A robust framework for the decision-making based on single-valued neutrosophic fuzzy soft expert setting

Yousef Al-Qudah [1,*]
[1]Department of Mathematics, Faculty of Arts and Science, Amman Arab University, Amman, Jordan
Email: alqudah@aau.edu.jo

Abstract

A soft expert set is widely used as a tool to solve and model the problems appearing in computer science and operations research. It aims to incorporate expert knowledge and uncertainty-handling capabilities into the analysis and decision-making processes. On the other hand, the idea of single neutrosophic sets (SVNSs) and fuzzy sets (FSs) are imported models for handling the uncertainty data. In this work, the authors combine the critical features of FSs and SVNSs under expert systems in one model. Accordingly, this model worked to provide decision-makers with more flexibility in the process of interpreting uncertain information. From a scientific point of view, the process of evaluating this high-performance SVNFSES disappears. Therefore, in this paper, we initiated a new approach known as single-valued neutrosophic fuzzy soft expert sets (SVNFSESs) as a new development in a fuzzy soft computing environment. We investigate some fundamental operations on SVNFSESS along with their basic properties. Also, we investigate AND and OR operations between two SVNFSESS as well as several numerical examples to clarify the above fundamental operations. Finally, we have given distance measures (DM) between two SVNFSESs to construct a new algorithm that is used to demonstrate the effectiveness of the method in handling some real-life applications.

Kewords: Neutrosophic sets; neutrosophic soft sets; Single-valued neutrosophic soft sets; expert soft set, optimization, Decision Making.

1 Introduction

Multi-attribute decision-making (MADM) is widely used as a tool to solve and model the problems appearing in computer science and operations research. Neutrosophic set theory[1] is an extension of classical set theory that aims to handle uncertainty, vagueness, and indeterminacy in a more comprehensive way. It was introduced by Florentin Smarandache in 1995 and has since found applications in various fields, including decision-making, expert systems, image processing, and artificial intelligence. A neutrosophic set is characterized by three components: the membership function, the non-membership function, and the indeterminacy function. These functions assign degrees of truth, falsity, and indeterminacy, respectively, to each element in the set. Unlike classical sets, which assign elements either a membership degree of 1 or 0, neutrosophic sets allow for partial membership, non-membership, or indeterminacy. The membership function represents the degree to which an element belongs to the set. It ranges between 0 and 1, where 0 indicates no membership, 1 indicates full membership, and values in between represent degrees of partial membership. The non-membership function represents the degree to which an element does not belong to the set. It also ranges between 0 and 1, where 0 indicates no non-membership, 1 indicates full non-membership, and values in between represent degrees of partial non-membership. The indeterminacy function represents the degree of indeterminacy or uncertainty associated with an element's membership or non-membership. It ranges between 0 and 1, where 0 represents complete determinacy, 1 represents complete indeterminacy, and values in between represent degrees of partial indeterminacy. Neutrosophic sets provide a more nuanced and flexible approach to handling

https://doi.org/10.54216/IJNS.230216

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

uncertainty and imprecision in various domains. They can capture situations where the degree of membership or non-membership is uncertain or ambiguous, allowing for more realistic modeling of complex and uncertain systems. Neutrosophic set theory has been applied in decision-making processes to handle conflicting and uncertain information. Neutrosophic logic and inference have been developed to reason with neutrosophic sets and make decisions based on the degrees of truth, falsity, and indeterminacy. Moreover, neutrosophic fuzzy sets combine neutrosophic sets and fuzzy sets,[2] incorporating degrees of truth, falsity, and indeterminacy into fuzzy logic systems. This hybrid approach enhances the modeling of uncertainty, vagueness, and ambiguity in decision-making processes. Overall, neutrosophic set theory offers a valuable framework for dealing with uncertainty and indeterminacy, providing a more comprehensive representation and analysis of complex systems and decision problems. Its applications continue to evolve, and researchers are exploring new extensions and variations to address specific challenges in different domains.

Soft set[3] theory is a mathematical framework that provides a flexible and intuitive way to handle uncertainty, vagueness, and imprecision in decision-making and data analysis. It was introduced by Molodtsov in 1999 as a generalization of classical set theory. In soft set theory, a soft set is defined as a collection of objects with a characteristic function that assigns degrees of membership to each object in a set. Unlike classical sets, where an object is either a member or non-member, soft sets allow for partial membership or degrees of uncertainty associated with membership. SSs have received wide attention from researchers around the world and they have introduced a lot of works for example, neutrosophic soft set(NSS),[4,5] interval-NSS[6,7] complex interval-NSS,[8] weighted Similarity Measure on neutrosophic soft set[9,10] and a lot of models employed in solving real-life applications see.[11–16] In other hande, Soft Expert Set[17–19] is a concept that combines elements of soft sets and expert systems. It aims to incorporate expert knowledge and uncertainty handling capabilities into the analysis and decision-making processes.

In traditional expert systems, human expertise is captured and represented using rules or knowledge bases. These systems rely on the knowledge and experience of domain experts to make informed decisions. However, they may not adequately handle uncertainties or imprecise information. With this design, many research works emerged that combine NS and SES like neutrosophic soft expert sets (NSESs),[20]interval-NSESs,[21]Generalized neutrosophic soft expert set,[22] neutrosophic soft expert graphs,[23] possibility neutrosophic soft expert sets[24,25] and a lot of models employed in solving real-life applications see.[26–28] In this work, we will collect a number of properties present in each of fuzzy set, neutrosophic set, and soft expert set under a single value in one model called Single value neutrosophic fuzzy soft expert set (SVNFSES). Based on this model, we present several distance measures and explain the mechanism for their use in solving a decision-making problem. This article is organized as follows: We review some basic definitions of the associated studies in Section 2. Section 3 presents the formulation of the SVNFSE-set and its operations. While in section 4, we demonstrate the set-theoretic operations of SVNSE-sets together with some propositions and examples. Section 5 discusses the applications of the SVNFSE-set in decision-making problems based on Distance Measure on SVNFSE-sets. Finally, conclusions and suggestions for further studies are pointed out in section 6.

2 Preliminaries

In this section, we recapitulate some of the ideas like FS, NS, SVNS, SS, and SVNSS that are considered beneficial in developing our new concept.

Definition 2.1. [2] Assume that $\hat{\mathbb{U}} = \{u_1, u_2, u_3, ..., u_n\}$ be a reference set. Then the FS formed as following structure:

$$\mathcal{Q} = \left\{ \left(u, \left\langle \ddot{\partial}^t_Q (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$$

where $\ddot{\partial}^t_Q (u_i)$ refer to true membership of object u_i in $\hat{\mathbb{U}}$ and persistent as a mapping:$\ddot{\partial}^t_Q : \hat{\mathbb{U}} \to [0, 1]$.

Definition 2.2. [2] Let $\mathcal{Q}_1 = \left\{ \left(u, \left\langle \ddot{\partial}^t_{Q_1} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ and $\mathcal{Q}_2 = \left\{ \left(\tau, \left\langle \ddot{\partial}^t_{Q_2} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ be two FS on reference set $\hat{\mathbb{U}}$. Then the fundamental operation on FSs defined as following:

https://doi.org/10.54216/IJNS.230216
Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

1.Union $Q_3 = \left\{ \left(u, \left\langle \max \left(\ddot{\partial}^t_{Q_1}(u_i), \ddot{\partial}^t_{Q_2}(u_i) \right) \right\rangle \right) \middle| u \in \hat{\mathbb{T}} \right\}$.

2.Intersection $Q_3 = \left\{ \left(\tau, \left\langle \min \left(\ddot{\partial}^t_{Q_1}(u_i), \ddot{\partial}^t_{Q_2}(u_i) \right) \right\rangle \right) \middle| \partial \in \hat{\mathbb{U}} \right\}$.

3.Complement $Q_1^c = \left\{ \left(u, \left\langle \left(1 - \ddot{\partial}^t_{Q_1}(u_i) \right) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$.

4.Subset $\mathcal{Q}_1 \subseteq \mathcal{Q}_2$ if $\ddot{\partial}^t_{Q_1}(u_i) \leq \ddot{\partial}^t_{Q_2}(u_i)$.

