Unveiling Neutrosophic Dimensions in the context of BF-algebras: Investigating Subalgebras, Ideals, and Homomorphisms

Satyanarayana B. [1], Rajani P. [2*], Ramesh D.[3], Ahmed Abdelhafeez[4,*], Abdelrhman Refaat[5], Khaled ELMenshawy[6,7]

[1]MathematicsDepartment, Acharya Nagarjuna University, Nagarjuna Nagar, India
[2]BS & H Department, SR Gudlavalleru Engineering College, Gudlavalleru, India
[3]Department of Engineering Mathematics, College of Engineering, Koneru Lakshmaih Educational Foundation, Vaddeswaram-522302, Guntur(DT), Andhra Pradesh, India
[4]Faculty of Information Systems and Computer Science, October 6th University, Giza, 12585, Egypt
[5]Faculty of Informatics, Midocean University by Everyone's Smart University, 3999, Fujairah, United Arab Emirates
[6]Faculty of Information Systems and Computer Science, October 6th University, Giza, 12585, Egypt;
[7]Faculty of Informatics, Midocean University by Everyone's Smart University, 3999, Fujairah, United Arab Emirates
Emails: drbsn63@yahoo.co.in; rajanipapers@gmail.com; ram.fuzzy@gmail.com; aahafeez.scis@o6u.edu.eg; abdelrhman.refaat@midocean.ae; Kmensh.csis@o6u.edu.eg; Khaled@midocean.ae
*Corresponding author: rajanipapers@gmail.com

Abstract

The foundational concepts of subalgebra, ideal, and homomorphism within the domain of BF-algebra were originally introduced by Andrzej Walendziak [A. Walendziak, On BF-Algebras, Math. Slovaca, 57(2) (2007), 119-128]. In this paper, we introduce innovative concepts related to Neutrosophic BF-Subalgebras and Neutrosophic BF-Ideals derived from the application of Subalgebra, Ideal, and Homomorphism principles to Neutrosophic sets. We explore the outcomes concerning a Neutrosophic BF-Ideal with respect to principle of homomorphism, homomorphic image of a Neutrosophic BF-Ideal satisfying the sup-inf property, and homomorphic pre-images within the context of a Neutrosophic set embedded in BF-algebra. The outcomes of the above study are equally applicable to Neutrosophic BF-Subalgebra Lastly, we delve into the conceptual understanding of a level set of a Neutrosophic BF-Ideal within a BF-algebra. In the future, the above study can be extended to address various types of implicative ideals and filters within BF-algebra. Moreover, these Neutrosophic BF-Subalgebras and Neutrosophic BF-Ideals can be applied to neutrosophic soft sets within the context of BF-algebra, which are used for various decision making techniques.

Keywords: BF-algebra; Ideals; Subalgebras; Homomorphism; Neutrosophic BF-Subalgebra; Neutrosophic BF-Ideal.

1. Introduction

Uncertainty is a pervasive characteristic encountered in numerous intricate systems and practical contexts, spanning a wide array of disciplines, including behavioral, biological, and chemical studies. In 1965, Lotfi A.Zadeh Pioneered the notion of theory of fuzzy sets to grapple with these uncertainties in real-world applications [34], represented as 'f,' A fuzzy set assigns values within the range [0,1] to elements from a non-empty set S and its adaptability renders it valuable across various engineering fields.

In 1990, F.Smarandache expanded the landscape of uncertainty modeling by introducing Neutrosophic logic and set theory, which evolved from intuitionistic fuzzy sets, paraconsistent sets, and intuitionistic sets [26,27]. Atanassov

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[2] emphasized the importance of the Level of inaccuracy or deviation from the truth(f) and provided insights into Sets in the context of intuitionistic fuzzy logic. Smarandache coined the term "Neutrosophic", representing a philosophy of neutral thought. The primary demarcation between 'fuzzy logic and sets ' or ‘Intuition-based fuzzy logic and sets and logic and sets based on the concept of ‘Neutrosophy’ hinges on the inclusion of a separate component representing the degree of uncertainty or impartiality. The distinctive attribute of the Neutrosophic set comprises three constituent elements (t, i, f) = (Veracity, uncertainity, non-Truthhood) and boasts manifold real-life applications across various disciplines [1,3,4,5,6,11,12,15,16,21].

Y. B. Jun, E. H. Roh, and H. S. Kim broadened the horizon by introducing BH-algebras in [9], encompassing BCK/BCI/B-algebras. In 2002, Neggers and Kim [17] ushered in a novel concept, B-algebra, as a broader concept encompassing BCK-algebras, yielding a plethora of results. In 2005, Wang et al. [32,33] Pioneered the idea concerning intervals in Neutrosophy sets, offering greater flexibility in comparison to Neutrosophic sets that have single values. Subsequently, Khan et al. [14] proposed the concept of Neutrosophic N-structures and their practical utilization within the context of semigroups in 2017 and finds utility in a multitude of logical algebraic context [10,18,23,28]. Walendziak [31] made further contributions to this realm by introducing BF-algebras, a generic form of B-Algebra, and exploring various characteristics, including ideals and normal ideals in BF-algebras.
Algebraic structures occupy a foundational role with extensive applications in diverse domains, providing a mathematical framework for modeling various phenomena. The integration of Neutrosophic sets has discovered applications in numerous logical algebras and practical scenarios [7,13,19,20,29].
The foundational concepts of subalgebras, ideals, and homomorphisms were introduced by Andrzej Walendziak [31].

- This paper introduces novel concepts within the framework of Neutrosophic sets (ƝS) in BF-algebras: Neutrosophic BF–subalgebra (ƝSA), Neutrosophic BF-Ideal (Ɲi), and Homomorphism principles to ƝS.
- We establish that a Ɲi with respect to a homomorphism is a Ɲi, a homomorphic image of a Ɲi satisfying the sup-inf property is a Ɲi, homomorphic pre-images of a ƝS within the context of BF-algebra is a ƝS, while examining a range of associated attributes. The results of these studies hold good for ƝSA also. And we delve into the concept of level sets within a ƝS in BF-algebras, analyzing its attributes.

The table provided showcases a collection of acronyms.

ƝS	Neutrosophic set
ƝSA	Neutrosophic BF–subalgebra
Ɲi	Neutrosophic BF-Ideal

2.Major contributions of the work

- In Section 3, a literature review of a few definitions with examples regarding subalgebra and ideal of BF-algebras is provided.
- In Section 4, we explored the idea of a ƝSA and Ɲi of BF-algebra are introduced and illustrated with examples .
- In Section 5, we discussed the relationships of **ƝSA** and **Ɲi** of BF-lgebra under homomorphism principles.
- In Section 6, we discussed the charateristcs of a level set of a ƝS within BF-algebra**.**

3. Preliminaries

Definition 1[31]: A BF-algebra is a structure $S:=(S\neq \emptyset, ϫ, 0)$ satisfying:

(I) $p ϫ p=0$, (1)
(II) $p ϫ 0=p$, (2)
(III) $0 ϫ (p ϫ q) = q ϫ p$, $\forall p,q\in S$ (3)

Example[31]: Let S = {0, 1, 2, 3} with the composition table

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ϰ	0	1	2	3
0	0	1	2	3
1	1	0	3	0
2	2	3	0	2
3	3	0	2	0

is a BF-algebra.

Definition 2[31]: Consider a BF-algebra S: =(S≠Ø,ϰ,0). Let M(≠Ø) contained within S is defined as a Subalgebra if ∀*p*, *q*∈M, *p*ϰ*q*∈M (4)

Example[31] : Let S = {0, 1, 2, 3} with the composition table

ϰ	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	1
3	3	1	1	0

is a BF-algebra then M = {0, 1} is a subalgebra within S.

Definition3[31]: Consider a BF-algebra S:=(S≠Ø,ϰ,0). LetM(≠Ø) contained within S is defined as an Ideal if

0∈ M (5)
∀*p*, *q*∈S, pϰ*q*∈M, *q* ∈M⇒*p* ∈ M (6)

Example[31]:M= {0,2,3} qualifies as an ideal in the framework of Definition 2, pertaining to a BF-algebra.

