Dynamics and Stability of Interconnected Systems: A Graph-Theoretic Neuromorphic Approach

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Abstract

We investigate the stability of huge, linked subsystems in separate nonlinear dynamical systems. These systems' properties depend on both their dynamics and their link structure. We examine two concepts of stability. The initial one is connection stability, where a complete system is robust in the meaning of Lyapunov given the uncertainty and temporal fluctuations in the linking lengths among systems. The next is the widely accepted idea of asymptotic stability of the entire system, which is predicated on the premise that all linkages are set at the nominal values. We propose graph-based characteristics of two types of a stable for the situation of homogenous subsystems by making linkages to spectrum graph theory, in particular the spectrum of the sign adjacency matrix. We also obtain constraints on the highest amplitude of the sign adjacency matrix of independent relevance via this method.

Keywords: Complete system; Adjacency matrix; Stability of dynamical systems; Graph.

1. Introduction

Complexity systems are made up of smaller linked subsystems, including the Website, the electric grid, and physical, environmental, and financial systems. The interaction among the dynamics and topologies of network infrastructures is an interesting area of study spanning various subjects. The behaviors of the total system are dictated through the dynamic of subsystems as well as the architecture of connections [1-8].

The stability of these structures is a subject of specific importance: Could the linkages and specific dynamics combined to lead the condition of an overall system to become untenable? Preliminary studies by May [9] examined randomized interaction patterns in migration patterns and showed that unstable happens if the connectivity power among subsystems goes past a threshold number. For further information as well as [10-12]. Decentralized control has additionally been studied in connection with the development of communication networks and response patterns for the stabilization of dynamical systems [13-15]. The sparseness patterns and related topological criteria necessary for a matrix to describe a Hurwitz system have been the focus of recent research [16-18].

In this article, we examine the stability of a network of separate linearization and provide topological requirements for the stability of the system as a whole. In particular, we focus on two stability concepts. First, often referred to as connecting stable, is a powerful idea of stabilization in which the system's status monotonically decreases to zero notwithstanding uncertainty or temporal fluctuations in the connectivity among the subsystems. The next is the common understanding of asymptotic stabilization, which states that known or time-invariant connection, the state's status is zero. As the first involves the second, the circumstances that are necessary for connected stability are necessary for Fundamental stabilization. Likewise, the prerequisites for connected stability equally hold for stability. So, to achieve the 2 types of stability inside the situation of linear systems, we offer pattern constraints for auto dynamics and linkage strength. If every one of the self-dynamics and linking energies is nonzero, and the sign structures of the linkages fit a precise definition of structural balance, the ideas of stabilization are comparable, which we will show. Whenever the linkages' signals are permitted to change, the difference between connecting and stable will become more obvious. To study this situation, we establish links to spectrum graphs specifically, the spectrum concept of adjacency matrices, to get constraints on the greatest amplitude for the system's dynamic matrices. The constraints that we discover have independent value for the investigation of directed graphs. This

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article consists of 6 parts, the first part is the introduction, the second part is the concepts that we need in this research, the third part consists of Stability of Copulative, and the fourth part consists of 4. A Requirement for Connecting Scalar Structures to be Connectively Stable and Requirements for Durability of Linked Scalar Structures According to Graphs in the fifth part and finally the conclusions in the sixth part.

2. Preliminaries

The symbol for the collection of positive real numbers is $\mathbb{R}_+$. The symbol for the item (k, l) in matrices H is h_{kl}. A Parallel $\parallel H \parallel$ is going to be employed to represent the norm of a matrices H. The eigenvalues for the symmetric $m\ x\ n$ matrix H will be arranged as $\xi_{min}(H)=\xi_1(H)\leq\xi_2(H)\leq\cdots\leq\xi_n(H)=\xi_{max}(H)$.

A matrix H has a spectral radius was being provided by $(H)\triangleq max_{1\leq k\leq m\times n}\left|\xi_k(H)\right|$. Given similar definitions as $H > 0, H < 0$, and $H = 0$, the symbol $H \geq 0$ for a matrix H denotes that every member of matrices is greater than 0. The matrices that are represented by the formula $|H|$ are the result of calculating the absolute value of each item in matrices H.

If an $m \times n$ matrices A that contains negative off-diagonal entries meets one of the counterparts listed below, it is referred to as an M-matrices [1].

