Mathematical Morphological Operations for Quadri -Partitioned Neutrosophic Set

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Abstract
This research aims to introduce a novel notion in mathematical morphological operations on a quadri-partitioned neutrosophic set, which is a particular case of the neutrosophic refined set. In neutrosophic theory, the set is divided into three parts: the true set, the false set, and the indeterminancy set. The indeterminacy is studied in depth in quadri neutrosophic. The primary intention is to reduce uncertainty. The suggested study extracts the core concepts of morphological operations and explains their algebraic properties. Some features of morphological operators linked to quadri-partitioned neutrosophic sets are also derived.

Keywords: Mathematical morphological operations; Quadri partitioned neutrosophic set.

1. Introduction

F. Smarandache created the neutrosophic set theory, which is applied in a variety of domains such as image processing, decision technology, algebraic structures, and more [6]. Neutrosophy introduces a novel concept that represents uncertainty about an eventand can handle some concerns that fuzzy logic cannot [2]. Using neutrosophic set theory, Eman.M. El-Nakeeb et al. established the essential idea of mathematical morphological operations. Basic concepts and algebraic properties are also covered. Chatterjee et al. (2016) introduced the quadri-partitioned neutrosophic set as a idea. It is an example of a neutrosophically polished set. The membership function of truth (T), the membership function of contradiction (C), the membership function of ignorance (U), and the membership function of falsity (F) define the quadr-partitioned neutrosophic set. This work presents a novel idea of mathematical morphological procedures based on a quadripartitioned neutrosophic set. The fundamental definitions are based on quadri-partitioned neutrosophic set theory. The algebraic properties are also discussed. Mathematics is increasingly being used in biology, genetics, and medicine to depict and grasp complex biological processes. Mathematical modelling facilitates the analysis of biological processes, the simulation of disease propagation, the study of population dynamics, and the development of treatment programmes. Statistics and probability theory are also used in data analysis and experimental design. In general, the mathematics notion in morphology works with shapes through the use of mathematical set theory. Georges Matheron and Jean Serra pioneered mathematical morphology in image processing in 1964. In image processing, the core morphological operators are erosion, dialation, opening, and closing [1]. Given that morphology is the study of shapes, the fundamental focus of mathematical morphology is the mathematical concept of using set theory to describe shapes. The core morphological operators dilation, erosion, opening, and closing comprise the principles of this image processing paradigm. [2]. A morphological operator transforms one image into another by using a user-selectable structural element. Morphology differs from typical linear image processing because its basic operations are non-linear and use a different form of algebra than linear

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algebra [3]. Initially, the theory was centred solely on the concept of sets and operations, which were designed particularly for binary cases. The theory of lattices was eventually enlarged to include grayscale images as a result of its progress, and as a result, an image processing illustration theory was provided in [4]. Mathematical morphology is still a challenging subject to study [5]. In this work morphological operators are discussed for the quadripartitioned neutrosophic sets.

2. Preliminaries

Definition 2.1 Neutrosophic Dilation The two sets of neutrosophic sets will be X and Y. When the neutrosophic set's dilation is given as [2],

$X \oplus Y = \langle T_{X\oplus Y}, I_{X\oplus Y}, F_{X\oplus Y} \rangle$ where for the each $a_1, a_2 \in Z^2$

$$T_{X\oplus Y}(a_2) = \operatorname*{Sup}_{a_1\in Z^2} \min\big(T_X(a_2+a_1), T_Y(a_1)\big)$$
$$I_{X\oplus Y}(a_2) = \operatorname*{Sup}_{a_1\in Z^2} \min\big(I_X(a_2+a_1), I_Y(a_1)\big)$$
$$F_{X\oplus Y}(a_2) = \inf_{a_1\in Z^2} \max\big(1-F_X(a_2+a_1), 1-F_Y(a_1)\big)$$

Definition 2.2 Neutrosophic Erosion Let X and Y be the two neutrosophic sets. Then the erosion of the neutrosophic set is defined as [2].

$X \ominus Y = \langle T_{X\ominus Y}, I_{X\ominus Y}, F_{X\ominus Y} \rangle$ where for the each $a_1, a_2 \in Z^2$

$$T_{X\ominus Y}(a_2) = \inf_{a_1\in Z^2} \max\big(T_X(a_2+a_1), 1-T_Y(a_1)\big)$$
$$I_{X\ominus Y}(a_2) = \inf_{a_1\in Z^2} \max\big(I_x(a_2+a_1), 1-I_Y(a_1)\big)$$
$$F_{X\ominus Y}(a_2) = \sup_{a_1\in Z^2} \min\big(1-F_X(a_2+a_1), F_Y(a_1)\big)$$

Definition 2.3 Neutrosophic Opening Let X and Y should be the two neutrosophic sets.Then its opening operation is defined as [2]

$X \circ Y = \langle T_{X\circ Y}, I_{X\circ Y}, F_{X\circ Y} \rangle$ where $a_1, a_2, a_3 \in Z^2$

$$T_{X\circ Y}(a_2) = \sup_{a_1\in Z^2} \min\left(\inf_{Z\in R^n} \max\left(T_X(a_2-a_1+a_3), 1-T_Y(a_3)\right), T_Y(a_1)\right)$$
$$I_{X\circ Y}(a_2) = \sup_{a_1\in Z^2} \min\left(\inf_{Z\in R^n} \max\left(I_X(a_2-a_1+a_3), 1-I_Y(c)\right), I_Y(a_1)\right)$$
$$F_{X\circ Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{Z\in R^n} \min\left(1-F_X(a_2-a_1+a_3), F_Y(a_1)\right), 1-F_Y(a_1)\right)$$

Definition 2.4 Neutrosophic Closing . Let X and Y be the two neutrosophic sets. Then its closing operation is defined as [2].

$X * Y = \langle T_{X*Y}, I_{X*Y}, F_{X*Y} \rangle$ where $a_1, a_2, a_3 \in Z^2$

$$T_{X*Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{z\in R^n} \min\left(T_X(a_2-a_1+a_3), T_Y(a_3)\right), 1-T_Y(a_1)\right)$$
$$I_{X*Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{z\in R^n} \min\left(I_X(a_2-a_1+a_3), I_Y(a_3)\right), 1-I_Y(a_1)\right)$$
$$F_{X*Y}(a_2) = \sup_{a_1\in Z^2} \min\left(\inf_{z\in R^n} \max\left(1-F_X(a_2-a_1+a_3), 1-F_Y(a_2-a_1+a_3)\right), F_Y(a_1)\right)$$

3.Mathematical Morphology

Morphological theory can benefit from any mathematical framework which deals with shapes, combinations of them, or how they have developed Mathematical morphology has drawn ideas and methods from a wide range of mathematical disciplines, including algebra (lattice theory), discrete geometry, geometrical probability, integral geometry, partial differential equations, and topology [8]. Mathematical morphology essentially uses sets to represent the parts of an image[14,15]. Standardised set symbols can be utilized to clarify image operations when the image is viewed as an universe as a whole with values denoting the pixels that comprise the image [7].

Definition 3.1: Quadri Partitioned Neutrosophic set .A Quadri Partitioned Neutrosophic set $\mathbb{Q}$ on the universal set **A** is defined as [11,13]:

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$\mathbb{Q} = < T_{\mathbb{Q}}, C_{\mathbb{Q}}, U_{\mathbb{Q}}, F_{\mathbb{Q}} >$ Where $T_{\mathbb{Q}}, C_{\mathbb{Q}}, U_{\mathbb{Q}}, F_{\mathbb{Q}} : \rightarrow [0,1]$ for **A** all a in **A**
$0 \leq +C_{\mathbb{Q}} + U_{\mathbb{Q}} + F_{\mathbb{Q}} \leq 4$ Here, $T_{\mathbb{Q}}(a)$ is the function of true membership, $C_{\mathbb{Q}}(a)$ is the function of contradiction membership, $U_{\mathbb{Q}}(a)$ is the function of ignorance membership, $F_{\mathbb{Q}}(a)$ is the function of false membership.

Definition 3.2:Complement of Quadri Partitioned Neutrosophic set . The Quadri partitioned neutrosophic set[QPN] complement $\mathbb{Q}$ is denoted as $\mathbb{Q}^c$ and is defined as [11] :
$\mathbb{Q}^c = \langle T^C_{\mathbb{Q}}, C^C_{\mathbb{Q}}, U^C_{\mathbb{Q}}, F^C_{\mathbb{Q}} \rangle$ Where $T^C_{\mathbb{Q}}, C^C_{\mathbb{Q}}, U^C_{\mathbb{Q}}, F^C_{\mathbb{Q}} : A \rightarrow [0,1]$ for all a in A. $T^C_{\mathbb{Q}} = 1 - T_{\mathbb{Q}}$, $C^C_{\mathbb{Q}} = 1 - C_{\mathbb{Q}}$, $U^C_{\mathbb{Q}} = 1 - U_{\mathbb{Q}}$, $F^C_{\mathbb{Q}} = 1 - F_{\mathbb{Q}}$. The empty set of Quadri partitioned neutrosophic set of A is defined as $\emptyset_{QPN} =< 0,0,1,1 >$ where $\underline{1}(a) = 1$ and $\underline{0}(a) = 0$, for all a ∈ A. The Quadri partitioned neutrosophic set of A is $1_{QPN} = \langle 1,1,0,0 \rangle$ where $\underline{1}(a) = 1$ and $\underline{0}(a) = 0$, for all a ∈ A.