Definition 4[8,22,31]: A mapping *f* from BF-algebras S:=(S≠Ø,ϰ,0) to S′:=(S′≠Ø,Δ,0′) is a homomorphism if f(pϰq) = f(p) Δ f(q), ∀ p,q∈S (7)

Example: LetS = (R, ϰ, 0) where ϰ is given byp ϰ q $= \begin{cases} p, & if\ q = 0 \\ q, & if\ p = 0 \\ 0, & otherwise \end{cases}$

where (R, ϰ, 0) is the set of real numbers is a BF-algebra[**1**] and (S′={0′,1′,2′},Δ,0′) is a BF-algebra[**31**] is defined as

Δ	0′	1′	2′
0′	0′	1′	2′
1′	1′	0′	0′
2′	2′	0′	0′

A mapping f:R→ S′as f(R-{0})=1′, f(0)=0′ is a homomorphism.
From here S:=(S≠Ø,ϰ,0) and S′:=(S′≠Ø,Δ,0′) are considered as BF-algebras

Note[22]: If *f* is a homomorphism from S to S′ then *f*(0) = 0′. (8)

Definition 5[8]: A mapping *f* from S to S′ is said to be an epimorphism if i) *f*(pϰq) = *f*(p) Δ *f*(q),∀ p, q∈S

and ii)f is onto (9)

4. Neutrosophic concept on BF-algebra [24,25,29,30]

Let γ(S, [0,1]) be the family of mappings from a set S to [0,1]. A NS over a universe S ≠Ø is

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$$S_{\aleph} = \frac{S}{(\rho_{\aleph}, I_{\aleph}, \aleph_{\aleph})} = \left\{\frac{p}{(\rho_{\aleph}(p), I_{\aleph}(p), \aleph_{\aleph}(p))} / p \in S\right\}$$

Where $\rho_{\aleph}$, $I_{\aleph}$ and $\aleph_{\aleph}$ are mappings on S which are truthfulness membership mapping, the unfixed membership mapping and fallaciousness membership mapping respectively on S.

A ƝS $S_{\aleph}$ over S holds if
$\forall p \in S$, $0 \leq \rho_{\aleph}(p) + I_{\aleph}(p) + \aleph_{\aleph}(p) \leq 1$

Note: From here , $\wedge\{p_1, p_2\} = \min\{p_1, p_2\}$ and $\vee\{p_1, p_2\} = \max\{p_1, p_2\}$.

Definition 6: AƝS $S_{\aleph}$ within S is called a ƝSA if

i) $\rho_{\aleph}(p \ast q) \geq \wedge\{\rho_{\aleph}(p), \rho_{\aleph}(q)\}$ (10)

ii) $I_{\aleph}(p \ast q) \geq \wedge\{I_{\aleph}(p), I_{\aleph}(q)\}$ (11)

iii) $\aleph_{\aleph}(p \ast q) \leq \vee\{\aleph_{\aleph}(p), \aleph_{\aleph}(q)\}$, $\forall p,q \in S$ (12)

Example: The following ƝS is defined within the context of BF-algebra example 3.13[**31**].

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$S_{\aleph}$	0	1	2
$\rho_{\aleph}$	0.453	0.177	0.081
$I_{\aleph}$	0.459	0.295	0.155
$\aleph_{\aleph}$	0.088	0.528	0.764

Definition7: A ƝS $S_{\aleph} = (\rho_{\aleph}, I_{\aleph}, \aleph_{\aleph})$within S is a Ɲi if

(1) $\rho_{\aleph}(0) \geq \rho_{\aleph}(p)$, $I_{\aleph}(0) \geq I_{\aleph}(p)$ and $\aleph_{\aleph}(0) \leq \aleph_{\aleph}(p)$,$\forall p \in S$ (13)

(2) $\rho_{\aleph}(p) \geq \wedge\{\rho_{\aleph}(p \ast q), \rho_{\aleph}(q)\}$ (14)

(3) $I_{\aleph}(p) \geq \wedge\{I_{\aleph}(p \ast q), I_{\aleph}(q)\}$ (15)

(4) $\aleph_{\aleph}(p) \leq \vee\{\aleph_{\aleph}(p \ast q), \aleph_{\aleph}(q)\}$, $\forall$ p, q $\in$ S (16)

Example: The following ƝS is defined within the context of BF-algebra example 3.2[**31**].

$S_{\aleph}$	0	1	2	3
$\rho_{\aleph}$	0.4	0.4	0.1	0.1
$I_{\aleph}$	0.4	0.4	0.1	0.1
$\aleph_{\aleph}$	0.2	0.2	0.3	0.3

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5.Applications of Homorphism principles on a ƝS

Definition8: Let ⱳ be a homomorphism from S to S′.For any ƝS, $S'_{\aleph}$=($\rho'_{\aleph}, I'_{\aleph}, \aleph'_{\aleph}$)in S',We define a new ƝS

$S_{\aleph}^{ⱳ} = (\rho_{\aleph}^{ⱳ}, I_{\aleph}^{ⱳ}, \mathbb{N}_{\aleph}^{ⱳ})$ within S as

$\rho_{\aleph}^{ⱳ}(p) = \rho'_{\aleph}(ⱳ(p))$ (17)

$I_{\aleph}^{ⱳ}(p) = I'_{\aleph}(ⱳ(p))$ (18)

$\aleph_{\aleph}^{ⱳ}(p) = \aleph'_{\aleph}(ⱳ(p))$ (19) $\forall\, p \in S$

Theorem 1: Let ⱳ be an epimorphism from S to S′.

a) If $S'_{\aleph}$=($\rho'_{\aleph}, I'_{\aleph}, \aleph'_{\aleph}$) represents a Ɲi within S′ then
$S_{\aleph}^{ⱳ} = (\rho_{\aleph}^{ⱳ}, I_{\aleph}^{ⱳ}, \aleph_{Ɲ}^{ⱳ})$ similarly characterizes as a Ɲi within S.
b) If $S_{\aleph}^{ⱳ} = (\rho_{\aleph}^{ⱳ}, I_{\aleph}^{ⱳ}, \aleph_{Ɲ}^{ⱳ})$ represents a Ɲi within S then
$S'_{\aleph}$=($\rho'_{\aleph}, I'_{\aleph}, \aleph'_{\aleph}$) similarly characterizes as a Ɲi within S′.

Proof: Let ⱳ be an epimorphism from S to S′

a) Suppose that $S'_{\aleph}$=($\rho'_{\aleph}, I'_{\aleph}, \aleph'_{\aleph}$)is Ɲi within S′

Also since ⱳ: S→ S' is onto

For any $p' \in S'$∃at least one p∈ S ∋ ⱳ(p) = p'

Let's Consider $\rho_{\aleph}^{ⱳ}(0) = \rho'_{\aleph}(ⱳ(0)) = \rho'_{\aleph}(0') \geq \rho'_{\aleph}(p') \geq \rho'_{\aleph}(ⱳ(\mathrm{p})) \geq \rho_{\aleph}^{ⱳ}(p)$by (17&8&13)

In simpler terms, $\rho_{\aleph}^{ⱳ}(0) \geq \rho_{\aleph}^{ⱳ}(p) \cdot \forall\, p \in S$

Let's Consider $I_{\aleph}^{ⱳ}(0) = I'_{\aleph}(ⱳ(0)) = I'_{\aleph}(0') \geq I'_{\aleph}(p') \geq I'_{\aleph}(ⱳ(p)) \geq I_{\aleph}^{ⱳ}(p)$by (18&8&13)

In simpler terms, $I_{\aleph}^{ⱳ}(0) \geq I_{\aleph}^{ⱳ}(p), \forall\, p \in S$

Let's Consider $\aleph_{\aleph}^{ⱳ}(0) = \aleph'_{\aleph}(ⱳ(0)) = \aleph'_{\aleph}(0') \leq \aleph'_{\aleph}(p') \leq \aleph'_{\aleph}(ⱳ(p)) \leq \aleph_{\aleph}^{ⱳ}(p)$by (19&8&13)

In simpler terms, $\aleph_{\aleph}^{ⱳ}(0) \leq \aleph_{\aleph}^{ⱳ}(p), \forall\, p \in S$

Let p∈ S *and* $q' \in S'$, *since* ⱳ *is onto then* ∃ *atleast one* $q \in S \ni ⱳ(q) = q'$and

$(i)\rho_{\aleph}^{ⱳ}(p) = \rho'_{\aleph}(ⱳ(p)) \geq \wedge \{\rho'_{\aleph}(ⱳ(p)\Delta q'), \rho'_{\aleph}(q')\}$ by(17 & 14)

$\geq \wedge \{\rho'_{\aleph}(ⱳ(p)\Delta ⱳ(q)), \rho'_{\aleph}(q')\}$

$\geq \wedge \{\rho'_{\aleph}(ⱳ((p \ast \mathrm{q})), \rho'_{\aleph}(ⱳ(q))\}$ by (7)

$\therefore \rho_{\aleph}^{ⱳ}(p) \geq \wedge \{\rho_{\aleph}^{ⱳ}(\mathrm{p} \ast \mathrm{q}), \rho_{\aleph}^{ⱳ}(\mathrm{q})\}, \forall\, \mathrm{p}, \mathrm{q} \in \mathrm{S}$ by (17)

$(ii)I_{\aleph}^{ⱳ}(\mathrm{p}) = I'_{\aleph}(ⱳ(\mathrm{p})) \geq \wedge \{I'_{\aleph}(ⱳ(\mathrm{p})\Delta \mathrm{q}'), I'_{\aleph}(\mathrm{q}')\}$ by (18 & 15)

$\geq \wedge \{I'_{\aleph}(ⱳ(\mathrm{p})\Delta ⱳ(\mathrm{q})), I'_{\aleph}(ⱳ(\mathrm{q}))\}$

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$\geq \wedge\{I'_{\aleph}(\omega(p ⵅ q)), I'_{\aleph}(\omega(q))\}$ by(7)