Definition 2.3. [1] Assume that $\hat{\mathbb{U}} = \{u_1, u_2, \tau_3, ..., u_n\}$ be a reference set $\hat{\mathbb{U}}$. Then the NS formed as following structure:

$$\hat{\hat{\mathcal{A}}}_{NS} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$$

where $\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i)$ refer to true membership,indeterminacy membership and falsehood membership of object u_i in $\hat{\mathbb{U}}$ and persistent as a mapping:$\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i) : \hat{\mathbb{U}} \to [0, 1]$.

Definition 2.4. [24] Let $\hat{\hat{\mathcal{A}}}_{SVNS} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ and $\hat{\hat{\mathcal{B}}}_{SVNS} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}}(u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ be two SVNS on reference set $\hat{\mathbb{U}}$ Then the fundamental operation on SVNS defined as following:

1.Union
$\hat{\hat{\mathcal{C}}}_{SVNS} = \left\{ \left(u, \left\langle \max \left[\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}}(u_i) \right], \min \left[\ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}}(u_i) \right], \min \left[\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}}(u_i) \right] \right\rangle \right) | u \in U \right\}$.

2.Intersection
$\hat{\hat{\mathcal{C}}}_{SVNS} = \left\{ \left(u, \left\langle \min \left[\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}}(u_i) \right], \max \left[\ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}}(u_i) \right], \max \left[\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i), \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}}(u_i) \right] \right\rangle \right) | u \in U \right\}$.

3.Complement
$\hat{\hat{\mathcal{A}}}^c_{SVNS} = \left\{ \left(u, \left\langle \left[\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i) \right], \left[1 - \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i) \right], \left[\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i) \right] \right\rangle \right) | u \in U \right\}$.

4.Subset
$\hat{\hat{\mathcal{A}}}_{SVNS} \subseteq \hat{\hat{\mathcal{B}}}_{sVNS}$ if$\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}}(u_i) \leq \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}}(u_i)$,$\ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}}(u_i) \geq \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}}(u_i)$,$\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}}(u_i) \geq \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}}(u_i)$.

Definition 2.5. [24] Let $\hat{\mathbb{U}} = \{u_1, u_2, u_3, ..., u_n\}$ and $\hat{\mathbb{E}} = \{z_1, z_2, z_3, ..., z_m\}$ be a reference set and attribute set, respectively. Then a SS over $\hat{\mathbb{U}}$ given as structures as follows:

$$\overset{\leftrightarrow}{S} = \left\{ \left(z, \left\langle \overset{\leftrightarrow}{S}(z_i) \right\rangle \right) \middle| z \in \hat{\mathbb{E}} \right\}$$

where the function $\overset{\leftrightarrow}{S}$ given by following mapping:

$$\overset{\leftrightarrow}{S} = E \to P^{(U)}$$

Here $P^{(U)}$ refer to collection of subsets of reference set $\hat{\mathbb{T}}$.

Definition 2.6. [24] A term $\mathcal{P}^{svnss}$ is said to be SVNSS on soft reference set $\left(\hat{\mathbb{U}}, \hat{\mathbb{E}}\right)$, where $\mathcal{P}^{svnss} : \hat{\mathbb{E}} \to SVN(P)^{(U)}$, such that $SVN(P)^{(U)}$ is a collection of all SVNS-subsets over $\hat{\mathbb{U}}$.

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

3 Structuring the concept of single value neutrosophic fuzzy soft expert set (SVNFSESS)

In this section, we introduce the concept of a SVNFES and define some properties of this model, namely, the null of the SVNFES , the absolute of the SVNFES , a subset of the SVNFES , and the equality of the SVNFES . Illustrated examples are also given.

Definition 3.1. The ordered pair (H, O) pointing to single-neutrosophic soft expert set (SNFSESS) on U. If

1. The mapping $\mathrm{H} : \mathrm{O} \rightarrow SVNFN^{\mathrm{U}}$ where $\mathrm{O} \subseteq \mathrm{Z} = \mathrm{M} \times \mathrm{N} \times \mathrm{Y}$, such that for all $z \in \mathrm{Z}$ then $z = (m \times n \times y = 0 \; or \; 1)$

2. Here $\mathrm{U} = \{u_1, u_2, u_3, ..., u_s\}, \mathrm{M} = \{m_1, m_2, m_3, ..., m_s\}, \mathrm{N} = \{n_1, n_2, n_3, ..., n_s\}$ represent to reference set, attribute set, set of experts respectively and $\mathrm{Y} = \{0, 1\}$.

3. A single neutrosophic soft set (PIVNSS) $\mathcal{H}$ on $\hat{\mathbb{U}}$ has the structures as follows:

$$\mathcal{H}^{svnfses} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\mathcal{H}} (z_i) (u_j), \ddot{\partial}^i_{\mathcal{H}} (z_i) (u_j), \ddot{\partial}^f_{\mathcal{H}} (z_i) (u_j) \right\rangle | u \in \hat{\mathbb{U}}, z \in \hat{\mathbb{Z}} \right\}$$

Where $\ddot{\partial}^t_{\mathcal{H}} (z_i) (u_j), \ddot{\partial}^i_{\mathcal{H}} (z_i) (u_j), \ddot{\partial}^f_{\mathcal{H}} (z_i) (u_j)$ represent of the three SVNFSES memberships as a single real number and $\mathcal{P}^{ivnss}$ refer to SVNSESN degree of element $u_i \in \hat{\mathbb{U}}$ to $\mathcal{H}^{svnses}$ who also can denoted by $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$

Example 3.2. Consider that a tourism company would like to evaluate a group of hotels it owns to see who is suitable. This evaluation was based on two experts working for the company. Now we assume that $\hat{\mathbb{U}}$ includes three hotels$\{u_1, u_2, u_3\}$, and the object is evaluated by two experts$\mathrm{N} = \{n_1, n_2\}$, and for the criteria that were adopted in this evaluation process, it can be represented by $\mathrm{M} = \{m_1, m_2, m_3\}$ such that $m_1 = Food \; services m_2 = Staff, m_3 = Number \; of \; rooms$. Now for $\mathrm{O} \subseteq \mathrm{Z} = \mathrm{M} \times \mathrm{N} \times \mathrm{Y}$, Now our concept presents the opinions of the two experts as follows:

$$\mathcal{H}(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3, 0.6, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8, 0.1, 0.6 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_2 = (m_1, n_2, 1)) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.4, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.1, 0.5, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.2, 0, 0.8 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_3 = (m_2, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.7, 0.6, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.6, 0.3, 0.1 \rangle} \right), \left(\frac{u_3}{\langle 0.2, 0.3, 0.5 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_4 = (m_2, n_2, 1)) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.3, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.6, 0.4, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.5, 0.4, 0.3 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_5 = (m_3, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.7, 0.5, 0.9 \rangle} \right), \left(\frac{u_2}{\langle 0.2, 0.6, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.3, 0.4, 0.6 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_6 = (m_3, n_2, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3, 0.6, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8, 0.1, 0.6 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_7 = (m_1, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3, 0.6, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8, 0.1, 0.6 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_8 = (m_1, n_2, 0)) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.6, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.7, 0.2, 0.5 \rangle} \right), \left(\frac{u_3}{\langle 0.1, 0.1, 0.3 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_9 = (m_2, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.5, 0.8 \rangle} \right), \left(\frac{u_2}{\langle 0.9, 0.8, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.1, 0.6, 0.9 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_{10} = (m_2, n_2, 0)) = \left\{ \left(\frac{u_1}{\langle 0.3, 0.4, 0.7 \rangle} \right), \left(\frac{u_2}{\langle 0.3, 0.4, 0.2 \rangle} \right), \left(\frac{u_3}{\langle 0.6, 0.6, 0.2 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_{11} = (m_3, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.4, 0.5, 0.6 \rangle} \right), \left(\frac{u_2}{\langle 0.1, 0.2, 0.8 \rangle} \right), \left(\frac{u_3}{\langle 0.2, 0.4, 0.7 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_{12} = (m_3, n_2, 0)) = \left\{ \left(\frac{u_1}{\langle 0.6, 0.8, 0.6 \rangle} \right), \left(\frac{u_2}{\langle 0.2, 0.5, 0.6 \rangle} \right), \left(\frac{u_3}{\langle 0.4, 0.9, 0.9 \rangle} \right) \right\}.$$

