On The 15-Plithogenic Square Real Matrices

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Abstract

This paper is dedicated to study the algebraic properties and meta-structures that are related to symbolic 15-plithogenic with symbolic plithogenic real entries, where symbolic 15-plithogenic eigenvectors and values will be discussed and presented in terms of theorems. As well as, the computation of determinants, inverses, and scalar values.

Keywords: symbolic 15-plithogenic matrix; symbolic 15-plithogenic eigenvalue; symbolic 15- plithogenic eigenvector.

1. Introduction

The symbolic n-plithogenic algebra began with the work of Smarandache [2], where he defined for the first time the applications of symbolic n-plithogenic sets in building algebraic generalizations of well-known algebraic structures.

The main difference between symbolic n-plithogenic algebraic structure and n-refined neutrosophic structure is the definition of the multiplication operation, where the multiplication between the sub-indices is defined as follows:

$P_iP_j = P_{\max(i,j)}$. For more details about similar systems of neutrosophic and refined neutrosophic matrices, see [12-16].

Many authors followed his steps, where symbolic 2-plithogenic rings were defined by Merkepci.et.al [1], and then they were used to find symbolic 2-plithogenic modules [3], and symbolic 3-plithogenic structures [4-6].

Recently, the symbolic n-plithogenic matrices have been introduced for different values of n, see [7-11, 17-18]. The algebraic properties of these matrices were studied widely, especially those which are related to the diagonalization problem such as eigenvalues, eigenvectors, and inverses.

In general, the symbolic n-plithogenic square real matrix is defined with the following formula:

$M = M_0 + \sum_{i=1}^{n} M_iP_i$, where M_i are m-square classical matrices with real entries.

This has motivated us to follow these efforts, where we show the concept of symbolic 15 plithogenic matrices with their elementary algebraic properties.

2. Main Discussion

Definition:

The square symbolic 15-plithogenic matrix is defined as follows:

$\mu = \mu_0 + \sum_{i=1}^{15} \mu_iP_i$; $(\mu_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 15-plithogenic matrix:

$$\mu = \begin{pmatrix} -3 & -9 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 5 & 1 \end{pmatrix} P_1 + \begin{pmatrix} 2 & -1 \\ 7 & 2 \end{pmatrix} P_2 + \begin{pmatrix} 6 & -8 \\ 9 & -6 \end{pmatrix} P_3 + \begin{pmatrix} 8 & 5 \\ 6 & 1 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 9 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 4 & -1 \\ 2 & -8 \end{pmatrix} P_{12} + \begin{pmatrix} 4 & -1 \\ 2 & -9 \end{pmatrix} P_{13} + \begin{pmatrix} 4 & -1 \\ 2 & -8 \end{pmatrix} P_{14} + \begin{pmatrix} 4 & -1 \\ 2 & -8 \end{pmatrix} P_{15}.$$

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Definition.

Let $\mu = \mu_0 + \sum_{i=1}^{15} \mu_i P_i$ be a symbolic 15-plithogenic matrix of size $n \times n$, hence:

$$\det \mu = \det(\mu_0) + \left[\det\left(\sum_{i=0}^{1} \mu_i\right) - \det(\mu_0)\right] P_1 + \left[\det\left(\sum_{i=0}^{2} \mu_i\right) - \det\left(\sum_{i=0}^{1} \mu_i\right)\right] P_2$$
$$+ \left[\det\left(\sum_{i=0}^{3} \mu_i\right) - \det\left(\sum_{i=0}^{2} \mu_i\right)\right] P_3 + \left[\det\left(\sum_{i=0}^{4} Z_i\right) - \det\left(\sum_{i=0}^{3} \mu_i\right)\right] P_4$$
$$+ \left[\det\left(\sum_{i=0}^{5} \mu_i\right) - \det\left(\sum_{i=0}^{4} \mu_i\right)\right] P_5 + \left[\det\left(\sum_{i=0}^{6} \mu_i\right) - \det\left(\sum_{i=0}^{5} \mu_i\right)\right] P_6$$
$$+ \left[\det\left(\sum_{i=0}^{7} \mu_i\right) - \det\left(\sum_{i=0}^{6} \mu_i\right)\right] P_7 + \left[\det\left(\sum_{i=0}^{8} \mu_i\right) - \det\left(\sum_{i=0}^{7} \mu_i\right)\right] P_8$$
$$+ \left[\det\left(\sum_{i=0}^{9} \mu_i\right) - \det\left(\sum_{i=0}^{8} \mu_i\right)\right] P_9 + \left[\det\left(\sum_{i=0}^{10} \mu_i\right) - \det\left(\sum_{i=0}^{9} \mu_i\right)\right] P_{10}$$
$$+ \left[\det\left(\sum_{i=0}^{11} \mu_i\right) - \det\left(\sum_{i=0}^{10} \mu_i\right)\right] P_{11} + \left[\det\left(\sum_{i=0}^{12} \mu_i\right) - \det\left(\sum_{i=0}^{11} \mu_i\right)\right] P_{12}$$
$$+ \left[\det\left(\sum_{i=0}^{13} \mu_i\right) - \det\left(\sum_{i=0}^{12} \mu_i\right)\right] P_{13} + \left[\det\left(\sum_{i=0}^{14} \mu_i\right) - \det\left(\sum_{i=0}^{13} \mu_i\right)\right] P_{14}$$
$$+ \left[\det\left(\sum_{i=0}^{15} \mu_i\right) - \det\left(\sum_{i=0}^{14} \mu_i\right)\right] P_{15}$$

Theorem1.