$\therefore I^{\omega}_{\aleph}(p) \geq \wedge\{I^{\omega}_{\aleph}(p ⵅ q), I^{\omega}_{\aleph}(q)\} \, \forall\, p, q \in S$by(18)

$(iii) \aleph^{\omega}_{\aleph}(p) = \aleph'_{\aleph}(\omega(p)) \leq \vee\{\aleph'_{\aleph}(\omega(p)\Delta q'), \aleph'_{\aleph}(q')\}$ by(19 & 16)

$\leq \vee\{\aleph'_{\aleph}(\omega(p)\Delta\omega(q)), \aleph'_{\aleph}(\omega(q))\}$

$\leq \vee\{\aleph'_{\aleph}(\omega(p ⵅ q)), \aleph'_{\aleph}(\omega(q))\}$ by(7)

$\therefore \aleph^{\omega}_{\aleph}(p) \leq \vee\{\aleph^{\omega}_{\aleph}(p ⵅ q), \aleph^{\omega}_{\aleph}(q)\} \, \forall\, p, q \in S$

Hence $S^{\omega}_{\aleph}$ is a Ɲi within S.

b) Suppose that $S^{\omega}_{\aleph}$ is a Ɲi within S,since ω is onto

$\forall p', q' \in S' \exists$ at least one p, q$\in S$ $be \ni \omega(p) = p'$ and $\omega(q) = q'$

Let us consider $\rho'_{\aleph}(\omega(0)) = \rho^{\omega}_{\aleph}(0) \geq \rho^{\omega}_{\aleph}(p) \geq \rho'_{\aleph}(\omega(p)) \forall\, p \in S$ by(17 & 13)

$I'_{\aleph}(\omega(0)) = I^{\omega}_{\aleph}(0) \geq I^{\omega}_{\aleph}(p) \geq I'_{\aleph}(\omega(p)) \forall\, p \in S$ by(18 & 13)

$\mathbb{N}'_{\aleph}(\omega(0)) = \aleph^{\omega}_{\aleph}(0) \leq \aleph^{\omega}_{\aleph}(p) \leq \aleph'_{\aleph}(\omega(p)) \forall\, p \in S$ by(19& 13)

and (i)$\rho'_{\aleph}(p') = \rho'_{\aleph}(\omega(p)) = \rho^{\omega}_{\aleph}(p) \geq \wedge\{\rho^{\omega}_{\aleph}(p ⵅ q), \rho^{\omega}_{\aleph}(q)\} \geq \wedge\{\rho'_{\aleph}(\omega(p ⵅ q)), \rho'_{\aleph}(\omega(q))\}$ by(17 & 14)

$\geq \wedge\{\rho'_{\aleph}(\omega(p)\Delta\omega(q)), \rho'_{\aleph}(\omega(q))\}$ by(7)

$\therefore \rho'_{\aleph}(p') \geq \wedge\{\rho'_{\aleph}(p'\Delta q'), \rho'_{\aleph}(q')\} \, \forall\, p', q' \in S'$

(ii)$I'_{\aleph}(p') = I'_{\aleph}(\omega(p)) = I^{\omega}_{\aleph}(p) \geq \wedge\{I^{\omega}_{\aleph}(p ⵅ q), I^{\omega}_{\aleph}(q)\} \geq \wedge\{I'_{\aleph}(\omega(p ⵅ q)), I'_{\aleph}(\omega(q))\}$ by(18& 15)

$\geq \wedge\{I'_{\aleph}(\omega(p)\Delta\omega(q)), I'_{\aleph}(\omega(q))\}$ by(7)

$\therefore I'_{\aleph}(p') \geq \wedge\{I'_{\aleph}(p'\Delta q'), I'_{\aleph}(q')\} \, \forall\, p', q' \in S'$

and (iii)$\aleph'_{\aleph}(p') = \aleph'_{\aleph}(\omega(p)) = \aleph^{\omega}_{\aleph}(p) \leq \vee\{\aleph^{\omega}_{\aleph}(p ⵅ q), \aleph^{\omega}_{\aleph}(q)\} \leq \vee\{\aleph'_{\aleph}(\omega(p ⵅ q)), \aleph'_{\aleph}(\omega(q))\}$ by(19& 16)

$\leq \vee\{\aleph'_{\aleph}(\omega(p)\Delta\omega(q)), \aleph'_{\aleph}(\omega(q))\}$ by(7)

$\therefore \aleph'_{\aleph}(p') \leq \vee\{\aleph'_{\aleph}(p'\Delta q'), \aleph'_{\aleph}(q)\} \, \forall\, p', q' \in S'$

Hence $S'_{\aleph}$ is a Ɲi within S′.

Proposition1: Every Ɲi over S is a ƝSA over S.

Proof: The proof follows a direct path by using (1), (2) and definition (7)

The subsequent illustration involving $S_{\aleph}$ demonstrates that the opposite of Proposition 1 might not be valid within the framework of BF-algebra as in Definition 1.

$S_{\aleph}$	0	1	2	3
$\rho_{\aleph}$	0.1	0	0.1	0.1

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$I_ℵ$	0.1	0.1	0.1	0.1
$ℵ_N$	0	0.1	0	0

$$\rho_ℵ(p) = \rho_ℵ(1) = 0 \ngeq \wedge\{\rho_ℵ(p \text{x} q) = \rho_ℵ(1\text{x}2) = \rho_ℵ(3) = 0.1, \rho_ℵ(q) = \rho_ℵ(2) = 0.1\}$$

Theorem 2: Let ω be an epimorphism from S to $S.'$

a) If $S'_ℵ$=($\rho'_ℵ, I'_ℵ, ℵ'_ℵ$) represents a NSA within S' then

$S^{ω}_ℵ = (\rho^{ω}_ℵ, I^{ω}_ℵ, ℵ^{ω}_N)$ similarly characterizes as a NSA within S .

b)If $S^{ω}_ℵ = (\rho^{ω}_ℵ, I^{ω}_ℵ, ℵ^{ω}_N)$ represents a NSA within S then

$S'_ℵ$=($\rho'_ℵ, I'_ℵ, ℵ'_ℵ$)similarly characterizes as a NSA within S'.

Proof: The proof follows a direct path, as indicated by Proposition 1 and theorem 1.

Definition 9: Let ω be a mapping on a set S and $S_ℵ = (\rho_ℵ, I_ℵ, ℵ_ℵ)$ be a NS within S then $\rho'_ℵ, I'_ℵ, ℵ'_ℵ$ on ω$(S)$is defined by $\rho'_ℵ(q)$= $Sup\rho_ℵ(p), p \in ω^{-1}(q)$

$I'_ℵ(q) = SupI_ℵ(p), p \in ω^{-1}(q)$

$ℵ'_ℵ(q) = inf ℵ_ℵ(p), p \in ω^{-I}(q), \forall q \in ω(S)$ is called the image of $S_ℵ$ $under$ ω.

If $\rho_ℵ, I_ℵ, ℵ_ℵ$ are fuzzy sets in ω (S), then the fuzzy set $\rho_ℵ = \rho'_ℵ\ oω$, $I_ℵ = I'_ℵ oω$ and $ℵ_ℵ = ℵ'_ℵ\ oω$ is the Pre-image of $\rho'_ℵ, I'_ℵ$ and $ℵ'_ℵ$ respectively covered by ω.

Example: Let's examine a BF-algebra S:=(S={0_s,1_s,2_s},Δ,0_s) that possesses the subsequent table..

Δ	0_s	1_s	2_s
0_s	0_s	1_s	2_s
1_s	1_s	0_s	0_s
2_s	2_s	0_s	0_s

and BF-algebra S′:=(S′={0_S′,1_S′,2_S′},Δ,0_S′)that possesses the subsequent table..

Δ	0_S′	1_S′	2_s
0_S′	0_S′	1_S′	2_S′
1_S′	1_S′	0_S′	0_S′
2_S′	2_S′	0_S′	0_S′

Define ω: S→ S'as ω(0_s)=0_S′, ω(1_s)=1_S′, ω(2_s)=1_S′

Define a NS $S'_ℵ$=($\rho'_ℵ, I'_ℵ, ℵ'_ℵ$) within S' as

$S'_ℵ$	0_S′	1_S′	2_S′

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$\rho'_{\aleph}$	0.1	0.4	0.3
$I'_{\aleph}$	0.6	0.1	0.1
$\aleph'_{\aleph}$	0.3	0.5	0.4

then $\omega^{-1}(S'_{\aleph})$ is a ℕS in S given by

$S_{\aleph}$	0_s	1_s	2_s
$\rho_{\aleph}$	0.1	0.4	0.4
$I_{\aleph}$	0.6	0.1	0.1
$\aleph_{\aleph}$	0.3	0.5	0.5

Definition10:A ℕS $S_{\aleph} = (\rho_{\aleph}, I_{\aleph}, \aleph_{\aleph})$ within S exhibits the 'sup-inf' property when, for any T subset of S, the following condition holds:$\exists p_o \in T \ni \rho_{\aleph}(p_o) = Sup\rho_{\aleph}(t), t \in T, I_{ℕ}(p_o) = SupI_{\aleph}(t), t \in T$ and

$$\aleph_{\aleph}(p_o) = inf\ \aleph_{\aleph}(t), t \in T$$

Theorem3: Let $\omega: S \to S'$ be an epimorphism.