Also $\mathcal{H}(z_i)$ can represent as a matrix as a following form:
$\mathcal{H}(z_i) =$

https://doi.org/10.54216/IJNS.230216
Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

$$\begin{pmatrix} ((0.2,0.5,0.3)) & ((0.3,0.6,0.7)) & ((0.8,0.1,0.6)) \\ ((0.5,0.4,0.2)) & ((0.1,0.5,0.7)) & ((0.2,0.0,0.8)) \\ ((0.7,0.6,0.2)) & ((0.6,0.3,0.1)) & ((0.2,0.3,0.5)) \\ ((0.5,0.3,0.2)) & ((0.6,0.4,0.7)) & ((0.5,0.4,0.3)) \\ ((0.7,0.5,0.9)) & ((0.2,0.6,0.7)) & ((0.3,0.4,0.6)) \\ ((0.2,0.5,0.3)) & ((0.3,0.6,0.7)) & ((0.8,0.1,0.6)) \\ ((0.4,0.6,0.2)) & ((0.7,0.2,0.5)) & ((0.1,0.1,0.3)) \\ ((0.4,0.5,0.8)) & ((0.9,0.8,0.7)) & ((0.1,0.6,0.9)) \\ ((0.3,0.4,0.7)) & ((0.3,0.4,0.2)) & ((0.6,0.6,0.2)) \\ ((0.4,0.5,0.6)) & ((0.1,0.2,0.8)) & ((0.2,0.4,0.7)) \\ ((0.6,0.8,0.6)) & ((0.2,0.5,0.6)) & ((0.4,0.9,0.9)) \end{pmatrix}$$

Definition 3.3. (Agree SVNFSES): Agree SVNFSES $\mathcal{H}_1$ represents agreement of all expert's opinions and is defined as follows:

$$\mathcal{H}_{\mathcal{O}} = \{H_O(z_i) : z_i \in M \times N \times \{1\}\}$$

Example 3.4. Take the terms $\mathcal{H}_{\mathcal{O}}(z_1)$ in Example 3.2. above

$$\mathcal{K}(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3,0.6,0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8,0.1,0.6 \rangle} \right) \right\}.$$

Definition 3.5. (Disagree SVNFSES): disagree SVNFSES $\mathcal{H}_0$ represents disagreement of all expert's opinions and is defined as follows:

$$\mathcal{H}_{\mathcal{O}} = \{H_O(z_i) : z_i \in M \times N \times \{0\}\}$$

Example 3.6. Take the terms $\mathcal{H}_{\mathcal{O}}(z_9)$ in Example 3.2. above

$$\mathcal{K}(z_9 = (m_2, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.4,0.5,0.8 \rangle} \right), \left(\frac{u_2}{\langle 0.9,0.8,0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.1,0.6,0.9 \rangle} \right) \right\}.$$

Definition 3.7. (SVNFSE-subset): Let $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ and $\mathcal{K}_{\mathrm{P}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then $\mathcal{H}_{\mathrm{O}}$ is said SVNFSE-subset of $\mathcal{K}_{\mathrm{P}}$ and denoted by $\mathcal{H}_{\mathrm{O}} \subseteq \mathcal{K}_{\mathrm{P}}$ if:

1. $\mathcal{H}_{\mathrm{O}}(u_i)$ is SVNFSE-subset of $\mathcal{K}_{\mathrm{P}}(u_i)$, $\forall u_i \in \hat{\mathbb{U}}$.

2.$\mathrm{O} \subseteq \mathrm{P}$.

Example 3.8. Take the terms $\mathcal{H}_{\mathcal{O}}(z_i)$ in Example 3.2. above and take

$$\mathcal{K}(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3,0.6,0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8,0.1,0.6 \rangle} \right) \right\}.$$
$$\mathcal{K}(z_9 = (m_2, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.4,0.5,0.8 \rangle} \right), \left(\frac{u_2}{\langle 0.9,0.8,0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.1,0.6,0.9 \rangle} \right) \right\}.$$

Now, its clear the two terms are $\subseteq \mathcal{H}_{\mathcal{O}}$.

Definition 3.9. (Equality of SVNFSE-sets): Let $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ and $\mathcal{K}_{\mathrm{P}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then$\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ is called equal of $\mathcal{K}_{\mathrm{P}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ and denoted by $\mathcal{H}_{\mathcal{O}} = \mathcal{K}_{\mathcal{P}}$ if:

1. $\mathcal{H}(u_i)$ is SVNFSE-subset of $\mathcal{K}(u_i)$ and $\mathcal{K}(u_i)$ is SVNFSE-subset of $\mathcal{H}(u_i)$, $\forall u_i \in \hat{\mathbb{U}}$.

2. $\mathcal{O}$ is subset of $\mathcal{P}$ and $\mathcal{P}$ is subset of $\mathcal{O}$, $\forall u_i \in \hat{\mathbb{U}}$.

Example 3.10. Conseder $\mathcal{H}_{\mathcal{O}}(z_i)$ in Example 3.2. above and take $\mathcal{H}_{\mathcal{O}} =$

$$\begin{pmatrix} (0.2, 0.50.3) & (0.3, 0.6, 0.8) & (0.8, 0.1, 0.6) \\ (0.5, 0.80.2) & (0.2, 0.4, 0.8) & (0.8, 0, 0.2) \\ (0.4, 0.50.3) & (0.9, 0.6, 0.8) & (0.8, 0.7, 0.7) \end{pmatrix} \text{ and}$$

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

$\mathcal{G}_{\mathcal{C}} =$

$$\begin{pmatrix} (0.1, 0.30.3) & (0.6, 0.8, 0.8) & (0.8, 0.1, 0.1) \\ (0.5, 0.80.2) & (0.2, 0.4, 0.8) & (0.8, 0, 0.2) \\ (0.4, 0.7, 0.8) & (0.4, 0.6, 0.8) & (0.8, 0.5, 0.4) \end{pmatrix} \text{ and}$$

$\mathcal{K}_{\mathcal{P}} =$

$$\begin{pmatrix} (0.2, 0.50.3) & (0.3, 0.6, 0.8) & (0.8, 0.1, 0.6) \\ (0.5, 0.80.2) & (0.2, 0.4, 0.8) & (0.8, 0, 0.2) \\ (0.4, 0.50.3) & (0.9, 0.6, 0.8) & (0.8, 0.7, 0.7) \end{pmatrix}$$

Then, its clear $\mathcal{H}_{\mathcal{O}} = \mathcal{K}_{\mathcal{P}}$ and $\mathcal{H}_{\mathcal{O}} \neq \mathcal{G}_{\mathcal{C}}$.

Definition 3.11. (null SVNFSE-set): Let $\mathcal{H}_{\rm O} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be SVNFSE-set on reference set $\hat{\mathbb{U}}$. Then we say that $\mathcal{H}_{\rm O}$ is " null SVNFSE-set" and denoted by $\widehat{\Phi}_{(0)}$ if $\mathcal{H}(u_i) = (0, 1, 1)$ and $\Theta(z_i) = 0, \forall u_i \in \hat{\mathbb{U}}$.

Example 3.12. Taking into account the matrix notation of $\mathcal{H}_{\mathcal{O}}$ as an Example 3.2, it can be observed that we possess.

$$\widehat{\Phi}_{(0)} = \begin{pmatrix} ((0,1,1)) & ((0,1,1)) & ((0,1,1)) \\ ((0,1,1)) & ((0,1,1)) & ((0,1,1)) \\ ((0,1,1)) & ((0,1,1)) & ((0,1,1)) \end{pmatrix}$$

Definition 3.13. (absolute SVNFES-set): Let $\mathcal{H}_{\rm O} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be SVNFSE-set on reference set $\hat{\mathbb{U}}$. Then we say that $\mathcal{H}_{\rm O}$ is " absolute SVNFES-set" and denoted by $\widehat{\Omega}_{(1)}$ if $\mathcal{H}(u_i) = (1, 0, 0)$ and $\Theta(z_i) = 1, \forall u_i \in \hat{\mathbb{U}}$.

