interval (0,1]

(ii) $nZ(z)$ equals zero if and only if z equals zero

(iii) $nZ(\sigma z)$ is less than or equal to $aR(\sigma)nZ(z)$

(iv) $nZ(z+w)$ is less than or equal to $nZ(z) \odot nZ(w)$.

This characterizes $(Z, nZ, \mathbb{Q})$ as a-fuzzy standardized area.

Observation 2.5: [15] Within an a-fuzzy standardized $area\ (Z, n, \odot)$, the equality $nZ(z-w)$ equals $nZ(w-z)$ for all $z, w \in Z$.

Observation 2.6: [15] Given a fuzzy standardized area $(Z, nZ, \odot)$ and a sequence (zk) in Z, the sequence is identified as fuzzily converging to the limit z as k approaches infinity if, for every ∈ (0,1), there exists N in N such that $nZ(zk - z)$ is less than s for all k greater than or equal to N. In this context, the series is acknowledged to fuzzily converge to the limit z.

Observation 2.7: [15] In an a-fuzzy standardized area $(Z, nZ, \odot)$, the open fuzzy ball is denoted by $fb(w, r)$, and the open closed fuzzy ball is denoted by $fb[w, r]$. The center of the open fuzzy ball is u, positioned in Z, and the radius of the open fuzzy ball is t, situated in $(Z, nZ, \odot)$.

Observation 2.8: [15] The function $z \mapsto nZ(z)$ manifests as a fuzzy cont. function defined from Z into R, provided that both $(Z, n, \odot)\ and\ (R, n, \odot)$ stand as a-fuzzy standardized areas, respectively.

Observation 2.9: [15] In an a-fuzzy standardized area $(Z, nZ, \odot)$, a sequence (zk) earns recognition as a fuzzy Cauchy sequence if, for every s within the interval (0,1), there exists N within the set N such that $nZ(zk - zm)$ is less than s for all k, m greater than or equal to N.

Observation 2.10: [15] Let's denote a set with fuzzy openness in the a-fuzzy standardized area $(Z, nZ, \odot)$ as W. It adheres to the condition where $fb(w, j)$ forms a part of W for any w within W and j within the interval (0,1). Additionally, if DC is perceived as fuzzily open, then $D \subseteq Z$ is acknowledged as fuzzily closed. Precisely, the fuzzy closure of D, symbolized by D Å, is delineated as the minimal fuzzy closed ensemble encompassing D.

Observation 2.11: [15] If we consider $\bar{B} = Z\ where\ B \subseteq Z$, then characterizing B as fuzzily dense in the a-fuzzy standardized area $(Z, nZ, \odot)$.

Observation 2.12: [15] The assortment $fb(z, j)$ is acknowledged as a set with fuzzy openness in the a-fuzzy standardized area $(Z, nZ, \odot)$.

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Observation 2.13: [15] An a-fuzzy standardized area $(Z, nZ, \circledcirc)$ reaches the state of being fuzzily include if, for any arbitrary fuzzy Cauchy sequence (zk) in Z, there exists $z \in Z\ with\ zk \to z$.

Observation 2.14: [15] In the a-fuzzy standardized area $(Z, nZ, \circledcirc), if\ zk \to z \in Z, then\ (zk)$ is classified as a fuzzy Cauchy sequence.

Observation 2.15: [15] Given any d \in D¯, there exists $(dk) \in D\ with\ dk \to d\ when\ D \subseteq Z, and\ (Z, nZ, \circledcirc)$ constitutes an a-fuzzy standardized area.

Observation 2.16: [16] Considering two fuzzy standardized areas $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc)$, the function $L: Z \to W$ gains the attribute of being fuzzily cont. at z∈Z if, for every r∈(0,1), there exists t∈(0,1) such that $nW[L(z) - L(y)]$ is less than r for any $y \in Z, and\ nZ(z - y)$ is less than t. Furthermore, L is recognized as fuzzily cont. on Z if it sustains fuzzily cont. behavior at each point on Z.

Observation 2.17: [16] For a set fuzzily closed in Z, denoted as $D = \{z \in Z: 0 < n(z) \leq 1\}$, achieving smallness necessitates Z to be of finite dimension, where $(Z, n, \circledcirc)$ is an a-fuzzy standardized area.