If ℕS $S_{\aleph} = (\rho_{\aleph}, I_{\aleph}, \aleph_{\aleph})$ is a ℕi within S that possesses the 'Sup-inf' property, then the image of $S_{ℕ}$under mapping ω also constitutes a ℕi within S'.

Proof: Let $\omega: S \to S'$ be an epimorphism

Suppose that $S_{\aleph} = (\rho_{\aleph}, I_{\aleph}, \aleph_{\aleph})$ is a ℕi within S that possesses the "Sup-inf" property

for any p$\in S, \rho_{\aleph}(0) \geq \rho_{\aleph}(p)$

$I_{\aleph}(0) \geq I_{\aleph}(p)$

$\aleph_{\aleph}(0) \leq \aleph_{\aleph}(p)$

The image of $S_{ℕ}$ covered by ω is denoted as $S'_{\aleph}$=($\rho'_{\aleph}, I'_{\aleph}, \aleph'_{\aleph}$) and is defined as

$\rho'_{\aleph} : S' \to [0,1]$ by $\rho'_{\aleph}(q') = Sup\ \rho_{\aleph}(p), p \in \omega^{-1}(q') \forall q' \in S'$

$I'_{\aleph} : S' \to [0,1]$ by $I'_{\aleph}(q') = Sup\ I_{\aleph}(p), p \in \omega^{-1}(q') \forall q' \in S'$

$\aleph'_{\aleph} : S' \to [0,1]$ by $\aleph'_{\aleph}(q') = inf\ \aleph_{\aleph}(p), p \in \omega^{-1}(q') \forall q' \in S'$

Since $S_{ℕ}$ is a ℕi within S

Thus $\rho'_{\aleph}(0') = Sup\ \rho_{\aleph}(p) = \rho_{\aleph}(0) \geq \rho_{\aleph}(p), p \in \omega^{-1}(0') \forall p \in S$

$$\Rightarrow \rho'_{\aleph}(0') \geq \rho_{\aleph}(p), \forall p \in S$$

In addition, $\rho'_{\aleph}(p') = Sup\ \rho_{\aleph}(p), p \in \omega^{-1}(p') \forall p' \in S'$

Hence $\rho'_{\aleph}(0') \geq Sup\ \rho_{\aleph}(p) = \rho'_{\aleph}(p'), p \in \omega^{-1}(p')$

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$\Rightarrow \rho'_{\aleph}(0') \geq \rho'_{\aleph}(p'), \forall\, p' \in S'$

$I'_{\aleph}(0') = Sup\, I_{\aleph}(p) = I_{\aleph}(0) \geq I_{\aleph}(p),\, p \in\ \omega^{-1}(0')\ \forall p \in S$

$\Rightarrow I'_{\aleph}(0') \geq I_{\aleph}(p), \forall\, p \in S$

In addition, $I'_{\aleph}(p') = Sup\, I_{N}(p), p \in \omega^{-1}(p') \forall\, p' \in S'$

Hence $I'_{\aleph}(0') \geq\ Sup\, I_{N}(p) = I'_{\aleph}(p'), p \in \omega^{-1}(p')$

$\Rightarrow I'_{\aleph}(0') \geq I'_{\aleph}(p'), \forall\, p' \in S'$

and $\aleph'_{\aleph}(0') = Inf\, \aleph_{\aleph}(p) = \aleph_{\aleph}(0) \leq \aleph_{\aleph}(p),\, p \in\ \omega^{-1}(0')\ \forall p \in S$

$\Rightarrow \aleph'_{\aleph}(0') \leq \aleph_{\aleph}(p), \forall p \in$S

In addition, $\mathbb{N}'_{N}(p') = Inf\ \aleph_{\aleph}(p), p \in \omega^{-1}(p') \forall\, p' \in S'$

Hence $\aleph'_{\aleph}(0') \leq\ Inf\ \aleph_{\aleph}(p) = \aleph'_{\aleph}(p'), p \in \omega^{-1}(p')$

$\Rightarrow\ \aleph'_{\aleph}(0') \leq\ \aleph'_{\aleph}(p'), \forall\, p' \in S'$

Since ω is onto, $\forall\, p', q', \in S' \exists$ at least one r, $q \in S \ni \omega(r) = p', \omega(q) =\ q'$

Hence $\rho'_{\aleph}(p') = \rho'_{\aleph}(\omega(p))$

$= Sup\ \rho_{\aleph}(t), t \in \omega^{-1}(\omega(p))$

$= \rho_{\aleph}(r)$

$\geq \wedge\{\rho_{\aleph}(r \times q), \rho_{\aleph}(q)\} \geq \wedge\{\rho'_{\aleph}(h(r \times q)), \rho'_{\aleph}(h(q))\}$

$\geq \wedge\{\rho'_{\aleph}(\omega(r)\Delta\omega(q)), \rho'_{\aleph}(\omega(q))\}$

$\therefore \rho'_{\aleph}(p') \geq \wedge\{\rho'_{\aleph}(p'\Delta q'), \rho'_{\aleph}(q')\} \forall p', q', \in S'$

$I'_{\aleph}(p') =\ I'_{\aleph}(\omega(p))$

$= Sup\ I_{\aleph}(t), t \in \omega^{-1}(\omega(p))$

$= I_{\aleph}(r)$

$\geq \wedge\{I_{\aleph}(r \times q), I_{\aleph}(q)\}$

$\geq \wedge\{I'_{\aleph}(\omega(r \times q)), I'_{\aleph}(\omega(q))\}$

$\geq \wedge\{I'_{\aleph}(\omega(r)\Delta\omega(q)), I'_{\aleph}(\omega(q))\}$

$\therefore I'_{\aleph}(p') \geq \wedge\{I'_{\aleph}(p'\Delta\, q'), I'_{\aleph}(q')\} \forall p', q', \in S'$

$\aleph'_{\aleph}(p') =\ \aleph'_{\aleph}(\omega(p))$

$= Sup\ \aleph_{\aleph}(t), t \in \omega^{-1}(\omega(p))$

$= \aleph_{\aleph}(r)$

$\leq \vee\{\aleph_{\aleph}(r \times q), \aleph_{\aleph}(q)\}$

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$\leq \vee\{\aleph'_{\aleph}(\omega(r \times q)), \aleph'_{\aleph}(\omega(q))\}$

$\leq \vee\{\aleph'_{\aleph}(\omega(r)\Delta\omega(q)), \aleph'_{\aleph}(\omega(q))\}$

$\therefore \aleph'_{\aleph}(p') \leq \vee\{\aleph'_{\aleph}(p'\Delta q'), \aleph'_{\aleph}(q')\}\forall p', q', \in S'$

Hence image of $S_{\aleph}$ under ω is a Ɲi within S'.

Theorem 4: Let $\omega: S \rightarrow S'$ be an epimorphism.

If ƝS $S_{\aleph} = (\rho_{\aleph}, I_{\aleph}, \aleph_{\aleph})$is a ƝSA within S that possesses the 'Sup-inf' property, then the image of $S_{\aleph}$ under mapping ω also constitutes a ƝSA within S'.

Proof: The proof follows a direct path, as indicated by Proposition 1 and theorem 3.

Definition11:(11.1)Let ω:S→S′ be a homomorphism and $S_{\aleph}$ be a ƝS within S then the homomorphic image of $S_{\aleph}$ covered by ω is denoted by $\omega(S_{\aleph})$ is a ƝS within S′ defined by ω $(S_{\aleph})$={(p′, $\omega(\rho_{\aleph})$(p′), $\omega(I_{\aleph})$(p′), $\omega(\aleph_{\aleph})$(p′))/p′∈S′}

where $\omega(\rho_{\aleph})(p') = \begin{cases} \sup\{(\rho_{\aleph})(p)/p \in \omega^{-1}(p')\}, & if\ \omega^{-1}(p') \neq \emptyset \\ 0, & otherwise \end{cases}$ (20)

$\omega(I_{\aleph})(p') = \begin{cases} \sup\{(I_{\aleph})(p)/p \in \omega(p')\}, & if\ \omega^{-1}(p') \neq \emptyset \\ 0, & otherwise \end{cases}$ (21)

and $\omega(\aleph_{\aleph})(p') = \begin{cases} \inf\{(\aleph_{\aleph})(p)/p \in \omega^{-1}(p')\}, & if\ \omega^{-1}(p') \neq \emptyset \\ 1, & otherwise \end{cases}$ (22)

(11.2) If $S'_{\aleph}$ is a ƝS within S′ then the homomorphic pre-image of $S'_{\aleph}$under the mapping ω forms a ƝS within S defined as by $\omega^{-1}(S'_{\aleph})$(p)= {(p, $\omega^{-1}(\rho'_{\aleph})$(p), $\omega^{-1}(I'_{\aleph})$(p), $\omega^{-1}(\aleph'_{\aleph})$(p))/ p ∈S}which can be expressed as{ (p, $\rho'_{\aleph}$ (ω(p)), $I'_{\aleph}$(ω(p)), $\aleph'_{\aleph}$(ω(p)) /p∈S }.