Example 3.14. Taking into account the matrix notation of $\mathcal{H}_{\rm O}$ as an Example 3.2, it can be observed that we possess.

$$\widehat{\Phi}_{(0)} = \begin{pmatrix} ((1,0,0)) & ((1,0,0)) & ((1,0,0)) \\ ((1,0,0)) & ((1,0,0)) & ((1,0,0)) \\ ((1,0,0)) & ((1,0,0)) & ((1,0,0)) \end{pmatrix}$$

Definition 3.15. (Complement operation of SVNFSE-set): Let $\mathcal{H}_{\rm O} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be SVNFSE-set on reference set $\hat{\mathbb{U}}$ and defied as follows

$$\mathcal{H}_{\rm O} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z \in \hat{\mathbb{Z}} \right\}$$

Then,complement operation of SVNFSES-set defined as follows

$$\mathcal{H}_{\rm O}{}^c = \left\{ \left(u, \left\langle \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j), 1 - \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z \in \hat{\mathbb{Z}} \right\}$$

Here: we follow the complement role of SVNS-complement.

Example 3.16. Take the terms $\mathcal{H}_{\mathcal{O}}(z_{i=1,9})$ in Example 3.2. above and take

$$\mathcal{H}(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3,0.6,0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8,0.1,0.6 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_9 = (m_2, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.4,0.5,0.8 \rangle} \right), \left(\frac{u_2}{\langle 0.9,0.8,0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.1,0.6,0.9 \rangle} \right) \right\}.$$

Then the complement of them given as following:

$$\mathcal{H}^c(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.3,0.5,0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.7,0.4,0.3 \rangle} \right), \left(\frac{u_3}{\langle 0.6,0.9,0.8 \rangle} \right) \right\}.$$
$$\mathcal{H}^c(z_9 = (m_2, n_1, 0)) = \left\{ \left(\frac{u_1}{\langle 0.8,0.5,0.4 \rangle} \right), \left(\frac{u_2}{\langle 0.7,0.2,0.9 \rangle} \right), \left(\frac{u_3}{\langle 0.9,0.4,0.1 \rangle} \right) \right\}.$$

https://doi.org/10.54216/IJNS.230216
Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

4 The set-theoretic operations pertaining to SVNSE-sets

Now, in this section, we introduce the set-theoretic operations pertaining to SVNSE-sets as well as some properties and numerical examples that illustrate how these tools work in algebraic environments.

Definition 4.1. (Union of SVNFSE-sets) Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle | u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$
and
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle | u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$
be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then,the union of SVNFSE-sets denoted by $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{K}_{\mathrm{P}}$ and defiend as following: $\mathcal{D}_{\mathrm{S}}=\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{K}_{\mathrm{P}}$, where $\hat{\cup}$ denotes SVNFSE-sets-union.

Example 4.2. Taking into account one part of SVNFSES $\mathcal{H}_{\mathrm{O}}$ as an Example 3.2, and $\mathcal{K}_{\mathrm{P}}$ given in an Example 3.4,

$$\mathcal{H}\left(z_{2}=\left(m_{1},n_{2},1\right)\right)=\left\{\left(\frac{u_{1}}{\langle0.5,0.4,0.2\rangle}\right),\left(\frac{u_{2}}{\langle0.1,0.5,0.7\rangle}\right),\ \left(\frac{u_{3}}{\langle0.2,0,0.8\rangle}\right)\right\}.$$

and

$$\mathcal{K}\left(z_{2}=\left(m_{1},n_{2},1\right)\right)=\left\{\left(\frac{u_{1}}{\langle0.4,0.5,0.8\rangle}\right),\left(\frac{u_{2}}{\langle0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_{3}}{\langle0.1,0.6,0.9\rangle}\right)\right\}.$$

then,the union of SVNFSE-sets can be possess as following :

$$\mathcal{D}\left(z_{2}=\left(m_{1},n_{2},1\right)\right)=\left\{\left(\frac{u_{1}}{\langle0.5,0.4,0.2\rangle}\right),\left(\frac{u_{2}}{\langle0.9,0.5,0.7\rangle}\right),\ \left(\frac{u_{3}}{\langle0.2,0,0.9\rangle}\right)\right\}.$$

Also $\mathcal{D}\left(u_{i}\right)$ can represent as a matrix as a following form:
$\mathcal{D}_{\Psi}=$

$$\left(((0.5,0.4,0.2))\quad((0.9,0.5,0.7))\quad((0.2,0,0.9))_{1\times3}\right)$$

Definition 4.3. (Intersection of PIVNS-sets) Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle | u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$
and
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle | u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$
be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then,the intersection of SVNFSE-sets denoted by $\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{K}_{\mathrm{P}}$ and defiend as following: $\mathcal{C}_{\mathrm{L}}=\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{K}_{\mathrm{P}}$, where $\hat{\cap}$ denotes SVNFSE-sets-intersection.

Example 4.4. Taking into account one part of SVNFSES $\mathcal{H}_{\mathrm{O}}$ as an Example 3.2, and $\mathcal{K}_{\mathrm{P}}$ given in an Example 3.4,

$$\mathcal{H}\left(z_{2}=\left(m_{1},n_{2},1\right)\right)=\left\{\left(\frac{u_{1}}{\langle0.5,0.4,0.2\rangle}\right),\left(\frac{u_{2}}{\langle0.1,0.5,0.7\rangle}\right),\ \left(\frac{u_{3}}{\langle0.2,0,0.8\rangle}\right)\right\}.$$

and

$$\mathcal{K}\left(z_{2}=\left(m_{1},n_{2},1\right)\right)=\left\{\left(\frac{u_{1}}{\langle0.4,0.5,0.8\rangle}\right),\left(\frac{u_{2}}{\langle0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_{3}}{\langle0.1,0.6,0.9\rangle}\right)\right\}.$$

then,the intersection of SVNFSE-sets can be possess as following :

$$\mathcal{C}\left(z_{2}=\left(m_{1},n_{2},1\right)\right)=\left\{\left(\frac{u_{1}}{\langle0.4,0.5,0.8\rangle}\right),\left(\frac{u_{2}}{\langle0.1,0.8,0.7\rangle}\right),\ \left(\frac{u_{3}}{\langle0.1,0,0.8\rangle}\right)\right\}.$$

Also $\mathcal{C}\left(u_{i}\right)$ can represent as a matrix as a following form:
$\mathcal{C}_{\mathcal{L}}=$

$$\left(((0.4,0.5,0.8))\quad((0.1,0.8,0.7))\quad((0.1,0,0.8))_{1\times3}\right)$$

https://doi.org/10.54216/IJNS.230216

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

Proposition 4.5. *Let*
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$
be a SVNFSE-set on reference set $\hat{\mathbb{U}}$. *Then,the following statements hold:*

1. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{H}_{\mathrm{O}}=\mathcal{H}_{\mathrm{O}}$.
2. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{H}_{\mathrm{O}}=\mathcal{H}_{\mathrm{O}}$.
3. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\widehat{\Phi}_{(0)}=\mathcal{H}_{\mathrm{O}}$.
4. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\widehat{\Phi}_{(0)}=\widehat{\Phi}_{(0)}$.
5. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\widehat{\Omega}_{(1)}=\widehat{\Omega}_{(1)}$.
6. $\mathcal{H}_{\Theta}\hat{\cap}\widehat{\Omega}_{(1)}=\mathcal{H}_{\mathrm{O}}$.

Proposition 4.6. *Let*
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$,
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$ *and*
$\mathcal{G}_{\mathrm{C}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{G}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{G}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{G}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$ *be three SVNFSE-sets on reference set* $\hat{\mathbb{U}}$. *Then,the following statements hold:*

1. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}=\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{H}_{\mathrm{O}}$.
2. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{G}_{\mathrm{C}}=\mathcal{G}_{\mathrm{C}}\hat{\cap}\mathcal{H}_{\mathrm{O}}$.
3. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\left(\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}\right)\hat{\cup}\mathcal{K}_{\mathrm{P}}$.
4. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\left(\mathcal{G}_{\mathrm{C}}\hat{\cap}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{G}_{\mathrm{C}}\right)\hat{\cap}\mathcal{K}_{\mathrm{P}}$.
5. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\left(\mathcal{G}_{\mathrm{C}}\hat{\cap}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}\right)\hat{\cap}\left(\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{K}_{\mathrm{C}}\right)$.
6. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\left(\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{G}_{\mathrm{C}}\right)\hat{\cup}\left(\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{K}_{\mathrm{P}}\right)$.