Let $\mu = \mu_0 + \sum_{i=1}^{15} \mu_i P_i$ be a symbolic 15-plithogenic matrix of size $n \times n$, hence:

1. μ is invertible if and only if $\det \mu$ is an invertible symbolic 15-plithogenic real number.
2. $\mu^{-1} = {\mu_0}^{-1} + [(\sum_{i=0}^{1} \mu_i)^{-1} - {\mu_0}^{-1}]P_1 + [(\sum_{i=0}^{2} \mu_i)^{-1} - (\sum_{i=0}^{1} \mu_i)^{-1}]P_2 + [(\sum_{i=1}^{3} \mu_i)^{-1} - (\sum_{i=0}^{2} \mu_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \mu_i)^{-1} - (\sum_{i=0}^{3} \mu_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} \mu_i)^{-1} - (\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} \mu_i)^{-1} - (\sum_{i=0}^{5} \mu_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} \mu_i)^{-1} - (\sum_{i=0}^{6} \mu_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \mu_i)^{-1} - (\sum_{i=0}^{7} \mu_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \mu_i)^{-1} - (\sum_{i=0}^{8} \mu_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \mu_i)^{-1} - (\sum_{i=0}^{9} \mu_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \mu_i)^{-1} - (\sum_{i=0}^{10} \mu_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \mu_i)^{-1} - (\sum_{i=0}^{11} \mu_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \mu_i)^{-1} - (\sum_{i=0}^{12} \mu_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \mu_i)^{-1} - (\sum_{i=0}^{13} \mu_i)^{-1}]P_{14} + \left[(\sum_{i=1}^{15} \mu_i)^{-1} - (\sum_{i=0}^{14} \mu_i)^{-1}\right] P_{15}$

Definition.

Let $q = q_0 + \sum_{i=1}^{15} q_i P_i$ be a symbolic 15-plithogenic real number and $\mu = \mu_0 + \sum_{i=1}^{15} \mu_i P_i$ be a symbolic 15-plithogenic square real matrix, then q is called symbolic 15-plithogenic eigen value if and only if $ZX = qX$.
X is called symbolic 15-plithogenic eigenvector.

Theorem2.

Let $o = o_0 + \sum_{i=1}^{15} o_i P_i \in 15 - SP_R$, $X = X_0 + \sum_{i=1}^{15} X_i P_i$ be a symbolic 15-plithogenic real vector, then o is eigen value of $\mu = \mu_0 + \sum_{i=1}^{15} \mu_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} o_i$ is eigen value of $\sum_{i=0}^{j} \mu_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 15$.

Theorem3.

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$$\mu^n = {\mu_0}^n + P_1\left[\left(\sum_{i=0}^{1}\mu_i\right)^n - {\mu_0}^n\right] + \left[\left(\sum_{i=0}^{2}\mu_i\right)^n - \left(\sum_{i=0}^{1}\mu_i\right)^n\right]P_2 + \left[\left(\sum_{i=1}^{3}\mu_i\right)^n - \left(\sum_{i=0}^{2}\mu_i\right)^n\right]P_3$$
$$+ \left[\left(\sum_{i=1}^{4}\mu_i\right)^n - \left(\sum_{i=0}^{3}\mu_i\right)^n\right]P_4 + \left[\left(\sum_{i=1}^{5}\mu_i\right)^n - \left(\sum_{i=0}^{4}\mu_i\right)^n\right]P_5 + \left[\left(\sum_{i=1}^{6}\mu_i\right)^n - \left(\sum_{i=0}^{5}\mu_i\right)^n\right]P_6$$
$$+ \left[\left(\sum_{i=1}^{7}\mu_i\right)^n - \left(\sum_{i=0}^{6}\mu_i\right)^n\right]P_7 + \left[\left(\sum_{i=1}^{8}\mu_i\right)^n - \left(\sum_{i=0}^{7}\mu_i\right)^n\right]P_8 + \left[\left(\sum_{i=1}^{9}\mu_i\right)^n - \left(\sum_{i=0}^{8}\mu_i\right)^n\right]P_9$$
$$+ \left[\left(\sum_{i=1}^{10}\mu_i\right)^n - \left(\sum_{i=0}^{9}\mu_i\right)^n\right]P_{10} + \left[\left(\sum_{i=1}^{11}\mu_i\right)^n - \left(\sum_{i=0}^{10}\mu_i\right)^n\right]P_{11}$$
$$+ \left[\left(\sum_{i=1}^{12}Z_i\right)^n - \left(\sum_{i=0}^{11}Z_i\right)^n\right]P_{12} + \left[\left(\sum_{i=1}^{13}\mu_i\right)^n - \left(\sum_{i=0}^{12}\mu_i\right)^n\right]P_{13}$$
$$+ \left[\left(\sum_{i=1}^{14}\mu_i\right)^n - \left(\sum_{i=0}^{13}\mu_i\right)^n\right]P_{14} + \left[\left(\sum_{i=1}^{15}\mu_i\right)^n - \left(\sum_{i=0}^{14}\mu_i\right)^n\right]P_{15}$$

