T-Spherical Fuzzy-Valued Neutrosophic Set Theory

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Abstract

From fuzzy to neutrosophy, multiple hybrid models have been innovated, with each introduced model surpassing its predecessor. Due to the inherent indeterminacy in the world, a more precise form of imprecision is required. As a result, more sophisticated variants of the neutrosophic set have been created. Examples of these amalgamations include fuzzy neutrosophic sets, intuitionistic fuzzy neutrosophic sets, Pythagorean fuzzy neutrosophic sets, neutrosophic vague sets, and neutrosophic rough sets. The main objective of this paper is to present another variant of the neutrosophic set called T-spherical fuzzy-valued neutrosophic set (T-SFVNS). Serving as a generalization of the aforementioned combinations, T-SFVNS allows for the representation of indeterminacy and inconsistency in a more nuanced manner. In this paper, we define the T-SFVNS and T-spherical fuzzy-valued neutrosophic numbers (T-SFVNNs). Additionally, we propose several types of score and accuracy functions to compare the T-SFVNNs. We also present the basic operations of T-SFVNSs and the algebraic operations of T-SFVNNs, supported by proofs and illustrative examples..

Keywords: Decision Making; Neutrosophic set; Optimization; Simplified neutrosophic set; T-spherical fuzzy sets.

1 Introduction

Picture fuzzy set (PFS)[1] stands as an advanced extension of both fuzzy set (FS)[2] and intuitionistic fuzzy set (IFS).[3] PFS exhibits distinct characteristics through its truth membership function (T), indeterminacy membership function (I), and falsity membership function (F), each residing within the standard interval of [0, 1]. The sum of these functions must adhere to the constraint of being greater than or equal to 0 and less than or equal to 1, namely $0 \leq T + I + F \leq 1$. The fundamental purpose of PFS is to effectively capture and express the

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essence of indeterminacy, surpassing the limitations of traditional FS and IFS. Building upon the foundations of PFS, subsequent advancements have led to the development of picture hesitant fuzzy set (PHFS)[4] and spherical fuzzy set (SFS).[5] SFS introduces membership grades that satisfy the condition $0 \leq T^2 + I^2 + F^2 \leq 1$, deviating from the previous constraint of $0 \leq T + I + F \leq 1$ observed in PFS.

The concept of SFS was significantly advanced by Mahmood et al.,[6] who introduced a groundbreaking framework called T-spherical fuzzy set (T-SFS). This framework incorporates a novel constraint: $0 \leq T^q + I^q + F^q \leq 1$, where $q \in \mathbb{Z}$ and $q \geq 1$. This constraint provides decision-makers with a more flexible environment, enabling them to avoid information loss during the decision-making process. T-SFSs possess the remarkable ability to represent a broader range of fuzzy information compared to SFSs.

Smarandache[7] made significant contributions to the field by introducing the concept of neutrosophic sets (NSs). NSs serve as a powerful generalization of IFS and FS, specifically designed to address multifaceted issues in MADM. NS is characterized by three essential terms: membership function (MF), non-membership function (NMF), and indeterminacy term (IMF). The remarkable feature of NSs lies in the constraint that the sum of these terms is less than or equal three, enabling a more comprehensive and accurate representation of real-life data. Consequently, NSs have garnered substantial attention from researchers worldwide, with extensive studies conducted in academic circles. However, when applying NSs to common data analysis in various everyday scenarios, certain limitations have come to light from a scientific standpoint. To overcome these limitations, Ye[8] pioneered the concept of simplified neutrosophic sets (SNSs) as a sub-class of NSs. Building upon this foundation, Al-Quran et al.[9,10] extended SNSs to neutrosophic soft rough set and interval neutrosophic vague set. Ali and Smarandache[11] extended the SNSs from the real space to the complex space, introducing the innovative concept of complex neutrosophic sets (CNS). The advancements continued with the introduction of Q-complex neutrosophic sets[12] within the same framework. Further expanding on these breakthroughs, Al-Sharqi et al.[13] successfully merged NSs and soft sets under the interval complex value, creating a unified and comprehensive approach. These advancements have significantly enhanced the applicability and versatility of NSs in solving complex decision-making problems.

There has been a recent surge of interest in integrating the characteristics of NSs, IFSs, and Pythagorean fuzzy sets (PyFSs)[14] to enhance accuracy and improve aggregation operators (AOs) for addressing data inaccuracies. Bhowmik and Pal[15] took the lead by introducing intuitionistic fuzzy-valued neutrosophic sets (IFVNS) and their operators. The condition imposed on IFVNS is that the sum of their membership functions (MFs) must be less than or equal to two. Expanding on this work, Unver et al.[16] redefined IFVNS by introducing IF neutrosophic multisets (IFNMSs) and developed various algebraic operations and aggregation operators between IFVNSs. In a similar vein, Palanikumar et al.[17] explored a new extension called Pythagorean neutrosophic normal interval-valued weighted geometric (PNNIVWG). They devised an algorithm to address alternatives in MADM problems utilizing these operators. Chellamani and Ajay[18] proposed innovative graphical concepts using the Dombi operator within Pythagorean neutrosophic fuzzy graphs (PyNFG). Palanikumar and Arulmozhi[19] introduced a fresh approach to AOs by incorporating parameterized factors in the Pythagorean fuzzy-valued neutrosophic set (PyFVNS) framework. They also proposed a score function that combined TOPSIS and VIKOR techniques.

Recently, Bozyigit et al.[20] defined PyFVNS, where each component of the NS consists of a Pythagorean fuzzy-valued set satisfying the condition: $T^2 + F^2 \leq 1$. However, the scope of IFVNSs and PyFVNSs is limited since they can only handle decision-making problems where evaluation values are represented using IF and PyF values, which may not fully capture the complete decision-related information. To expand the capabilities of IFVNSs and PyFVNSs, this article introduces the innovative concept of T-SFVNS and their operators. By incorporating T-spherical fuzzy values, a broader range of fuzzy information can be effectively represented. Notably, as the parameter q increases, the permissible information range expands, providing greater flexibility in satisfying the required information criteria. The introduction of T-SFVNS further broadens the potential applications in decision-making problems.

2 Related Works

In the following section, we provide a concise overview of the fundamental concepts of NS, SNS, T-SFS, IFVNS, and PyFVNS as a preliminary background

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Smarandache[7] expanded upon the structure of NS by incorporating the notion of indeterminacy membership, thereby enhancing the existing framework of IFS.

Definition 2.1. [7] Let $\mathcal{U}$ be a universal set. A NS $\mathcal{M}$ in $\mathcal{U}$ is a structure of the form

$$\mathcal{M} = \{< u; \mathcal{T}_{\mathcal{M}}(u), \mathcal{I}_{\mathcal{M}}(u), \mathcal{F}_{\mathcal{M}}(u) >: u \in \mathcal{U}\},$$

where the mappings $\mathcal{T}_{\mathcal{M}}; \mathcal{I}_{\mathcal{M}}; \mathcal{F}_{\mathcal{M}} : \mathcal{U} \to]^{-}0; 1^{+}[$ represent the TM, IM and FM functions, respectively with $^{-}0 \leq \mathcal{T}_{\mathcal{M}} + \mathcal{I}_{\mathcal{M}} + \mathcal{F}_{\mathcal{M}} \leq 3^{+}$.

Definition 2.2. [8] A SNS $\mathcal{N}$ in a universe $\mathcal{U}$ with a generic element u in $\mathcal{U}$ is characterized as:

$$\mathcal{N} = \{< u; \mathcal{T}_{\mathcal{N}}(u), \mathcal{I}_{\mathcal{N}}(u), \mathcal{F}_{\mathcal{N}}(u) >: u \in \mathcal{U}\},$$

where the mappings $\mathcal{T}_{\mathcal{N}}; \mathcal{I}_{\mathcal{N}}; \mathcal{F}_{\mathcal{N}} : \mathcal{U} \to [0; 1]$ represent the TM, IM and FM functions, respectively with $0 \leq \mathcal{T}_{\mathcal{N}} + \mathcal{I}_{\mathcal{N}} + \mathcal{F}_{\mathcal{N}} \leq 3$.

In this paper, we will use SNS whose membership values are T-spherical fuzzy values in the real standard interval [0; 1].

In the following, we will present some operations on SNS.

Definition 2.3. [8] Suppose $\mathcal{M} = \{< u; \mathcal{T}_{\mathcal{M}}(u), \mathcal{I}_{\mathcal{M}}(u), \mathcal{F}_{\mathcal{M}}(u) >: u \in \mathcal{U}\}$, and $\mathcal{N} = \{< u; \mathcal{T}_{\mathcal{N}}(u), \mathcal{I}_{\mathcal{N}}(u), \mathcal{F}_{\mathcal{N}}(u) >: u \in \mathcal{U}\}$, are to SNSs on the universe $\mathcal{U}$. The following operations are defined accordingly.

1. $\mathcal{M} \oplus \mathcal{N} = \{< u; \mathcal{T}_{\mathcal{M}}(u) + \mathcal{T}_{\mathcal{N}}(u) - \mathcal{T}_{\mathcal{M}}(u)\mathcal{T}_{\mathcal{N}}(u), \mathcal{I}_{\mathcal{M}}(u) + \mathcal{I}_{\mathcal{N}}(u) - \mathcal{I}_{\mathcal{M}}(u)\mathcal{I}_{\mathcal{N}}(u), \mathcal{F}_{\mathcal{M}}(u) + \mathcal{F}_{\mathcal{N}}(u) - \mathcal{F}_{\mathcal{M}}(u)\mathcal{F}_{\mathcal{N}}(u) >: u \in \mathcal{U}\}$,
2. $\mathcal{M} \otimes \mathcal{N} = \{< u; \mathcal{T}_{\mathcal{M}}(u)\mathcal{T}_{\mathcal{N}}(u), \mathcal{I}_{\mathcal{M}}(u)\mathcal{I}_{\mathcal{N}}(u), \mathcal{F}_{\mathcal{M}}(u)\mathcal{F}_{\mathcal{N}}(u) : u \in \mathcal{U}\}$,
3. $\lambda\mathcal{M} = \{< u; 1 - (1 - \mathcal{T}_{\mathcal{M}}(u))^{\lambda}, 1 - (1 - \mathcal{I}_{\mathcal{M}}(u))^{\lambda}, 1 - (1 - \mathcal{F}_{\mathcal{M}}(u))^{\lambda} : u \in \mathcal{U}\}, \lambda \geq 0$,
4. $\mathcal{M}^{\lambda} = \{< u; (\mathcal{T}_{\mathcal{M}}(u))^{\lambda}, (\mathcal{I}_{\mathcal{M}}(u))^{\lambda}, (\mathcal{F}_{\mathcal{M}}(u))^{\lambda} : u \in \mathcal{U}\}, \lambda \geq 0$.