Proof. **1.** We take the left side $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}=$
$H_{O}=\left\{\left(u,\left\langle\ddot{\partial}_{H}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right)|u\in U,z\in Z\right\}\bigcup$
$G_{C}=\left\{\left(u,\left\langle\ddot{\partial}_{G}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right)|u\in U,z\in Z\right\}$
$=\left\{\left(u,\left\langle\max\left(\ddot{\partial}_{H}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{t}\left(z_{i}\right)\left(u_{j}\right)\right),\min\left(\ddot{\partial}_{H}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{i}\left(z_{i}\right)\left(u_{j}\right)\right),\min\left(\ddot{\partial}_{H}^{f}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{f}\left(z_{i}\right)\left(u_{j}\right)\right)\right\rangle\right)\right\}$
$=\left\{\left(u,\left\langle\max\left(\ddot{\partial}_{G}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{t}\left(z_{i}\right)\left(u_{j}\right)\right),\min\left(\ddot{\partial}_{G}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{i}\left(z_{i}\right)\left(u_{j}\right)\right),\min\left(\ddot{\partial}_{G}^{f}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{f}\left(z_{i}\right)\left(u_{j}\right)\right)\right\rangle\right)\right\}$
$=G_{C}=\left\{\left(u,\left\langle\ddot{\partial}_{G}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{G}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right)|u\in U,z\in Z\right\}\bigcup$
$H_{O}=\left\{\left(u,\left\langle\ddot{\partial}_{H}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{H}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right)|u\in U,z\in Z\right\}$.
$=\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{H}_{\mathrm{O}}$.

□

Note: *The rest of the proof is similar to proof method 1*

Proposition 4.7. *Let*
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$,
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle\right|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$ *be two SVNFSE-sets on reference set* $\hat{\mathbb{U}}$. *Then,the following statements hold:*

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

1. $({\mathcal{H}_{\mathrm{O}}}^c)^c = \mathcal{H}_{\mathrm{O}}$.
2. $\left(\mathcal{H}_{\mathrm{O}} \hat{\cup} \mathcal{K}_{\mathrm{P}}\right)^c = {\mathcal{H}_{\mathrm{O}}}^c \hat{\cap} {\mathcal{K}_{\mathrm{P}}}^c$.
3. $\left(\mathcal{H}_{\mathrm{O}} \hat{\cap} \mathcal{K}_{\mathrm{P}}\right)^c = {\mathcal{H}_{\mathrm{O}}}^c \hat{\cup} {\mathcal{K}_{\mathrm{P}}}^c$.

Here, paragraphs 2 and 3 refer to De Morgan's law.

Proof. **1.** $\mathcal{H}_{\mathrm{O}} = \left\{ u_j \left(\ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j) \ , \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j) \, , \, \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$H^c_O = \left\{ u_j \left(\ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \ , 1 - \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j) \, , \, \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$(\mathcal{H}^c_O)^c = \left\{ u_j \left(\ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j) \ , 1 - \left(1 - \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j)\right) , \, \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$= \left\{ u_j \left(\ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j) \ , \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j) \, , \, \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$= \mathcal{H}_{\mathcal{O}}$

Note: The rest of proof is similar to proof method 1 □

Definition 4.8. **(*AND* of SVNFSE-sets):** Let $\mathcal{H}_{\mathcal{O}}and\mathcal{G}_{\mathcal{C}}$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then the "*AND*" operation of both $\mathcal{H}_{\mathcal{O}}$ and $\mathcal{G}_{\mathcal{C}}$ defined as $\mathcal{H}_{\mathcal{O}} \wedge \mathcal{G}_{\mathcal{C}} = \mathcal{R}_{\mathcal{L}}$ such that $\mathcal{R}_{\mathcal{L}}(u_i, u_j)(z_i) = \mathcal{H}_{\mathcal{O}}(u_j)(z_i) \cap \mathcal{G}_{\mathcal{C}}(u_j)(z_i)$ Here $\cap$ refer to the intersection of SVNS.

Example 4.9. Taking into account the SVNFSE $\mathcal{H}_{\mathcal{O}}$ as an Example 3.2, and $\mathcal{G}_{\mathcal{C}}$ given in an Example 3.4, then, **(*AND* of SVNFSE-sets):** can be possess as following :

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_1) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right) \right\}.$$

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_2) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right) \right\}.$$

Definition 4.10. **(*OR* of SVNFSE-sets):** Let $\mathcal{H}_{\mathcal{O}}and\mathcal{G}_{\mathcal{C}}$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then the "*OR*" operation of both $\mathcal{H}_{\mathcal{O}}$ and $\mathcal{G}_{\mathcal{C}}$ defined as $\mathcal{H}_{\mathcal{O}} \vee \mathcal{G}_{\mathcal{C}} = \mathcal{F}_{\mathcal{V}}$ such that $\mathcal{F}_{\mathcal{V}}(u_i, u_j)(z_i) = \mathcal{H}_{\mathcal{O}}(u_j)(z_i) \cap \mathcal{G}_{\mathcal{C}}(u_j)(z_i)$ Here $\cup$ refer to the union of SVNS.

Example 4.11. Taking into account the SVNFSE $\mathcal{H}_{\mathcal{O}}$ as an Example 3.2, and $\mathcal{G}_{\mathcal{C}}$ given in an Example 3.4, then, **(*OR* of SVNFSE-sets):** can be possess as following :

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_1) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right) \right\}.$$

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_2) = \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right), \left(\frac{u_1}{\langle 0.2,0.5,0.1 \rangle} \right) \right\}.$$

5 Distance Measure on SVNFSE-sets

In this part, we introduce and study the distance measure (DM) of SVNFSESs in order to calculate the ratio of DM between two SVNFSESs. After that, we will employ these DMs in one application in DM problem by proposing an algorithm shown in Figure1.

https://doi.org/10.54216/IJNS.230216

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

Definition 5.1. Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$,
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$ and
$\mathcal{G}_{\mathrm{C}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{G}}^{t}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{G}}^{i}\left(z_{i}\right)\left(u_{j}\right),\ddot{\partial}_{\mathcal{G}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle|u\in\hat{\mathbb{U}},z_{i}\in\hat{\mathbb{Z}}\right\}$ be three SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then, a function $\mathbb{D}$:SVNFSES×SVNFSES $\rightarrow[0,1]$ is called distance measure (SVNFSES($\hat{\mathbb{U}}$)) if $\mathbb{D}$ fulfilled the following notes:

i. $\mathbb{D}(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}})\geq 0$, and $\mathbb{D}(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}})=0$, iff both $\mathcal{H}_{\mathrm{O}}=\mathcal{G}_{\mathrm{C}}$.

ii. $\mathbb{D}(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}})=\mathbb{D}(\mathcal{G}_{\mathrm{C}},\mathcal{H}_{\mathrm{O}})$

iii. $\mathbb{D}\left(\mathcal{H}_{\mathrm{O}},\mathcal{K}_{\mathrm{P}}\right)\leq\mathbb{D}\left(\mathcal{H}_{\mathrm{O}},\mathcal{K}_{\mathrm{P}}\right)+\mathbb{D}\left(\mathcal{G}_{\mathrm{C}},\mathcal{K}_{\mathrm{P}}\right)$ (triangle inequality).