4. Mathematical Morphology for Quadri-Partitioned Neutrosophic

Here is the idea of quadri-partitioned neutrosophic morphology with the base of the morphological operator using fuzzy set operators and neutrosophic operators. The context of an quadri-partitioned neutrosophic set can therefore be used with these terms.Here the set of all quadri partitioned neutrosophoic subset of A by $\mathbb{Q}(A)$.In the following definitions, E is consider as Euclidean space and two quadri partitioned neutrosophic subsets of A: $X,Y \in \mathbb{Q}$.

Definition 4.1: The structuring element Y mirror reflection in its origin is defined as [2] : $-Y = \langle -T_Y, -C_Y, -U_Y, -F_Y \rangle$, where $-T_Y(a_1) = T_Y(-a_1), -C_Y(a_1) = C_Y(-a_1), -U_Y(a_1) = U_Y(-a_1)$, and $-F_Y(a_1) = F_Y(-a_1)$. The translation of X into E for each p by $p \in Z^2$ is $X_p = \langle T_{Xp}, C_{Xp}, U_{Xp}, F_{Xp} \rangle$ where $T_{Xp}(a_1) = T_{Xp}(a_1 + p), C_{Xp} = C_{Gp}(a_1 + p), U_{Xp} = U_{Xp}(a_1 + p)$, and $F_{Xp} = F_{Xp}(a_1 + p)$. Most of the Quadri partitioned neutrosophic morphological operations can be derived from the combining theoretical operations of Quadri partitioned neutrosophic set with dialation and erosion which are the two basic operators.

5.Quadri partitioned Neutrosophic Morphological Operations

The quadri partitioned neutrosophy notion is introduced to morphology by the degree to which the structuring component fits within the image at the four levels of trueness, contradiction, ignorance, and falseness. In terms of their membership, contradiction, ignorance, and non-membership functions, the operations of quadri-partitioned neutrosophic erosion, dilation, opening, and closing of a quadri-partitioned neutrosophic image by a quadri-partitioned neutrosophic structuring element have never before been defined to our knowledge.

5.1 The Operation of Dilation

Dilation operations are described as the structuring of component X on image Y and the movement of the element through the image in a manner similar to convolution. The dilation operator's two main inputs are the image that needs to be dilated and a set of coordinates called a structuring element, which can be thought of as a kernel. This structural element determines the precise dilation impact on the input image [9,10]. Different types of neutrosophic operations are explained [16-22].

Definition 5.2: Quadri partitioned neutrosophic dilation. Let X and Y be the two neutrosophic sets that are quadri-partitioned. The quadri partitioned neutrosophic set's dilation is therefore defined as
$X \oplus Y = \langle T_{X\oplus Y}, C_{X\oplus Y}, U_{X\oplus Y}, F_{X\oplus Y} \rangle$ where for the each $a_1, a_2 \in Z^2$

$$T_{X\oplus Y}(a_2) = \sup_{a_1 \in Z^2} \min\left(T_X(a_2 + a_1), T_Y(a_1)\right)$$

$$C_{X\oplus Y}(a_2) = \sup_{a_1 \in Z^2} \min\left(C_X(a_2 + a_1), C_Y(a_1)\right)$$

$$U_{X\oplus Y}(a_2) = \inf_{a_1 \in Z^2} \max\left(1 - U_X(a_2 + a_1), 1 - U_Y(a_1)\right)$$

$$F_{X\oplus Y}(a_2) = \inf_{a_1 \in Z^2} \max\left(1 - F_X(a_2 + a_1), 1 - F_Y(a_1)\right)$$

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5.3 Numerical Example for Quadri partitioned neutrosophic dilation

Let X= {⟨a, 0.7,0.2,0.3,0.5 ⟩ , ⟨b, 0.9,0.1,0.6,0,4 ⟩, ⟨c, 0.4,0.3.0.2.0.8 ⟩} and Y={⟨u, 0.6,0.1,0.2,0.4 ⟩, ⟨v, 0.8,0.3,0.5,0.6 ⟩, ⟨w, 0.3,0.2,0.4,0.7 ⟩} be the two neutrosophic sets that are quadri-partitioned. Consider z = {0,1,2} ,then the quadri partitioned neutrosophic set's dilation is

$$T_{X\oplus Y}(a_2) = \sup_{a_1\in Z^2} \min\left(T_X(a_2+a_1), T_Y(a_1)\right)$$

$$T_{X\oplus Y}(2) = \sup_{a_1\in Z^2} \left(\min\left(T_X(2+a_1), T_Y(a_1)\right)\right.$$

$$= \sup_{a_1\in Z^2} \left(\min\left(T_X(2+0), T_Y(0)\right), \min\left(T_X(2+1), T_Y(1)\right), \min\left(T_X(2+2), T_Y(2)\right)\right)$$

$$= \sup_{a_1\in Z^2} \left(\min\left(T_X(2), T_Y(0)\right), \min\left(T_X(3), T_Y(1)\right), \min\left(T_X(4), T_Y(2)\right)\right)$$

$$= \sup_{a_1\in Z^2} \left(\min(0.4,0.6), \min(0.4,0.8), \min(0.4,0.3)\right)$$

$$= \sup_{a_1\in Z^2} (0.4,0.4,0.3)$$

$$= 0.4$$

$$C_{X\oplus Y}(a_2) = \sup_{a_1\in Z^2} \left(\min\left(C_X(a_2+a_1), C_Y(a_1)\right)\right.$$

$$C_{X\oplus Y}(2) = \sup_{a_1\in Z^2} \left(\min\left(C_X(2+a_1), C_Y(a_1)\right)\right.$$

$$= \sup_{a_1\in Z^2} \left(\min\left(C_X(2+0), C_Y(0)\right), \min\left(C_X(2+1), C_Y(1)\right), \min\left(C_X(2+2), C_Y(2)\right)\right)$$

$$= \sup_{a_1\in Z^2} \left(\min\left(C_X(2), C_Y(0)\right), \min\left(C_X(3), C_Y(1)\right), \min\left(C_X(4), C_Y(2)\right)\right)$$

$$= \sup_{a_1\in Z^2} \left(\min(0.3,0.1), \min(0.3,0.3), \min(0.3,0.2)\right)$$

$$= \sup_{a_1\in Z^2} (0.1,0.3,0.2)$$

$$= 0.3$$

$$U_{X\oplus Y}(a_2) = \inf_{a_1\in Z^2} \max\left(1-U_X(a_2+a_1), 1-U_Y(a_1)\right)$$

$$U_{X\oplus Y}(2) = \inf_{a_1\in Z^2} \left(\max\left(1-U_X(2+0), 1-U_Y(0)\right), \left(\max\left(1-U_X(2+1), 1-U_Y(1)\right)\left(\max(1-U_X(2+2), 1 - U_Y(2))\right)\right)\right.$$

$$= \inf_{a_1\in Z^2} \left(\max\left(1-U_X(2), 1-U_Y(0)\right), \max\left(1-U_X(3), 1-U_Y(1)\right), \max(1-U_X(4), 1-U_Y(2))\right)$$

$$= \inf_{a_1\in Z^2} \left(\max(1-0.2, 1-0.2), \max(1-0.2, 1-0.5), \max(1-0.2, 1-0.4)\right)$$

$$= \inf_{a_1\in Z^2} (0.8,0.8,0.8)$$

$$=0.8$$

$$F_{X\oplus Y}(a_2) = \inf_{a_1\in Z^2} \max\left(1-F_X(a_2+a_1), 1-F_Y(a_1)\right)$$

$$F_{X\oplus Y}(2) = \inf_{a_1\in Z^2} \left(\max\left(1-F_X(2+0), 1-F_Y(0)\right), \left(\max\left(1-F_X(2+1), 1-F_Y(1)\right)\left(\max(1-F_X(2+2), 1 - F_Y(2))\right)\right)\right.$$

$$= \inf_{a_1\in Z^2} \left(\max\left(1-F_X(2), 1-F_Y(0)\right), \max\left(1-F_X(3), 1-F_Y(1)\right), \max(1-F_X(4), 1-F_Y(2))\right)$$

$$= \inf_{a_1\in Z^2} \left(\max(1-0.8, 1-0.4), \max(1-0.8, 1-0.6), \max(1-0.8, 1-0.7)\right)$$

$$= \inf_{a_1\in Z^2} (0.6,0.4,0.3)$$

$$=0.3$$

5.4 The Operation of Erosion

Since the process of erosion is identical to that of dilatation, the pixels are changed to "white" rather than "black." The image that is to be degraded and a structural element are the erosion operator's two primary inputs. This structural element determines the precise impact of the erosion on the supplied image. For grey- scale images, erosion is defined mathematically as follows [2] :

Definition 5.5: Quadri partitioned neutrosophic Erosion. Let X and Y be the two Quadri partitioned neutrosophic sets. Then the erosion of the quadri partitioned neutrosophic set is defined as

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$X \ominus Y = \langle T_{X\ominus Y}, C_{X\ominus Y}, U_{X\ominus Y}, F_{X\ominus Y} \rangle$ where for the each $a_1, a_2 \in Z^2$