Theorem 5 : Let ω be a homomorphism from S to S′. If $S_{\aleph}$ be a ƝS within S and $S'_{\aleph}$ be a ƝS within S′ then

1) ω $(\omega^{-1}(S'_{\aleph})) = S'_{\aleph}$ and 2) $\omega^{-1}(\omega(S_{\aleph})) = S_{\aleph}$.

Proof: Let ω be a homomorphism from S to S′.

1)Let's examine the equation $\omega(\omega^{-1}(\rho'_{\aleph})(p'))$=sup{ $\omega^{-1}(\rho'_{\aleph})$(p)/p∈ ω^{-1}(p′)}as given by equation (20). This equation simplifies to sup{$\rho'_{\aleph}$(ω(p)/p∈S and ω(p)= p′}, which ultimately equals $\rho'_{\aleph}$ (p′), ∀p′∈S′. Similarly for $\omega(\omega^{-1}(I'_{\aleph})(p'))$= $I'_{\aleph}$(p′), ∀p′∈S′by equation (21) and also examine the equation $\omega(\omega^{-1}(\aleph'_{\aleph})(p'))$ = inf{ $\omega^{-1}(\aleph'_{\aleph})$(p) / p∈ ω^{-1}(p′)} as given by equation(22) which simplifies to inf{$\aleph'_{\aleph}$(ω(p)) / p∈ S and ω(p)= p′}= $\aleph'_{\aleph}$(p′), ∀p′∈S′. Hence (1) proved.

2) Let's consider $\omega^{-1}(\omega(\rho_{\aleph}))$(p) = $\omega(\rho_{\aleph})h$(p)= sup{$\rho_{\aleph}$(p)/p∈ ω^{-1}(p′)} =$\rho_{\aleph}$(p)

$\omega^{-1}(\omega(I_{\aleph}))$(p) = $\omega(I_{\aleph})$ ω(p) = sup{$I_{\aleph}$(p)/p∈ ω^{-1}(p′)}=$I_{\aleph}$(p)

$\omega^{-1}(\omega(\aleph_{\aleph}))$(p) = $\omega(\aleph_{\aleph})\omega(p)$= inf{$\aleph_{\aleph}$(p) / p∈$h^{-1}$(p′)}=$\aleph_{\aleph}$(p). Hence, (2) proved.

Theorem6 : Let ω be an epimorphism from S to S′ then $S'_{\aleph}$ is a Ɲi within S′ iff the pre-image $\omega^{-1}(S'_{\aleph})$ of $S'_{\aleph}$ is a Ɲi within S.

Proof: Let ω be an epimorphism from S to S′

Suppose that S'_N is a Ɲi within S′

Since ω is onto,∀p', q'∈S′, ∃ atleast one p, q ∈ S be ∋ ω(p)= p' and ω(q)=q'

Let's consider $\omega^{-1}(\rho'_{\aleph})(0)$= $\rho'_{\aleph}$(ω(0))= $\rho'_{\aleph}$(0′)≥ $\rho'_{\aleph}$ (p′) ≥ $\rho'_{\aleph}$ (ω(p)) by(8 & 13)

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In simpler terms, $\omega^{-1}(\rho'_{\aleph})(0) \geq \omega^{-1}(\rho'_{\aleph})(p)\ \forall p \in S$

Let's Consider $\omega^{-1}(I'_{\aleph})(0) = I'_{\aleph}(\omega(0)) = I'_{\aleph}(0') \geq I'_{\aleph}(p') \geq I'_{\aleph}(\omega(p))$ by(8 & 13)

In simpler terms, $\omega^{-1}(I'_{\aleph})(0) \geq \omega^{-1}(I'_{\aleph})(p)\ \forall p \in S$

Let's Consider $\omega^{-1}(\aleph'_{\aleph})(0) = \aleph'_{\aleph}(\omega(0)) = \aleph'_{\aleph}(0') \leq \aleph'_{\aleph}(p') \leq \aleph'_{\aleph}(\omega(p))$ by(8 & 13)

In simpler terms, $\omega^{-1}(\aleph'_{\aleph})(0) \leq \omega^{-1}(\aleph'_{\aleph})(p)\ \forall p \in S$ and

(i) $\omega^{-1}(\rho'_{\aleph})(p) = \rho'_{\aleph}(\omega(p)) \geq \wedge\{\rho'_{\aleph}(\omega(p)\,\Delta\,\omega(q)), \rho'_{\aleph}(\omega(q))\}$ by (definition 11.2 & 14)

$\geq \wedge\{\rho'_{\aleph}(\omega(p \ast q)), \rho'_{\aleph}(\omega(q))\}$ by(7)

$\therefore \omega^{-1}(\rho'_{\aleph})(p) \geq \wedge\{\omega^{-1}(\rho'_{\aleph}(p \ast q)), \omega^{-1}(\rho'_{\aleph}(q))\} \forall p,q \in S$

(ii) $\omega^{-1}(I'_{\aleph})(p) = I'_{\aleph}(\omega(p)) \geq \wedge\{I'_{\aleph}(\omega(p)\Delta\omega(q)), I'_{\aleph}(\omega(q))\}$ by(definition 11.2 &15)

$\geq \wedge\{I'_{\aleph}(\omega(p \ast q)), I'_{\aleph}(\omega(q))\}$ by(7)

$\therefore \omega^{-1}(I'_{\aleph})(p) \geq \wedge\{\omega^{-1}(I'_{\aleph}(p \ast q)), \omega^{-1}(I'_{\aleph}(q))\} \forall p,q \in S$

(iii) $\omega^{-1}(\aleph'_{\aleph})(p) = \aleph'_{\aleph}(\omega(p)) \leq \vee\{\aleph'_{\aleph}(\omega(p)\,\Delta\,\omega(q)), \aleph'_{\aleph}(\omega(q))\}$by (definition 11.2 & 16)

$\leq \vee\{\aleph'_{\aleph}(\omega(p \ast q)), \aleph'_{\aleph}(\omega(q))\}$ by(7)

$\therefore \omega^{-1}(\aleph'_{\aleph})(p) \leq \vee\{\omega^{-1}(\aleph'_{\aleph}(p \ast q)), \omega^{-1}(\aleph'_{\aleph}(q))\} \forall p,q \in S$

Hence, $\omega^{-1}(S'_N)$ is a Ni within S.

Conversely, suppose that $\omega^{-1}(S'_{\aleph})$ is a Ni within S

since ω is onto $\forall p', q' \in S', \exists$ atleast one $p, q \in S$ be $\ni$ $\omega(p) = p'$ and $\omega(q) = q'$

Let's consider $(\rho'_{\aleph})(0') = \rho'_{\aleph}(\omega(0)) = \omega^{-1}(\rho'_{\aleph})(0) \geq \omega^{-1}(\rho'_{\aleph})(p) \geq \rho'_{\aleph}(\omega(p))$ by (8 & definition 11.2)

In simpler terms, $(\rho'_{\aleph})(0') \geq (\rho'_{\aleph})(p') \forall p \in S$

Let's consider $(I'_{\aleph})(0') = I'_{\aleph}(\omega(0)) = \omega^{-1}(I'_{\aleph})(0) \geq \omega^{-1}(I'_{\aleph})(p) \geq I'_{\aleph}(\omega(p))$ by (8 & definition 11.2)

In simpler terms, $(I'_{\aleph})(0') \geq (I'_{\aleph})(p') \forall p \in S$

Let's consider $(\aleph'_{\aleph})(0') = \aleph'_{\aleph}(\omega(0)) = \omega^{-1}(\aleph'_{\aleph})(0) \leq \omega^{-1}(\aleph'_{\aleph})(p) \leq \aleph'_{\aleph}(\omega(p))$ by (8 & definition 11.2)

In simpler terms, $(\aleph'_{\aleph})(0') \leq (\aleph'_{\aleph})(p') \forall p \in S$ and

(i) $(\rho'_{\aleph})(p') = \rho'_{\aleph}(\omega(p))$

$= \omega^{-1}(\rho'_{\aleph})(p)$ by (definition 11.2)

$\geq \wedge\{\omega^{-1}(\rho'_{\aleph}(p \ast q)), \omega^{-1}(\rho'_{\aleph}(q))\}$ by (14)

$\geq \wedge\{\rho'_{\aleph}(\omega(p \ast q)), \rho'_{\aleph}(\omega(q))\}$by (definition 11.2)
$\geq \wedge\{\rho'_{\aleph}(\omega(p)\,\Delta\,\omega(q)), \rho'_{\aleph}(\omega(q))\}$ by(7)

$\geq \wedge\{\rho'_{\aleph}(p'\Delta q'), \rho'_{\aleph}(q')\}$

$\therefore (\rho'_{\aleph})(p') \geq \wedge\{\rho'_{\aleph}(p'\Delta q'), \rho'_{\aleph}(q')\}\ \ \forall p,q \in S$