Theorem4.

Let $\mu = \mu_0 + \sum_{i=1}^{15}\mu_i P_i$ be a square 15-plithogenic invertible real matrix, then:

1). $\det(\mu^{-1}) = (\det\mu)^{-1}$

2). $\det\mu^t = \det\mu$

3). $\det(\mu.C) = \det\mu.\det C\,; C = C_0 + \sum_{i=1}^{15}C_i P_i$.

Definition.

Let $\mu = \mu_0 + \sum_{i=1}^{15}\mu_i P_i$ be a symbolic 15-plithogenic real square matrix, then:

μ is called orthogonal if and only if $\mu^t = \mu^{-1}$.

Theorem5.

μ is orthogonal if and only if $\sum_{i=0}^{j}\mu_i\,;\ 0 \le j \le 15$ are orthogonal.

Proof of theorem1.

1). Let $\mu = \mu_0 + \sum_{i=1}^{15}\mu_i P_i$, then Z is invertible if and only if there exists $T = T_0 + \sum_{i=1}^{15}T_i P_i$ such that:

$\mu \times T = U_{n\times n}$, hence:

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$$\begin{cases} \mu_0 T_0 = U_{n\times n} \\ \sum_{i=0}^{1}\mu_i \sum_{i=0}^{1} T_i - \mu_0 T_0 = O_{n\times n} \\ \sum_{i=0}^{2}\mu_i \sum_{i=0}^{2} T_i - \sum_{i=0}^{1}\mu_i \sum_{i=0}^{1} T_i = O_{n\times n} \\ \sum_{i=0}^{3}\mu_i \sum_{i=0}^{3} T_i - \sum_{i=0}^{2}\mu_i \sum_{i=0}^{2} T_i = O_{n\times n} \\ \sum_{i=0}^{4}\mu_i \sum_{i=0}^{4} T_i - \sum_{i=0}^{3}\mu_i \sum_{i=0}^{3} T_i = O_{n\times n} \\ \sum_{i=0}^{5}\mu_i \sum_{i=0}^{5} T_i - \sum_{i=0}^{4}\mu_i \sum_{i=0}^{4} T_i = O_{n\times n} \\ \sum_{i=0}^{6}\mu_i \sum_{i=0}^{6} T_i - \sum_{i=0}^{5}\mu_i \sum_{i=0}^{5} T_i = O_{n\times n} \\ \sum_{i=0}^{7}\mu_i \sum_{i=0}^{7} T_i - \sum_{i=0}^{6}\mu_i \sum_{i=0}^{6} T_i = O_{n\times n} \\ \sum_{i=0}^{8}\mu_i \sum_{i=0}^{8} T_i - \sum_{i=0}^{7}\mu_i \sum_{i=0}^{7} T_i = O_{n\times n} \\ \sum_{i=0}^{9}\mu_i \sum_{i=0}^{9} T_i - \sum_{i=0}^{8}\mu_i \sum_{i=0}^{8} T_i = O_{n\times n} \\ \sum_{i=0}^{10}\mu_i \sum_{i=0}^{10} T_i - \sum_{i=0}^{9}\mu_i \sum_{i=0}^{9} T_i = O_{n\times n} \\ \sum_{i=0}^{11}\mu_i \sum_{i=0}^{11} T_i - \sum_{i=0}^{10}\mu_i \sum_{i=0}^{10} T_i = O_{n\times n} \\ \sum_{i=0}^{12}\mu_i \sum_{i=0}^{12} T_i - \sum_{i=0}^{11}\mu_i \sum_{i=0}^{11} T_i = O_{n\times n} \\ \sum_{i=0}^{13}\mu_i \sum_{i=0}^{13} T_i - \sum_{i=0}^{12}\mu_i \sum_{i=0}^{12} T_i = O_{n\times n} \\ \sum_{i=0}^{14}\mu_i \sum_{i=0}^{14} T_i - \sum_{i=0}^{13}\mu_i \sum_{i=0}^{13} T_i = O_{n\times n} \\ \sum_{i=0}^{15}\mu_i \sum_{i=0}^{15} T_i - \sum_{i=0}^{14}\mu_i \sum_{i=0}^{14} T_i = O_{n\times n} \end{cases}$$

This implies that:

$$\begin{cases} \mu_0 T_0 = U_{n\times n} \\ \sum_{i=0}^{j}\mu_i \sum_{i=0}^{j} T_i = U_{n\times n} \end{cases} ;\ 1 \le j \le 15$$

Hence $\det\left(\sum_{i=0}^{j}\mu_i\right) \neq 0$ for all $1 \le j \le 15$, so that $\det(\mu)$ is invertible in $15 - SP_R$.