Now based on definition 5.1, we will define the following distance measures:

1.Hamming distance
$\mathbb{D}_{SVNFSES}^{H}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right)=\frac{1}{3}\sum_{i}^{n}\sum_{j}^{m}\frac{\left[\left|\Delta_{ij}\ddot{\partial}_{\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}}^{t}\right|+\left|\Delta_{ij}\ddot{\partial}_{\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}}^{i}\right|+\left|\Delta_{ij}\ddot{\partial}_{\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}}^{f}\right|\right]}{3}$ where

$$\begin{aligned}\Delta_{ij}\ddot{\partial}_{\mathcal{H},\mathcal{G}}^{t}&=\left(\ddot{\partial}_{\mathcal{H}_{\mathrm{O}}}^{t}-\ddot{\partial}_{\mathcal{G}_{\mathrm{C}}}^{t}\right),\\ \Delta_{ij}\ddot{\partial}_{\mathcal{H},\mathcal{G}}^{i}&=\left(\ddot{\partial}_{\mathcal{H}_{\mathrm{O}}}^{i}-\ddot{\partial}_{\mathcal{G}_{\mathrm{C}}}^{i}\right),\\ \Delta_{ij}\ddot{\partial}_{\mathcal{H},\mathcal{G}}^{f}&=\left(\ddot{\partial}_{\mathcal{H}_{\mathrm{O}}}^{f}-\ddot{\partial}_{\mathcal{G}_{\mathrm{C}}}^{f}\right),\end{aligned}$$

2.Normalized Hamming distance

$\mathbb{D}_{SVNFSES}^{NH}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right)=\frac{D_{SVNFSES}^{H}(\mathcal{P}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}})}{mn}$.

3. Euclidean distance

$D_{SVNFSES}^{E}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right)=\sqrt{\frac{1}{3}\sum_{i}^{n}\sum_{j}^{m}\frac{\left[\left|\Delta_{ij}\ddot{\partial}_{\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}}^{t}\right|+\left|\Delta_{ij}\ddot{\partial}_{\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}}^{i}\right|+\left|\Delta_{ij}\ddot{\partial}_{\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}}^{f}\right|\right]}{6}}$
where

$$\begin{aligned}\Delta_{ij}\ddot{\partial}_{\mathcal{H},\mathcal{G}}^{t}&=\left(\ddot{\partial}_{\mathcal{H}_{\mathrm{O}}}^{t}-\ddot{\partial}_{\mathcal{G}_{\mathrm{C}}}^{t}\right),\\ \Delta_{ij}\ddot{\partial}_{\mathcal{H},\mathcal{G}}^{i}&=\left(\ddot{\partial}_{\mathcal{H}_{\mathrm{O}}}^{i}-\ddot{\partial}_{\mathcal{G}_{\mathrm{C}}}^{i}\right),\\ \Delta_{ij}\ddot{\partial}_{\mathcal{H},\mathcal{G}}^{f}&=\left(\ddot{\partial}_{\mathcal{H}_{\mathrm{O}}}^{f}-\ddot{\partial}_{\mathcal{G}_{\mathrm{C}}}^{f}\right),\end{aligned}$$

4. Normalized Euclidean distance

$\mathbb{D}_{SVNFSES}^{NE}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{O}}\right)=\frac{D_{PIVNSS}^{E}(\mathcal{P}_{\mathrm{O}},\mathcal{G}_{\mathrm{O}})}{\sqrt{mn}}$.

Theorem 5.2. *The function* $\mathbb{D}_{SVNFSES}^{H}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right),\mathbb{D}_{SVNFSES}^{NH}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right),D_{SVNFSES}^{E}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right)$ *and* $\mathbb{D}_{SVNFSES}^{NE}\left(\mathcal{H}_{\mathrm{O}},\mathcal{G}_{\mathrm{C}}\right)$ *:SVNFSES*$\left(\hat{\mathbb{U}}\right)\rightarrow R^{+}$ *that given by Definition 5.1.respectively are metrics, where* R^{+} *is the set all non-negative real-numbers.*

https://doi.org/10.54216/IJNS.230216

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

Proof. The proof is straightforward. □

Sometimes users resort to adjusting the resulting values when using the tools given above, using weight values that are specified by the user. Now we will redefine the above tools, but combined with weight values.

Now based on definition 5.1, we will define the following distance measures:

1. Weighted Hamming distance:

$D^{wH}_{SVNFSES}(\mathcal{H}_{\mathcal{O}}, \mathcal{G}_{\mathcal{C}}) =$

$\left\{ \frac{1}{3} \sum_{i}^{n} \sum_{j}^{m} w_i \left[\left| \Delta_{ij} \ddot{\partial}^t_{\mathcal{P}_{\mathcal{O}}, \mathcal{G}_{\mathcal{C}}} \right| + \left| \Delta_{ij} \ddot{\partial}^i_{\mathcal{H}_{\mathcal{O}}, \mathcal{G}_{\mathcal{O}}} \right| + \left| \Delta_{ij} \ddot{\partial}^f_{\mathcal{H}_{\mathcal{O}}, \mathcal{G}_{\mathcal{C}}} \right| \right]^{\gamma} \right\}^{\frac{1}{\gamma}}$

where

$$\Delta_{ij}\ddot{\partial}^t_{H_{\mathcal{O}},G_{\mathcal{C}}} = \left|\ddot{\partial}^t_{P_{\mathcal{O}}} - \ddot{\partial}^t_{G_{\mathcal{C}}}\right|^{\gamma}$$
$$\Delta_{ij}\ddot{\partial}^i_{H_{\mathcal{O}},G_{\mathcal{C}}} = \left|\ddot{\partial}^i_{P_{\mathcal{O}}} - \ddot{\partial}^i_{G_{\mathcal{C}}}\right|^{\gamma}$$
$$\Delta_{ij}\ddot{\partial}^f_{H_{\mathcal{O}},G_{\mathcal{C}}} = \left|\ddot{\partial}^f_{P_{\mathcal{O}}} - \ddot{\partial}^f_{G_{\mathcal{C}}}\right|^{\gamma}$$

2. Weighted Normalized Hamming distance:

$\mathbb{D}^{wNH}_{SVNFSES}(\mathcal{H}_{\mathcal{O}}, \mathcal{G}_{\mathcal{C}}) = \frac{D^{wH}_{SVNFSES}(\mathcal{H}_{\mathcal{O}}, \mathcal{G}_{\mathcal{C}})}{mn}.$

6 Application of SVNFSE-sets in real-life situations

In this section of the current research, we will create a new algorithm based on the tools presented in this work to solve one of the decision-making problems (to help a couple choose a new home in one of the residential complexes). This algorithm will present its steps in Figure 1 as following:

6.1 Statement of the problem

Assume that the married couple, Mr. Xu and Mrs. Xu wants to purchase a house in one of the low-cost residential complexes. In the low-cost residential complexes, there are two houses that represent by reference set $\hat{\mathbb{U}} = \{u_1, u_2, u_3\}$.The two couples in their selection focus on observing the attributes that can be represented by the following attribute set $\hat{\mathbb{M}} = \{m_1, m_2, m_3, \}$ such that $m_1 =$House area,$m_2 =$House price, and $m_3 =$ Materials used in building the house .In this scenario, the couple resorts to two experts $\{n_1, n_2\}$ in real estate issues for the purpose of consultation. Here we will analyze the expert's opinions by building two models $(SVNFSES - memberships)$ from our proposed evaluation of this evaluation as a follows:

Step 1. Build three SVNFSES models $\mathcal{K}_{\mathcal{P}}$ represents optimal evaluation and $\mathcal{H}_{\mathcal{O}}$ and $\mathcal{G}_{\mathcal{C}}$ represents the opinion of the first expert (n_1) and the second expert (n_2) for three houses (u_1), (u_2) and (u_3) :

$\mathcal{H}_{\mathcal{O}} =$

$\Big\{\mathcal{H}(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.3 \rangle} \right) \right. .$

$\mathcal{H}(z_2 = (m_1, n_2, 1)) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.4, 0.2 \rangle} \right) \right\}.$

https://doi.org/10.54216/IJNS.230216
Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