Observation 2.18: [16] The fuzzy boundedness of the relation $L: D(L) \to Y$ is determined by the existence of s∈ $(0,1)\ with\ nY[L(z)] < snZ(z)\ in\ any\ z \in D(L), while\ (Z, nZ, \circledcirc)\ and\ (Y, nY, \circledcirc)\ d$enote two a-fuzzy standardized areas.

Notation: [16] For two a-fuzzy standardized areas $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc), afb(Z, W) = \{L: Z \to Z$, L being a linear fuzzy bounded function$\}$.

Observation 2.19: [16] Considering a-fuzzy standard regions $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc), let\ nafb(Z, W)(L)$ equal the sum of both $D(L)\ and\ nW(Lz)$ for all L that belongs to the set of $afb(Z, W)$. In this case, the expression $[afb(Z, W), nafb(Z, W), \circledcirc]$ represents an a-fuzzy standardised area.

Observation 2.20: [16] Formula (1) can be reformulated as $nW[L(u)] < nafb(Z, W)[L]nZ(u)$ [Formula (2)].

Observation 2.21: [16] This means that if W is fuzzy include, then $afb(Z, W)$ is also fuzzy include, if $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc)$ are two a-fuzzy standardised areas represented by the variables.

Observation 2.22: [16] For two a-fuzzy standardized areas $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc), with\ L: D(L) \to W$ being a linear function where $D(T) \subseteq W$, L is fuzzy cont. if and only if L is fuzzy bounded.

Observation 2.23: [16] An a-fuzzy standardized area $(Z, n, \circledcirc)$ attains the status of being fuzzy small if a finite set $\{U1, U2, U3, \dots, Uk\} \subseteq \Omega\ exists, such\ that\ Z = \cup j = 1kUj$.

Observation 2.24: [16] The a-fuzzy standardized area $(Z, n, \circledcirc)$ is fuzzy small if and only if, for any arbitrary sequence $(zk)\ in\ Z$, there exists a subsequence $(zkj)\ with\ zkj \to z \in Z$.

Observation 2.25: [16] The fuzzy include a-fuzzy standardised area is represented by $(Z, nZ, \circledcirc)$, and if M is a subset of Z, then W is said to be fuzzy include if and only if it is also fuzzy closed.

Observation 2.26: [16] Z is recognized as fuzzy totally bounded if $(Z, n, \circledcirc)$ is a fuzzy small a-fuzzy standardized area.

Observation 2.27: [18] Considering two a-fuzzy standardized areas $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc)\ with\ dimZ = k$, every linear function $L: Z \to W$ is deemed fuzzy bounded.

Observation 2.28: [19] If $(Z, nZ, \circledcirc)\ and\ (W, nW, \circledcirc)$ are two a-fuzzy standardized areas, W is fuzzy include, and assuming $L: Z \to W$ is a fuzzy bounded linear function, then L has an extension $S: (D(L))^{-} W$ such that S is a fuzzy bounded linear function with $nafb(Z, W)[L] = nafb(Z, W)[S]$.

3. The attributes of the spectral of linear functions

In this section, we delve into the spectral features of a fuzzy contractive linear function denoted as $T: Z \to Z$, acting on an a-fuzzified standardized area (Z, v, Δ). To embark on this exploration, we introduce the transformation $T\beta = (T - \beta I), where\ \beta \in C$ [as indicated in Formula (3)], and draw upon foundational principles from spectral analysis, as previously established. The spectral investigation of fuzzy contractive linear functions unfolds as a natural expansion of Eigen measurement exploration applied to finite matrices, echoing multiple facets of the situation encountered in finite-dimensional contexts.

Our initial finding focuses on Eigen measurements, revealing that the point spectrum of a fuzzy small linear function is not intricately complex. This theorem carries more profound implications than initially apparent. In the subsequent section, we will demonstrate that every spectral measurement a not equal to 0 that a fuzzy small linear function may or may not possess is, in fact, an Eigen measurement. This observation underscores the similarity between the spectrum of a fuzzy small linear function and that of a function in a finite-dimensional area to a considerable degree.