$T_{X\ominus Y}(a_2) = \inf_{a_1 \in Z^2} \max(T_X(a_2 + a_1), 1 - T_Y(a_1))$

$C_{X\ominus Y}(a_2) = \inf_{a_1 \in Z^2} \max(C_X(a_2 + a_1), 1 - C_Y(a_1))$

$U_{X\ominus Y}(a_2) = \sup_{a_1 \in Z^2} \min(1 - U_X(a_2 + a_1), U_Y(a_1))$

$F_{X\ominus Y}(a_1) = \sup_{a_1 \in Z^2} \min(1 - F_X(a_2 + a_1), F_Y(a_1))$

5.6 Numerical Example for Quadri partitioned neutrosophic Erosion

Let $X = \{ \langle a, 0.7, 0.2, 0.3, 0.5 \rangle, \langle b, 0.9, 0.1, 0.6, 0.4 \rangle, \langle c, 0.4, 0.3.0.2.0.8 \rangle \}$ and $Y = \{ \langle u, 0.6, 0.1, 0.2, 0.4 \rangle, \langle v, 0.8, 0.3, 0.5, 0.6 \rangle, \langle w, 0.3, 0.2, 0.4, 0.7 \rangle \}$ be the two neutrosophic sets that are quadri-partitioned.Consider z = {0,1,2} ,then the quadri partitioned neutrosophic set's erosion is

$T_{X\ominus Y}(a_2) = \inf_{a_1 \in Z^2} \max\ (T_X(a_2 + a_1), 1 - T_Y(a_1))$

$T_{X\ominus Y}(2) = \inf_{a_1 \in Z^2} \{\max\ (T_X(2+0), 1 - T_Y(0)), \max(T_X(2+1), 1 - T_Y(1)), \max(T_X(2+2), 1 - T_Y(2))\}$

$= \inf_{a_1 \in Z^2} \{\max\ (T_X(2), 1 - T_Y(0)), \max(T_X(3), 1 - T_Y(1)), \max(T_X(4), 1 - T_Y(2))\}$

$= \inf_{a_1 \in Z^2} \{\max(0.4,0.4), \max(0.4,0.2), \max(0.4,0.7)\}$

$= \inf_{a_1 \in Z^2} \{0.4, 0.4, 0.7\}$

$= 0.4$

$C_{X\ominus Y}(2) = \inf_{a_1 \in Z^2} \{\max\ (C_X(2+0), 1 - C_Y(0)), \max(C_X(2+1), 1 - C_Y(1)), \max(C_X(2+2), 1 - C_Y(2))\}$

$= \inf_{a_1 \in Z^2} \{\max\ (C_X(2), 1 - C_Y(0)), \max(C_X(3), 1 - C_Y(1)), \max(C_X(4), 1 - C_Y(2))\}$

$= \inf_{a_1 \in Z^2} \{\max(0.3,0.9), \max(0.3,0.7), \max(0.4,0.8)\}$

$= \inf_{a_1 \in Z^2} \{0.9, 0.7, 0.8\}$

$= 0.7$

$U_{X\ominus Y}(a_2) = \sup_{a_1 \in Z^2} \min(1 - U_X(a_2 + a_1), U_Y(a_1))$

$U_{X\ominus Y}(2) = \sup_{a_1 \in Z^2} \{\min(1 - U_X(2+0), U_Y(0)), \min(1 - U_X(2+1), U_Y(1)), \min(1 - U_X(2+2), U_Y(2))\}$

$= \sup_{a_1 \in Z^2} \{\min(1 - U_X(2), U_Y(0)), \min(1 - U_X(3), U_Y(1)), \min(1 - U_X(4), U_Y(2))\}$

$= \sup_{a_1 \in Z^2} \{\min(0.8, 0.2), \min(0.8,0.5), \min(0.8, 0.4)\}$

$= \sup_{a_1 \in Z^2} \{0.2, 0.5, 0.4\}$

$=0.5$

$F_{X\ominus Y}(a_2) = \sup_{a_1 \in Z^2} \min(1 - F_X(a_2 + a_1), F_Y(a_1))$

$F_{X\ominus Y}(2) = \sup_{a_1 \in Z^2} \{\min(1 - F_X(2+0), F_Y(0)), \min(1 - F_X(2+1), F_Y(1)), \min(1 - F_X(2+2), F_Y(2))\}$

$= \sup_{a_1 \in Z^2} \{\min(1 - F_X(2), F_Y(0)), \min(1 - F_X(3), F_Y(1)), \min(1 - F_X(4), F_Y(2))\}$

$= \sup_{a_1 \in Z^2} \{\min(0.2,0.4), \min(0.2,0.6), \min(0.2,0.7)\}$

$= \sup_{a_1 \in Z^2} \{0.2, 0.2, 0.2\}$

$= 0.2$

6.The Opening Operation and Closing Operation

Dilation and erosion, the two main procedures, can work together to produce more complex sequences. Opening and closure are the most crucial of all for morphological filtration. Erosion and dilatation using the same structural part are considered opening operations. The opening operator requires two basic inputs: a structural element and an image that has to be opened. The grey-level opening is produced by grey-level dilatation after grey-level erosion [12].

Definition 6.1 : Quadri partitioned Neutrosophic Opening. Let X and Y be the two Quadri partitioned neutrosophic sets. Then its opening operation is described as

$X \circ Y = \langle T_{X\circ Y}, C_{X\circ Y}, U_{X\circ Y}, F_{X\circ Y} \rangle$ where $a_1, a_2, a_3 \in Z^2$

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$$T_{X\circ Y}(a_2) = \sup_{a_1\in Z^2} \min\left(\inf_{Z\in R^n} \max\left(T_X(a_2 - a_1 + a_3), 1 - T_Y(a_3)\right), T_Y(a_1)\right)$$
$$C_{X\circ Y}(a_2) = \sup_{a_1\in Z^2} \min\left(\inf_{Z\in R^n} \max\left(C_X(a_2 - a_1 + a_3), 1 - C_Y(a_3)\right), C_Y(a_1)\right)$$
$$U_{X\circ Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{Z\in R^n} \min\left(1 - U_X(a_2 - a_1 + a_3), U_Y(a_1)\right), 1 - U_Y(a_1)\right)$$
$$F_{X\circ Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{Z\in R^n} \min\left(1 - F_X(a_2 - a_1 + a_3), F_Y(a_1)\right), 1 - F_Y(a_1)\right)$$

Definition 6.2: Quadri partitioned Neutrosophic Closing. Let X and Y be the two Quadri partitioned neutrosophic sets. Then its closing operation is defined as

$X * Y = \langle T_{M*N}, C_{M*N}, U_{M*N}, F_{M*N} \rangle$ where $a_1, a_2, a_3 \in Z^2$

$$T_{X*Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{Z\in R^n} \min\left(T_X(a_2 - a_1 + a_3), T_Y(a_3)\right), 1 - T_Y(a_1)\right)$$
$$C_{X*Y}(a_2) = \inf_{a_1\in Z^2} \max\left(\sup_{Z\in R^n} \min\left(C_X(a_2 - a_1 + a_3), C_Y(a_3)\right), 1 - C_Y(a_1)\right)$$
$$U_{X*Y}(a_2) = \sup_{a_1\in Z^2} \min\left(\inf_{Z\in R^n} \max\left(1 - U_X(a_2 - a_1 + a_3), 1 - U_Y(a_3)\right), U_Y(a_1)\right)$$
$$F_{X*Y}(a_2) = \sup_{a_1\in Z^n} \min\left(\inf_{Z\in R^n} \max\left(1 - F_X(a_2 - a_1 + a_3), 1 - F_Y(a_3)\right), F_Y(a_1)\right)$$

7. Algebraic Properties in Quadri partitioned neutrosophic

7.1 Proposition Duality Theorem for Dilation. Let X and Y be the two Quadri partitioned neutrosophic sets. Quadri-partitioned neutrosophic erosion and dilation are dual operations.That is ,

$(X^C \oplus Y)^C = \langle T_{(X^C\oplus Y)^C}, C_{(X^C\oplus Y)^C}, U_{(X^C\oplus Y)^C}, F_{(X^C\oplus Y)^C} \rangle$ where for each $a_1, a_2 \in Z^2$

Proof:Let

$$T_{(X^C\oplus Y)^C}(a_2) = 1 - T_{(X^C\oplus Y)}(a_2)$$
$$= 1 - \sup_{a_1\in Z^2} \min\left(T_{X^C}(a_2 + a_1), T_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2}\left[1 - \min(T_{X^C}(a_2 + a_1), T_Y(a_1))\right]$$
$$= \inf_{a_1\in Z^2}\left[\max(1 - T_{X^C}(a_2 + a_1), 1 - T_Y(a_1))\right]$$
$$= \inf_{a_1\in Z^2}\max\left[(T_X(a_2 + a_1), 1 - T_Y(a_1))\right]$$
$$= T_{X\ominus Y}(a_2)$$

$$C_{(X^C\oplus Y)^C}(a_2) = 1 - C_{(X^C\oplus Y)}(a_2)$$
$$= 1 - \sup_{a_1\in Z^2} \min\left(C_{X^C}(a_2 + a_1), C_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2}\left[1 - \min(C_{X^C}(a_2 + a_1), C_Y(a_1))\right]$$
$$= \inf_{a_1\in Z^2}\left[\max(1 - C_{X^C}(a_2 + a_1), 1 - C_Y(a_1))\right]$$
$$= \inf_{a_1\in Z^2}\max\left[(C_X(a_2 + a_1), 1 - C_Y(a_1))\right]$$
$$= C_{X\ominus Y}(a_2)$$

$$U_{(X^C\oplus Y)^C}(a_2) = 1 - U_{(X^C\oplus Y)}(a_2)$$
$$= 1 - \inf_{a_1\in Z^2} \max\left(\left(1 - U_{X^C}(a_2 + a_1), 1 - U_Y(a_1)\right)\right)$$
$$= \sup_{a_1\in Z^2}\left[1 - \max(1 - U_{X^C}(a_2 + a_1), 1 - U_Y(a_1))\right]$$
$$= \sup_{a_1\in Z^2}\min\left[(1 - U_X(a_2 + a_1), U_Y(a_1))\right]$$
$$= U_{X\ominus Y}(a_2)$$

$$F_{(X^C\oplus Y)^C}(a_2) = 1 - F_{(X^C\oplus Y)}(a_2)$$
$$= 1 - \inf_{a_1\in Z^2} \max\left(1 - F_{X^C}(b + a_1), 1 - F_Y(a_1)\right)$$