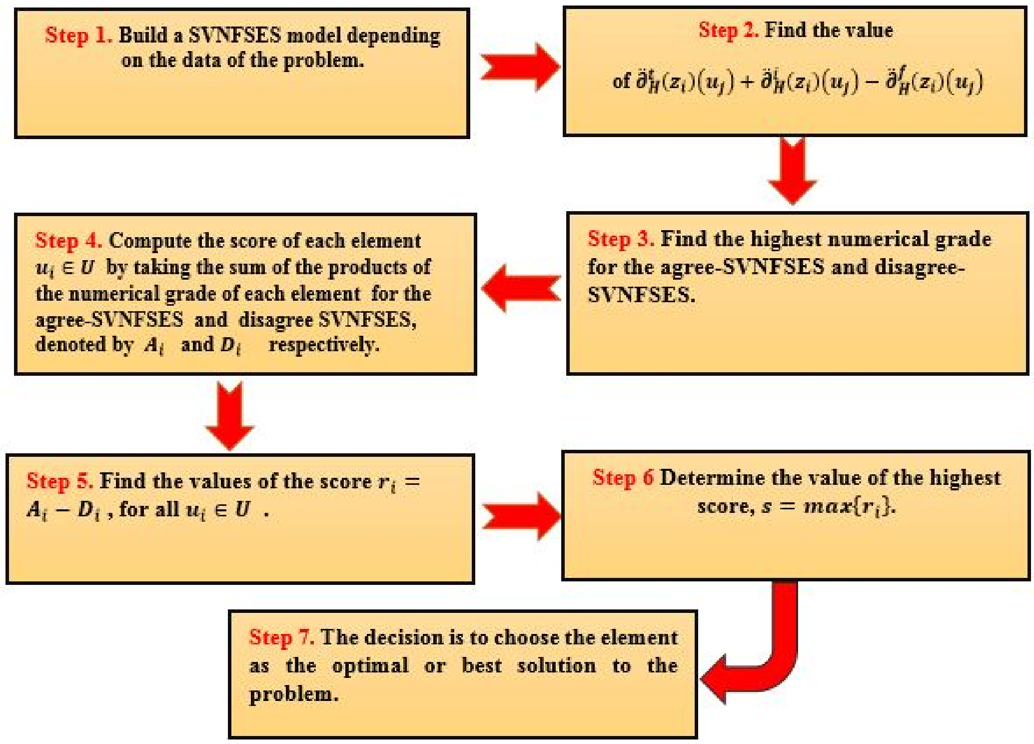

Figure 1: The propose algorithm based on DM technical

$$
\begin{aligned}
&\mathcal{H}\left(z_3=(m_2,n_1,1)\right)=\left\{\left(\tfrac{u_1}{\langle 0.7,0.6,0.2\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_4=(m_2,n_2,1)\right)=\left\{\left(\tfrac{u_1}{\langle 0.5,0.3,0.2\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_5=(m_3,n_1,1)\right)=\left\{\left(\tfrac{u_1}{\langle 0.7,0.5,0.9\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_6=(m_3,n_2,1)\right)=\left\{\left(\tfrac{u_1}{\langle 0.2,0.5,0.3\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_7=(m_1,n_1,0)\right)=\left\{\left(\tfrac{u_1}{\langle 0.2,0.5,0.3\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_8=(m_1,n_2,0)\right)=\left\{\left(\tfrac{u_1}{\langle 0.4,0.6,0.2\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_9=(m_2,n_1,0)\right)=\left\{\left(\tfrac{u_1}{\langle 0.4,0.5,0.8\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_{10}=(m_2,n_2,0)\right)=\left\{\left(\tfrac{u_1}{\langle 0.3,0.4,0.7\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_{11}=(m_3,n_1,0)\right)=\left\{\left(\tfrac{u_1}{\langle 0.4,0.5,0.6\rangle}\right)\right\}.\\
&\mathcal{H}\left(z_{12}=(m_3,n_2,0)\right)=\left\{\left(\tfrac{u_1}{\langle 0.6,0.8,0.6\rangle}\right)\right\}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{G}_{\mathcal{C}}=\\
&\Big\{\mathcal{G}\left(z_1=(m_1,n_1,1)\right)=\left\{\left(\tfrac{u_2}{\langle 0.1,0.4,0.2\rangle}\right)\right\}.\\
&\mathcal{G}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\tfrac{u_2}{\langle 0.8,0.7,0.1\rangle}\right)\right\}.\\
&\mathcal{G}\left(z_3=(m_2,n_1,1)\right)=\left\{\left(\tfrac{u_2}{\langle 0.9,0.2,0.1\rangle}\right)\right\}.\\
&\mathcal{G}\left(z_4=(m_2,n_2,1)\right)=\left\{\left(\tfrac{u_2}{\langle 0.6,0.3,0.4\rangle}\right)\right\}.\\
&\mathcal{G}\left(z_5=(m_3,n_1,1)\right)=\left\{\left(\tfrac{u_2}{\langle 0.4,0.5,0.9\rangle}\right)\right\}.\\
&\mathcal{G}\left(z_6=(m_3,n_2,1)\right)=\left\{\left(\tfrac{u_2}{\langle 0.3,0.5,0.6\rangle}\right)\right\}.\\
&\mathcal{G}\left(z_7=(m_1,n_1,0)\right)=\left\{\left(\tfrac{u_2}{\langle 0.2,0.5,0.3\rangle}\right)\right\}.
\end{aligned}
$$

https://doi.org/10.54216/IJNS.230216

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

$\mathcal{G}\left(z_8=(m_1,n_2,0)\right)=\left\{\left(\frac{u_2}{\langle 0.9,0.1,0.2\rangle}\right)\right\}.$
$\mathcal{G}\left(z_9=(m_2,n_1,0)\right)=\left\{\left(\frac{u_2}{\langle 0.6,0.4,0.8\rangle}\right)\right\}.$
$\mathcal{G}\left(z_{10}=(m_2,n_2,0)\right)=\left\{\left(\frac{u_2}{\langle 0.6,0.4,0.7\rangle}\right)\right\}.$
$\mathcal{G}\left(z_{11}=(m_3,n_1,0)\right)=\left\{\left(\frac{u_2}{\langle 0.5,0.6,0.2\rangle}\right)\right\}.$
$\mathcal{G}\left(z_{12}=(m_3,n_2,0)\right)=\left\{\left(\frac{u_2}{\langle 0.7,0.4,0.2\rangle}\right)\right\}\Big\}$

$\mathcal{Y}_{\mathcal{P}}=$
$\Big\{\mathcal{Y}\left(z_1=(m_1,n_1,1)\right)=\left\{\left(\frac{u_3}{\langle 0.4,0.2,0.5\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_3}{\langle 0.2,0.3,0.3\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_3=(m_2,n_1,1)\right)=\left\{\left(\frac{u_3}{\langle 0.3,0.5,0.4\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_4=(m_2,n_2,1)\right)=\left\{\left(\frac{u_3}{\langle 0.1,0.3,0.6\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_5=(m_3,n_1,1)\right)=\left\{\left(\frac{u_3}{\langle 0.4,0.5,0.4\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_6=(m_3,n_2,1)\right)=\left\{\left(\frac{u_3}{\langle 0.6,0.5,0.6\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_7=(m_1,n_1,0)\right)=\left\{\left(\frac{u_3}{\langle 0.8,0.5,0.4\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_8=(m_1,n_2,0)\right)=\left\{\left(\frac{u_3}{\langle 0.2,0.1,0.9\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_9=(m_2,n_1,0)\right)=\left\{\left(\frac{u_3}{\langle 0.9,0.4,0.6\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_{10}=(m_2,n_2,0)\right)=\left\{\left(\frac{u_3}{\langle 0.3,0.4,0.3\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_{11}=(m_3,n_1,0)\right)=\left\{\left(\frac{u_3}{\langle 0.5,0.6,0.2\rangle}\right)\right\}.$
$\mathcal{Y}\left(z_{12}=(m_3,n_2,0)\right)=\left\{\left(\frac{u_3}{\langle 0.2,0.4,0.5\rangle}\right)\right\}\Big\}$

Step 2. Table 1 present the values that we got it in step 2.