2). It holds directly as follows:

$\sum_{i=0}^{j} T_i = \left(\sum_{i=0}^{j}\mu_i\right)^{-1}$ for $1 \le j \le 15$, hence:

$$\mu^{-1} = {\mu_0}^{-1} + [(\textstyle\sum_{i=0}^{1}\mu_i)^{-1} - {\mu_0}^{-1}]P_1 + [(\sum_{i=0}^{2}\mu_i)^{-1} - (\mu)^{-1}]P_2 + [(\sum_{i=1}^{3}\mu_i)^{-1} - (\sum_{i=0}^{2}\mu_i)^{-1}]P_3 + [(\sum_{i=1}^{4}\mu_i)^{-1} - (\sum_{i=0}^{3}\mu_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5}\mu_i\right)^{-1} - (\sum_{i=0}^{4}\mu_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6}\mu_i)^{-1} - \left(\sum_{i=0}^{5}\mu_i\right)^{-1}\right]P_6 + [(\sum_{i=1}^{7}\mu_i)^{-1} - (\sum_{i=0}^{6}\mu_i)^{-1}]P_7 + [(\sum_{i=1}^{8}\mu_i)^{-1} - (\sum_{i=0}^{7}\mu_i)^{-1}]P_8 + [(\sum_{i=1}^{9}\mu_i)^{-1} - (\sum_{i=0}^{8}\mu_i)^{-1}]P_9 +$$

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$[(\sum_{i=1}^{10}\mu_i)^{-1}-(\sum_{i=0}^{9}\mu_i)^{-1}]P_{10}+[(\sum_{i=1}^{11}\mu_i)^{-1}-(\sum_{i=0}^{10}\mu_i)^{-1}]P_{11}+[(\sum_{i=1}^{12}\mu_i)^{-1}-(\sum_{i=0}^{11}\mu_i)^{-1}]P_{12}+[(\sum_{i=1}^{13}\mu_i)^{-1}-(\sum_{i=0}^{12}\mu_i)^{-1}]P_{13}+[(\sum_{i=1}^{14}\mu_i)^{-1}-(\sum_{i=0}^{13}\mu_i)^{-1}]P_{14}+\left[\left(\sum_{i=1}^{15}\mu_i\right)^{-1}-(\sum_{i=0}^{14}\mu_i)^{-1}\right]P_{15}.$

Proof of theorem2.

It is clear that o is an eigen value of Z with X as an eigen vector if and only if:
$\mu.X=o.X$, which is equivalent to:

$$\begin{cases}\mu_0X_0=o_0X_0\\ \sum_{i=0}^{j}\mu_i\sum_{i=0}^{j}X_i=\sum_{i=0}^{j}o_i\sum_{i=0}^{j}X_i\end{cases};\ 1\le j\le 15$$

Which is equivalent to:
$\sum_{i=0}^{j}o_i$ is an eigen value of $\sum_{i=0}^{j}\mu_i$ with $\sum_{i=0}^{j}X_i$ as an eigen vector for all $1\le j\le 15$.

Proof of theorem4.

1). $det\mu^{-1}=det(\mu_0{}^{-1})+P_1[det(\sum_{i=0}^{1}\mu_i)^{-1}-det(\mu_0{}^{-1})]+[det(\sum_{i=0}^{2}\mu_i)^{-1}-det(\sum_{i=0}^{1}\mu_i)^{-1}]P_2+[det(\sum_{i=1}^{3}\mu_i)^{-1}-det(\sum_{i=0}^{2}\mu_i)^{-1}]P_3+[det(\sum_{i=1}^{4}\mu_i)^{-1}-det(\sum_{i=0}^{3}\mu_i)^{-1}]P_4+\left[det\left(\sum_{i=1}^{5}\mu_i\right)^{-1}-det(\sum_{i=0}^{4}\mu_i)^{-1}\right]P_5+\left[det(\sum_{i=1}^{6}\mu_i)^{-1}-det\left(\sum_{i=0}^{5}\mu_i\right)^{-1}\right]P_6+[det(\sum_{i=1}^{7}\mu_i)^{-1}-det(\sum_{i=0}^{6}\mu_i)^{-1}]P_7+[det(\sum_{i=1}^{8}\mu_i)^{-1}-det(\sum_{i=0}^{7}\mu_i)^{-1}]P_8+[det(\sum_{i=1}^{9}\mu_i)^{-1}-det(\sum_{i=0}^{8}\mu_i)^{-1}]P_9+[det(\sum_{i=1}^{10}\mu_i)^{-1}-det(\sum_{i=0}^{9}\mu_i)^{-1}]P_{10}+[det(\sum_{i=1}^{11}\mu_i)^{-1}-det(\sum_{i=0}^{10}\mu_i)^{-1}]P_{11}+[det(\sum_{i=1}^{12}\mu_i)^{-1}-det(\sum_{i=0}^{11}\mu_i)^{-1}]P_{12}+[det(\sum_{i=1}^{13}\mu_i)^{-1}-det(\sum_{i=0}^{12}\mu_i)^{-1}]P_{13}+[det(\sum_{i=1}^{14}\mu_i)^{-1}-det(\sum_{i=0}^{13}\mu_i)^{-1}]P_{14}+\left[det\left(\sum_{i=1}^{15}\mu_i\right)^{-1}-det(\sum_{i=0}^{14}\mu_i)^{-1}\right]P_{15}=(det\mu)^{-1}.$