1 It has a vector $k \in \mathbb{R}_+^n$ where $Hk > 0$ occurs.

2 Every amplitude of H has a positive number component.

Let's define $G = \{\mathrm{M}, \mathrm{N}\}$ to describe a graph with $\mathrm{M} = \{m_1, m_2, \dots, m_n\}$ is the collection of vertices and $\mathrm{N} \subset \mathrm{M} \times \mathrm{M}$ for edge collection. We also suppose that $(m_k, m_k) \notin \mathrm{N}$ with all $m_k \in \mathrm{M}$. If $(m_k, m_l) \in \mathrm{N} \Leftrightarrow (m_l, m_k) \in \mathrm{N}$, the graphs are considered to be undirected. The collection $P_k(G)=\{m_l \in M/(m_l,m_k) \in N\}$ gives the neighbors of the vertex $m_k \in \mathrm{M}$. The equations $d_k(G) \triangleq |\mathrm{P}_k(G)|$give the grade of the vertices $m_k \in \mathrm{M}$. We shall stop relying on G when discussing the neighborhood and grade of a certain vertex once the situation is obvious. A cycle inside the graphs is a collection of vertices in the following order: $m_{k_1}, m_{k_2}, \dots, m_{k_k}, m_{k_1}$ Every couple of the following vertices in the series has an edge leading to the initial vertices to the next vertices.

A Graph $\mathrm{P} = \{\dot{\mathcal{V}}, \dot{\mathrm{N}}\}$, where $\dot{M} \subseteq M$ and $\dot{N} \subseteq N \cap (\dot{M} \times \dot{M})$; then subgraph is derived if equivalence occurs in the latter formula. In the graph G, a clique is a generated subgraph $\mathrm{P} = \{\dot{\mathcal{V}}, \dot{\mathrm{N}}\}$ of G where $\dot{N}$ is made up of edges connecting every couple of distinct vertex in $\dot{M}$.

The balanced adjacency matrices of graphs with the mathematical formula G is a matrix $H \in \mathbb{R}^{m\times n}$, whose item $h_{k\,l} = 0$ is nonzero if $(m_k, m_l) \notin \mathrm{N}$ and zero otherwise. A balanced adjacency matrix is referred to as an indicated adjacency matrix if the nonzero members take values from $\{-1,1\}$. The traditional adjacency matrices can be obtained if all nonzero members $= 1$. Weight graphs having value $h_{k\,l}$ on edge (m_k, m_l) is connected to each balanced adjacency matrix; a signed graph has edges that can have either a negative or positive value. If a given graph's value multiplied over every cycle equals positive, the graphs are considered to be structurally stable. Whenever there is no identity in the graphs, the diagonal members of the adjacency matrix are 0. The collection of eigenvalues in the adjacency matrices H makes the spectrum of mathematical graphs G. Graphs are covered in greater detail in common literature like [19-20].

3. Stability of Copulative

Assume a separate nonlinear system named Ψ that consists of Ω subsystems, every one of which is of type

$$x_k(w+1)=H_{kk}x_k(w)+\sum_{\substack{l=1\\ l\neq k}}^{\Omega} e_{kl}H_{kl}x_l(w)+\Phi(w), 1\leq i\leq\Omega, \tag{1}$$

So, with dynamical matrices H_{kk} in $\mathbb{R}^{m_1\times n_1}$ and the order to provide clarity $x_k \in \mathbb{R}^{m_1}$. The connectivity between subsystems l and k is denoted by $H_{kk}x_k(w)$. The value of the variable $e_{kl} \in [0,1]$ reflects an ambiguous coupling strength that ranges from 0 (without linking) to 1 and $\Phi(w)$ is a non-linear function. The vector $x=[x_1^{T+1} \quad x_2^{T+1} \quad \cdots \quad x_\Omega^{T+1}]^{T+1}$ indicates the condition of the mathematical system.

Definition 3.1. If the system Ψ is stable in the Lyapunov concept for any $e_{kl} \in [0,1]$, $k, l \in \{1,2, \dots, \Omega\}$, then it is copulative stable. The inequality may be employed to limit the connection values for the subsystem given (1).

$$\left\|\sum_{l=1}^{\Omega} e_{kl}H_{kl}x_l(w)+\Phi(w)\right\|_2\leq\sum_{j=1}^{N} e_{kl}\|H_{kl}\|_2\|x_l\|_2+\|\Phi(w)\|_2. \tag{2}$$

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It's indeed easy to identify a Lyapunov function with the type if H_{kk} is consistent.

$$m_k(x_k)=\left(x_k^{T+1}P_kx_k\right)^{3/2}+\left(\Phi(w)\right)^{3/2}, \forall k\in\{1,2,\ldots,\Omega\} \quad (3)$$

where H is a positive definite matrix that satisfies:

$$H_{kk}^{T}P_kH_{kk}-P_i=-F_i \quad (4)$$

for a certain real nonnegative matrix F_i. The magnitude of turmoil that may be accepted even while causing the Lyapunov function to drop at every period is defined as a resilience constraint for every subsystem depending on its Lyapunov function to handle cumulative turmoil like linkage.