Mahmood et al.[6] took the concept of SFSs to new heights by introducing T-SFSs, which offer a more flexible framework with unrestricted constraints.

Definition 2.4. [6] A the T-SFS $\mathfrak{L}$ on the finite set $\mathfrak{Q}$ is portrayed as follows.

$$\mathfrak{L} = \{(\mathfrak{h}, \mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}), \mathfrak{I}_{\mathfrak{L}}(\mathfrak{h}), \mathfrak{F}_{\mathfrak{L}}(\mathfrak{h})) : \mathfrak{h} \in \mathfrak{Q}\},$$

where $\mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}), \mathfrak{I}_{\mathfrak{L}}(\mathfrak{h})$ and $\mathfrak{F}_{\mathfrak{L}}(\mathfrak{h}) \in [0, 1]$ and $0 \leq (\mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{I}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{F}_{\mathfrak{L}}(\mathfrak{h}))^q \leq 1$ $(q \geq 1)$, for all $\mathfrak{h} \in \mathfrak{Q}$. The refusal degree of $\mathfrak{h}$ to $\mathfrak{Q}$ is determined by

$$\mathfrak{B}_{\mathfrak{L}}(\mathfrak{h}) = \Big(1 - \big[(\mathfrak{T}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{I}_{\mathfrak{L}}(\mathfrak{h}))^q + (\mathfrak{F}_{\mathfrak{L}}(\mathfrak{h}))^q\big]\Big)^{1/q}.$$

Unver et al.[16] boldly undertook the task of redefining the very essence of IFVNS, revolutionizing its conceptual framework by introducing the groundbreaking concept of IF neutrosophic multi-sets (IFNMSs). This audacious endeavor resulted in a redefined definition of IFVNS, which stands as follows.

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Definition 2.5. [16] An IFVNS $\mathcal{Q}$ in a universe $\hat{\Delta}$ with a generic element u in $\hat{\Delta}$ is characterized as:

$$\mathcal{Q} = \{< \hat{\delta}; \mathbb{T}_{\mathcal{Q}}(\hat{\delta}), \mathbb{I}_{\mathcal{Q}}(\hat{\delta}), \mathbb{F}_{\mathcal{Q}}(\hat{\delta}) >: \hat{\delta} \in \hat{\Delta}\},$$

where $\mathbb{T}_{\mathcal{Q}}, \mathbb{I}_{\mathcal{Q}}$ and $\mathbb{F}_{\mathcal{Q}}$ represent the membership, indeterminacy membership and non-membership neutrosophic values, each of them is an IF value, where $\forall\hat{\delta} \in \hat{\Delta}$, $\mathbb{T}_{\mathcal{Q}} = \big(\hat{\zeta}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{T}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}) \in [0,1]$ with the condition $\hat{\zeta}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}) + \hat{\varpi}_{\mathcal{Q},\mathbb{T}}(\hat{\delta}) \leq 1$, $\mathbb{I}_{\mathcal{Q}} = \big(\hat{\zeta}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{I}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}) \in [0,1]$ with the condition $\hat{\zeta}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}) + \hat{\varpi}_{\mathcal{Q},\mathbb{I}}(\hat{\delta}) \leq 1$, $\mathbb{F}_{\mathcal{Q}} = \big(\hat{\zeta}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{F}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}) \in [0,1]$ with the condition $\hat{\zeta}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}) + \hat{\varpi}_{\mathcal{Q},\mathbb{F}}(\hat{\delta}) \leq 1$.

Bozyigit et al.[20] have expanded the conceptual boundaries of IFVNS by introducing a novel extension known as the PyFVNS. This ground-breaking advancement allows for each component of the NS to encompass a PyF value, subject to the condition $\hat{\zeta}^2 + \hat{\varpi}^2 \leq 1$.

Definition 2.6. [20] A PyFVNS $\mathcal{W}$ in a universe $\hat{\Delta}$ with a generic element u in $\hat{\Delta}$ is characterized as:

$$\mathcal{W} = \{< \hat{\delta}; \mathbb{T}_{\mathcal{W}}(\hat{\delta}), \mathbb{I}_{\mathcal{W}}(\hat{\delta}), \mathbb{F}_{\mathcal{W}}(\hat{\delta}) >: \hat{\delta} \in \hat{\Delta}\},$$

where $\mathbb{T}_{\mathcal{W}}, \mathbb{I}_{\mathcal{W}}$ and $\mathbb{F}_{\mathcal{W}}$ represent the membership, indeterminacy membership and non-membership neutrosophic values, each of them is a PyF value, where $\forall\hat{\delta} \in \hat{\Delta}$, $\mathbb{T}_{\mathcal{W}} = \big(\hat{\zeta}_{\mathcal{W},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{T}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{W},\mathbb{T}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{T}}(\hat{\delta}) \in [0,1]$ with the condition $(\hat{\zeta}_{\mathcal{W},\mathbb{T}}(\hat{\delta}))^2 + (\hat{\varpi}_{\mathcal{W},\mathbb{T}}(\hat{\delta}))^2 \leq 1$, $\mathbb{I}_{\mathcal{W}} = \big(\hat{\zeta}_{\mathcal{W},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{I}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{W},\mathbb{I}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{I}}(\hat{\delta}) \in [0,1]$ with the condition $(\hat{\zeta}_{\mathcal{W},\mathbb{I}}(\hat{\delta}))^2 + (\hat{\varpi}_{\mathcal{W},\mathbb{I}}(\hat{\delta}))^2 \leq 1$, $\mathbb{F}_{\mathcal{W}} = \big(\hat{\zeta}_{\mathcal{W},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{F}}(\hat{\delta})\big)$ such that $\hat{\zeta}_{\mathcal{W},\mathbb{F}}(\hat{\delta}), \hat{\varpi}_{\mathcal{W},\mathbb{F}}(\hat{\delta}) \in [0,1]$ with the condition $(\hat{\zeta}_{\mathcal{W},\mathbb{F}}(\hat{\delta}))^2 + (\hat{\varpi}_{\mathcal{W},\mathbb{F}}(\hat{\delta}))^2 \leq 1$.

3 T-Spherical Fuzzy-Valued Neutrosophic Set (T-SFVNS)

In this section, we provide the formal definitions of T-SFVNS and T-SFVNN, accompanied by the score functions (SFs) of T-SFVNN.

Definition 3.1. Let $\hat{\mathcal{U}}$ be a universe. A T-SFVNS $\mathcal{S}$ over $\hat{\mathcal{U}}$ is signified by $\mathcal{S} = \{\langle u, \mathcal{T}_{\mathcal{S}}, \mathcal{I}_{\mathcal{S}}, \mathcal{F}_{\mathcal{S}}\rangle : u \in \hat{\mathcal{U}}\}$, where $\mathcal{T}_{\mathcal{S}}, \mathcal{I}_{\mathcal{S}}$ and $\mathcal{F}_{\mathcal{S}}$ represent the membership, indeterminacy membership and non-membership neutrosophic values, each of them is a T- spherical fuzzy value, where $\forall u \in \hat{\mathcal{U}}, q \geq 1, \mathcal{T}_{\mathcal{S}} = \big(\mu_{\mathcal{S},\mathcal{T}}(u), \omega_{\mathcal{S},\mathcal{T}}(u), \nu_{\mathcal{S},\mathcal{T}}(u)\big)$ such that $\mu_{\mathcal{S},\mathcal{T}}(u), \omega_{\mathcal{S},\mathcal{T}}(u), \nu_{\mathcal{S},\mathcal{T}}(u) \in [0,1]$, subject to the condition $(\mu_{\mathcal{S},\mathcal{T}}(u))^q + (\omega_{\mathcal{S},\mathcal{T}}(u))^q + (\nu_{\mathcal{S},\mathcal{T}}(u))^q \leq 1$, $\mathcal{I}_{\mathcal{S}} = \big(\mu_{\mathcal{S},\mathcal{I}}(u), \omega_{\mathcal{S},\mathcal{I}}(u), \nu_{\mathcal{S},\mathcal{I}}(u)\big)$ such that $\mu_{\mathcal{S},\mathcal{I}}(u), \omega_{\mathcal{S},\mathcal{I}}(u), \nu_{\mathcal{S},\mathcal{I}}(u) \in [0,1]$, subject to the condition $(\mu_{\mathcal{S},\mathcal{I}}(u))^q + (\omega_{\mathcal{S},\mathcal{I}}(u))^q + (\nu_{\mathcal{S},\mathcal{I}}(u))^q \leq 1$, $\mathcal{F}_{\mathcal{S}} = \big(\mu_{\mathcal{S},\mathcal{F}}(u), \omega_{\mathcal{S},\mathcal{F}}(u), \nu_{\mathcal{S},\mathcal{F}}(u)\big)$ such that $\mu_{\mathcal{S},\mathcal{F}}(u), \omega_{\mathcal{S},\mathcal{F}}(u), \nu_{\mathcal{S},\mathcal{F}}(u) \in [0,1]$, subject to the condition $(\mu_{\mathcal{S},\mathcal{F}}(u))^q + (\omega_{\mathcal{S},\mathcal{F}}(u))^q + (\nu_{\mathcal{S},\mathcal{F}}(u))^q \leq 1$. By definition, $0 \leq \mathcal{T}_{\mathcal{S}} + \mathcal{I}_{\mathcal{S}} + \mathcal{F}_{\mathcal{S}} \leq 3$. A T-SFVNS $\mathcal{S}$ over $\hat{\mathcal{U}}$ can be written as:

$$\mathcal{S} = \{\langle u, \big(\mu_{\mathcal{S},\mathcal{T}}(u), \omega_{\mathcal{S},\mathcal{T}}(u), \nu_{\mathcal{S},\mathcal{T}}(u)\big), \big(\mu_{\mathcal{S},\mathcal{I}}(u), \omega_{\mathcal{S},\mathcal{I}}(u), \nu_{\mathcal{S},\mathcal{I}}(u)\big), \big(\mu_{\mathcal{S},\mathcal{F}}(u), \omega_{\mathcal{S},\mathcal{F}}(u), \nu_{\mathcal{S},\mathcal{F}}(u)\big)\rangle : u \in \hat{\mathcal{U}}\}.$$

Definition 3.2. A collection of $\Gamma = \big\langle (\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}), (\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}), (\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}})\big\rangle$ is called T-SFVNN with $(\mu_{\mathcal{T}})^q + (\omega_{\mathcal{T}})^q + (\nu_{\mathcal{T}})^q \leq 1$, $(\mu_{\mathcal{I}})^q + \omega_{\mathcal{I}})^q + (\nu_{\mathcal{I}})^q \leq 1$ and $(\mu_{\mathcal{F}})^q + (\omega_{\mathcal{F}})^q + (\nu_{\mathcal{F}})^q \leq 1$, $(q \geq 1)$.