2). $\mu^t=\mu_0{}^t+\mu_1{}^tP_1+\mu_2{}^tP_2+\mu_3{}^tP_3+\mu_4{}^tP_4+\mu_5{}^tP_5+\mu_6{}^tP_6+\mu_7{}^tP_7+\mu_8{}^tP_8+\mu_9{}^tP_9+\mu_{10}{}^tP_{10}+\mu_{11}{}^tP_{11}+\mu_{12}{}^tP_{12}+\mu_{13}{}^tP_{13}+\mu_{14}{}^tP_{14}+\mu_{15}{}^tP_{15}.$

$det\mu^t=det(\mu_0{}^t)+[det(\sum_{i=0}^{1}\mu_i{}^t)-det(\mu_0{}^t)]P_1+[det(\sum_{i=0}^{2}\mu_i{}^t)-det(\sum_{i=0}^{1}\mu_i{}^t)]P_2+[det(\sum_{i=0}^{3}\mu_i{}^t)-det(\sum_{i=0}^{2}\mu_i{}^t)]P_3+[det(\sum_{i=0}^{4}\mu_i{}^t)-det(\sum_{i=0}^{3}\mu_i{}^t)]P_4+\left[det\left(\sum_{i=0}^{5}\mu_i{}^t\right)-det\ (\sum_{i=0}^{4}\mu_i{}^t)\right]P_5+\left[det(\sum_{i=0}^{6}\mu_i{}^t)-det\ (\sum_{i=0}^{5}Z_i{}^t)\right]P_6+[det(\sum_{i=0}^{7}\mu_i{}^t)-det\ (\sum_{i=0}^{6}\mu_i{}^t)]P_7+[det(\sum_{i=0}^{8}\mu_i{}^t)-det\ (\sum_{i=0}^{7}\mu_i{}^t)]P_8+[det(\sum_{i=0}^{9}\mu_i{}^t)-det\ (\sum_{i=0}^{8}\mu_i{}^t)]P_9+[det(\sum_{i=0}^{10}\mu_i{}^t)-det\ (\sum_{i=0}^{9}\mu_i{}^t)]P_{10}+[det(\sum_{i=0}^{11}\mu_i{}^t)-det\ (\sum_{i=0}^{10}\mu_i{}^t)]P_{11}+[det(\sum_{i=0}^{12}\mu_i{}^t)-det\ (\sum_{i=0}^{11}\mu_i{}^t)]P_{12}+[det(\sum_{i=0}^{13}\mu_i{}^t)-det\ (\sum_{i=0}^{12}\mu_i{}^t)]P_{13}+[det(\sum_{i=0}^{14}\mu_i{}^t)-det\ (\sum_{i=0}^{13}\mu_i{}^t)]P_{14}+\left[det\left(\sum_{i=0}^{15}\mu_i{}^t\right)-det\ (\sum_{i=0}^{14}\mu_i{}^t)\right]P_{15}=\det(\mu_0)+[\det(\sum_{i=0}^{1}\mu_i)-\det(\mu_0)]P_1+[\det(\sum_{i=0}^{2}\mu_i)-\det(\sum_{i=0}^{1}\mu_i)]P_2+[\det(\sum_{i=0}^{3}\mu_i)-\det(\sum_{i=0}^{2}\mu_i)]P_3+[\det(\sum_{i=0}^{4}\mu_i)-\det(\sum_{i=0}^{3}\mu_i)]P_4+\left[\det\left(\sum_{i=0}^{5}\mu_i\right)-\det(\sum_{i=0}^{4}\mu_i)\right]P_5+\left[\det(\sum_{i=0}^{6}\mu_i)-\det\left(\sum_{i=0}^{5}\mu_i\right)\right]P_6+[\det(\sum_{i=0}^{7}\mu_i)-\det(\sum_{i=0}^{6}\mu_i)]P_7+[\det(\sum_{i=0}^{8}\mu_i)-\det(\sum_{i=0}^{7}\mu_i)]P_8+[\det(\sum_{i=0}^{9}\mu_i)-\det(\sum_{i=0}^{8}\mu_i)]P_9+[\det(\sum_{i=0}^{10}\mu_i)-\det(\sum_{i=0}^{9}\mu_i)]P_{10}+[\det(\sum_{i=0}^{11}\mu_i)-\det(\sum_{i=0}^{10}\mu_i)]P_{11}+[\det(\sum_{i=0}^{12}\mu_i)-\det(\sum_{i=0}^{11}\mu_i)]P_{12}+[\det(\sum_{i=0}^{13}\mu_i)-\det(\sum_{i=0}^{12}\mu_i)]P_{13}+[\det(\sum_{i=0}^{14}\mu_i)-\det(\sum_{i=0}^{13}\mu_i)]P_{14}+\left[\det\left(\sum_{i=0}^{15}\mu_i\right)-\det(\sum_{i=0}^{14}\mu_i)\right]P_{15}=det\mu.$