Furthermore, the theorem establishes that if a fuzzy small linear function on an a-fuzzy standardized area possesses an infinite set of Eigen measurements, it is possible to organize these Eigen measurements into a sequence that converges fuzzily to zero.

Theorem 3.1: Consider the a-fuzzy standardized area $(Z, n, \circledcirc)$ with a fuzzy small linear function $S: Z \rightarrow Z$. We aim to establish that the set of Eigen measurements of S is countable, and the only possible limit point is α=0.

Proof: It is sufficient to demonstrate that for every $k > 0$, the set of all $\mu \in \sigma p(S)$ such that $aC(\mu) \geq 1/k$ is finite. Suppose, for contradiction, that for some $k0 > 0$, there exists an infinite sequence (αk) of distinct Eigen measurements with $aC(\alpha k) \geq 1/k0. Let\ Szk = \alpha kzk\ for\ some\ zk = 0.$

Since the set of all zk is linearly independent, let $Nk = span\{z1, z2, \dots, zk\}$. Then, for every $\in Nm$, there exists a unique representation $z = \gamma 1z1 + \gamma 2z2 + \cdots + \gamma kzk. Applying$ $(S - \alpha kI)$ and utilizing $Szj = \alpha jzj$, we observe that zk no longer appears on the right side. Hence, $(S - \alpha kI) \in N(k-1)$, where the $Nk's$ are fuzzy closed.

Now, consider a sequence $(ym)\ such\ that\ yk \in Nk\ and\ n(yk - z) \geq 1/2\ for\ all\ z \in Nk$. We have $nafb(Z, Z)(Syk - Sym) \geq 1/(2k0)\ for\ k > m.$ Consequently, (Syk) has no convergent subsequence, contradicting the fuzzy smallness of S since (ym) is fuzzy bounded.

By adding and subtracting a term, we express $(Syk - Sym) = \alpha nyn - z{\sim}$, where $z{\sim}=\alpha nyn-Syk+Sym$. If $m<k$, we aim to show that $z{\sim} \in N(k-1). Since\ m \leq (k-1), we\ have\ ym \in Nm \subseteq N(k-1) = span\{z1, z2, \dots, zk\}.$ Hence, $Sym \in N(k-1)$. Since $Szj = \alpha jzj$ by (2), we can write $\alpha kyk - Syk = -((S - \alpha kI)yk) \in N(k-1)$. Together, this implies $z{\sim} \in N(k-1)$, and thus, $z = \alpha k - 1z{\sim} \in N(k-1)$.

Therefore, $n(\alpha kyk - z{\sim}) \leq aC(\alpha k)n(yk - z), but\ n(yk - z) \geq 1/2\ for\ all\ z \in N(k-1). So, aC(\alpha k)n(yk - z) \geq aC(\alpha k) \cdot 1/2 \geq 1/(2k0). Consequently, n(\alpha kyk - z{\sim}) \geq 1/(2k0). From\ (6), this\ yields\ (5).$

Hence, the assumption of an infinite sequence of Eigen measurements satisfying $aC(\alpha k) \geq 1/k0$ for some $k0 > 0$ must be false, and the theorem is proved.

The coming result demonstrates the composition of a fuzzy small linear function and a fuzzy bounded linear function.

Lemma 3.2: In the context of an a-fuzzy standardized area denoted as $(Z, n, \circledcirc)$, suppose we have a fuzzy small linear function $S1: Z \rightarrow Z$ and a fuzzy bounded linear function $S2: Z \rightarrow Z$. Under these conditions, it can be asserted that both S1S2 and S2S1 are characterized as fuzzy small functions.

Proof: Consider a set $B \subseteq Z$, characterized as a fuzzy bounded set. Given the fuzzy bounded nature of S2, it follows that $S2(B)$ also qualifies as a fuzzy bounded set. Notably, $S1[S2(B)]$ exhibits relative fuzzy smallness owing to the fuzzy small properties of S1. Consequently, $S1S2$ is identified as a fuzzy small function.

Next, we establish the fuzzy smallness of $S2S1$. Take (zk) as an arbitrary fuzzy bounded sequence in Z. Consequently, $(S1zk)$ possesses a fuzzy convergent subsequence denoted as (S1zkm), and the subsequent sequence $(S2S1zkm)$ is fuzzily convergent. Thus, $S2S1$ emerges as a fuzzy small function.