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(ii) $(I'_{\aleph})(p') = I'_{\aleph}(\omega(p))$

$= \omega^{-1}(I'_{\aleph})(p)$ by (definition 11.2)

$\geq \wedge\{\omega^{-1}(I'_{\aleph}(p \,x q)), \omega^{-1}(I'_{\aleph}(q))\}$ by (15)

$\geq \wedge\{I'_{\aleph}(\omega(p \,x q)), I'_{\aleph}(\omega(q))\}$ by (definition 11.2)

$\geq \wedge\{I'_{\aleph}(\omega(p)\,\Delta\,\omega(q)), I'_{\aleph}(\omega(q))\}$ by (7)

$\geq \wedge\{I'_{\aleph}(p'\Delta q'), I'_{\aleph}(q')\}$

$\therefore (I'_{\aleph})(p') \geq \wedge\{I'_{\aleph}(p'\Delta q'), I'_{\aleph}(q')\}$, $\forall p,q \in S$

(iii) $(\aleph'_{\aleph})(p') = \aleph'_{\aleph}(\omega(p))$

$= \omega^{-1}(\aleph'_{\aleph})(p)$ by (definition 11.2)

$\leq \vee\{\omega^{-1}(\aleph'_{\aleph}(p \,x q)), \omega^{-1}(\aleph'_{\aleph}(q))\}$by (16)

$\leq \vee\{\aleph'_{\aleph}(\omega(p \,x q)), \aleph'_{\aleph}(\omega(q))\}$by (definition 11.2)

$\leq \vee\{\aleph'_{\aleph}(\omega(p)\,\Delta\,\omega(q)), \aleph'_{\aleph}(\omega(q))\}$(by 7)

$\leq \vee\{\aleph'_{\aleph}(p'\Delta q'), \aleph'_{\aleph}(q')\}$

$\therefore (\aleph'_{\aleph})(p') \leq \vee\{\aleph'_{\aleph}(p'\Delta q'), \aleph'_{\aleph}(q')\}$ $\forall p,q \in S$

Hence, $S'_{\aleph}$ is a Ɲi within S′.

Theorem7 : Let ω be an epimorphism from Sto S′ then $S'_{\aleph}$ is a ƝSA within S′ iff the pre-image $\omega^{-1}(S'_{\aleph})$ of $S'_{\aleph}$ is a ƝSA within S.

Proof: The proof follows a direct path, as indicated by Proposition 1 and theorem 6.

6.Level set of a Neutrosophic set

Definition 12: Let $S_{\aleph}$ be a ƝS within S and let $£_1, £_2, £_3 \in [0,1]$ be *such that* $0 \leq £_1 + £_2 + £_3 \leq 1$ then the set $S_{\aleph}^{(£_1,£_2,£_3)} = \{p \in S / \rho_{\aleph}(p) \geq £_1, I_{\aleph}(p) \geq £_2, and\ \aleph_{\aleph}(p) \leq £_3\}$ is called as $(£_1, £_2, £_3)$ level subset of $S_{\aleph}$.

Theorem8: Let a ƝS $S_{\aleph}$ be a Ni within S, then $S_{\aleph}^{(£_1,£_2,£_3)}$ is an ideal within S, $\forall\ (£_1, £_2, £_3) \in Im(\rho_{\aleph}) \times Im(I_{\aleph}) \times Im(\aleph_{\aleph}) with\ 0 \leq £_1 + £_2 + £_3 \leq 1$.

Proof: Let a ƝS $S_{\aleph}$ be a Ɲi within S.

let $q \in S_{\aleph}^{(£_1,£_2,£_3)} \Rightarrow \rho_{\aleph}(q) \geq £_1, I_{\aleph}(q) \geq £_2,\ and\ \aleph_{\aleph}(q) \leq £_3$

since $S_{\aleph}$ is a Ɲi within S $\Rightarrow \rho_{\aleph}(0) \geq \rho_{\aleph}(q) \geq £_1, I_{\aleph}(0) \geq I_{\aleph}(q) \geq £_2 and\ \aleph_{\aleph}(0) \leq \aleph_{\aleph}(q) \leq £_3$

$\Rightarrow 0 \in S_{\aleph}^{(£_1,£_2,£_3)}$.

Let $p, q \in S\ be\ \ni p \,x\, q\ and\ q \in S_{\aleph}^{(£_1,£_2,£_3)}$

$\Rightarrow \rho_{\aleph}(p x q) \geq £_1 and\ \rho_{\aleph}(q) \geq £_1$

$I_{\aleph}(p x q) \geq £_2 and\ I_{\aleph}(q) \geq £_2$

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$\aleph_{\aleph}(p x q) \leq £_3 and\ \aleph_{\aleph}(q) \leq £_3$

Since $S_{\aleph}$ is a Ɲi, then (i)$\rho_{\aleph}(p) \geq \wedge\{\rho_{\aleph}(p x q), \rho_{\aleph}(q)\} \geq \wedge\{£_1, £_1\}$

$\Rightarrow \rho_{\aleph}(p) \geq £_1, \forall\, p \in S$

(ii) $I_{\aleph}(p) \geq \wedge\{I_{\aleph}(p x q), I_{\aleph}(q)\} \geq \wedge\{£_2, £_2\}$

$\Rightarrow I_{\aleph}(p) \geq £_2, \forall\, p \in S$

and (iii) $\aleph_{\aleph}(p) \leq \vee\{\aleph_{\aleph}(p x q), \aleph_{\aleph}(q)\} \leq \vee\{£_3, £_3\}$

$\Rightarrow \aleph_{\aleph}(p) \leq £_3, \forall\, p \in S$

$\Rightarrow p \in S_{\aleph}^{(£_1,£_2,£_3)}$

Hence, $S_{\aleph}^{(£_1,£_2,£_3)}$ forms an ideal within S.

Theorem 9: Consider a ƝS $S_{\aleph}$ within the set S, where $S_{\aleph}^{(£_1,£_2,£_3)}$ represents an ideal within S. If $(£_1, £_2, £_3) \in Im(\rho_{\aleph}) \times Im(I_{\aleph}) \times Im(\aleph_{\aleph})\ with\ 0 \leq £_1 + £_2 + £_3 \leq 1$ then $S_{\aleph}$ is a Ɲi within S.

Proof: Suppose that $S_{\aleph}$ is a ƝS and $S_{\aleph}^{(£_1,£_2,£_3)}$ is an ideal within S.

Let $S_{\aleph}$(p) = $(£_1, £_2, £_3)$∀p∈S

$\Rightarrow \boldsymbol{\rho}_{\aleph}(p) = £_1$, $\boldsymbol{I}_{\aleph}(p) = £_2$ & $\aleph_{\aleph}(p) = £_3$, ∀p∈S

Since $0 \in \boldsymbol{S}_{\aleph}^{(£_1,£_2,£_3)}$

$\Rightarrow \boldsymbol{\rho}_{\aleph}(0) \geq £_1 = \boldsymbol{\rho}_{\aleph}(p)$

$\boldsymbol{I}_{\aleph}(0) \geq £_2 = \boldsymbol{I}_{\aleph}(p)$

$\aleph_{\aleph}(0) \leq £_3 = \aleph_{\aleph}(p)$, ∀p∈S

Let p,q∈S be ∋$\boldsymbol{S}_{\aleph}(p x q) = (£_{11}, £_{21}, £_{31})$

$\Rightarrow \boldsymbol{\rho}_{\aleph}(p x q) = £_{11}$, $\boldsymbol{I}_{\aleph}(p x q) = £_{21}$, $\aleph_{\aleph}(p x q) = £_{31}$

$\Rightarrow p x q \in \boldsymbol{S}_{\aleph}^{(£_1,£_2,£_3)}$ and $\boldsymbol{S}_{\aleph}(q) = (£_{12}, £_{22}, £_{32})$

$\boldsymbol{\rho}_{\aleph}(q) = £_{12}$, $\boldsymbol{I}_{\aleph}(q) = £_{22}$ & $\aleph_{\aleph}(q) = £_{32}$, then q $\in \boldsymbol{S}_{\aleph}^{(£_{12},£_{22},£_{32})}$

Let us assume that $£_{11} \leq £_{12}$

$\Rightarrow £_{21} \leq £_{22}$ and $£_{31} \geq £_{32}$

⇒without loss of generality, it follows that $\boldsymbol{S}_{\aleph}^{(£_{12},£_{22},£_{32})} \subseteq \boldsymbol{S}_{\aleph}^{(£_{11},£_{21},£_{31})}$

⇒So that $p x q \in \boldsymbol{S}_{\aleph}^{(£_{11},£_{21},£_{31})}$ and $q \in \boldsymbol{S}_{\aleph}^{(£_{11},£_{21},£_{31})}$

$\Rightarrow p \in \boldsymbol{S}_{\aleph}^{(£_{11},£_{21},£_{31})}$, since $\boldsymbol{S}_{\aleph}^{(£_{11},£_{21},£_{31})}$ is an ideal of S.