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$= \sup_{a_1 \in Z^2} [1 - \max(1 - F_{X^C}(a_2 + a_1), 1 - F_Y(a_1))]$

$= \sup_{a_1 \in Z^2} [\min(1 - F_X(a_2 + a_1), F_Y(a_1))]$

$= F_{X \ominus Y}(a_2)$

$\langle T_{(X^C \oplus Y)^C}, C_{(X^C \oplus Y)^C}, U_{(X^C \oplus Y)^C}, F_{(X^C \oplus Y)^C} \rangle = \langle T_{X \ominus Y}, C_{X \ominus Y}, U_{X \ominus Y}, F_{X \ominus Y} \rangle$

Hence the elements of two sets(True, contradiction, ignorance, falsity), one generated from operations on $(X^C \oplus Y)$, and the other from operations on $X \ominus Y$ are equivalent.

7.2 Proposition of Duality Theorem for closing

The two quadri-partitioned neutrosophic sets M and N shall be dual operations, including quadri-partitioned neutrosophic opening and quadri-partitioned neutrosophic closing..That is, $(X^C * Y)^C = \langle T_{(X^C * Y)^C}, C_{(X^C * Y)^C}, U_{(X^C * Y)^C}, F_{(X^C * Y)^C} \rangle$

Proof: Let

$T_{(X^C * Y)^C}(a_2) = 1 - T_{X^C * Y}(a_2)$

$T_{(X^C * Y)^C}(a_2) = 1 - \inf_{a_1 \in Z^2} \max[\sup_{Z \in R^n} \min(T_{X^C}(a_2 - a_1 + a_3), T_Y(a_3)), 1 - T_Y(a_1)]$

$= \sup_{a_1 \in Z^2} \min[(1 - \sup_{Z \in R^n} \min(T_{X^C}(a_2 - a_1 + a_3), T_Y(a_3)), T_Y(a_1)]$

$= \sup_{a_1 \in Z^2} \min[\inf_{Z \in R^n} \max(1 - T_{X^C}(a_2 - a_1 + a_3), 1 - T_Y(a_3)), T_Y(a_1)]$

$= \sup_{a_1 \in Z^2} \min[\inf_{Z \in R^n} \max(T_X(a_2 - a_1 + a_3), 1 - T_Y(a_3)), T_Y(a_1)]$

$= T_{X \circ Y}(a_2)$

$C_{(X^C * Y)^C}(b) = 1 - C_{X^C * Y}(a_2)$

$C_{(X^C * Y)^C}(b) = 1 - \inf_{a_1 \in Z^2} \max[\sup_{Z \in R^n} \min(C_{X^C}(a_2 - a_1 + a_3), C_Y(a_3)), 1 - C_Y(a_1)]$

$= \sup_{a_1 \in Z^2} \min[(1 - \sup_{Z \in R^n} \min(C_{X^C}(a_2 - a_1 + a_3), C_Y(a_3)), C_Y(a_1)]$

$= \sup_{a_1 \in Z^2} \min[\inf_{Z \in R^n} \max(1 - C_{X^C}(a_2 - a_1 + a_3), 1 - C_Y(a_3)), C_Y(a_1)]$

$= \sup_{a_1 \in Z^2} \min[\inf_{Z \in R^n} \max(C_X(a_2 - a_1 + a_3), 1 - C_Y(a_3)), C_Y(a_1)]$

$= C_{X \circ Y}(a_2)$

$U_{(X^C * Y)^C}(a_2) = 1 - U_{X^C * Y}(a_2)$

$= 1 - \sup_{a_1 \in Z^2} \min[\inf_{Z \in R^n} \max(1 - U_{X^C}(a_2 - a_1 + a_3), 1 - U_Y(a_3)), U_Y(a_1)]$

$= \inf_{a_1 \in Z^2} \max[1 - \inf_{Z \in R^n} \max(1 - U_{X^C}(a_2 - a_1 + a_3), 1 - U_Y(a_3)), U_Y(a_1)]$

$= \inf_{a_1 \in Z^2} \max[\sup_{Z \in R^n} \min(1 - U_X(a_2 - a_1 + a_3), U_Y(a_3)), 1 - U_Y(a_1)]$

$= U_{X \circ Y}(a_2)$

$F_{(X^C * Y)^C}(b) = 1 - F_{X^C * Y}(a_2)$

$= 1 - \sup_{a_1 \in Z^2} \min[\inf_{Z \in R^n} \max(1 - F_{X^C}(a_2 - a_1 + a_3), 1 - F_Y(a_3)), F_Y(a_1)]$

$= \inf_{a_1 \in Z^2} \max[1 - \inf_{Z \in R^n} \max(1 - F_{X^C}(a_2 - a_1 + a_3), 1 - F_Y(a_3)), F_Y(a_1)]$

$= \inf_{a_1 \in Z^2} \max[\sup_{Z \in R^n} \min(1 - F_X(a_2 - a_1 + a_3), F_Y(a_3)), 1 - F_Y(a_1)]$

$= F_{X \circ Y}(a_2)$

$\langle T_{(X^C * Y)^C}, C_{(X^C * Y)^C}, U_{(X^C * Y)^C}, F_{(X^C * Y)^C} \rangle = \langle T_{X \circ Y}, C_{X \circ Y}, U_{X \circ Y}, F_{X \circ Y} \rangle$

This completes the proposition.

Lemma 1: for any $X \in \mathbb{Q}(A)$ [any $\mathbb{Q}(A)$ quadri partitioned neutrosophic set], and the quadri partitioned neutrosophic universal set $1_{\mathbb{Q}}$, have that $X \oplus 1_{\mathbb{Q}} \subseteq X$, $X \oplus 1_{\mathbb{Q}} = \langle T_{X \oplus 1_{\mathbb{Q}}}, C_{X \oplus 1_{\mathbb{Q}}}, U_{X \oplus 1_{\mathbb{Q}}}, F_{X \oplus 1_{\mathbb{Q}}} \rangle$

Proof : Let

$T_{X \oplus 1_{\mathbb{Q}}}(a_2) = \sup_{a_1 \in Z^2} \min(T_X(a_2 + a_1), 1)$

$= \sup_{a_1 \in Z^2} (T_X(a_2 + a_1))$

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$= T_X(a_2)$

$C_{X\oplus 1_{\mathbb{Q}}}(a_2) = \sup_{a_1 \in Z^2} \min(C_X(a_2+a_1), 1)$

$= \sup_{a_1 \in Z^2} (C_X(a_2+a_1))$

$= C_X(a_2)$

$U_{X\oplus 1_{\mathbb{Q}}}(a_2) = \inf_{a_1 \in Z^2} \max(1-U_X(a_2+a_1), 1-0)$

$= \underline{1}(a_2)$

$F_{X\oplus 1_{\mathbb{Q}}}(a_2) = \inf_{a_1 \in Z^2} \max(1-F_X(a_2+a_1), 1-0)$

$= \underline{1}(a_2)$

$\langle T_X, C_X, \underline{1}, \underline{1} \rangle \subseteq \langle T_X, C_X, U_X, F_X \rangle = X$

This proved the lemma.

Lemma 2: for any $X \in \mathbb{Q}(A)$ [any $\mathbb{Q}(A)$ quadri partitioned neutrosophic set], and the quadri partitioned neutrosophic empty set $0_{\mathbb{Q}}$, have that $X \oplus 0_{\mathbb{Q}} \subseteq X$, $X \oplus 0_{\mathbb{Q}} = \langle T_{X\oplus 0_{\mathbb{Q}}}, C_{X\oplus 0_{\mathbb{Q}}}, U_{X\oplus 0_{\mathbb{Q}}}, F_{X\oplus 0_{\mathbb{Q}}} \rangle$

Proof : Let

$T_{X\oplus 0_{\mathbb{Q}}}(a_2) = \sup_{a_1 \in Z^2} \min(T_X(a_2+a_1), 0)$

$= \underline{0}(a_2)$

$C_{X\oplus 1_{\mathbb{Q}}}(a_2) = \sup_{a_1 \in Z^2} \min(C_X(a_2+a_1), 0)$

$= \underline{0}(a_2)$

$U_{X\oplus 0_{\mathbb{Q}}}(a_2) = \inf_{a_1 \in Z^2} \max(1-U_X(a_2+a_1), 1-1)$

$= \inf_{a_1 \in Z^2} (1-U_X(a_2+a_1))$

$= U_{X^c}(a_2)$

$F_{X\oplus 0_{\mathbb{Q}}}(a_2) = \inf_{a_1 \in Z^2} \max(1-F_X(a_2+a_1), 1-1)$

$= \inf_{a_1 \in Z^2} (1-F_X(b+a))$

$= F_{X^c}(a_2)$

$\langle \underline{0}, \underline{0}, U_{X^c}, F_{X^c} \rangle \subseteq \langle T_{X^c}, C_{X^c}, U_{X^c}, F_{X^c} \rangle = X^C$

Hence proved the lemma.