Definition 3.2. For every subsystem, the durability condition of a Lyapunov (3) is provided as:

$$\omega_m(F_k)=\frac{\psi_{max}(F_k)+1}{\psi_{min}^{3/2}(P_k)\psi_{min}^{3/2}(P_k-F_k)+\psi_{min}(P_k)+1}. \quad (5)$$

Remark 3.3. For the scalar scheme, stability $x_k[w+2]=h_{kk}x_{k+1}[w+1]$. For $F_k=1$, the durability constraint (5) is $\omega_m(2)=2-|h_{kk}|$.

Considering a Lyapunov function which is built in respect of a Lyapunov that applies to every several subsystems to examine connection stability across the whole system.

$$m(x)=\sum_{k=1}^{\Omega}\zeta_km_k(x_{k+1}), \quad \text{For every k, } \zeta_k \text{ is positive.} \quad (6)$$

Proposition 3.4. Because every subsystem's Lyapunov function has the type (3), the reduction rate of the Lyapunov result produced in (6) can indeed be topmost by

$$m(x(w+2))-m(x(w+1))\leq-\eta^{T+1}\Psi\dot{\rho}(x(w+1)), \quad \forall x(w+1)\in R^n$$

with

$$\eta=\left[\zeta_1\psi_{max}^{3/2}(P_1) \quad \psi_2\lambda_{max}^{3/2}(P_2) \quad \cdots \quad \zeta_\Omega\psi_{max}^{3/2}(P_\Omega)\right]^T, \ \Psi_{kl}=\begin{cases}\omega_m(F_k) & if\ k=l\\ -\|H_{kl}\|_2 & if\ k\neq l\end{cases},$$

$$\dot{\rho}(x(w+1))=[\|x_1(w+1)\|_2 \quad \|x_2(w+1)\|_2 \quad \cdots \quad \|x_\Omega(w+1)\|_2]^{T+1} \quad (7)$$

The durability destined omega $\omega_m(F_k)$ of every sub-system I inside the matrix Ψ described by the above-said lemma is determined by matrix A ii of each subsystem I from (4) and (5), and the power of the worst- situation for the linkage is taken into account via $l\|H_{kl}\|_2$. The first attribute of M-matrices defined in part 2 is instantly determined depending on the characterization given previously, yielding the next conclusion.

4. A Requirement for Connecting Scalar Structures to be Connectively Stable.

Assume that the mathematical system Ψ is made up of $\Omega=m+1$ linked scalar components of the following shape: where $1\leq i\leq m+1$,

$$x_k[w+2]=m_{kk}x_k[w+1]+\sum_{\substack{k=1\\k\neq i}}^{m+1}e_{kl}h_{kl}x_{kl}[w+1]+\Phi(w), \quad (8)$$

with, for every $k,l\in\{1,2,\ldots,m+1\}$., $h_{kl}\in\mathbb{R}$. $h_{kl}=0$ if subsystems l also isn't linked to subsystems k. As previously, when Ψ is stable in the meaning of Lyapunov for any $e_{kl}\in[0,1]$, then it is interconnection consistent. Ψ would be deemed stable if it remains constant whenever $e_{kl}=1$ for all $k,l\in\{1,2,\ldots,m+1\}$.

Utilizing Remark 3.3, and supposing that $|m_{kl}|\leq 1$ over all k the matrices Θ in (7) for the aforementioned connected components are provided as

$$\Theta_{kl}=\begin{cases}1-|m_{kk}+1| & if\ k=l\\ -|m_{kl}+1| & otherwise.\end{cases}$$

Suppose Φ represents the $m\times n$ matrices, where $(k.l)$ is equivalent to m_{kl} from (8). The result is $K=I-2|\Phi|$. According to Proposition 3.4, K 's Θ-matrix status is an essential requirement forconnection durability. A required and adequate requirement for K as an Θ -matrix would be that every one of the eigenvalues of $K=I-2|\Phi|$ contains nonnegative components, according to the second feature of Θ -matrices in Part 2. Thus $2|\Phi|$ seems to have a positive integer eigenvalue with the biggest value among all of the eigenvalues since it is a nonnegative matrix. Consequently, iff all of the eigenvalues of $2|\Phi|$ have magnitudes smaller than 1, every one of K 's eigenvalues will also have nonnegative portions.