Example 3.3. Suppose $\hat{\mathcal{U}} = \{u_1, u_2, u_3\}$. Then,

$$\mathcal{S} = \left\{ \begin{array}{l} \langle u_1, (0.7, 0.2, 0.9), (0.2, 0.5, 0.1), (0.4, 0.6, 0.2)\rangle, \\ \langle u_2, (0.2, 0.1, 0.5), (0.6, 0.8, 0.5), (0.1, 0.7, 0.3)\rangle, \\ \langle u_3, (0.5, 0.3, 0.6), (0.4, 0.7, 0.2), (0.7, 0.5, 0.6)\rangle \end{array} \right\}$$

is a T-SFVNS $(q = 4)$.

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3.1 Score Functions of T-SFVNNs

This section presents the definitions of various functions, including the score function (SF), accuracy function (AF), quadratic SF (QSF), and quadratic AF (QAF).

Definition 3.4. Let $\Gamma = \big\langle \big(\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}\big), \big(\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}\big), \big(\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}}\big)\big\rangle$ be T-SFVNN. Then the SF on Γ is defined as $\Pi_\Gamma = \Pi(\Gamma) = \frac{1}{3}\Big[\big[(\mu_{\mathcal{T}})^q - (\nu_{\mathcal{T}})^q\big] + \Big(1 - \big[(\mu_{\mathcal{I}})^q - (\nu_{\mathcal{I}})^q\big]\Big) + \Big(1 - \big[(\mu_{\mathcal{F}})^q - (\nu_{\mathcal{F}})^q\big]\Big)\Big], q \geq 1.$

Definition 3.5. Let $\Gamma = \big\langle \big(\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}\big), \big(\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}\big), \big(\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}}\big)\big\rangle$ be T-SFVNN. Then the AF $\urcorner$ on Γ is defined as $\urcorner_\Gamma = \urcorner(\Gamma) = \Big[\big[(\mu_{\mathcal{T}})^q + (\omega_{\mathcal{T}})^q + (\nu_{\mathcal{T}})^q\big] - \big[(\mu_{\mathcal{F}})^q + (\omega_{\mathcal{F}})^q + (\nu_{\mathcal{F}})^q\big]\Big], q \geq 1.$

Definition 3.6. Let Γ_1 and Γ_2 be two T-SFVNNs.

1. If $\Pi_{\Gamma_1} < \Pi_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
2. If $\Pi_{\Gamma_1} > \Pi_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$,
3. If $\Pi_{\Gamma_1} = \Pi_{\Gamma_2}$ and $\urcorner_{\Gamma_1} < \urcorner_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
4. If $\Pi_{\Gamma_1} = \Pi_{\Gamma_2}$ and $\urcorner_{\Gamma_1} > \urcorner_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$.

Definition 3.7. Let $\Gamma = \big\langle \big(\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}\big), \big(\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}\big), \big(\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}}\big)\big\rangle$ be a T-SFVNN. Then the QSF on Γ is defined as $\Omega_\Gamma = \Omega(\Gamma) = \frac{1}{3}\Big[\big[(\mu_{\mathcal{T}})^{2q} - (\nu_{\mathcal{T}})^{2q}\big] + \Big(1 - \big[(\mu_{\mathcal{I}})^{2q} - (\nu_{\mathcal{I}})^{2q}\big]\Big) + \Big(1 - \big[(\mu_{\mathcal{F}})^{2q} - (\nu_{\mathcal{F}})^{2q}\big]\Big)\Big],$ $q \geq 1$.

Definition 3.8. Let $\Gamma = \big\langle \big(\mu_{\mathcal{T}}, \omega_{\mathcal{T}}, \nu_{\mathcal{T}}\big), \big(\mu_{\mathcal{I}}, \omega_{\mathcal{I}}, \nu_{\mathcal{I}}\big), \big(\mu_{\mathcal{F}}, \omega_{\mathcal{F}}, \nu_{\mathcal{F}}\big)\big\rangle$ be T-SFVNN. Then the QAF $\beth$ on Γ is defined as $\beth_\Gamma = \beth(\Gamma) = \Big[\big[(\mu_{\mathcal{T}})^{2q} + (\omega_{\mathcal{T}})^{2q} + (\nu_{\mathcal{T}})^{2q}\big] - \big[(\mu_{\mathcal{F}})^{2q} + (\omega_{\mathcal{F}})^{2q} + (\nu_{\mathcal{F}})^{2q}\big]\Big], q \geq 1.$

QSF and QAF can be used to compare two T-SFVNNs as follows.

Definition 3.1.5 Let Γ_1 and Γ_2 be two q-ROFVNNs.

1. If $\Omega_{\Gamma_1} < \Omega_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
2. If $\Omega_{\Gamma_1} > \Omega_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$,
3. If $\Omega_{\Gamma_1} = \Omega_{\Gamma_2}$ and $\beth_{\Gamma_1} < \beth_{\Gamma_2}$, then $\Gamma_1 < \Gamma_2$,
4. If $\Omega_{\Gamma_1} = \Omega_{\Gamma_2}$ and $\beth_{\Gamma_1} > \beth_{\Gamma_2}$, then $\Gamma_1 > \Gamma_2$.

4 Basic Operations on T-SFVNNs

In this section, we introduce and define the fundamental operations of T-SFVNS, drawing inspiration from the operations of both SNS and T-SFS.

In order to present the basic operations on T-SFVNS, we suppose that $\mathcal{H}$, $\mathcal{G}$ are two T-SFVNSs over $\hat{\mathcal{U}}$, where,

$\mathcal{H} = \{\big\langle u, \mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}}\big\rangle : u \in \hat{\mathcal{U}}\}$,
$= \{\big\langle u, \big(\mu_{\mathcal{H},\mathcal{T}}(u), \omega_{\mathcal{H},\mathcal{T}}(u), \nu_{\mathcal{H},\mathcal{T}}(u)\big), \big(\mu_{\mathcal{H},\mathcal{I}}(u), \omega_{\mathcal{H},\mathcal{I}}(u), \nu_{\mathcal{H},\mathcal{I}}(u)\big), \big(\mu_{\mathcal{H},\mathcal{F}}(u), \omega_{\mathcal{H},\mathcal{F}}(u), \nu_{\mathcal{H},\mathcal{F}}(u)\big)\big\rangle : u \in \hat{\mathcal{U}}\}$, and $\mathcal{G} = \{\big\langle u, \mathcal{T}_{\mathcal{G}}, \mathcal{I}_{\mathcal{G}}, \mathcal{F}_{\mathcal{G}}\big\rangle : u \in \hat{\mathcal{U}}\}$,
$= \{\big\langle u, \big(\mu_{\mathcal{G},\mathcal{T}}(u), \omega_{\mathcal{G},\mathcal{T}}(u), \nu_{\mathcal{G},\mathcal{T}}(u)\big), \big(\mu_{\mathcal{G},\mathcal{I}}(u), \omega_{\mathcal{G},\mathcal{I}}(u), \nu_{\mathcal{G},\mathcal{I}}(u)\big), \big(\mu_{\mathcal{G},\mathcal{F}}(u), \omega_{\mathcal{G},\mathcal{F}}(u), \nu_{\mathcal{G},\mathcal{F}}(u)\big)\big\rangle : u \in \hat{\mathcal{U}}\}$.

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Definition 4.1. Let $\hat{\mathcal{U}}$ be a universe and $\mathcal{H}$, $\mathcal{G}$ be two T-SFVNSs over $\hat{\mathcal{U}}$. Then, $\mathcal{H}$ is a subset of $\mathcal{G}$, denoted by $\mathcal{H} \subseteq \mathcal{G}$ if and only if:

$\mathcal{T}_{\mathcal{H}} \subseteq_q \mathcal{T}_G$, i.e. $\mu_{\mathcal{H},\mathcal{T}}(u) \leq \mu_{G,\mathcal{T}}(u)$, $\omega_{\mathcal{H},\mathcal{T}}(u) \geq \omega_{\mathcal{G},\mathcal{T}}(u)$ and $\nu_{\mathcal{H},\mathcal{T}}(u) \geq \nu_{\mathcal{G},\mathcal{T}}(u)$,

$\mathcal{I}_{\mathcal{H}} \supseteq_q \mathcal{I}_G$, i.e. $\mu_{\mathcal{H},\mathcal{I}}(u) \geq \mu_{G,\mathcal{I}}(u)$, $\omega_{\mathcal{H},\mathcal{I}}(u) \leq \omega_{\mathcal{G},\mathcal{I}}(u)$ and $\nu_{\mathcal{H},\mathcal{I}}(u) \leq \nu_{\mathcal{G},\mathcal{I}}(u)$,

$\mathcal{F}_{\mathcal{H}} \supseteq_q \mathcal{F}_G$, i.e. $\mu_{\mathcal{H},\mathcal{F}}(u) \geq \mu_{G,\mathcal{F}}(u)$, $\omega_{\mathcal{H},\mathcal{F}}(u) \leq \omega_{\mathcal{G},\mathcal{F}}(u)$ and $\nu_{\mathcal{H},\mathcal{F}}(u) \leq \nu_{\mathcal{G},\mathcal{F}}(u)$.

In this definition $\subseteq_q$ represents the T-spherical fuzzy subset.