8. Quadri partitioned neutrosophic Morphological Operations

Here, quadri partitioned neutrosophic basic properties of dilation, erosion, opening, and closing morphological operators are investigated, which are defined in Section 7.

8.1 Quadri partitioned neutrosophic Dilation Properties

Proposition 8.1 . The following given properties are satisfied by the quadri-partitioned neutrosophic dilation: ∀ X,Y $\in \mathbb{Q}(Z^2)$

a) Commutative : $X \oplus Y = Y \oplus X$

b) Associativity : $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$

c) Monotonicity : (both arguments are increasing)

i) if $X \subseteq Y \Rightarrow \langle T_{X\oplus Z}, C_{X\oplus Z}, U_{X\oplus Z}, F_{X\oplus Z} \rangle \subseteq \langle T_{Y\oplus Z}, C_{Y\oplus Z}, U_{Y\oplus Z}, F_{Y\oplus Z} \rangle$,then

$T_{X\oplus Z} \subseteq T_{Y\oplus Z}$, $C_{X\oplus Z} \subseteq C_{Y\oplus Z}$, $U_{X\oplus Z} \supseteq U_{Y\oplus Z}$ and $F_{X\oplus Z} \supseteq F_{Y\oplus Z}$

ii) if $X \subseteq Y \Rightarrow \langle T_{Z\oplus X}, C_{Z\oplus X}, U_{Z\oplus X}, F_{Z\oplus X} \rangle \subseteq \langle T_{Z\oplus Y}, C_{Z\oplus Y}, U_{Z\oplus Y}, F_{Z\oplus Y} \rangle$,then

$T_{Z\oplus X} \subseteq T_{Z\oplus Y}$, $C_{Z\oplus X} \subseteq C_{Z\oplus Y}$, $U_{Z\oplus X} \supseteq U_{Z\oplus Y}$ and $F_{Z\oplus X} \supseteq F_{Z\oplus Y}$

Proof:

The proof of a) ,b), and c) are obvious.

Proposition 8.2 . For any family $(X_i | i \in I)$ in $\mathbb{Q}(Z^2)$ and $A \in \mathbb{Q}(Z^2)$

i) $\langle T_{\bigcap_{i\in I} X_i \oplus Y}, C_{\bigcap_{i\in I} X_i \oplus Y}, U_{\bigcap_{i\in I} X_i \oplus Y}, F_{\bigcap_{i\in I} X_i \oplus Y} \rangle \subseteq \langle T_{\bigcap_{i\in I}(X_i \oplus Y)}, C_{\bigcap_{i\in I}(X_i \oplus Y)}, U_{\bigcap_{i\in I}(X_i \oplus Y)}, F_{\bigcap_{i\in I}(X_i \oplus Y)} \rangle$,then

$T_{\bigcap_{i\in I} X_i \oplus Y} \subseteq T_{\bigcap_{i\in I}(X_i \oplus Y)}$, $C_{\bigcap_{i\in I} X_i \oplus Y} \subseteq C_{\bigcap_{i\in I}(X_i \oplus Y)}$, $U_{\bigcap_{i\in I} X_i \oplus Y} \supseteq U_{\bigcup_{i\in I}(X_i \oplus Y)}$ and $F_{\bigcap_{i\in I} X_i \oplus Y} \supseteq F_{\bigcup_{i\in I}(X_i \oplus Y)}$

ii) if $\langle T_{Y\oplus \bigcap_{i\in I} X_i}, C_{Y\oplus \bigcap_{i\in I} X_i}, U_{Y\oplus \bigcap_{i\in I} X_i}, F_{Y\oplus \bigcap_{i\in I} X_i} \rangle \subseteq \langle T_{\bigcap_{i\in I}(Y\oplus X_i)}, C_{\bigcap_{i\in I}(Y\oplus X_i)}, U_{\bigcap_{i\in I}(Y\oplus X_i)}, F_{\bigcap_{i\in I}(Y\oplus X_i)} \rangle$,then

$T_{Y\oplus \bigcap_{i\in I} X_i} \subseteq T_{\bigcap_{i\in I}(Y\oplus X_i)}$, $C_{Y\oplus \bigcap_{i\in I} X_i} \subseteq C_{\bigcap_{i\in I}(Y\oplus X_i)}$, $U_{Y\oplus \bigcap_{i\in I} X_i} \supseteq U_{\bigcup_{i\in I}(Y\oplus X_i)}$ and $F_{Y\oplus \bigcap_{i\in I} X_i} \supseteq F_{\bigcup_{i\in I}(Y\oplus X_i)}$

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Proof : Let

i) $\langle T_{\bigcap_{i\in I} X_i \oplus Y}, C_{\bigcap_{i\in I} X_i \oplus Y}, U_{\bigcap_{i\in I} X_i \oplus Y}, F_{\bigcap_{i\in I} X_i \oplus Y}\rangle \subseteq \langle T_{\bigcap_{i\in I}(X_i \oplus Y)}, C_{\bigcap_{i\in I}(X_i \oplus Y)}, U_{\bigcap_{i\in I}(X_i \oplus Y)}, F_{\bigcap_{i\in I}(X_i \oplus Y)}\rangle$

$$T_{\bigcap_{i\in I} X_i \oplus Y}(a_2) = \operatorname*{Sup}_{a_1\in Z^2} \min\left(T_{\bigcap_{i\in I} X_i}(a_2+a_1), T_Y(a_1)\right)$$
$$= \operatorname*{Sup}_{a_1\in Z^2} \min\left(\inf_{i\in I} T_{X_i}(a_2+a_1), T_Y(a_1)\right)$$
$$= \operatorname*{Sup}_{a_1\in Z^2} \inf_{i\in I}\left(\min T_{X_i}(a_2+a_1), T_Y(a_1)\right)$$
$$\leq \inf_{i\in I} \operatorname*{Sup}_{a_1\in Z^2}\left(\min T_{X_i}(a_2+a_1), T_Y(a_1)\right)$$
$$\leq T_{\bigcap_{i\in I} X_i \oplus Y}(a_2)$$
$$\leq T_{\bigcap_{i\in I}(X_i \oplus Y)}$$

$$C_{\bigcap_{i\in I} X_i \oplus Y}(a_2) = \operatorname*{Sup}_{a_1\in Z^2} \min\left(C_{\bigcap_{i\in I} X_i}(a_2+a_1), C_Y(a_1)\right)$$
$$= \operatorname*{Sup}_{a_1\in Z^2} \min\left(\inf_{i\in I} C_{X_i}(a_2+a_1), C_Y(a_1)\right)$$
$$= \operatorname*{Sup}_{a_1\in Z^2} \inf_{i\in I}\left(\min C_{X_i}(a_2+a_1), C_Y(a_1)\right)$$
$$\leq \inf_{i\in I} \operatorname*{Sup}_{a_1\in Z^2}\left(\min C_{X_i}(a_2+a_1), C_Y(a_1)\right)$$
$$\leq C_{\bigcap_{i\in I} X_i \oplus Y}(a_2)$$
$$\leq C_{\bigcap_{i\in I}(X_i \oplus Y)}$$

$$U_{\bigcap_{i\in I} X_i \oplus Y}(a_2) = \inf_{a_1\in Z^2} \max\left(1 - U_{\bigcap_{i\in I} X_i}(a_2+a_1), 1 - U_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2} \max\left(1 - \inf_{i\in I} U_{X_i}(a_2+a_1), 1 - U_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2} \max\left(\sup_{i\in I}(1 - U_{X_i}(a_2+a_1), 1 - U_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2} \sup_{i\in I}\left(\max U_{X_i}(a_2+a_1), 1 - U_Y(a_1)\right)$$
$$\geq \operatorname*{Sup}_{i\in I} \inf_{a_1\in Z^2}\left(\max U_{X_i}(a_2+a_1), 1 - U_Y(a_1)\right)$$
$$\geq \bigcup_{i\in I} \inf_{a_1\in Z^2}\left(\max U_{X_i}(a_2+a_1), 1 - U_Y(a_1)\right)$$
$$\geq U_{\bigcup_{i\in I}(X_i \oplus Y)}(a_2)$$

$$F_{\bigcap_{i\in I} X_i \oplus Y}(a_2) = \inf_{a_1\in Z^2} \max\left(1 - F_{\bigcap_{i\in I} M_i}(a_2+a_1), 1 - F_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2} \max\left(1 - \inf_{i\in I} F_{X_i}(a_2+a_1), 1 - F_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2} \max\left(\sup_{i\in I}(1 - F_{X_i}(a_2+a_1), 1 - F_Y(a_1)\right)$$
$$= \inf_{a_1\in Z^2} \sup_{i\in I}\left(\max F_{X_i}(a_2+a_1), 1 - F_Y(a_1)\right)$$
$$\geq \operatorname*{Sup}_{i\in I} \inf_{a_1\in Z^2}\left(\max F_{X_i}(a_2+a_1), 1 - F_Y(a_1)\right)$$
$$\geq \bigcup_{i\in I} \inf_{a_1\in Z^2}\left(\max F_{X_i}(a_2+a_1), 1 - F_Y(a_1)\right)$$
$$\geq F_{\bigcup_{i\in I}(X_i \oplus Y)}(a_2)$$

Hence proved (i)

ii) The proof of (ii) is similar to (i).