The initial claim in this section, suggesting the spectral theory of fuzzy small linear functions to be akin to that of linear functions on finite-dimensional areas, finds support in a key characteristic: for any nonzero Eigen measurement of a fuzzy small linear function, the corresponding Eigen area may exhibit either finite or infinite dimensionality.

Theorem 3.3: Let $(Z, n, \circledcirc)$ be an a-fuzzy standardized area, and let $S: Z \rightarrow Z$ be a fuzzy small linear function. Then, for every $\alpha = 0$, the null area $N(S\alpha)$ of $S\alpha = (S - \alpha I)$ is finite-dimensional.

Proof: To establish the fuzzy smallness of the null area, we show that the fuzzy closed ball $B = \{z \in Z: n(z) \leq 1\}$ in $N(S\alpha)$ is fuzzy small. Consider a sequence (zk) in B. Since $n(zk) \leq 1, (zk)$ is fuzzy bounded. Furthermore, (Szk) has a fuzzy convergent subsequence $(Szkm)$. Now, $zk \in B \subseteq N(S\alpha)\ implies\ S\alpha zk = (S -$

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$\alpha I)zk = Szk - \alpha zk = 0, so\ zk = \alpha - 1Szk\ because\ \alpha = 0$. Consequently, $(zkm) = (\alpha - 1Szkm)$ also fuzzy converges to a limit in *B* since *B* is fuzzy closed. Thus, *B* is fuzzy small because (*zk*) was arbitrarily chosen from *B*. This proves t$hat\ dimN(S\alpha) < \infty$.

Corollary 3.4: Let (*Z*,*n*,⊚) be an a-fuzzy standardized area, and let *S*:*Z*→*Z* be a fuzzy small linear function. Then, for every *α*= $0, dimN(S\alpha k) < \infty\ fork \in N\ and\ \{0\} = N(S\alpha 0) \subseteq N(S\alpha 1) \subseteq N(S\alpha 2)$

Proof: Since *Sα* is linear, it maps 0 to 0. Therefore, $S\alpha k(z) = 0\ implies\ S\alpha k + 1(z) = 0,$ and follows. To prove (7), we utilize the binomial theorem:

$$S\alpha k = (S - \alpha I)k = m = 0\sum k(mk)Sm - 1(-\alpha)k - m = -\alpha I + Sm = 1\sum k(mk)Sm(-\alpha)k - m$$

This can be expressed as $S\alpha k = W - \mu I,$ where $\mu = -(-\alpha)\ and\ W = SS1 = S1S\ with\ S1$ denoting the sum on the right. Since *S* is fuzzy small and *S*1 is fuzzy bounded, *W* is fuzzy small by Lemma 3.2. Therefore, applying Theorem 8.3.3 to $W - \mu I$ yields. Next, we delve into the ranges of $S\alpha, S\alpha 2, \ldots$ for fuzzy small linear function *Sα* and any *α*=0. The result is an extension of this finding to $S\alpha k\ for\ k \in N$.

Theorem 3.5: Consider an a-fuzzy standardized area $(Z, n, \circledcirc). If\ S: Z \to Z\ i$s a fuzzy small linear function, then for every $\alpha = 0, the\ range\ of\ S\alpha = (S - \alpha I)$ is fuzzy closed.

Proof: Assume, for the sake of contradiction, that *R*(*Sα*) is not fuzzy closed. Proceeding according to the coming steps:

(i) We consider y in $(S\_alpha\ (z))^{-} where\ y \in / S\alpha(z), and\ a\ sequence\ (S\alpha zk)$ that fuzzily converges to y. We show that $zk \in / N(S\alpha)$, but $N(S\alpha)$ contains a sequence (xk) such that $n(zk - xk) < 2\delta k\ , where\ \delta k$ is the fuzzy distance from $zk\ to\ N(S\alpha)$.

(ii) We show that $ak \to 0, where\ ak = n(zk - xk)$.

(iii) We obtain the anticipated contradiction by considering the sequence $(wk)\ where\ wk = ak - 1(zk - xk)$.