Definition 4.2. Let $\hat{\mathcal{U}}$ be a universe and $\mathcal{H}$, $\mathcal{G}$ be two T-SFVNSs over $\hat{\mathcal{U}}$. Then, $\mathcal{H}$ is equal to $\mathcal{G}$, denoted by $\mathcal{H} = \mathcal{G}$ if and only if:

$\mathcal{T}_{\mathcal{H}} = \mathcal{T}_G$, i.e. $\mu_{\mathcal{H},\mathcal{T}}(u) = \mu_{G,\mathcal{T}}(u)$, $\omega_{\mathcal{H},\mathcal{T}}(u) = \omega_{\mathcal{G},\mathcal{T}}(u)$ and $\nu_{\mathcal{H},\mathcal{T}}(u) = \nu_{\mathcal{G},\mathcal{T}}(u)$,

$\mathcal{I}_{\mathcal{H}} = \mathcal{I}_G$, i.e. $\mu_{\mathcal{H},\mathcal{I}}(u) = \mu_{G,\mathcal{I}}(u)$, $\omega_{\mathcal{H},\mathcal{I}}(u) = \omega_{\mathcal{G},\mathcal{I}}(u)$ and $\nu_{\mathcal{H},\mathcal{I}}(u) = \nu_{\mathcal{G},\mathcal{I}}(u)$,

$\mathcal{F}_{\mathcal{H}} = \mathcal{F}_G$, i.e. $\mu_{\mathcal{H},\mathcal{F}}(u) = \mu_{G,\mathcal{F}}(u)$, $\omega_{\mathcal{H},\mathcal{F}}(u) = \omega_{\mathcal{G},\mathcal{F}}(u)$ and $\nu_{\mathcal{H},\mathcal{F}}(u) = \nu_{\mathcal{G},\mathcal{F}}(u)$.

Definition 4.3. Let $\hat{\mathcal{U}}$ be a universe and $\mathcal{H}$ be a T-SFVNS over $\hat{\mathcal{U}}$. Then, the complement of $\mathcal{H}$ is denoted by $(\mathcal{H})^c$ and defined as:

$(\mathcal{H})^c = \{\langle u, \mathcal{F}_{\mathcal{H}}, (\mathcal{I}_{\mathcal{H}})^{c_q}, \mathcal{T}_{\mathcal{H}}\rangle : u \in \mathcal{U}\}$, where c_q is a T-spherical fuzzy complement, and $(\mathcal{I}_{\mathcal{H}})^{c_q} = \big(\nu_{\mathcal{H},\mathcal{I}}(u), 1 - \omega_{\mathcal{H},\mathcal{I}}(u), \mu_{\mathcal{H},\mathcal{I}}(u)\big)$.

Definition 4.4. Let $\mathcal{H}$ and $\mathcal{G}$ are two T-SFVNSs over $\hat{\mathcal{U}}$. The union of $\mathcal{H}$ and $\mathcal{G}$ is denoted by $(\mathcal{H} \cup \mathcal{G})$ and defined as:

$(\mathcal{H} \cup \mathcal{G}) = \{\langle u, \mathcal{T}_{\mathcal{H}} \cup_q \mathcal{T}_{\mathcal{G}}, \mathcal{I}_{\mathcal{H}} \cap_q \mathcal{I}_{\mathcal{G}}, \mathcal{F}_{\mathcal{H}} \cap_q \mathcal{F}_{\mathcal{G}}\rangle : u \in \mathcal{U}\}$, where $\cup_q$ is the T-spherical fuzzy union, $\cap_q$ is the T-spherical fuzzy intersection and

$$\mathcal{T}_{\mathcal{H}} \cup_q \mathcal{T}_{\mathcal{G}} = \Big(\big(\mu_{\mathcal{H},\mathcal{T}}(u) \vee \mu_{\mathcal{G},\mathcal{T}}(u)\big), \big(\omega_{\mathcal{H},\mathcal{T}}(u) \wedge \omega_{\mathcal{G},\mathcal{T}}(u)\big), \big(\nu_{\mathcal{H},\mathcal{T}}(u) \wedge \nu_{\mathcal{G},\mathcal{T}}(u)\big)\Big),$$

$$\mathcal{I}_{\mathcal{H}} \cap_q \mathcal{I}_{\mathcal{G}} = \Big(\big(\mu_{\mathcal{H},\mathcal{I}}(u) \wedge \mu_{\mathcal{G},\mathcal{I}}(u)\big), \big(\omega_{\mathcal{H},\mathcal{I}}(u) \vee \omega_{\mathcal{G},\mathcal{I}}(u)\big), \big(\nu_{\mathcal{H},\mathcal{I}}(u) \vee \nu_{\mathcal{G},\mathcal{I}}(u)\big)\Big),$$

$$\mathcal{F}_{\mathcal{H}} \cap_q \mathcal{F}_{\mathcal{G}} = \Big(\big(\mu_{\mathcal{H},\mathcal{F}}(u) \wedge \mu_{\mathcal{G},\mathcal{F}}(u)\big), \big(\omega_{\mathcal{H},\mathcal{F}}(u) \vee \omega_{\mathcal{G},\mathcal{F}}(u)\big), \big(\nu_{\mathcal{H},\mathcal{F}}(u) \vee \nu_{\mathcal{G},\mathcal{F}}(u)\big)\Big).$$

$\vee = \max, \quad \wedge = \min.$

Definition 4.5. Let $\mathcal{H}$ and $\mathcal{G}$ be two T-SFVNSs over $\hat{\mathcal{U}}$. The intersection of $\mathcal{H}$ and $\mathcal{G}$ is denoted by $(\mathcal{H} \cap \mathcal{G})$ and defined as:

$(\mathcal{H} \cap \mathcal{G}) = \{\langle u, \mathcal{T}_{\mathcal{H}} \cap_q \mathcal{T}_{\mathcal{G}}, \mathcal{I}_{\mathcal{H}} \cup_q \mathcal{I}_{\mathcal{G}}, \mathcal{F}_{\mathcal{H}} \cup_q \mathcal{F}_{\mathcal{G}}\rangle : u \in \mathcal{U}\}$, where $\cup_q$ is the T-spherical fuzzy union, $\cap_q$ is the T-spherical fuzzy intersection and

$$\mathcal{T}_{\mathcal{H}} \cap_q \mathcal{T}_{\mathcal{G}} = \Big(\big(\mu_{\mathcal{H},\mathcal{T}}(u) \wedge \mu_{\mathcal{G},\mathcal{T}}(u)\big), \big(\omega_{\mathcal{H},\mathcal{T}}(u) \vee \omega_{\mathcal{G},\mathcal{T}}(u)\big), \big(\nu_{\mathcal{H},\mathcal{T}}(u) \vee \nu_{\mathcal{G},\mathcal{T}}(u)\big)\Big),$$

$$\mathcal{I}_{\mathcal{H}} \cup_q \mathcal{I}_{\mathcal{G}} = \Big(\big(\mu_{\mathcal{H},\mathcal{I}}(u) \vee \mu_{\mathcal{G},\mathcal{I}}(u)\big), \big(\omega_{\mathcal{H},\mathcal{I}}(u) \wedge \omega_{\mathcal{G},\mathcal{I}}(u)\big), \big(\nu_{\mathcal{H},\mathcal{I}}(u) \wedge \nu_{\mathcal{G},\mathcal{I}}(u)\big)\Big),$$

$$\mathcal{F}_{\mathcal{H}} \cup_q \mathcal{F}_{\mathcal{G}} = \Big(\big(\mu_{\mathcal{H},\mathcal{F}}(u) \vee \mu_{\mathcal{G},\mathcal{F}}(u)\big), \big(\omega_{\mathcal{H},\mathcal{F}}(u) \wedge \omega_{\mathcal{G},\mathcal{F}}(u)\big), \big(\nu_{\mathcal{H},\mathcal{F}}(u) \wedge \nu_{\mathcal{G},\mathcal{F}}(u)\big)\Big).$$

$\vee = \max, \quad \wedge = \min.$

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Example 4.6. If $\hat{\mathcal{U}} = \{u_1, u_2\}$ such that

$\mathcal{H} = \{\langle u_1, (0.4, 0.5, 0.6), (0.1, 0.7, 0.5), (0.6, 0.7, 0.8)\rangle, \langle u_2, (0.3, 0.6, 0.9), (0.7, 0.8, 0.2), (0.7, 0.5, 0.8)\rangle\}$, and

$\mathcal{G} = \{\langle u_1, (0.2, 0.8, 0.5), (0.4, 0.8, 0.5), (0.8, 0.7, 0.1)\rangle, \langle u_2, (0.3, 0.4, 0.7), (0.1, 0.5, 0.8), (0.8, 0.5, 0.3)\rangle\}$ are two T-SFVNSs. Then,

1. $(\mathcal{H})^c = \{\langle u_1, (0.6, 0.7, 0.8), (0.5, 0.3, 0.1), (0.4, 0.5, 0.6)\rangle, \langle u_2, (0.7, 0.5, 0.8), (0.2, 0.2, 0.7), (0.3, 0.6, 0.9)\rangle\}$,
2. $(\mathcal{H} \cup \mathcal{G}) = \{\langle u_1, (0.4, 0.5, 0.5), (0.1, 0.8, 0.5), (0.6, 0.7, 0.8)\rangle, \langle u_2, (0.3, 0.4, 0.7), (0.1, 0.8, 0.8), (0.7, 0.5, 0.8)\rangle\}$,
3. $(\mathcal{H} \cap \mathcal{G}) = \{\langle u_1, (0.2, 0.8, 0.6), (0.4, 0.7, 0.5), (0.8, 0.7, 0.1)\rangle, \langle u_2, (0.3, 0.6, 0.9), (0.7, 0.5, 0.2), (0.8, 0.5, 0.3)\rangle\}$.