Proposition 8.3 : For any family $(X_i | i \in I)$ in $\mathbb{Q}(Z^2)$ and $Y \in \mathbb{Q}(Z^2)$

i)if $\langle T_{\bigcup_{i\in I} X_i \oplus Y}, C_{\bigcup_{i\in I} X_i \oplus Y}, U_{\bigcup_{i\in I} X_i \oplus Y}, F_{\bigcup_{i\in I} X_i \oplus Y}\rangle \supseteq \langle T_{\bigcup_{i\in I}(X_i \oplus Y)}, C_{\bigcup_{i\in I}(X_i \oplus Y)}, U_{\bigcup_{i\in I}(X_i \oplus Y)}, F_{\bigcup_{i\in I}(X_i \oplus Y)}\rangle$,then

$T_{\bigcup_{i\in I} X_i \oplus Y} \supseteq T_{\bigcup_{i\in I}(X_i \oplus Y)}$, $C_{\bigcup_{i\in I} X_i \oplus Y} \supseteq C_{\bigcup_{i\in I}(X_i \oplus Y)}$, $U_{\bigcup_{i\in I} X_i \oplus Y} \subseteq U_{\bigcap_{i\in I}(X_i \oplus Y)}$ and $F_{\bigcup_{i\in I} X_i \oplus Y} \subseteq F_{\bigcap_{i\in I}(X_i \oplus Y)}$

ii) *if* $\langle T_{Y\oplus \bigcup_{i\in I} X_i}, C_{Y\oplus \bigcup_{i\in I} X_i}, U_{Y\oplus \bigcup_{i\in I} X_i}, F_{Y\oplus \bigcup_{i\in I} X_i}\rangle \supseteq \langle T_{\bigcup_{i\in I}(X_i \oplus Y)}, C_{\bigcup_{i\in I}(X_i \oplus Y)}, U_{\bigcup_{i\in I}(X_i \oplus Y)}, F_{\bigcup_{i\in I}(X_i \oplus Y)}\rangle$,then

$T_{Y\oplus \bigcup_{i\in I} X_i} \supseteq T_{\bigcup_{i\in I}(X_i \oplus Y)}$, $C_{Y\oplus \bigcup_{i\in I} X_i} \supseteq C_{\bigcup_{i\in I}(X_i \oplus Y)}$, $U_{Y\oplus \bigcup_{i\in I} X_i} \subseteq U_{\bigcap_{i\in I}(X_i \oplus Y)}$ and $F_{Y\oplus \bigcup_{i\in I} X_i} \subseteq F_{\bigcap_{i\in I}(X_i \oplus Y)}$

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Proof : Let (ii)

$$\langle T_{Y\oplus \underset{i\in I}{\cup} X_i}, C_{Y\oplus \underset{i\in I}{\cup} X_i}, U_{Y\oplus \underset{i\in I}{\cup} X_i}, F_{Y\oplus \underset{i\in I}{\cup} X_i}\rangle \supseteq \langle T_{\underset{i\in I}{\cup}(X_i\oplus Y)}, C_{\underset{i\in I}{\cup}(X_i\oplus Y)}, U_{\underset{i\in I}{\cup}(X_i\oplus Y)}, F_{\underset{i\in I}{\cup}(X_i\oplus Y)}\rangle$$

$$\begin{aligned}
T_{Y\oplus \underset{i\in I}{\cup} X_i}(a_2) &= \underset{a_1\in Z^2}{\text{Sup}} \min\left(T_Y(a_2+a_1), T_{\underset{i\in I}{\cup} X_i}(a_1)\right)\\
&= \underset{a_1\in Z^2}{\text{Sup}} \min\left(T_Y(a_2+a_1), \underset{i\in I}{\text{Sup}}\, T_{X_i}(a_1)\right)\\
&\geq \underset{a_1\in Z^2}{\text{Sup}}\left(\underset{i\in I}{\text{Sup}} \min T_Y(a_2+a_1), T_{X_i}(a_1)\right)\\
&\geq \underset{i\in I}{\cup}\left(\underset{a_1\in Z^2}{\text{Sup}}\, min T_Y(a_2+a_1), T_{X_i}(a_1)\right)\\
&\geq \underset{i\in I}{\cup} T_{(X_i\oplus Y)}(a_2+a_1)\\
&\geq T_{\underset{i\in I}{\cup}(Y\oplus X_i)}(a_2)
\end{aligned}$$

$$\begin{aligned}
C_{Y\oplus \underset{i\in I}{\cup} X_i}(a_2) &= \underset{a_1\in Z^2}{\text{Sup}} \min\left(C_Y(a_2+a_1), C_{\underset{i\in I}{\cup} X_i}(a_1)\right)\\
&= \underset{a_1\in Z^2}{\text{Sup}} \min\left(C_Y(a_2+a_1), \underset{i\in I}{\text{Sup}}\, C_{X_i}(a_1)\right)\\
&\geq \underset{a_1\in Z^2}{\text{Sup}}\left(\underset{i\in I}{\text{Sup}} \min C_Y(a_2+a_1), C_{X_i}(a_1)\right)\\
&\geq \underset{i\in I}{\cup}\left(\underset{a_1\in Z^2}{\text{Sup}}\, min C_Y(a_2+a_1), C_{X_i}(a_1)\right)\\
&\geq \underset{i\in I}{\cup} C_{(X_i\oplus Y)}(a_2+a_1)\\
&\geq C_{\underset{i\in I}{\cup}(Y\oplus X_i)}(a_2)
\end{aligned}$$

$$\begin{aligned}
U_{Y\oplus \underset{i\in I}{\cup} X_i}(a_2) &= \underset{a_1\in Z^2}{\inf} \max\left(1-U_Y(a_2+a_1), 1-U_{\underset{i\in I}{\cup} X_i}(a_1)\right)\\
&= \underset{a_1\in Z^2}{\inf} \max\left(1-U_Y(a_2+a_1), 1-\underset{i\in I}{\text{Sup}}\, U_{X_i}(a_1)\right)\\
&= \underset{a_1\in Z^2}{\inf} \max\left(1-U_Y(a_2+a_1), \underset{i\in I}{\inf}\,(1-U_{X_i}(a_1))\right)\\
&\leq \underset{a_1\in Z^2}{\inf}\,\underset{i\in I}{\inf}\left(\max(1-U_Y(a_2+a_1), 1-U_{X_i}(a_1))\right)\\
&\leq \underset{i\in I}{\inf}\,\underset{a_1\in Z^2}{\inf}\left(\max(1-U_Y(a_2+a_1), 1-U_{X_i}(a_1))\right)\\
&\leq \underset{i\in I}{\cap}\,\underset{a_1\in Z^2}{\inf} max\left(1-U_Y(a_2+a_1), 1-U_{X_i}(a_1)\right)\\
&\leq U_{\underset{i\in I}{\cap}(Y\oplus X_i)}(a_2)
\end{aligned}$$

$$\begin{aligned}
F_{Y\oplus \underset{i\in I}{\cup} X_i}(a_2) &= \underset{a_1\in Z^2}{\inf} \max\left(1-F_Y(a_2+a_1), 1-F_{\underset{i\in I}{\cup} X_i}(a_1)\right)\\
&= \underset{a_1\in Z^2}{\inf} \max\left(1-F_Y(a_2+a_1), 1-\underset{i\in I}{\text{Sup}}\, F_{X_i}(a_1)\right)\\
&= \underset{a_1\in Z^2}{\inf} \max\left(1-F_Y(a_2+a_1), \underset{i\in I}{\inf}\,(1-F_{X_i}(a))\right)\\
&\leq \underset{a_1\in Z^2}{\inf}\,\underset{i\in I}{\inf}\left(\max(1-F_Y(a_2+a_1), 1-F_{X_i}(a_1))\right)\\
&\leq \underset{i\in I}{\inf}\,\underset{a_1\in Z^2}{\inf}\left(\max(1-F_Y(a_2+a_1), 1-F_{x_i}(a_1))\right)\\
&\leq \underset{i\in I}{\cap}\,\underset{a_1\in Z^2}{\inf} max\left(1-F_Y(a_2+a_1), 1-F_{X_i}(a_1)\right)\\
&\leq F_{\underset{i\in I}{\cap}(Y\oplus X_i)}(a_2)
\end{aligned}$$

Hence proved the part (ii)

The proof of the (i) similar to (ii)

9. Properties of Quadri partitioned neutrosophic erosion

Proposition 9. 1: The quadri-partitioned neutrosophic erosion satisfies the property of monotonicity for all $X,Y,Z \in \mathbb{Q}(Z^2)$

i)if $X\subseteq Y \Rightarrow \langle T_{X\ominus Z}, C_{X\ominus Z}, U_{X\ominus Z}, F_{X\ominus Z}\rangle \subseteq \langle T_{Y\ominus Z}, C_{Y\ominus Z}, U_{Y\ominus Z}, F_{Y\ominus Z}\rangle$,then

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$T_{X\ominus Z} \subseteq T_{Y\ominus Z}$, $C_{X\ominus Z} \subseteq C_{Y\ominus Z}$, $U_{X\ominus Z} \supseteq U_{Y\ominus Z}$ *and* $F_{X\ominus Z} \supseteq F_{Y\ominus Z}$
ii) if $X\subseteq Y \Longrightarrow \langle T_{Z\ominus X}, C_{Z\ominus X}, U_{Z\ominus X}, F_{Z\ominus X}\rangle \supseteq \langle T_{Z\ominus Y}, C_{Z\ominus Y}, U_{Z\ominus Y}, F_{Z\ominus Y}\rangle$,then
$T_{Z\ominus X} \supseteq T_{Z\ominus Y}, C_{Z\ominus X} \supseteq C_{Z\ominus Y}, U_{Z\ominus X} \subseteq U_{Z\ominus Y}$ and $F_{Z\ominus X} \subseteq F_{Z\ominus Y}$

Note that : The erosion operator, unlike the dilation operator, does not meet commutatively and associativity.