$\Rightarrow \boldsymbol{\rho}_{\aleph}(p) \geq £_{11} = \wedge\{\boldsymbol{\rho}_{\aleph}(p x q), \boldsymbol{\rho}_{\aleph}(q)\}$

$\boldsymbol{I}_{\aleph}(p) \geq £_{21} = \wedge\{\boldsymbol{I}_{\aleph}(p x q), \boldsymbol{I}_{\aleph}(q)\}$

and $\aleph_{\aleph}(p) \leq £_{31} = \vee\{\aleph_{\aleph}(p x q), \aleph_{\aleph}(q)\}$

Hence, $\boldsymbol{S}_{\aleph}$ is a Ɲi within S.

Note: $\boldsymbol{S}_{\aleph}^{(£_1,£_2,£_3)} = \{\mathbf{p} \in S \,/\, \boldsymbol{\rho}_{\aleph}(p) \geq £_1, \boldsymbol{I}_{\aleph}(p) \geq £_2, \aleph_{\aleph}(p) \leq £_3\}$

$= \{\mathbf{p} \in S \,/\, \boldsymbol{\rho}_{\aleph}(p) \geq £_1\} \cap \{\mathbf{p} \in S \,/\, \boldsymbol{I}_{\aleph}(p) \geq £_2\} \cap \{\mathbf{p} \in \mathbf{S} \,/\, \aleph_{\aleph}(p) \leq £_3\}$

$\Rightarrow \boldsymbol{S}_{\aleph}^{(£_1,£_2,£_3)} = \boldsymbol{\rho}(\boldsymbol{\rho}_{\aleph}; £_1) \cap I(\boldsymbol{I}_{\aleph}; £_2) \cap \aleph(\aleph_{\aleph}; £_3)$.

Corollary1: Let $\boldsymbol{S}_{\aleph}$ be a ƝS within S then $\boldsymbol{S}_{\aleph}$ is a Ɲi within S iff $\boldsymbol{\rho}(\boldsymbol{\rho}_{\aleph}; £_1)$, $I(\boldsymbol{I}_{\aleph}; £_2)$, $\aleph(\aleph_{\aleph}; £_3)$ are ideals within S $\forall £_1 \in [0, \boldsymbol{\rho}_{\aleph}(0)], £_2 \in [0, \boldsymbol{I}_{\aleph}(0)]$ and $£_3 \in [\aleph_{\aleph}(0), 1]$ with $0 \leq £_1 + £_2 + £_3 \leq 1$.

Theorem 10: Consider M(≠Ø) contained within S and a ƝS $\boldsymbol{S}_{\aleph}$ defined within S as follows:

$$\boldsymbol{\rho}_{\aleph}(p) = \begin{cases} £_{\rho}, & \textbf{if } \mathbf{p} \in \mathbf{M} \\ £_1, & \textbf{otherwise} \end{cases}$$

$$\boldsymbol{I}_{\aleph}(p) = \begin{cases} £_{I}, & \textbf{if } \mathbf{p} \in \mathbf{M} \\ £_2, & \textbf{otherwise} \end{cases} \quad \text{and}$$

$$\aleph_{\aleph}(p) = \begin{cases} £_{\aleph}, & \textbf{if } \mathbf{p} \in \mathbf{M} \\ £_3, & \textbf{otherwise} \end{cases}$$

∀p∈S where $0 \leq £_{\rho} \leq £_1$, $0 \leq £_{I} \leq £_2$, $0 \leq £_{\aleph} \leq £_3$ and $£_{\rho} + £_{I} + £_{\aleph} \leq 1$, $£_1 + £_2 + £_3 \leq 1$

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In this context, $\boldsymbol{S}_{\aleph}$ is Ɲi within S while M constitutes an ideal within S.
Proof: Let's assume that a S_{N} is a Ɲi within S and M is a subset within S.
Let p∈ M.
$\Rightarrow \boldsymbol{\rho}_{\aleph}(0) \geq \boldsymbol{\rho}_{\aleph}(p) = £_{\boldsymbol{\rho}}$
$\boldsymbol{I}_{\aleph}(0) \geq \boldsymbol{I}_{\aleph}(p) = £_{I}$
$\aleph_{\aleph}(0) \leq \aleph_{\aleph}(p) = £_{\aleph}$
⇒0∈ M
Let p,q∈ S be ∃pӿq∈M and q∈M, since $\boldsymbol{S}_{\aleph}$ is a Ɲi within S
Then (i) $\boldsymbol{\rho}_{\aleph}(p) \geq \wedge\{\boldsymbol{\rho}_{\aleph}(pӿq), \boldsymbol{\rho}_{\aleph}(q)\} \geq \wedge\{£_{\boldsymbol{\rho}}, £_{\boldsymbol{\rho}}\} = £_{\boldsymbol{\rho}}$
$\boldsymbol{I}_{\aleph}(p) \geq \wedge\{\boldsymbol{I}_{\aleph}(pӿq), \boldsymbol{I}_{\aleph}(q)\} \geq \wedge\{£_{I}, £_{I}\} = £_{I}$
and $\aleph_{\aleph}(p) \leq \vee\{\aleph_{\aleph}(pӿq), \aleph_{\aleph}(q)\} \leq \vee\{£_{\aleph}, £_{\aleph}\} = £_{\aleph}$
⇒p∈ M.
Hence, M constitutes an ideal within S.
Conversely suppose that M constitutes an ideal within S.
(i) Let p∈S.
Case (i): if p ∈ M then $\boldsymbol{\rho}_{\aleph}(p) = £\, \boldsymbol{\rho}$
$\boldsymbol{I}_{\aleph}(p) = £_{I}$
$\aleph_{\aleph}(p) = £_{\aleph}$, ∀p∈S
Since 0 ∈ M.
$\Rightarrow \boldsymbol{\rho}_{\aleph}(0) = £_{\boldsymbol{\rho}}$
$\boldsymbol{I}_{\aleph}(0) = £_{I}$
$\aleph_{\aleph}(0) = £_{\aleph}$,
Hence, $\boldsymbol{\rho}_{\aleph}(0) = \boldsymbol{\rho}_{\aleph}(p)$
$\boldsymbol{I}_{\aleph}(0) = \boldsymbol{I}_{\aleph}(p)$
$\aleph_{\aleph}(0) = \aleph_{\aleph}(p)$
Case (ii): If $\boldsymbol{p}$∉M then
$\Rightarrow \boldsymbol{\rho}_{\aleph}(p) = £_{1}$
$\boldsymbol{I}_{\aleph}(p) = £_{2}$
$\aleph_{\aleph}(p) = £_{3}$,
Now $\rho_{\aleph}(0) = £_{\rho} < £_{1} = \rho_{\aleph}(p)$

$I_{\aleph}(0) = £_{I} < £_{2} = I_{\aleph}(p)$

$\aleph_{\aleph}(0) = £_{\aleph} < £_{3} = \aleph_{\aleph}(p)$

Hence, in either case

$\rho_{\aleph}(0) \geq \rho_{\aleph}(p)$, $I_{\aleph}(0) \geq I_{\aleph}(p)$ and $\aleph_{\aleph}(0) \leq \aleph_{\aleph}(p)$,∀p∈S

(ii) Let p, q∈S be ∃pӿq∈S and q∈S

Case (i) : If pӿq∈M and q∈M

Since M constitutes an ideal within S

⇒p∈M

(i) $\rho_{\aleph}(p) = £_{\boldsymbol{\rho}} = \wedge\{£_{\boldsymbol{\rho}}, £_{\boldsymbol{\rho}}\}$
$\geq \wedge\{\rho_{\aleph}(pӿ q), \rho_{\aleph}(q)\}$
(ii) $I_{\aleph}(p) = £_{I} = \wedge\{£_{I}, £_{I}\}$
$\geq \wedge\{I_{\aleph}(pӿq), I_{\aleph}(q)\}$

and(iii) $\aleph_{\aleph}(p) = £_{\aleph} = \vee\{£_{\aleph}, £_{\aleph}\}$

$\leq \vee\{\aleph_{\aleph}(pӿ q), \aleph_{\aleph}(q)\}$

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Case (ii) : If p ⨰q∈M and q∉M

⇒p∉M

(i) $\rho_ℵ$(p) = £1= ∧{£ρ,£1}

≥∧{$\rho_ℵ$(p⨰ q), $\rho_ℵ$(q)}

(ii) $I_ℵ$(p) = £2= ∧ {£I,£2}
≥∧ {$I_ℵ$(p⨰ q), $I_ℵ$(q)}

and (iii) $ℵ_ℵ$(p) = £3= v{£ℵ, £3}

≤v{$ℵ_ℵ$(p⨰ q), $ℵ_ℵ$(q)}

Case (iii) : If p ⨰q∉M and q∈M

⇒p∉M

(i) $\rho_ℵ$(p) = £1 =∧ {£1, £ρ}

≥∧ {$\rho_ℵ$(p⨰ q), $\rho_ℵ$(q)}
(ii) $I_ℵ$(p) = £2 = ∧ {£2,£I}
≥∧ {$I_ℵ$(p⨰ q), $I_ℵ$(q)} and

(iii) $ℵ_ℵ$(p) = £3 = v{£3, £ℵ}

≤v{$ℵ_ℵ$(p⨰ q), $ℵ_ℵ$(q)}

Case (iv) : If p ⨰q∉M and q∉M

⇒p∉M

(i) $\rho_ℵ$(p) = £1 =∧ {£1 ,£1}

≥∧ {$\rho_ℵ$(p⨰ q), $\rho_ℵ$(q)}
(ii) $I_ℵ$(p) = £2 =∧ {£2 ,£2}
≥∧ {$I_ℵ$(p⨰ q), $I_ℵ$(q)}

and (iii) $ℵ_ℵ$(p) = £3 = v{£3 , £3}

≤ v{$ℵ_ℵ$(p⨰q), $ℵ_ℵ$(q)}, ∀p,q∈S

Hence, $S_ℵ$ is a Ɲi within S.