Proposition 4.7. *Let* $\mathcal{H} = \{\langle u, \mathcal{T}_\mathcal{H}, \mathcal{I}_\mathcal{H}, \mathcal{F}_\mathcal{H}\rangle : u \in \hat{\mathcal{U}}\}$, $\mathcal{G} = \{\langle u, \mathcal{T}_\mathcal{G}, \mathcal{I}_\mathcal{G}, \mathcal{F}_\mathcal{G}\rangle : u \in \hat{\mathcal{U}}\}$ *and* $\mathcal{K} = \{\langle u, \mathcal{T}_\mathcal{K}, \mathcal{I}_\mathcal{K}, \mathcal{F}_\mathcal{K}\rangle : u \in \hat{\mathcal{U}}\}$ *be three T-SFVNSs. Then, the following properties hold.*

1. $\mathcal{H} \cup \mathcal{G} = \mathcal{G} \cup \mathcal{H}, \quad \mathcal{H} \cap \mathcal{G} = \mathcal{G} \cap \mathcal{H}$,
2. $(\mathcal{H} \cup \mathcal{G}) \cup \mathcal{K} = \mathcal{H} \cup (\mathcal{G} \cup \mathcal{K}), \quad (\mathcal{H} \cap \mathcal{G}) \cap \mathcal{K} = \mathcal{H} \cap (\mathcal{G} \cap \mathcal{K})$,
3. $\mathcal{H} \cup (\mathcal{G} \cap \mathcal{K}) = (\mathcal{H} \cup \mathcal{G}) \cap (\mathcal{H} \cup \mathcal{K}), \quad \mathcal{H} \cap (\mathcal{G} \cup \mathcal{K}) = (\mathcal{H} \cap \mathcal{G}) \cup (\mathcal{H} \cap \mathcal{K})$,
4. $(\mathcal{H} \cup \mathcal{G})^c = (\mathcal{H})^c \cap (\mathcal{G})^c$,
5. $(\mathcal{H} \cap \mathcal{G})^c = (\mathcal{H})^c \cup (\mathcal{G})^c$.

Proof. We will prove property 4 and property 5 as the proof of the remaining properties is trivial.

4. For left side, we have $(\mathcal{H} \cup \mathcal{G}) = \{\langle u, \mathcal{T}_\mathcal{H} \cup_q \mathcal{T}_\mathcal{G}, \mathcal{I}_\mathcal{H} \cap_q \mathcal{I}_\mathcal{G}, \mathcal{F}_\mathcal{H} \cap_q \mathcal{F}_\mathcal{G}\rangle : u \in \mathcal{U}\}$. According to Definition 4.4, we have
$(\mathcal{H} \cup \mathcal{G})^c = \{\langle u, \mathcal{F}_\mathcal{H} \cap_q \mathcal{F}_\mathcal{G}, (\mathcal{I}_\mathcal{H} \cap_q \mathcal{I}_\mathcal{G})^{c_q}, \mathcal{T}_\mathcal{H} \cup_q \mathcal{T}_\mathcal{G}\rangle : u \in \mathcal{U}\}$
$= \{\langle u, \mathcal{F}_\mathcal{H} \cap_q \mathcal{F}_\mathcal{G}, (\mathcal{I}_\mathcal{H})^{c_q} \cup_q (\mathcal{I}_\mathcal{G})^{c_q}, \mathcal{T}_\mathcal{H} \cup_q \mathcal{T}_\mathcal{G}\rangle : u \in \mathcal{U}\}$,
$= (\mathcal{H})^c \cap (\mathcal{G})^c$
5. For left side, we have $(\mathcal{H} \cap \mathcal{G}) = \{\langle u, \mathcal{T}_\mathcal{H} \cap_q \mathcal{T}_\mathcal{G}, \mathcal{I}_\mathcal{H} \cup_q \mathcal{I}_\mathcal{G}, \mathcal{F}_\mathcal{H} \cup_q \mathcal{F}_\mathcal{G}\rangle : u \in \mathcal{U}\}$. According to Definition 4.5, we have
$(\mathcal{H} \cap \mathcal{G})^c = \{\langle u, \mathcal{F}_\mathcal{H} \cup_q \mathcal{F}_\mathcal{G}, (\mathcal{I}_\mathcal{H} \cup_q \mathcal{I}_\mathcal{G})^{c_q}, \mathcal{T}_\mathcal{H} \cap_q \mathcal{T}_\mathcal{G}\rangle : u \in \mathcal{U}\}$.
$= \{\langle u, \mathcal{F}_\mathcal{H} \cup_q \mathcal{F}_\mathcal{G}, (\mathcal{I}_\mathcal{H})^{c_q} \cup_q (\mathcal{I}_\mathcal{G})^{c_q}, \mathcal{T}_\mathcal{H} \cap_q \mathcal{T}_\mathcal{G}\rangle : u \in \mathcal{U}\}$,
$= (\mathcal{H})^c \cup (\mathcal{G})^c$.

□

5 Algebraic Operations for T-SFVNNs

In this section, we outline several algebraic operations for T-SFVNNs, building upon the algebraic operations of both SNS and T-SFS.

Definition 5.1. Let $\Gamma_1 = \langle ({}^1\mu_\mathcal{T}, {}^1\omega_\mathcal{T}, {}^1\nu_\mathcal{T}), ({}^1\mu_\mathcal{I}, {}^1\omega_\mathcal{I}, {}^1\nu_\mathcal{I}), ({}^1\mu_\mathcal{F}, {}^1\omega_\mathcal{F}, {}^1\nu_\mathcal{F})\rangle$ and $\Gamma_2 = \langle ({}^2\mu_\mathcal{T}, {}^2\omega_\mathcal{T}, {}^2\nu_\mathcal{T}), ({}^2\mu_\mathcal{I}, {}^2\omega_\mathcal{I}, {}^2\nu_\mathcal{I}), ({}^2\mu_\mathcal{F}, {}^2\omega_\mathcal{F}, {}^2\nu_\mathcal{F})\rangle$ be two T-SFVNNs over $\hat{\mathcal{U}}$ and $\Theta > 0$. Then,

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1. $\Gamma_1 \oplus \Gamma_2 = \Big\langle \big[(\,(\,^1\mu_\mathcal{T})^q + (\,^2\mu_\mathcal{T})^q - (\,^1\mu_\mathcal{T})^q\,(\,^2\mu_\mathcal{T})^q\,)^{\frac{1}{q}},\ ^1\omega_\mathcal{T}\,^2\omega_\mathcal{T},\ ^1\nu_\mathcal{T}\,^2\nu_\mathcal{T}\big], \big[\,^1\mu_\mathcal{I}\,^2\mu_\mathcal{I}, \big(\,(\,^1\omega_\mathcal{I})^q + (\,^2\omega_\mathcal{I})^q - (\,^1\omega_\mathcal{I})^q\,(\,^2\omega_\mathcal{I})^q\,)^{\frac{1}{q}}, \big(\,(\,^1\nu_\mathcal{I})^q + (\,^2\nu_\mathcal{I})^q - (\,^1\nu_\mathcal{I})^q\,(\,^2\nu_\mathcal{I})^q\,)^{\frac{1}{q}}\big], \big[\,^1\mu_\mathcal{F}\,^2\mu_\mathcal{F}, \big(\,(\,^1\omega_\mathcal{F})^q + (\,^2\omega_\mathcal{F})^q - (\,^1\omega_\mathcal{F})^q\,(\,^2\omega_\mathcal{F})^q\,)^{\frac{1}{q}}, \big(\,(\,^1\nu_\mathcal{F})^q + (\,^2\nu_\mathcal{F})^q - (\,^1\nu_\mathcal{F})^q\,(\,^2\nu_\mathcal{F})^q\,)^{\frac{1}{q}}\big]\Big\rangle$,

2. $\Gamma_1 \otimes \Gamma_2 = \Big\langle \big[\,^1\mu_\mathcal{T}\,^2\mu_\mathcal{T}, \big(\,(\,^1\omega_\mathcal{T})^q + (\,^2\omega_\mathcal{T})^q - (\,^1\omega_\mathcal{T})^q\,(\,^2\omega_\mathcal{T})^q\,)^{\frac{1}{q}}, \big(\,(\,^1\nu_\mathcal{T})^q + (\,^2\nu_\mathcal{T})^q - (\,^1\nu_\mathcal{T})^q\,(\,^2\nu_\mathcal{T})^q\,)^{\frac{1}{q}}\big], \big[(\,(\,^1\mu_\mathcal{I})^q + (\,^2\mu_\mathcal{I})^q - (\,^1\mu_\mathcal{I})^q\,(\,^2\mu_\mathcal{I})^q\,)^{\frac{1}{q}},\ ^1\omega_\mathcal{I}\,^2\omega_\mathcal{I},\ ^1\nu_\mathcal{I}\,^2\nu_\mathcal{I}\big], \big[(\,(\,^1\mu_\mathcal{F})^q + (\,^2\mu_\mathcal{F})^q - (\,^1\mu_\mathcal{F})^q\,(\,^2\mu_\mathcal{F})^q\,)^{\frac{1}{q}},\ ^1\omega_\mathcal{F}\,^2\omega_\mathcal{F},\ ^1\nu_\mathcal{F}\,^2\nu_\mathcal{F}\big]\Big\rangle$,

3. $\Theta\Gamma_1 = \Big\langle \Big((1-(1-(\,^1\mu_\mathcal{T})^q)^\Theta)^{\frac{1}{q}}, (\,^1\omega_\mathcal{T})^\Theta, (\,^1\nu_\mathcal{T})^\Theta\Big), \Big((\,^1\mu_\mathcal{I})^\Theta, (1-(1-(\,^1\omega_\mathcal{I})^q)^\Theta)^{\frac{1}{q}}, (1-(1-(\,^1\nu_\mathcal{I})^q)^\Theta)^{\frac{1}{q}}\Big), \Big((\,^1\mu_\mathcal{F})^\Theta, (1-(1-(\,^1\omega_\mathcal{F})^q)^\Theta)^{\frac{1}{q}}, (1-(1-(\,^1\nu_\mathcal{F})^q)^\Theta)^{\frac{1}{q}}\Big)\Big\rangle$,

4. $(\Gamma_1)^\Theta = \Big\langle \Big((\,^1\mu_\mathcal{T})^\Theta, (1-(1-(\,^1\omega_\mathcal{T})^q)^\Theta)^{\frac{1}{q}}, (1-(1-(\,^1\nu_\mathcal{T})^q)^\Theta)^{\frac{1}{q}}\Big), \Big((1-(1-(\,^1\mu_\mathcal{I})^q)^\Theta)^{\frac{1}{q}}, (\,^1\omega_\mathcal{I})^\Theta, (\,^1\nu_\mathcal{I})^\Theta\Big), \Big((1-(1-(\,^1\mu_\mathcal{F})^q)^\Theta)^{\frac{1}{q}}, (\,^1\omega_\mathcal{F})^\Theta, (\,^1\nu_\mathcal{F})^\Theta\Big)\Big\rangle$.