3). we have:

$\mu.C=\mu_0C_0+[\sum_{i=0}^{1}\mu_i\sum_{i=0}^{1}C_i-\mu_0C_0]P_1+[\sum_{i=0}^{2}\mu_i\sum_{i=0}^{2}C_i-\sum_{i=0}^{1}\mu_i\sum_{i=0}^{1}C_i]P_2+[\sum_{i=0}^{3}\mu_i\sum_{i=0}^{3}C_i-\sum_{i=0}^{2}\mu_i\sum_{i=0}^{2}C_i]P_3+[\sum_{i=0}^{4}\mu_i\sum_{i=0}^{4}C_i-\sum_{i=0}^{3}\mu_i\sum_{i=0}^{3}B_i]P_4+[\sum_{i=0}^{5}\mu_i\sum_{i=0}^{5}C_i-\sum_{i=0}^{4}\mu_i\sum_{i=0}^{4}C_i]P_5+[\sum_{i=0}^{6}\mu_i\sum_{i=0}^{6}C_i-\sum_{i=0}^{5}\mu_i\sum_{i=0}^{5}C_i]P_6+[\sum_{i=0}^{7}\mu_i\sum_{i=0}^{7}C_i-\sum_{i=0}^{6}\mu_i\sum_{i=0}^{6}C_i]P_7+[\sum_{i=0}^{8}\mu_i\sum_{i=0}^{8}C_i-\sum_{i=0}^{7}\mu_i\sum_{i=0}^{7}C_i]P_8+[\sum_{i=0}^{9}\mu_i\sum_{i=0}^{9}C_i-\sum_{i=0}^{8}\mu_i\sum_{i=0}^{8}C_i]P_9+[\sum_{i=0}^{10}\mu_i\sum_{i=0}^{10}C_i-\sum_{i=0}^{9}\mu_i\sum_{i=0}^{9}C_i]P_{10}+[\sum_{i=0}^{11}\mu_i\sum_{i=0}^{11}C_i-\sum_{i=0}^{10}\mu_i\sum_{i=0}^{10}C_i]P_{11}+[\sum_{i=0}^{12}\mu_i\sum_{i=0}^{12}C_i-\sum_{i=0}^{11}\mu_i\sum_{i=0}^{11}C_i]P_{12}+[\sum_{i=0}^{13}\mu_i\sum_{i=0}^{13}C_i-\sum_{i=0}^{12}\mu_i\sum_{i=0}^{12}C_i]P_{13}+[\sum_{i=0}^{14}\mu_i\sum_{i=0}^{14}C_i-\sum_{i=0}^{13}\mu_i\sum_{i=0}^{13}C_i]P_{14}+[\sum_{i=0}^{15}\mu_i\sum_{i=0}^{15}C_i-\sum_{i=0}^{14}\mu_i\sum_{i=0}^{14}C_i]P_{15}.$