Details:

(i) Suppose that $S\alpha(Z)$ is not fuzzy closed. Then, there exists y \in (S_\alpha(Z))¯such that $y \in / S\alpha(Z)$ and a sequence $(zk) \in Z\ such\ that\ yk = S\alpha zk \to y$ (Formula 9). Since *Sα*(*Z*) is a vector area, $0 \in S\alpha(Z)$. But $y \in / S\alpha(Z)$, implying *y*=0. This implies that $yk = 0\ and\ zk \in / N(S\alpha)$ for sufficiently large *k*. Without loss of generality, we may assume that this holds for all *k*. Since *N*(*Sα*) is fuzzy closed, the fuzzy distance $\delta k\ from\ zk\ to\ N(S\alpha)$ is positive, $i.e., \delta k = infx \in N(S\alpha)n(zk - x) > 0. By\ the\ definition\ of\ an\ infimum, there\ is\ a\ sequence\ (xk)\ in\ N(S\alpha)\ such\ that\ ak = n(zk - xk) < 2\delta k$ (Formula 10).

(ii) We show that $ak = n(zk - xk) \to 0\ as\ k \to \infty$ (Formula 11). Suppose Formula 11 does not hold. Then, $(zk - xk)$ has a fuzzy bounded subsequence. Since *S* is fuzzy small, it follows that $(S(zk - xk))$ has a fuzzy convergent subsequence. Now, *Sα*=(*S*−*αI*) with *α*=0, and we have *I*=*α*−1(*S*−*Sα*). Using Sαxk = 0 (because xk ∈ N(Sα)), we obtain $(zk - xk) = \alpha - 1(S - S\alpha)(zk - xk) = \alpha - 1[S(zk - xk) - S\alpha zk]$. Since $(S(zk - xk))$ has a fuzzy convergent subsequence and (*Sαzk*) fuzzily converges by Formula 8, we conclude that (*zk*−*xk*) has a fuzzy convergent subsequence, say $(zkm - xkm) \to v$. Since *S* is fuzzy small, this implies that *S* is fuzzy cont., and so is *Sα*. Hence, $S\alpha(zkm - xkm) \to S\alpha v. But\ S\alpha xkm = 0\ because\ xkm \in N(S\alpha)$, so that by Formula 8, we also have $S\alpha(zkm - xkm) = S\alpha zkm \to$*y*. Hence, $y \in S\alpha z,$ contradicting $y \in / S\alpha z$ as assumed at the beginning of part (1) of the proof. This contradiction resulted from our assumption that Formula 11 does not hold, so that Formula 11 is now proved.

(iii) Using ak as defined in Formula 11 and setting $wk = ak - 1(zk - xk)$ (Formula 12), and using $S\alpha xk = 0\ and\ (S\alpha zk)$ fuzzily converges, it follows that $S\alpha wk = ak - 1S\alpha zk \to 0$ (Formula 13). Again using $I = \alpha - 1(S - S\alpha)$, we obtain $wk = \alpha - 1(Swk - S\alpha wk)$ (Formula 14). Since *S* is fuzzy small and (*wk*) is fuzzy bounded, (Swk) fuzzily converges by Formula 11. Hence, Formula 14 shows that (*wk*) has a fuzzy convergent subsequence, say *wki* → *w* (Formula 15). A comparison with Formula 11 implies $S\alpha w = 0$. Hence, $w \in N(S\alpha)$. Since *xk*∈*N*(*Sα*), also $uk = xk + akw \in N(S\alpha)$. Hence, for the fuzzy distance from *zk* to *uk*, we must have $n(zk - uk) \geq \delta k$. Writing *uk* out and using Formula 11 and Formula 9, we thus obtain $\delta k \leq n(zk - xk - akw) = akn(wk - w) < 2\delta kn(wk - w)$. Dividing by $2\delta k > 0, we\ have\ 1/2 < n(wk - w),$ contradicting Formula 15, and the proof is including.

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4. Conclusion

In this work, we have presented a novel approach for the a-fuzzy standardized area and its fundamental characteristics with basic attributes of fuzzy compact and fuzzy bounded linear functions. Also, we have discussed some of the elementary properties of the spectral of fuzzy compact linear functions in terms of theorems and related proofs. In the future, we aim to achieve more and more to get better comprehension of the a-fuzzy compact linear spaces and functions.

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