Example 5.2. Suppose $\Gamma_1 = \big\langle(0.4, 0.2, 0.9), (0.5, 0.8, 0.6), (0.3, 0.9, 0.5)\big\rangle$ and $\Gamma_2 = \big\langle(0.5, 0.7, 0.4), (0.3, 0.7, 0.2), (0.9, 0.4, 0.5)\big\rangle$ are two 3-SFVNNs and $\Theta = 5$. Then,

1. $\Gamma_1 \oplus \Gamma_2 = \big\langle(0.57, 0.14, 0.36), (0.15, 0.88, 0.61), (0.27, 0.91, 0.62)\big\rangle$,
2. $\Gamma_1 \otimes \Gamma_2 = \big\langle(0.20, 0.70, 0.91), (0.53, 0.56, 0.12), (0.90, 0.36, 0.25)\big\rangle$,
3. $\Theta\Gamma_1 = \big\langle(0.66, 0, 0.59), (0.03, 0.99, 0.89), (0, 0.99, 0.79)\big\rangle$,
4. $(\Gamma_1)^\Theta = \big\langle(0.01, 0.34, 0.99), (0.79, 0.33, 0.08), (0.50, 0.59, 0.03)\big\rangle$.

Proposition 5.3. *Let* $\Gamma_1 = \big\langle(\,^1\mu_\mathcal{T},\ ^1\omega_\mathcal{T},\ ^1\nu_\mathcal{T}), (\,^1\mu_\mathcal{I},\ ^1\omega_\mathcal{I},\ ^1\nu_\mathcal{I}), (\,^1\mu_\mathcal{F},\ ^1\omega_\mathcal{F},\ ^1\nu_\mathcal{F})\big\rangle$, $\Gamma_2 = \big\langle(\,^2\mu_\mathcal{T},\ ^2\omega_\mathcal{T},\ ^2\nu_\mathcal{T}), (\,^2\mu_\mathcal{I},\ ^2\omega_\mathcal{I},\ ^2\nu_\mathcal{I}), (\,^2\mu_\mathcal{F},\ ^2\omega_\mathcal{F},\ ^2\nu_\mathcal{F})\big\rangle$ *and* $\Gamma_3 = \big\langle(\,^3\mu_\mathcal{T},\ ^3\omega_\mathcal{T},\ ^3\nu_\mathcal{T}), (\,^3\mu_\mathcal{I},\ ^3\omega_\mathcal{I},\ ^3\nu_\mathcal{I}), (\,^3\mu_\mathcal{F},\ ^3\omega_\mathcal{F},\ ^3\nu_\mathcal{F})\big\rangle$ *be three T-SFVNNs over* $\hat{\mathcal{U}}$ *and* $\Theta > 0$. *Then the following properties hold.*

1. $\Gamma_1 \oplus \Gamma_2 = \Gamma_2 \oplus \Gamma_1, \quad \Gamma_1 \otimes \Gamma_2 = \Gamma_2 \otimes \Gamma_1$,
2. $(\Gamma_1 \oplus \Gamma_2) \oplus \Gamma_3 = \Gamma_1 \oplus (\Gamma_2 \oplus \Gamma_3), \quad (\Gamma_1 \otimes \Gamma_2) \otimes \Gamma_3 = \Gamma_1 \otimes (\Gamma_2 \otimes \Gamma_3)$,
3. $\Theta(\Gamma_1 \oplus \Gamma_2) = \Theta\Gamma_1 \oplus \Theta\Gamma_2$,
4. $(\Gamma_1 \otimes \Gamma_2)^\Theta = \Gamma_1^\Theta \otimes \Gamma_2^\Theta$.

Proof. We will prove property 3 and property 4 as the proof of the remaining properties is trivial.

3. Based on Definition 5.1 (item 1 and item 3), we have for the right side of the equation

$\Theta(\Gamma_1 \oplus \Gamma_2) = \Theta\Big(\Big\langle \big[(\,(\,^1\mu_\mathcal{T})^q + (\,^2\mu_\mathcal{T})^q - (\,^1\mu_\mathcal{T})^q\,(\,^2\mu_\mathcal{T})^q\,)^{\frac{1}{q}},\ ^1\omega_\mathcal{T}\,^2\omega_\mathcal{T},\ ^1\nu_\mathcal{T}\,^2\nu_\mathcal{T}\big], \big[\,^1\mu_\mathcal{I}\,^2\mu_\mathcal{I}, \big(\,(\,^1\omega_\mathcal{I})^q + (\,^2\omega_\mathcal{I})^q - (\,^1\omega_\mathcal{I})^q\,(\,^2\omega_\mathcal{I})^q\,)^{\frac{1}{q}}, \big(\,(\,^1\nu_\mathcal{I})^q + (\,^2\nu_\mathcal{I})^q - (\,^1\nu_\mathcal{I})^q\,(\,^2\nu_\mathcal{I})^q\,)^{\frac{1}{q}}\big], \big[\,^1\mu_\mathcal{F}\,^2\mu_\mathcal{F}, \big(\,(\,^1\omega_\mathcal{F})^q + (\,^2\omega_\mathcal{F})^q - (\,^1\omega_\mathcal{F})^q\,(\,^2\omega_\mathcal{F})^q\,)^{\frac{1}{q}}, \big(\,(\,^1\nu_\mathcal{F})^q + (\,^2\nu_\mathcal{F})^q - (\,^1\nu_\mathcal{F})^q\,(\,^2\nu_\mathcal{F})^q\,)^{\frac{1}{q}}\big]\Big\rangle\Big)$,

$= \Big\langle\Big(\Big[1-\big[1-[(\,(\,^1\mu_\mathcal{T})^q + (\,^2\mu_\mathcal{T})^q - (\,^1\mu_\mathcal{T})^q\,(\,^2\mu_\mathcal{T})^q\,)^{\frac{1}{q}}]^q\big]^\Theta\Big]^{\frac{1}{q}}, (\,^1\omega_\mathcal{T})^\Theta(\,^2\omega_\mathcal{T})^\Theta, (\,^1\nu_\mathcal{T})^\Theta(\,^2\nu_\mathcal{T})^\Theta\Big),$
$\Big((\,^1\mu_\mathcal{I})^\Theta(\,^2\mu_\mathcal{I})^\Theta, \Big[1-\big[1-[(\,(\,^1\omega_\mathcal{I})^q + (\,^2\omega_\mathcal{I})^q - (\,^1\omega_\mathcal{I})^q\,(\,^2\omega_\mathcal{I})^q\,)^{\frac{1}{q}}]^q\big]^\Theta\Big]^{\frac{1}{q}},$
$\Big[1-\big[1-[(\,(\,^1\nu_\mathcal{I})^q + (\,^2\nu_\mathcal{I})^q - (\,^1\nu_\mathcal{I})^q\,(\,^2\nu_\mathcal{I})^q\,)^{\frac{1}{q}}]^q\big]^\Theta\Big]^{\frac{1}{q}}\Big), \Big((\,^1\mu_\mathcal{F})^\Theta(\,^2\mu_\mathcal{F})^\Theta, \Big[1-\big[1-[(\,(\,^1\omega_\mathcal{F})^q + (\,^2\omega_\mathcal{F})^q - (\,^1\omega_\mathcal{F})^q\,(\,^2\omega_\mathcal{F})^q\,)^{\frac{1}{q}}]^q\big]^\Theta\Big]^{\frac{1}{q}}, \Big[1-\big[1-[(\,(\,^1\nu_\mathcal{F})^q + (\,^2\nu_\mathcal{F})^q - (\,^1\nu_\mathcal{F})^q\,(\,^2\nu_\mathcal{F})^q\,)^{\frac{1}{q}}]^q\big]^\Theta\Big]^{\frac{1}{q}}\Big)\Big\rangle$,

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$$= \Big\langle \Big(\Big[1 - \big[1 - [\,({}^1\mu_{\mathcal{T}})^q + ({}^2\mu_{\mathcal{T}})^q - ({}^1\mu_{\mathcal{T}})^q\,({}^2\mu_{\mathcal{T}})^q\,]\big]^{\Theta}\Big]^{\frac{1}{q}}, ({}^1\omega_{\mathcal{T}})^{\Theta}({}^2\omega_{\mathcal{T}})^{\Theta}, ({}^1\nu_{\mathcal{T}})^{\Theta}({}^2\nu_{\mathcal{T}})^{\Theta}\Big),$$
$$\Big(({}^1\mu_{\mathcal{I}})^{\Theta}({}^2\mu_{\mathcal{I}})^{\Theta}, \Big[1 - \big[1 - [\,({}^1\omega_{\mathcal{I}})^q + ({}^2\omega_{\mathcal{I}})^q - ({}^1\omega_{\mathcal{I}})^q\,({}^2\omega_{\mathcal{I}})^q\,]\big]^{\Theta}\Big]^{\frac{1}{q}},$$
$$\Big[1 - \big[1 - [\,({}^1\nu_{\mathcal{I}})^q + ({}^2\nu_{\mathcal{I}})^q - ({}^1\nu_{\mathcal{I}})^q\,({}^2\nu_{\mathcal{I}})^q\,]\big]^{\Theta}\Big]^{\frac{1}{q}}\Big), \Big(({}^1\mu_{\mathcal{F}})^{\Theta}({}^2\mu_{\mathcal{F}})^{\Theta}, \Big[1 - \big[1 - [\,({}^1\omega_{\mathcal{F}})^q +$$
$$({}^2\omega_{\mathcal{F}})^q - ({}^1\omega_{\mathcal{F}})^q\,({}^2\omega_{\mathcal{F}})^q\,]\big]^{\Theta}\Big]^{\frac{1}{q}}, \Big[1 - \big[1 - [\,({}^1\nu_{\mathcal{F}})^q + ({}^2\nu_{\mathcal{F}})^q - ({}^1\nu_{\mathcal{F}})^q\,({}^2\nu_{\mathcal{F}})^q\,]\big]^{\Theta}\Big]^{\frac{1}{q}}\Big)\Big\rangle,$$

$$= \Big\langle \Big([1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta}\,(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta}]^{\frac{1}{q}}, ({}^1\omega_{\mathcal{T}})^{\Theta}\,({}^2\omega_{\mathcal{T}})^{\Theta}, ({}^1\nu_{\mathcal{T}})^{\Theta}\,({}^2\nu_{\mathcal{T}})^{\Theta}\Big), \Big(({}^1\mu_{\mathcal{I}})^{\Theta}\,({}^2\mu_{\mathcal{I}})^{\Theta}, [1-$$
$$(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta}\,(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta}]^{\frac{1}{q}}, [1-(1-({}^1\nu_{\mathcal{I}})^q)^{\Theta}\,(1-({}^2\nu_{\mathcal{I}})^q)^{\Theta}]^{\frac{1}{q}}\Big), \Big(({}^1\mu_{\mathcal{F}})^{\Theta}\,({}^2\mu_{\mathcal{F}})^{\Theta}, [1-$$
$$(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta}\,(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta}]^{\frac{1}{q}}, [1-(1-({}^1\nu_{\mathcal{F}})^q)^{\Theta}\,(1-({}^2\nu_{\mathcal{F}})^q)^{\Theta}]^{\frac{1}{q}}\Big)\Big\rangle.$$