$det(\mu.C)=det(\mu_0C_0)+[det(\sum_{i=0}^{1}\mu_i\sum_{i=0}^{1}C_i)-det(\mu_0C_0)]P_1+[det(\sum_{i=0}^{2}\mu_i\sum_{i=0}^{2}C_i)-det(\sum_{i=0}^{1}Z_i\sum_{i=0}^{1}C_i)]P_2+[det(\sum_{i=0}^{3}\mu_i\sum_{i=0}^{3}C_i)-det(\sum_{i=0}^{2}\mu_i\sum_{i=0}^{2}C_i)]P_3+[det(\sum_{i=0}^{4}\mu_i\sum_{i=0}^{4}C_i)-det(\sum_{i=0}^{3}\mu_i\sum_{i=0}^{3}C_i)]P_4+\left[det\left(\sum_{i=0}^{5}\mu_i\sum_{i=0}^{5}C_i\right)-det\ (\sum_{i=0}^{4}\mu_i\sum_{i=0}^{4}C_i)\right]P_5+\left[det(\sum_{i=0}^{6}\mu_i\sum_{i=0}^{6}C_i)-det\ \left(\sum_{i=0}^{5}\mu_i\sum_{i=0}^{5}C_i\right)\right]P_6+[det(\sum_{i=0}^{7}\mu_i\sum_{i=0}^{7}C_i)-det\ (\sum_{i=0}^{6}\mu_i\sum_{i=0}^{6}C_i)]P_7+[det(\sum_{i=0}^{8}\mu_i\sum_{i=0}^{8}C_i)-det\ (\sum_{i=0}^{7}\mu_i\sum_{i=0}^{7}C_i)]P_8+[det(\sum_{i=0}^{9}\mu_i\sum_{i=0}^{9}C_i)-det\ (\sum_{i=0}^{8}\mu_i\sum_{i=0}^{8}C_i)]P_9+[det(\sum_{i=0}^{10}\mu_i\sum_{i=0}^{10}C_i)-det\ (\sum_{i=0}^{9}\mu_i\sum_{i=0}^{9}C_i)]P_{10}+[det(\sum_{i=0}^{11}\mu_i\sum_{i=0}^{11}C_i)-det\ (\sum_{i=0}^{10}\mu_i\sum_{i=0}^{10}C_i)]P_{11}+[det(\sum_{i=0}^{12}\mu_i\sum_{i=0}^{12}C_i)-det\ (\sum_{i=0}^{11}\mu_i\sum_{i=0}^{11}C_i)]P_{12}+[det(\sum_{i=0}^{13}\mu_i\sum_{i=0}^{13}C_i)-det\ (\sum_{i=0}^{12}\mu_i\sum_{i=0}^{12}C_i)]P_{13}+[det(\sum_{i=0}^{14}\mu_i\sum_{i=0}^{14}C_i)-det\ (\sum_{i=0}^{13}\mu_i\sum_{i=0}^{13}C_i)]P_{14}+\left[det\left(\sum_{i=0}^{15}\mu_i\sum_{i=0}^{15}C_i\right)-det\ (\sum_{i=0}^{14}\mu_i\sum_{i=0}^{14}C_i)\right]P_{15}=det(\mu_0)det(C_0)+\left[det(\sum_{i=0}^{j}\mu_i).det(\sum_{i=0}^{j}C_i)-det(\sum_{i=1}^{j-1}\mu_{i-1}).det(\sum_{i=1}^{j-1}C_{i-1})\right]P_i=det(\mu)det(C);1\le j\le 15.$

Proof of theorem5.

μ is orthogonal if and only if $\mu^t=\mu^{-1}$, hence:

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${\mu_0}^t + \sum_{i=1}^{15} {\mu_i}^t P_i = {Z_0}^{-1} + [(\mu)^{-1} - {\mu_0}^{-1}]P_1 + [(\sum_{i=0}^{2} \mu_i)^{-1} - (\sum_{i=0}^{1} \mu_i)^{-1}]P_2 + [(\sum_{i=1}^{3} `\mu_i)^{-1} - (\sum_{i=0}^{2} \mu_i)^{-1}]P_3 + [(\sum_{i=1}^{4} \mu_i)^{-1} - (\sum_{i=0}^{3} \mu_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} \mu_i)^{-1} - (\sum_{i=0}^{4} \mu_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} \mu_i)^{-1} - (\sum_{i=0}^{5} \mu_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} \mu_i)^{-1} - (\sum_{i=0}^{6} \mu_i)^{-1}]P_7 + [(\sum_{i=1}^{8} \mu_i)^{-1} - (\sum_{i=0}^{7} \mu_i)^{-1}]P_8 + [(\sum_{i=1}^{9} \mu_i)^{-1} - (\sum_{i=0}^{8} \mu_i)^{-1}]P_9 + [(\sum_{i=1}^{10} \mu_i)^{-1} - (\sum_{i=0}^{9} \mu_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} \mu_i)^{-1} - (\sum_{i=0}^{10} \mu_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} \mu_i)^{-1} - (\sum_{i=0}^{11} \mu_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} \mu_i)^{-1} - (\sum_{i=0}^{12} \mu_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} \mu_i)^{-1} - (\sum_{i=0}^{13} \mu_i)^{-1}]P_{14} + \left[(\sum_{i=1}^{15} \mu_i)^{-1} - (\sum_{i=0}^{14} \mu_i)^{-1}\right] P_{15}$, thus:

$$
\begin{cases}
{\mu_0}^t = {\mu_0}^{-1} \\
{\mu_1}^t = \left(\sum_{i=0}^{1} \mu_i\right)^{-1} - {\mu_0}^{-1} \\
{\mu_2}^t = \left(\sum_{i=0}^{2} \mu_i\right)^{-1} - \left(\sum_{i=0}^{1} \mu_i\right)^{-1} \\
{\mu_3}^t = \left(\sum_{i=0}^{3} \mu_i\right)^{-1} - \left(\sum_{i=0}^{2} \mu_i\right)^{-1} \\
{\mu_4}^t = \left(\sum_{i=0}^{4} \mu_i\right)^{-1} - \left(\sum_{i=0}^{3} \mu_i\right)^{-1} \\
{\mu_5}^t = \left(\sum_{i=0}^{5} \mu_i\right)^{-1} - \left(\sum_{i=0}^{4} \mu_i\right)^{-1} \\
{\mu_6}^t = \left(\sum_{i=0}^{6} \mu_i\right)^{-1} - \left(\sum_{i=0}^{5} \mu_i\right)^{-1} \\
{\mu_7}^t = \left(\sum_{i=0}^{7} \mu_i\right)^{-1} - \left(\sum_{i=0}^{6} \mu_i\right)^{-1} \\
{\mu_8}^t = \left(\sum_{i=0}^{8} Z_i\right)^{-1} - \left(\sum_{i=0}^{7} Z_i\right)^{-1} \\
{\mu_9}^t = \left(\sum_{i=0}^{9} \mu_i\right)^{-1} - \left(\sum_{i=0}^{8} \mu_i\right)^{-1} \\
{\mu_{10}}^t = \left(\sum_{i=0}^{10} \mu_i\right)^{-1} - \left(\sum_{i=0}^{9} \mu_i\right)^{-1} \\
{\mu_{11}}^t = \left(\sum_{i=0}^{11} \mu_i\right)^{-1} - \left(\sum_{i=0}^{10} \mu_i\right)^{-1} \\
{\mu_{12}}^t = \left(\sum_{i=0}^{12} \mu_i\right)^{-1} - \left(\sum_{i=0}^{11} \mu_i\right)^{-1} \\
{\mu_{13}}^t = \left(\sum_{i=0}^{13} \mu_i\right)^{-1} - \left(\sum_{i=0}^{12} \mu_i\right)^{-1} \\
{\mu_{14}}^t = \left(\sum_{i=0}^{14} \mu_i\right)^{-1} - \left(\sum_{i=0}^{13} \mu_i\right)^{-1} \\
{\mu_{15}}^t = \left(\sum_{i=0}^{15} \mu_i\right)^{-1} - \left(\sum_{i=0}^{14} \mu_i\right)^{-1}
\end{cases}
$$

This implies that:

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$$\begin{cases} {\mu_0}^t = {\mu_0}^{-1} \\ \sum_{i=0}^{1} {\mu_i}^t = (\sum_{i=0}^{1} \mu_i)^{-1} \\ \sum_{i=0}^{2} {\mu_i}^t = (\sum_{i=0}^{2} \mu_i)^{-1} \\ \sum_{i=0}^{3} {\mu_i}^t = (\sum_{i=0}^{3} \mu_i)^{-1} \\ \sum_{i=0}^{4} {\mu_i}^t = (\sum_{i=0}^{4} \mu_i)^{-1} \\ \sum_{i=0}^{5} {\mu_i}^t = (\sum_{i=0}^{5} \mu_i)^{-1} \\ \sum_{i=0}^{6} {\mu_i}^t = (\sum_{i=0}^{6} \mu_i)^{-1} \\ \sum_{i=0}^{7} {\mu_i}^t = (\sum_{i=0}^{7} \mu_i)^{-1} \\ \sum_{i=0}^{8} {\mu_i}^t = (\sum_{i=0}^{8} \mu_i)^{-1}, \\ \sum_{i=0}^{9} {\mu_i}^t = (\sum_{i=0}^{9} \mu_i)^{-1} \\ \sum_{i=0}^{10} {\mu_i}^t = (\sum_{i=0}^{10} \mu_i)^{-1} \\ \sum_{i=0}^{11} {\mu_i}^t = (\sum_{i=0}^{11} \mu_i)^{-1} \\ \sum_{i=0}^{12} {\mu_i}^t = (\sum_{i=0}^{12} \mu_i)^{-1} \\ \sum_{i=0}^{13} {\mu_i}^t = (\sum_{i=0}^{13} \mu_i)^{-1} \\ \sum_{i=0}^{14} {\mu_i}^t = (\sum_{i=0}^{14} \mu_i)^{-1} \\ \sum_{i=0}^{15} {\mu_i}^t = (\sum_{i=0}^{15} \mu_i)^{-1} \end{cases}$$

3. Conclusion

in this work, we have found the algebraic properties of the symbolic 15-plithogenic matrices, where we have established many theorems that describe the algebraic behavior of these matrices, such as determinants, inverses, and eigenvalues. Also, the relationships between symbolic 15-plithogenic matrices and their classical components are presented.

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