Corollary 2: Let M(≠Ø) be a subset within S then $S_ℵ$ within Sdefined by

$\rho_ℵ(p) = \begin{cases} \mathbf{1}, & \textbf{if p} \in \mathbf{M} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$

$I_ℵ(p) = \begin{cases} \mathbf{1}, & \textbf{if p} \in \mathbf{M} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$ and

$ℵ_ℵ(p) = \begin{cases} \mathbf{1}, & \textbf{if p} \in \mathbf{M} \\ \mathbf{0}, & \textbf{otherwise} \end{cases}$ ∀p,q∈S

Then the following conditions are interchangeable:

1. $S_ℵ$ is a Ɲi within S
2. M is an ideal within S.

Proposition 2: Let $S_ℵ$ be a Ɲi within S and $(£_{11}, £_{21}, £_{31})$, $(£_{12}, £_{22}, £_{32})$∈ Im($\rho_ℵ$) x Im($I_ℵ$) x Im ($ℵ_ℵ$) with

0 ≤$£_{1i} + £_{2i} + £_{3i}$ ≤ 1 for i = 1,2, then $S_ℵ^{(£_{11}, £_{21}, £_{31})} = S_ℵ^{(£_{12}, £_{22}, £_{32})}$ iff $(£_{11}, £_{21}, £_{31}) = (£_{12}, £_{22}, £_{32})$.

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Proof: If $(£_{11}, £_{21}, £_{31})$ = $(£_{12}, £_{22}, £_{32})$ then clearly $S_{ℵ}^{(£_{11}, £_{21}, £_{31})}$=$S_{ℵ}^{(£_{12}, £_{22}, £_{32})}$.

Assume that $S_{ℵ}^{(£_{11}, £_{21}, £_{31})}$=$S_{ℵ}^{(£_{12}, £_{22}, £_{32})}$.

Since $(£_{11}, £_{21}, £_{31})$, $(£_{12}, £_{22}, £_{32})$∈ Im($\rho_ℵ$) x Im($I_ℵ$) x Im($ℵ_ℵ$) then ∃p∈S ∋$\rho_ℵ$(p)=$£_{11}$, $I_ℵ$(p)=$£_{21}$ and $ℵ_ℵ$(p)=$£_{31}$, it follows that p∈$S_{ℵ}^{(£_{11}, £_{21}, £_{31})}$=$S_{ℵ}^{(£_{12}, £_{22}, £_{32})}$ so that

$£_{11}$=$\rho_ℵ$(p) ≥$£_{12}$, $£_{21}$=$I_ℵ$ (p)≥$£_{22}$ and $£_{31}$=$ℵ_ℵ$(p)≤$£_{32}$.

Similarly, we have $£_{11}$≤$£_{12}$, $£_{21}$≤$£_{22}$ and $£_{31} \geq £_{32}$

Hence, $(£_{11}, £_{21}, £_{31})$ = $(£_{12}, £_{22}, £_{32})$

Theorem 11: Let $S_ℵ = (\rho_ℵ, I_ℵ, ℵ_ℵ)$ be a NS within S and Im($S_ℵ$) = {(ξ_0, μ_0, κ_0), (ξ_1, μ_1, κ_1), …(ξ_k, μ_k, κ_k)} where (ξ_i, μ_i, κ_i) <(ξ_j, μ_j, κ_j) where i> j

Let {M_r|r=0,…,k} be a family of Ideals within S ∃M_0 ⊂ M_1 ⊂… M_k = S and $S_ℵ$ (M_r^*) = (ξ_r, μ_r, κ_r) that is

$\xi_{S_N}(M_r^*)=\xi_r$, $\mu_{S_N}(M_r^*)=\mu_r$ where M_r^* =M_r \M_{r-1} and M_{-1} = Ø.

For r = 0,..,k then $S_ℵ$ is Ni within S.

Proof : Since 0 ∈ M_0 , we have $\rho_ℵ$ (0) = ξ_0 ≥ $\rho_ℵ$ (p), $I_ℵ$ (0) = μ_0 ≥ $I_ℵ$ (p), $ℵ_ℵ$(0) = κ_0 ≤ $ℵ_ℵ$(p), ∀p∈S.

Let p,q∈S

Case (i) :If p ⁎ q ∈M_r^* and q ∈M_r^* = M_r \$M_{r-1}$and M_r is a Ideal

⇒p ∈M_r

Thus $\rho_ℵ$ (p) ≥ ξ_r = ∧{$\rho_ℵ$(p⁎ q), $\rho_ℵ$(q)}
$I_ℵ$(p) ≥ μ_r=∧{$I_ℵ$(p⁎ q), $I_ℵ$(q)}
$ℵ_ℵ$(p) ≤ κ_r= v{$ℵ_ℵ$(p⁎q), $ℵ_ℵ$(q)}∀p,q∈S

Case (ii) :If p ⁎ q ∉M_r^* and q ∉M_r^* then the following cases will arise

Sub-case (i) : If p ⁎ q∈ S\$M_r$and q∈S\$M_r$

Sub-case (ii) : If p ⁎ q∈$M_{r-1}$and q∈M_{r-1}

Sub-case (iii) : If p ⁎ q∈ S\$M_r$and q∈$M_{r-1}$

Sub-case (iv) : If p ⁎ q∈M_{r-1} and q∈S\M_r

But in either case

$\rho_ℵ$(p) ≥∧{$\rho_ℵ$(p⁎ q), $\rho_ℵ$(q)}
$I_ℵ$(p) ≥∧{$I_ℵ$(p⁎ q), $I_ℵ$(q)} and
$ℵ_ℵ$(p) ≤v{$ℵ_ℵ$(p⁎q), $ℵ_ℵ$(q)}∀p,q∈S

Case (iii) : If p ⁎ q ∈M_r^* and q ∉M_r^* that either q ∈M_{r-1}^* or q ∈S\$M_r$. It follows that either p∈M_r or p∈S\$M_r$. Thus

$\rho_ℵ$(p) ≥∧{$\rho_ℵ$(p⁎ q), $\rho_ℵ$(q)}
$I_ℵ$(p) ≥∧{$I_ℵ$(p⁎ q), $I_ℵ$(q)} and
$ℵ_ℵ$(p) ≤v{$ℵ_ℵ$(p⁎q), $ℵ_ℵ$(q)}, ∀p,q∈S

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Case (iv) :If p ϫ q ∈ M_r^* and q ∈M_r^*then by similar process

$\rho_{\aleph}$(p) ≥∧{$\rho_{\aleph}$(pϫ q), $\rho_{\aleph}$(q)}
$I_{\aleph}$(p) ≥∧{$I_{\aleph}$(pϫ q), $I_{\aleph}$(q)} and
$\aleph_{\aleph}$(p) ≤v{$\aleph_{\aleph}$(pϫq), $\aleph_{\aleph}$(q)}, ∀p,q∈S

Thus $S_{\aleph}$ is Ɲi within S.

7. Conclusions

This paper has undertaken an exploration and analysis of key concepts within BF-algebras, specifically focusing on the notions of ƝS, ƝSAs, and Ɲis. We have delved into their characteristics and properties, shedding light on homomorphic images of ƝS and homomorphic pre-images of ƝS, as well as their association with ƝSAs and Ɲis. Notably, we have established that the level set of ƝS within a BF-algebra qualifies as an ideal when ƝS is a Ɲi. Following conclusions can be drawn from the study:

- It was established that a Ɲi with respect to a homomorphism is a Ɲi .
- A homomorphic image of a Ɲi satisfying the sup-inf property is a Ɲi.
- Homomorphic pre-images of a ƝS within the context of BF-algebra is a ƝS, while examining a range of associated attributes.
- The results of these studies hold good for NSA also.
- Attributes of level set of a ƝS wthin BF-algebras have also been analyzed.

Looking ahead, our future research endeavours will encompass the examination of Ɲeutrosophic BF-soft sets and its attributes, and a comprehensive investigation into various types of ideals and filters within the realm of BF-algebras.

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