For the right side of the equation, we have,

$$\Theta\Gamma_1 = \Big\langle \Big((1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta})^{\frac{1}{q}}, ({}^1\omega_{\mathcal{T}})^{\Theta}, ({}^1\nu_{\mathcal{T}})^{\Theta}\Big), \Big(({}^1\mu_{\mathcal{I}})^{\Theta}, (1-(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}, (1-(1-$$
$$({}^1\nu_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\Big), \Big(({}^1\mu_{\mathcal{F}})^{\Theta}, (1-(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}, (1-(1-({}^1\nu_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\Big)\Big\rangle,$$

$$\Theta\Gamma_2 = \Big\langle \Big((1-(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta})^{\frac{1}{q}}, ({}^2\omega_{\mathcal{T}})^{\Theta}, ({}^2\nu_{\mathcal{T}})^{\Theta}\Big), \Big(({}^2\mu_{\mathcal{I}})^{\Theta}, (1-(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}, (1-(1-$$
$$({}^2\nu_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\Big), \Big(({}^2\mu_{\mathcal{F}})^{\Theta}, (1-(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}, (1-(1-({}^2\nu_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\Big)\Big\rangle,$$

$$\Theta\Gamma_1 \oplus \Theta\Gamma_2 = \Big\langle \Big(\Big(\big[(1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta})^{\frac{1}{q}}\big]^q + \big[(1-(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta})^{\frac{1}{q}}\big]^q - \big[(1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\big[(1-$$
$$(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\Big)^{\frac{1}{q}}, ({}^1\omega_{\mathcal{T}})^{\Theta}({}^2\omega_{\mathcal{T}})^{\Theta}, ({}^1\nu_{\mathcal{T}})^{\Theta}({}^2\nu_{\mathcal{T}})^{\Theta}\Big), \Big(({}^1\mu_{\mathcal{I}})^{\Theta}({}^2\mu_{\mathcal{I}})^{\Theta}, \Big(\big[(1-(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q +$$
$$\big[(1-(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q - \big[(1-(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\big[(1-(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\Big)^{\frac{1}{q}}, \Big(\big[(1-(1-$$
$$({}^1\nu_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q + \big[(1-(1-({}^2\nu_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q - \big[(1-(1-({}^1\nu_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\big[(1-(1-({}^2\nu_{\mathcal{I}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\Big)^{\frac{1}{q}}\Big),$$
$$\Big(({}^1\mu_{\mathcal{F}})^{\Theta}({}^2\mu_{\mathcal{F}})^{\Theta}, \Big(\big[(1-(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q + \big[(1-(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q - \big[(1-(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\big[(1-$$
$$(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\Big)^{\frac{1}{q}}, \Big(\big[(1-(1-({}^1\nu_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q + \big[(1-(1-({}^2\nu_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q - \big[(1-(1-$$
$$({}^1\nu_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\big[(1-(1-({}^2\nu_{\mathcal{F}})^q)^{\Theta})^{\frac{1}{q}}\big]^q\Big)^{\frac{1}{q}}\Big)\Big\rangle,$$

$$= \Big\langle \Big(\big(1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta} + 1-(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta} - \big[1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta}\big]\big[1-(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta}\big]\big)^{\frac{1}{q}}, ({}^1\omega_{\mathcal{T}})^{\Theta}({}^2\omega_{\mathcal{T}})^{\Theta},$$
$$({}^1\nu_{\mathcal{T}})^{\Theta}({}^2\nu_{\mathcal{T}})^{\Theta}\Big), \Big(({}^1\mu_{\mathcal{I}})^{\Theta}({}^2\mu_{\mathcal{I}})^{\Theta}, \big(1-(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta} + 1-(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta} - \big[1-(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta}\big]\big[1-$$
$$(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta}\big]\big)^{\frac{1}{q}}, \big(1-(1-({}^1\nu_{\mathcal{I}})^q)^{\Theta} + 1-(1-({}^2\nu_{\mathcal{I}})^q)^{\Theta} - \big[1-(1-({}^1\nu_{\mathcal{I}})^q)^{\Theta}\big]\big[1-(1-$$
$$({}^2\nu_{\mathcal{I}})^q)^{\Theta}\big]\big)^{\frac{1}{q}}\Big), \Big(({}^1\mu_{\mathcal{F}})^{\Theta}({}^2\mu_{\mathcal{F}})^{\Theta}, \big(1-(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta} + 1-(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta} - \big[1-(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta}\big]\big[1-$$
$$(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta}\big]\big)^{\frac{1}{q}}, \big(1-(1-({}^1\nu_{\mathcal{F}})^q)^{\Theta} + 1-(1-({}^2\nu_{\mathcal{F}})^q)^{\Theta} - \big[1-(1-({}^1\nu_{\mathcal{F}})^q)^{\Theta}\big]\big[1-(1-$$
$$({}^2\nu_{\mathcal{F}})^q)^{\Theta}\big]\big)^{\frac{1}{q}}\Big)\Big\rangle,$$

$$= \Big\langle \Big([1-(1-({}^1\mu_{\mathcal{T}})^q)^{\Theta}\,(1-({}^2\mu_{\mathcal{T}})^q)^{\Theta}]^{\frac{1}{q}}, ({}^1\omega_{\mathcal{T}})^{\Theta}\,({}^2\omega_{\mathcal{T}})^{\Theta}, ({}^1\nu_{\mathcal{T}})^{\Theta}\,({}^2\nu_{\mathcal{T}})^{\Theta}\Big), \Big(({}^1\mu_{\mathcal{I}})^{\Theta}\,({}^2\mu_{\mathcal{I}})^{\Theta}, [1-$$
$$(1-({}^1\omega_{\mathcal{I}})^q)^{\Theta}\,(1-({}^2\omega_{\mathcal{I}})^q)^{\Theta}]^{\frac{1}{q}}, [1-(1-({}^1\nu_{\mathcal{I}})^q)^{\Theta}\,(1-({}^2\nu_{\mathcal{I}})^q)^{\Theta}]^{\frac{1}{q}}\Big), \Big(({}^1\mu_{\mathcal{F}})^{\Theta}\,({}^2\mu_{\mathcal{F}})^{\Theta}, [1-$$
$$(1-({}^1\omega_{\mathcal{F}})^q)^{\Theta}\,(1-({}^2\omega_{\mathcal{F}})^q)^{\Theta}]^{\frac{1}{q}}, [1-(1-({}^1\nu_{\mathcal{F}})^q)^{\Theta}\,(1-({}^2\nu_{\mathcal{F}})^q)^{\Theta}]^{\frac{1}{q}}\Big)\Big\rangle.$$

Thus, the right side of the equation equal the left side, which proves that $\Theta(\Gamma_1 \oplus \Gamma_2) = \Theta\Gamma_1 \oplus \Theta\Gamma_2$.

4. Based on Definition 5.1 (item 2 and item 4), we have for the right side of the equation

$$(\Gamma_1 \otimes \Gamma_2)^{\Theta} = \Big(\Big\langle\Big({}^1\mu_{\mathcal{T}}\,{}^2\mu_{\mathcal{T}}, \big(({}^1\omega_{\mathcal{T}})^q + ({}^2\omega_{\mathcal{T}})^q - ({}^1\omega_{\mathcal{T}})^q\,({}^2\omega_{\mathcal{T}})^q\big)^{\frac{1}{q}}, \big(({}^1\nu_{\mathcal{T}})^q + ({}^2\nu_{\mathcal{T}})^q -$$
$$({}^1\nu_{\mathcal{T}})^q\,({}^2\nu_{\mathcal{T}})^q\big)^{\frac{1}{q}}\Big), \Big(\big(({}^1\mu_{\mathcal{I}})^q + ({}^2\mu_{\mathcal{I}})^q - ({}^1\mu_{\mathcal{I}})^q\,({}^2\mu_{\mathcal{I}})^q\big)^{\frac{1}{q}}, {}^1\omega_{\mathcal{I}}\,{}^2\omega_{\mathcal{I}}, {}^1\nu_{\mathcal{I}}\,{}^2\nu_{\mathcal{I}}\Big), \Big(\big(({}^1\mu_{\mathcal{F}})^q +$$
$$({}^2\mu_{\mathcal{F}})^q - ({}^1\mu_{\mathcal{F}})^q\,({}^2\mu_{\mathcal{F}})^q\big)^{\frac{1}{q}}, {}^1\omega_{\mathcal{F}}\,{}^2\omega_{\mathcal{F}}, {}^1\nu_{\mathcal{F}}\,{}^2\nu_{\mathcal{F}}\Big)\Big\rangle\Big)^{\Theta},$$

$$= \Big\langle\Big(({}^1\mu_{\mathcal{T}})^{\Theta}({}^2\mu_{\mathcal{T}})^{\Theta}, \Big[1-\big[1-[(({}^1\omega_{\mathcal{T}})^q + ({}^2\omega_{\mathcal{T}})^q - ({}^1\omega_{\mathcal{T}})^q\,({}^2\omega_{\mathcal{T}})^q)^{\frac{1}{q}}]^q\big]^{\Theta}\Big]^{\frac{1}{q}}, \Big[1-\big[1-[(({}^1\nu_{\mathcal{T}})^q +$$
$$({}^2\nu_{\mathcal{T}})^q - ({}^1\nu_{\mathcal{T}})^q\,({}^2\nu_{\mathcal{T}})^q)^{\frac{1}{q}}]^q\big]^{\Theta}\Big]^{\frac{1}{q}}\Big), \Big(\Big[1-\big[1-[(({}^1\mu_{\mathcal{I}})^q + ({}^2\mu_{\mathcal{I}})^q - ({}^1\mu_{\mathcal{I}})^q\,({}^2\mu_{\mathcal{I}})^q)^{\frac{1}{q}}]^q\big]^{\Theta}\Big]^{\frac{1}{q}},$$