Definition 4.1. The mathematical system Ψ in equation (8) is interconnection robust if

$$2\xi+\eta\psi_{min}(|H|)\leq 1 \quad (9)$$

We will apply techniques in spectrum graphs to connect the illustration of a graph to conditions because |H| denotes the unbalanced adjacency matrix for connection (9). The following is an example of a traditional constraint.

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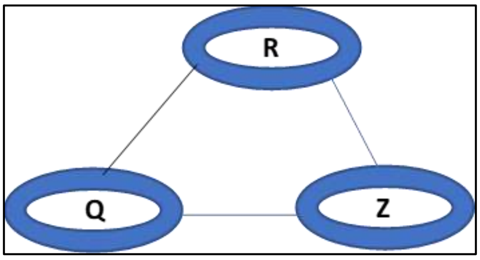

Figure 1: Signed graph in with k = 3 or complete graph with degree 3

Proposition 4.2. Suppose G be a graph having minimum and average degrees, respectively. The unbiased adjacency matrix's greatest amplitude then is calculated. An appeases

$$min\{D_{av}, \sqrt{D_{min}}\} \geq \psi_{min}(H) \geq \min_{(m_k,m_l)\in N} \sqrt{D_k D_l} \quad (10)$$

The next graph-theoretic adequate requirement can be obtained right away by applying the previous constraints in combination with Definition 4.1.

Proposition 4.3. Let the graph G associated with system Ψ in (8) have $d_{\max}$. Then Ψ is connectively stable if

$$2\xi + \eta \min_{(m_k,m_l)\in N} \sqrt{D_k D_l} \leq 1 \quad (11)$$

In the following part, we shall present limitations on the spectra of K that are required for connected durability in addition to stability.

5. Requirements for Durability of Linked Scalar Structures According to Graphs

The adjacency matrix H in its basic indicated version, $\{-1,0,1,2\}^{m\times n}$, will indeed be examined. The spectrum diameter of K is denoted as start matrices

$$\begin{aligned} \sigma(K) &\triangleq min\{\psi_{min}(K), |\psi_{max}(K)| + \Phi(w)\} \\ &= min\{\xi + \eta\lambda_{min}(H), |\xi - \eta|\psi_{max}(H)\| + \Phi(w)\}. \end{aligned} \quad (12)$$

As a consequence, we also will provide a little finding on the range of signed adjacency matrix to connect those criteria to graph attributes, and then we'll use these findings to deduce stabilization system-specific features (8). The following result provides a graph-theoretic lower bound on the spectral radius of any signed adjacency matrices.

Proposition 5.1. We already have the following for an unsigned symmetrical adjacency matrix

$$(H+1): \sigma(H+1) \geq \sqrt{D_{min}+1} \quad (13)$$

where $D_{\min}$ is the supporting graph's minimum degree.

Proof: Firstly, take notice that $\sigma(H^2+1) = \sigma^2(H+1)$. Remember that $(k+1)$-th diagonal item of H^2+1 is the squared of a 2-norm of a $(k+1)$-th row of $(H+1)$; this simply represents the number of positive integers in the $(k+1)$-th row, corresponding to the grade of vertex m_k. Considering that vertex m_l has the greatest standard, consider e_l be a vector having all of its components being zero but for one in the $(l+1)$-th location, which is 1. Thus, we obtain

$$\sigma^2(H+1) = \sigma(H^2+1) \leq \frac{e_l^{T+1}(H^2+1)e_l}{e_l^{T+1}e_l} = e_l^{T+1}(H^2+1)e_l = D_{min},$$

that results in the required constraint.

Definition 5.2. The least number of edges that must be eliminated from a sign graph to get it balanced is known as the frustration factor.

Proposition 5.2. Suppose that H is a linked signing adjacency matrix for given graphs G with frustration value k. 's biggest component fulfills.

$$\psi_{min}(|H+1|) \leq \psi_{min}(H+1) \leq \psi_{min}(|H+1|) - 2\sqrt{2(\theta+1)}. \quad (14)$$

Proof: If a graph G does have a frustration indicator of $\theta+1$ and an adjacency matrices $H+1$, an orthogonal commonality resulting matrix Ψ exists with every orthogonal component inside the range of $\{-1,0,1,2\}$ so that the matrices $(H \dot{+} 1) \triangleq \Psi H \Psi$ has precisely $2(\theta+1)$ negativity. It is obvious that the spectra of $(H+1)$ and $(H \dot{+} 1)$ are identical. Thus, we may put $(H \dot{+} 1) = |\dot{H}+1| - 2(H_1+1)$, and (H_1+1) is the unbalanced adjacency matrices for a graph having $k+1$ edges, where negative elements of $(H \dot{+} 1)$ are the edges of the graphs. Taking note of the fact that $(H \dot{+} 1)| = |H+1|$ and implementing Weyl's inequality (22) to the aforementioned formula results in