Proposition 9. 2: For any family $(X_i|i \in I)$ in $\mathbb{Q}(Z^2)$ and $N\in \mathbb{Q}(Z^2)$

(i) if $\langle T_{\bigcap_{i\in I} X_i\ominus Y}, C_{\bigcap_{i\in I} X_i\ominus Y}, U_{\bigcap_{i\in I} X_i\ominus Y}, F_{\bigcap_{i\in I} X_i\ominus Y}\rangle \subseteq \langle T_{\bigcap_{i\in I}(X_i\ominus Y)}, C_{\bigcap_{i\in I}(X_i\ominus Y)}, U_{\bigcap_{i\in I}(X_i\ominus Y)}, F_{\bigcap_{i\in I}(X_i\ominus Y)}\rangle$,then

$T_{\bigcap_{i\in I} X_i\ominus Y} \subseteq T_{\bigcap_{i\in I}(X_i\ominus Y)}$, $C_{\bigcap_{i\in I} X_i\ominus Y} \subseteq C_{\bigcap_{i\in I}(X_i\ominus Y)}$, $U_{\bigcap_{i\in I} X_i\ominus Y} \supseteq U_{\bigcap_{i\in I}(X_i\ominus Y)}$ *and* $F_{\bigcap_{i\in I} X_i\ominus Y} \supseteq F_{\bigcap_{i\in I}(X_i\ominus Y)}$

(ii) if $\langle T_{Y\ominus \bigcap_{i\in I} X_i}, C_{Y\ominus \bigcap_{i\in I} X_i}, U_{Y\ominus \bigcap_{i\in I} X_i}, F_{Y\ominus \bigcap_{i\in I} X_i}\rangle \supseteq \langle T_{\bigcap_{i\in I}(Y\ominus X_i)}, C_{\bigcap_{i\in I}(Y\ominus X_i)}, U_{\bigcap_{i\in I}(Y\ominus X_i)}, F_{\bigcap_{i\in I}(Y\ominus X_i)}\rangle$,then

$T_{Y\ominus \bigcap_{i\in I} X_i} \supseteq T_{\bigcup_{i\in I}(Y\ominus X_i)}, C_{Y\ominus \bigcap_{i\in I} X_i} \supseteq C_{\bigcup_{i\in I}(Y\ominus X_i)}, U_{Y\ominus \bigcap_{i\in I} X_i} \subseteq U_{\bigcap_{i\in I}(Y\ominus X_i)}$ and $F_{Y\ominus \bigcap_{i\in I} X_i} \subseteq F_{\bigcap_{i\in I}(Y\ominus X_i)}$

Proof :Let (i)

$\langle T_{\bigcap_{i\in I} X_i\ominus Y}, C_{\bigcap_{i\in I} X_i\ominus Y}, U_{\bigcap_{i\in I} X_i\ominus Y}, F_{\bigcap_{i\in I} X_i\ominus Y}\rangle \subseteq \langle T_{\bigcap_{i\in I}(X_i\ominus Y)}, C_{\bigcap_{i\in I}(X_i\ominus Y)}, U_{\bigcap_{i\in I}(X_i\ominus Y)}, F_{\bigcap_{i\in I}(X_i\ominus Y)}\rangle$

$$
\begin{aligned}
T_{\bigcap_{i\in I} X_i\ominus Y}(a_2) &= \inf_{a_1\in Z^2} max\left(T_{\bigcap_{i\in I} X_i}(a_2+a_1), 1-T_Y(a_1)\right)\\
&= \inf_{a_1\in Z^2} max\left(\inf_{i\in I} T_{X_i}(a_2+a_1), 1-T_Y(a_1)\right)\\
&\le \inf_{a_1\in Z^2}\inf_{i\in I}\left(\max\left(T_{X_i}(a_2+a_1), 1-T_Y(a_1)\right)\right)\\
&\le \bigcap_{i\in I}\inf_{a_1\in Z^2}\max\left(T_{X_i}(a_2+a_1), 1-T_Y(a_1)\right)\\
&\le \bigcap_{i\in I} T_{X_i\ominus Y}(a_2)
\end{aligned}
$$

$$
\begin{aligned}
C_{\bigcap_{i\in I} X_i\ominus Y}(b) &= \inf_{a_1\in Z^2} \max\left(C_{\bigcap_{i\in I} M_i}(a_2+a_1), 1-C_Y(a_1)\right)\\
&= \inf_{a_1\in Z^2} \max\left(\inf_{i\in I} C_{X_i}(a_2+a_1), 1-C_Y(a_1)\right)\\
&\le \inf_{a_1\in Z^2}\inf_{i\in I}\left(\max\left(C_{X_i}(a_2+a_1), 1-C_Y(a_1)\right)\right)\\
&\le \bigcap_{i\in I}\inf_{a_1\in Z^2}\max\left(C_{X_i}(a_2+a_1), 1-C_Y(a_1)\right)\\
&\le \bigcap_{i\in I} C_{X_i\ominus Y}(a_2)
\end{aligned}
$$

$$
\begin{aligned}
U_{\bigcap_{i\in I} X_i\ominus Y}(a_2) &= \sup_{a_1\in Z^2} min\left(1-U_{\bigcap_{i\in I} X_i}(a_2+a_1), U_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2} min\left(1-\inf_{i\in I} U_{X_i}(a_2+a_1), U_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2} min\left(\sup_{i\in I}\left(1-U_{X_i}(a_2+a_1)\right), U_Y(a_1)\right)\\
&\ge \sup_{a_1\in Z^2}\sup_{i\in I}\left(\min\left(1-U_{X_i}(a_2+a_1), U_Y(a_1)\right)\right)\\
&\ge \bigcup_{i\in I}\sup_{a_1\in Z^2} min\left(1-U_{X_i}(a_2+a_1), U_Y(a_1)\right)\\
&\ge U_{\bigcup_{i\in I}(X_i\ominus Y)}(a_2)
\end{aligned}
$$

$$
\begin{aligned}
F_{\bigcap_{i\in I} X_i\ominus Y}(a_2) &= \sup_{a_1\in Z^2} min\left(1-F_{\bigcap_{i\in I} X_i}(a_2+a_1), F_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2} min\left(1-\inf_{i\in I} F_{X_i}(a_2+a_1), F_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2} min\left(\sup_{i\in I}\left(1-F_{X_i}(a_2+a_1)\right), F_Y(a_1)\right)\\
&\ge \sup_{a_1\in Z^2}\sup_{i\in I}\left(\min\left(1-F_{X_i}(a_2+a_1), F_Y(a_1)\right)\right)\\
&\ge \bigcup_{i\in I}\sup_{a_1\in Z^2} \min\left(1-F_{X_i}(a_2+a_1), F_Y(a_1)\right)\\
&\ge F_{\bigcup_{i\in I}(X_i\ominus Y)}(a_2)
\end{aligned}
$$

Hence proved part (i).

The proof of (ii) is similar to (i).