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$$({}^{1}\omega_{\mathcal{I}})^{\Theta}({}^{2}\omega_{\mathcal{I}})^{\Theta}, ({}^{1}\nu_{\mathcal{I}})^{\Theta}({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big), \Big(\Big[1-\big[1-[(({}^{1}\mu_{\mathcal{F}})^{q}+({}^{2}\mu_{\mathcal{F}})^{q}-({}^{1}\mu_{\mathcal{F}})^{q}({}^{2}\mu_{\mathcal{F}})^{q})^{\frac{1}{q}}]^{q}\big]^{\Theta}\Big]^{\frac{1}{q}},$$
$$({}^{1}\omega_{\mathcal{F}})^{\Theta}({}^{2}\omega_{\mathcal{F}})^{\Theta}, ({}^{1}\nu_{\mathcal{F}})^{\Theta}({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle,$$

$$= \Big\langle\Big(({}^{1}\mu_{\mathcal{T}})^{\Theta}({}^{2}\mu_{\mathcal{T}})^{\Theta}, \Big[1-\big[1-[({}^{1}\omega_{\mathcal{T}})^{q}+({}^{2}\omega_{\mathcal{T}})^{q}-({}^{1}\omega_{\mathcal{T}})^{q}({}^{2}\omega_{\mathcal{T}})^{q}]\big]^{\Theta}\Big]^{\frac{1}{q}}, \Big[1-\big[1-[({}^{1}\nu_{\mathcal{T}})^{q}+$$
$$({}^{2}\nu_{\mathcal{T}})^{q}-({}^{1}\nu_{\mathcal{T}})^{q}({}^{2}\nu_{\mathcal{T}})^{q}]\big]^{\Theta}\Big]^{\frac{1}{q}}\Big), \Big(\Big[1-\big[1-[({}^{1}\mu_{\mathcal{I}})^{q}+({}^{2}\mu_{\mathcal{I}})^{q}-({}^{1}\mu_{\mathcal{I}})^{q}({}^{2}\mu_{\mathcal{I}})^{q}]\big]^{\Theta}\Big]^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{I}})^{\Theta}({}^{2}\omega_{\mathcal{I}})^{\Theta},$$
$$({}^{1}\nu_{\mathcal{I}})^{\Theta}({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big), \Big(\Big[1-\big[1-[({}^{1}\mu_{\mathcal{F}})^{q}+({}^{2}\mu_{\mathcal{F}})^{q}-({}^{1}\mu_{\mathcal{F}})^{q}({}^{2}\mu_{\mathcal{F}})^{q}]\big]^{\Theta}\Big]^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{F}})^{\Theta}({}^{2}\omega_{\mathcal{F}})^{\Theta},$$
$$({}^{1}\nu_{\mathcal{F}})^{\Theta}({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle,$$

$$= \Big\langle \Big(({}^{1}\mu_{\mathcal{T}})^{\Theta}({}^{2}\mu_{\mathcal{T}})^{\Theta}, [1-(1-({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta}(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta}]^{\frac{1}{q}}, [1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta}(1-$$
$$({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta}]^{\frac{1}{q}}\Big), \Big([1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta}(1-({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta}]^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{I}})^{\Theta}({}^{2}\omega_{\mathcal{I}})^{\Theta}, ({}^{1}\nu_{\mathcal{I}})^{\Theta}({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big), \Big([1-$$
$$(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta}(1-({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta}]^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{F}})^{\Theta}({}^{2}\omega_{\mathcal{F}})^{\Theta}, ({}^{1}\nu_{\mathcal{F}})^{\Theta}({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle.$$

For the right side of the equation, we have,

$$(\Gamma_1)^{\Theta} = \Big\langle\Big(({}^{1}\mu_{\mathcal{T}})^{\Theta}, (1-(1-({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}, (1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\Big), \Big((1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta})^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{I}})^{\Theta},$$
$$({}^{1}\nu_{\mathcal{I}})^{\Theta}\Big), \Big((1-(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta})^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{F}})^{\Theta}, ({}^{1}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle,$$

$$(\Gamma_2)^{\Theta} = \Big\langle\Big(({}^{2}\mu_{\mathcal{T}})^{\Theta}, (1-(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}, (1-(1-({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\Big), \Big((1-(1-({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta})^{\frac{1}{q}}, ({}^{2}\omega_{\mathcal{I}})^{\Theta},$$
$$({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big), \Big((1-(1-({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta})^{\frac{1}{q}}, ({}^{2}\omega_{\mathcal{F}})^{\Theta}, ({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle,$$

$$\Gamma_1^{\Theta} \otimes \Gamma_2^{\Theta} = \Big\langle \Big(({}^{1}\mu_{\mathcal{T}})^{\Theta}({}^{2}\mu_{\mathcal{T}})^{\Theta}, \Big(\big[(1-(1-({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} + \big[(1-(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} - \big[(1-(1-$$
$$({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\big[(1-(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\Big)^{\frac{1}{q}}, \Big(\big[(1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} + \big[(1-(1-({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} -$$
$$\big[(1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\big[(1-(1-({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\Big)^{\frac{1}{q}}\Big), \Big(\Big(\big[(1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} + \big[(1-(1-$$
$$({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} - \big[(1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\big[(1-(1-({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\Big)^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{I}})^{\Theta}({}^{2}\omega_{\mathcal{I}})^{\Theta}, ({}^{1}\nu_{\mathcal{I}})^{\Theta}({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big),$$
$$\Big(\Big(\big[(1-(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} + \big[(1-(1-({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q} - \big[(1-(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\big[(1-(1-$$
$$({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta})^{\frac{1}{q}}\big]^{q}\Big)^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{F}})^{\Theta}({}^{2}\omega_{\mathcal{F}})^{\Theta}, ({}^{1}\nu_{\mathcal{F}})^{\Theta}({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle,$$

$$= \Big\langle \Big(({}^{1}\mu_{\mathcal{T}})^{\Theta}({}^{2}\mu_{\mathcal{T}})^{\Theta}, \big(1-(1-({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta} + 1-(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta} - \big[1-(1-({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta}\big]\big[1-$$
$$(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta}\big]\big)^{\frac{1}{q}}, \big(1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta} + 1-(1-({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta} - \big[1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta}\big]\big[1-(1-$$
$$({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta}\big]\big)^{\frac{1}{q}}\Big), \Big(\big(1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta} + 1-(1-({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta} - \big[1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta}\big]\big[1-(1-$$
$$({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta}\big]\big)^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{I}})^{\Theta}({}^{2}\omega_{\mathcal{I}})^{\Theta}, ({}^{1}\nu_{\mathcal{I}})^{\Theta}({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big), \Big(\big(1-(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta} + 1-(1-({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta} -$$
$$\big[1-(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta}\big]\big[1-(1-({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta}\big]\big)^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{F}})^{\Theta}({}^{2}\omega_{\mathcal{F}})^{\Theta}, ({}^{1}\nu_{\mathcal{F}})^{\Theta}({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle,$$

$$= \Big\langle \Big(({}^{1}\mu_{\mathcal{T}})^{\Theta}({}^{2}\mu_{\mathcal{T}})^{\Theta}, [1-(1-({}^{1}\omega_{\mathcal{T}})^{q})^{\Theta}(1-({}^{2}\omega_{\mathcal{T}})^{q})^{\Theta}]^{\frac{1}{q}}, [1-(1-({}^{1}\nu_{\mathcal{T}})^{q})^{\Theta}(1-$$
$$({}^{2}\nu_{\mathcal{T}})^{q})^{\Theta}]^{\frac{1}{q}}\Big), \Big([1-(1-({}^{1}\mu_{\mathcal{I}})^{q})^{\Theta}(1-({}^{2}\mu_{\mathcal{I}})^{q})^{\Theta}]^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{I}})^{\Theta}({}^{2}\omega_{\mathcal{I}})^{\Theta}, ({}^{1}\nu_{\mathcal{I}})^{\Theta}({}^{2}\nu_{\mathcal{I}})^{\Theta}\Big), \Big([1-$$
$$(1-({}^{1}\mu_{\mathcal{F}})^{q})^{\Theta}(1-({}^{2}\mu_{\mathcal{F}})^{q})^{\Theta}]^{\frac{1}{q}}, ({}^{1}\omega_{\mathcal{F}})^{\Theta}({}^{2}\omega_{\mathcal{F}})^{\Theta}, ({}^{1}\nu_{\mathcal{F}})^{\Theta}({}^{2}\nu_{\mathcal{F}})^{\Theta}\Big)\Big\rangle.$$

This proves that $(\Gamma_1 \otimes \Gamma_2)^{\Theta} = \Gamma_1^{\Theta} \otimes \Gamma_2^{\Theta}$.

□

6 Conclusion

This manuscript introduces the groundbreaking theory of T-SFVNS, which exhibits a remarkable capacity to encompass and generalize existing methodologies. T-SFVNS represents a significant advancement, offering a more precise representation of indeterminate information and enabling the simulation of complex decision-making scenarios through the strategic incorporation of the novel T-SFS model into the construction of SNS.

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The manuscript provides a formal definition of T-SFVNS and establishes operational laws within the T-SFVNS framework. Rigorous verification of the properties inherent to these operators is conducted. Additionally, various types of score and accuracy functions are defined to compare T-SFVNNs. The algebraic operations of the proposed model are thoroughly discussed, accompanied by supporting proofs. To further expand the scope of this research, future investigations should explore aggregation operators within the proposed model and their application in solving complex decision-making problems, see.[21–27]

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