Table 1: The values of $\ddot{\partial}^t(z_i)+\ddot{\partial}^i(z_i)-\ddot{\partial}^f(z_i)$

z_i	u_1	u_2	u_3
z_1	0.4	0.3	0.1
z_2	0.7	1.4	0.2
z_3	1.1	1	0. 4
z_4	0.6	0.5	-0.2
z_5	0.3	0	0.5
z_6	1	0.2	0. 5
z_7	1	0.4	0.9
z_8	0.8	0.8	-0.6
z_9	0.1	0.2	0.7
z_{10}	0	0.3	0.4
z_{11}	0.3	0.9	0.9
z_{12}	0.8	0.9	0.1

Step 7. The value of $s=1.3$, therefor the two couple will choose house u_1.

https://doi.org/10.54216/IJNS.230216

Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

Table 2: Numerical Grade for Agree-SENSES

z_i	u_i	**Highest Numeric Grade**
z_1	u_1	0.4
z_2	u_2	1.4
z_3	u_1	1.1
z_4	u_1	0.6
z_5	u_3	0.5
z_6	u_1	1
Scour (u_1)=0.4+1.1+0.6+1=3.1		
Scour (u_2)=1.4		
Scour (u_3)=0.5		

Table 3: Numerical Grade for Disagree-SENSES

z_i	u_i	**Highest Numeric Grade**
z_7	u_1	1
z_8	u_1,u_2	0.8
z_9	u_3	0.7
z_{10}	u_3	0.4
z_{11}	u_2,u_3	0.9
z_{12}	u_2	0.9
Scour (u_1)=1+0.8=1.8		
Scour (u_2)=0.8+0.9+0.9=2.6		
Scour (u_3)=0.7+0.4+0.9=2		

Table 4: $r_i = A_i - D_i$

A_i	D_i	r_i
3.1	1.8	1.3
1.4	2.6	-0.4
0.5	2	-1.5

https://doi.org/10.54216/IJNS.230216
Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023

7 Conclusion

n this work, the notion of a SVNFSES as combining the critical features of FSs and SVNSs under expert systems in one model or as an extension to SES is introduced. The basic properties of this model namely null, absolute, subset, equality, and complement are presented. Also, the basic set theory like union, intersection, OR, and AND operations as well as some properties on SVNFSESs are discussed. Finally, we presented a decision-making method based on SVNFSES and gave an application of this method to solve a decision-making problem. For future research work, users can combine these tools with other fuzzy algebraic tools see[27, 28]

References

[1] F. Smarandache, Neutrosophic probability, set, and logic, American Research Press: Rehoboth, IL, USA, 1998.

[2] L.A. Zadeh,Fuzzy sets, Information and Control, vol 8, pp. 338–353.1965.

[3] D. Molodtsov, Soft set theory, first results, Computers and Mathematics with Applications, vol. 37, no.4-5, pp. 19-31, 1999.

[4] Saqlain, M., Jafar, M. N., & Riaz, M. (2020). A new approach of neutrosophic soft set with generalized fuzzy TOPSIS in application of smart phone selection. Neutrosophic Sets and Systems, 32, 306.

[5] Z. bin M. Rodzi et al., "Integrated Single-Valued Neutrosophic Normalized Weighted Bonferroni Mean (SVNNWBM)-DEMATEL for Analyzing the Key Barriers to Halal Certification Adoption in Malaysia," Int. J. Neutrosophic Sci., vol. 21, no. 3, pp. 106–114, 2023.

[6] Deli, I. Interval-valued neutrosophic soft sets and its decision making. International Journal of Machine Learning and Cybernetics, 8, 2017.

[7] F. Al-Sharqi, A.G. Ahmad, A. Al-Quran, Interval complex neutrosophic soft relations and their application in decision-making, Journal of Intelligent and Fuzzy Systems, vol. 43 , pp. 745-771, 2022.

[8] Binu, R., & Isaac, P. (2019, September). Weighted Similarity Measure and Decision Making in Clinical Application of Neutrosophic Soft Set. In 2019 International Conference on Data Science and Engineering (ICDSE) (pp. 61-65). IEEE.

[9] Broumi, S., Sahin, R., & Smarandache, F. (2014). Generalized interval neutrosophic soft set and its decision making problem. Journal of new results in science, 3(7), 29-47.

[10] Quek, S. G., Garg, H., Selvachandran, G., Palanikumar, M., Arulmozhi, K., & Smarandache, F. (2023). VIKOR and TOPSIS framework with a truthful-distance measure for the (t, s)-regulated interval-valued neutrosophic soft set. Soft Computing, 1-27.

[11] Al-Hijjawi, S., & Alkhazaleh, S. (2023). A generalized effective neurosophic soft set and its applications. AIMS Mathematics, 18(12), 29628-29666.

[12] Al-Qudah, Y., Hassan, M., & Hassan, N. (2019). Fuzzy parameterized complex multi-fuzzy soft expert set theory and its application in decision-making. Symmetry, 11(3), 358.

[13] F. Al-Sharqi, A. Al-Quran, M. U. Romdhini, Decision-making techniques based on similarity measures of possibility interval fuzzy soft environment, Iraqi Journal for Computer Science and Mathematics, vol. 4, pp.18–29, 2023.

[14] Zail, S. H., Abed, M. M., Faisal, A. S. Neutrosophic BCK-algebra and Ω-BCK-algebra. International Journal of Neutrosophic Science, 19(3), 8-15, 2022.

[15] Alkhazaleh, S., & Salleh, A. R. (2011). Soft Expert Sets. Adv. Decis. Sci., 2011, 757868-1.

[16] F. Al-Sharqi, M. U. Romdhini, A. Al-Quran, Group decision-making based on aggregation operator and score function of Q-neutrosophic soft matrix, Journal of Intelligent and Fuzzy Systems, vol. 45, pp.305–321, 2023.

[17] A. Al-Quran, F. Al-Sharqi, K. Ullah, M. U. Romdhini, M. Balti and M. Alomai, Bipolar fuzzy hypersoft set and its application in decision making, International Journal of Neutrosophic Science, vol. 20, no. 4, pp. 65-77, 2023.

[18] Sahin, M., Alkhazaleh, S., & Ulucay, V. (2015). Neutrosophic soft expert sets. Applied mathematics, 6(1), 116.

[19] F. Al-Sharqi, A.G. Ahmad, A. Al-Quran, Interval-valued neutrosophic soft expert set from real space to complex space, Computer Modeling in Engineering and Sciences, vol. 132(1), pp. 267–293, 2022.

[20] Uluçay, V., Şahin, M., & Hassan, N. (2018). Generalized neutrosophic soft expert set for multiple-criteria decision-making. Symmetry, 10(10), 437.

[21] Uluçay, V., & Şahin, M. (2020). Decision-making method based on neutrosophic soft expert graphs. In Neutrosophic Graph Theory and Algorithms (pp. 33-76). IGI Global.

[22] F. Al-Sharqi, Y. Al-Qudah and N. Alotaibi, Decision-making techniques based on similarity measures of possibility neutrosophic soft expert sets. Neutrosophic Sets and Systems, 55(1) (2023), 358-382.

[23] Abu Qamar, M., Hassan, N. (2018). Generalized Q-neutrosophic soft expert set for decision under uncertainty. Symmetry, 10(11), 621.

[24] Alhazaymeh, K., Al-Qudah, Y., Hassan, N., Nasruddin, A. M. (2020). Cubic vague set and its application in decision making. Entropy, 22(9), 963.

[25] Jamiatun Nadwa Ismail et al. The Integrated Novel Framework: Linguistic Variables in Pythagorean Neutrosophic Set with DEMATEL for Enhanced Decision Support. Int. J. Neutrosophic Sci., vol. 21, no. 2, pp. 129-141, 2023.

[26] M Palanikumar, K Arulmozhi, A Iampan, S Broumi, Medical diagnosis decision making using type-II generalized Pythagorean neutrosophic interval valued soft sets.International Journal of Neutrosophic Science (IJNS) 20 (1), 2023.

[27] Y. Al-Qudah and F. Al-Sharqi, Algorithm for decision-making based on similarity measures ofpossibility interval-valued neutrosophic soft setting settings. Int. J. Neutrosophic Sci., vol. 22, no. 3, pp. 69-83, 2023.

[28] M. U. Romdhini, F. Al-Sharqi, A. Nawawi, A. Al-Quran and H. Rashmanlou, Signless Laplacian Energyof Interval-Valued Fuzzy Graph and its Applications, Sains Malaysiana 52(7), 2127-2137, 2023

https://doi.org/10.54216/IJNS.230216
Received: June 19, 2023 Revised: September 17, 2023 Accepted: December 23, 2023