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$$\psi_{min}(|H+1|)\leq\psi_{min}(\dot{H+1}) \leq\psi_{min}((\dot{H+1}))-2\psi_{min}(H_1+1)$$
$$=\psi_{min}(|H+1|)-2\psi_{min}(H_1+1).$$

Utilizing the knowledge about graphs having aleph edges, $\psi_{min}(H+1)\geq\sqrt{2(\aleph+1)}$ for an unbalanced adjacency matrices $(H+1)$ leads to $\psi_{min}(H_1+1)\geq\sqrt{2(\theta+1)}$ that produces the desired outcome.

Proposition 5.3. Let the maximum and minimum coterie sizes, respectively, be $\phi_+$and ϕ_-, for a given adjacency matrices H and a corresponding given graph G. Next, we get

$$\psi_{max}(H+1)\geq\phi_+-1,\ \psi_{min}(H+1)\leq 1-\phi_-. \quad (15)$$

Proof: H can be transformed into the following via differencing the rows and columns:

$$\begin{bmatrix}(\gamma-I-2)_{\phi_+\times\phi_+} & \therefore \\ \therefore & \therefore\end{bmatrix},$$

with ∴ stands to represent the remaining adjacency matrices components and γ is $m\times n$ matrices made up entirely of 1s. Suppose

$$x=\frac{1}{\sqrt{(\phi_++1)}}[1 \quad \cdots \quad 1 \quad 0 \quad \cdots \quad 0]^{T+1}$$

where the first ϕ_+ item is 1 and the remaining ϕ_+ components are 0. Thus, we get

$$\psi_{max}(H+1)\geq\frac{x^{T+1}(H+1)x}{x^{T+1}x}=x^{T+1}(H+1)x=\frac{\phi_+(\phi_+-2)}{\phi_+}=\phi_+-2.$$

Applying the same logic, we get to $\psi_{\min}(H+1)\leq 2-\phi_-$.

Proposition 5.4. Consider a signed graph G with the associated signed adjacency matrix H. Let D_-be the largest number of negative elements in any row of H. Then Think about a given graph G and its corresponding sign adjacency matrices $(H+1)$. Let D -represent the $(H+1)$ row with the most positive components. Consequently

$$\psi_{min}|(H+1)|\leq\psi_{min}(H+1)\leq\psi_{min}(|H+1|)-2(D_-+1). \quad (16)$$

Proof: Observe that it's possible to express $H+1=|H+1|-2$, for any sign adjacency matrices $H+1$, and $U+1$ is the unregistered adjacency matrices and $U_{kl}=1$ *iff* $H_{kl}=-1$. Thus, we obtain:

$$\psi_{min}(H+1)\leq\psi_{min}|(H+1)|-2\psi_{min}(U+1)$$
$$\leq\psi_{min}/(H+1)/-2D_{min}(U+1)$$

In which the final action is taken after (10). The subgraph in G with the maximum grade of $D-$ that is generated via selecting just the positive edges matches to adjacency matrices $U+1$, which produces the desired outcome.

6. Conclusion

We investigated a collection of nonlinear system dynamics linked by a graph. In the interest of getting the system to just be stable, in the connected meaning, we put graph restrictions on the dynamical and linkage strength for the homogeneity scalar sub-system. We created limitations on the maximum amplitude of the indicated adjacency matrix to account for the relevance of indications of linkages and offered circumstances when the required and adequate criteria for connecting stable coincide. The application of these findings to more different systems is an interesting and difficult field for future work.

The future work based on the findings you mentioned could involve several aspects. Here are a few potential directions for further research:

1. Extension to Non-Homogeneous Systems: Investigate the impact of graph restrictions and dynamical and linkage strength on non-homogeneous scalar sub-systems. Explore how these limitations affect the stability and dynamics of such systems.
2. Generalization to Nonlinear Systems: Explore the applicability of the findings to nonlinear systems beyond scalar sub-systems. Investigate the effects of graph restrictions and linkage strength on the stability of higher-dimensional nonlinear systems.
3. Optimal Graph Topology: Investigate methods for determining the optimal graph topology that maximizes stability in a network of interconnected systems. Consider different optimization criteria, such as minimizing the maximum amplitude of the adjacency matrix or maximizing the connectivity while ensuring stability.

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