Proposition 9.3: For any family $(X_i|i \in I)$ in $\mathbb{Q}(Z^2)$ and $Y\in \mathbb{Q}(Z^2)$

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(i) if $\langle T_{\underset{i\in I}{\cup} X_i\ominus Y}, C_{\underset{i\in I}{\cup} X_i\ominus Y}, U_{\underset{i\in I}{\cup} X_i\ominus Y}, F_{\underset{i\in I}{\cup} X_i\ominus Y}\rangle \supseteq \langle T_{\underset{i\in I}{\cup}(X_i\ominus Y)}, C_{\underset{i\in I}{\cup}(X_i\ominus Y)}, U_{\underset{i\in I}{\cup}(X_i\ominus Y)}, F_{\underset{i\in I}{\cup}(X_i\ominus Y)}\rangle$,then
$T_{\underset{i\in I}{\cup} X_i\ominus Y} \supseteq T_{\underset{i\in I}{\cup}(X_i\ominus Y)}$, $C_{\underset{i\in I}{\cup} X_i\ominus Y} \supseteq C_{\underset{i\in I}{\cup}(X_i\ominus Y)}$, $U_{\underset{i\in I}{\cup} X_i\ominus Y} \subseteq U_{\underset{i\in I}{\cap}(X_i\ominus Y)}$ and $F_{\underset{i\in I}{\cup} X_i\ominus Y} \subseteq F_{\underset{i\in I}{\cap}(X_i\ominus Y)}$
ii) if $\langle T_{Y\ominus \underset{i\in I}{\cup} X_i}, C_{Y\ominus \underset{i\in I}{\cup} X_i}, U_{Y\ominus \underset{i\in I}{\cup} X_i}, F_{Y\ominus \underset{i\in I}{\cup} X_i}\rangle \subseteq \langle T_{\underset{i\in I}{\cup}(Y\ominus X_i)}, C_{\underset{i\in I}{\cup}(Y\ominus X_i)}, U_{\underset{i\in I}{\cup}(Y\ominus X_i)}, F_{\underset{i\in I}{\cup}(Y\ominus X_i)}\rangle$,then
$T_{Y\ominus \underset{i\in I}{\cup} X_i} \subseteq T_{\underset{i\in I}{\cap}(Y\ominus X_i)}$, $C_{Y\ominus \underset{i\in I}{\cup} X_i} \subseteq C_{\underset{i\in I}{\cap}(Y\ominus X_i)}$, $U_{Y\ominus \underset{i\in I}{\cup} X_i} \supseteq U_{\underset{i\in I}{\cup}(Y\ominus X_i)}$ and $F_{Y\ominus \underset{i\in I}{\cup} X_i} \supseteq F_{\underset{i\in I}{\cup}(Y\ominus X_i)}$

Proof :Let

$$\langle T_{\underset{i\in I}{\cup} X_i\ominus Y}, C_{\underset{i\in I}{\cup} X_i\ominus Y}, U_{\underset{i\in I}{\cup} X_i\ominus Y}, F_{\underset{i\in I}{\cup} X_i\ominus Y}\rangle \supseteq \langle T_{\underset{i\in I}{\cup}(X_i\ominus Y)}, C_{\underset{i\in I}{\cup}(X_i\ominus Y)}, U_{\underset{i\in I}{\cup}(X_i\ominus Y)}, F_{\underset{i\in I}{\cup}(X_i\ominus Y)}\rangle$$

$$\begin{aligned}
T_{\underset{i\in I}{\cup} X_i\ominus Y}(a_2) &= \inf_{a_1\in Z^2} max\left(T_{\underset{i\in I}{\cup} X_i}(a_2+a_1), T_Y(a_1)\right)\\
&= \inf_{a_1\in Z^2} max\left(\sup_{i\in I} T_{X_i}(a_2+a_1), T_Y(a_1)\right)\\
&= \inf_{a_1\in Z^2}\sup_{i\in I}\left(max\ T_{X_i}(a_2+a_1), T_Y(a_1)\right)\\
&\ge \underset{i\in I}{\cup}\inf_{a_1\in Z^2} \max\left(T_{X_i}(a_2+a_1), T_Y(a_1)\right)\\
&\ge \underset{i\in I}{\cup} T_{X_i\ominus Y}(a_2)\\
&\ge T_{\underset{i\in I}{\cup}(X_i\ominus Y)}(a_2)
\end{aligned}$$

$$\begin{aligned}
C_{\underset{i\in I}{\cup} X_i\ominus Y}(a_2) &= \inf_{a_1\in Z^2} max\left(C_{\underset{i\in I}{\cup} X_i}(a_2+a_1), C_Y(a_1)\right)\\
&= \inf_{a_1\in Z^2} max\left(\sup_{i\in I} C_{X_i}(a_2+a_1), C_Y(a_1)\right)\\
&= \inf_{a_1\in Z^2}\sup_{i\in I}\left(max\ C_{X_i}(a_2+a_1), C_Y(a_1)\right)\\
&\ge \underset{i\in I}{\cup}\inf_{a_1\in Z^2} max\left(C_{X_i}(a_2+a_1), C_Y(a_1)\right)\\
&\ge \underset{i\in I}{\cup} C_{X_i\ominus Y}(a_2)\\
&\ge C_{\underset{i\in I}{\cup}(X_i\ominus Y)}(a_2)
\end{aligned}$$

$$\begin{aligned}
U_{\underset{i\in I}{\cup} X_i\ominus Y}(a_2) &= \sup_{a_1\in Z^2} min\left(1-U_{\underset{i\in I}{\cup} X_i}(a_2+a_1), U_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2} min\left(1-\sup_{i\in I} U_{X_i}(a_2+a_1), U_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2} min\left(\inf_{i\in I}(1-U_{X_i}(a_2+a_1)), U_Y(a_1)\right)\\
&= \sup_{a_1\in Z^2}\inf_{i\in I}\left(\min(1-U_{X_i}(a_2+a_1), U_Y(a_1))\right)\\
&\le \inf_{i\in I}\sup_{a_1\in Z^n}\left(\min(1-U_{X_i}(a_2+a_1), U_Y(a_1))\right)\\
&\le \underset{i\in I}{\cap}\sup_{a_1\in Z^n} min\left(1-U_{X_i}(a_2+a_1), U_Y(a_1)\right)\\
&\le U_{\underset{i\in I}{\cap}(X_i\ominus Y)}(a_2)
\end{aligned}$$

$$\begin{aligned}
F_{\underset{i\in I}{\cup} X_i\ominus Y}(b) &= \sup_{a_1\in Z^n} min\left(1-F_{\underset{i\in I}{\cup} X_i}(a_2+a_1), F_Y(a_1)\right)\\
&= \sup_{a_1\in Z^n} min\left(1-\sup_{i\in I} F_{X_i}(a_2+a_1), F_Y(a_1)\right)\\
&= \sup_{a_1\in Z^n} min\left(\inf_{i\in I}(1-F_{X_i}(a_2+a_1)), F_Y(a_1)\right)\\
&= \sup_{a_1\in Z^n}\inf_{i\in I}\left(\min(1-F_{X_i}(a_2+a_1), F_Y(a))\right)\\
&\le \inf_{i\in I}\sup_{a_1\in Z^n}\left(\min(1-F_{X_i}(a_2+a_1), F_Y(a_1))\right)\\
&\le \underset{i\in I}{\cap}\sup_{a_1\in Z^n} \min\left(1-F_{X_i}(a_2+a_1), F_Y(a_1)\right)\\
&\le F_{\underset{i\in I}{\cap}(X_i\ominus Y)}(a_2)
\end{aligned}$$

Hence proved part (i).
The proof of (ii) is similar to (i).

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10. Quadri Partitioned Neutrosophic Closing Properties

The following properties are satisfied by the quadripartitioned neutrosophic closing.

Proposition 10.1: The Quadri Partitioned Neutrosophic closing satisfies :

Monotonicity , for all X, Y,Z $\in \mathbb{Q}(Z^2)$

i) if $X \subseteq Y \Longrightarrow \langle T_{X*Z}, C_{X*Z}, U_{X*Z}, F_{X*Z} \rangle \subseteq \langle T_{Y*Z}, C_{Y*Z}, U_{Y*Z}, F_{Y*Z} \rangle$,then

$T_{X*Z} \subseteq T_{Y*Z}$, $C_{X*Z} \subseteq C_{Y*Z}$, $U_{X*Z} \supseteq U_{Y*Z}$ *and* $F_{X*Z} \supseteq F_{Y*Z}$

Proposition10. 2: For any family $(X_i|i \in I)$ in $\mathbb{Q}(Z^2)$ and Y$\in \mathbb{Q}(Z^2)$

if $\langle T_{\bigcap_{i\in I} X_i*Y}, C_{\bigcap_{i\in I} X_i*Y}, U_{\bigcap_{i\in I} X_i*Y}, F_{\bigcap_{i\in I} X_i*Y} \rangle \subseteq \langle T_{\bigcap_{i\in I}(X_i*Y)}, C_{\bigcap_{i\in I}(X_i*Y)}, U_{\bigcap_{i\in I}(X_i*Y)}, F_{\bigcap_{i\in I}(X_i*Y)} \rangle$,then

$T_{\bigcap_{i\in I} X_i*Y} \subseteq T_{\bigcap_{i\in I}(X_i*Y)}$, $C_{\bigcap_{i\in I} X_i*Y} \subseteq C_{\bigcap_{i\in I}(X_i*Y)}$, $U_{\bigcap_{i\in I} X_i*Y} \supseteq U_{\bigcup_{i\in I}(X_i*Y)}$ *and* $F_{\bigcap_{i\in I} X_i*Y} \supseteq F_{\bigcup_{i\in I}(X_i*Y)}$

Proposition 10.3: For any family $(X_i|i \in I)$ in $\mathbb{Q}(Z^2)$ and Y$\in \mathbb{Q}(Z^2)$

if $\langle T_{\bigcup_{i\in I} X_i*Y}, C_{\bigcup_{i\in I} X_i*Y}, U_{\bigcup_{i\in I} X_i*Y}, F_{\bigcup_{i\in I} X_i*Y} \rangle \supseteq \langle T_{\bigcup_{i\in I}(X_i*Y)}, C_{\bigcup_{i\in I}(X_i*Y)}, U_{\bigcup_{i\in I}(X_i*Y)}, F_{\bigcup_{i\in I}(X_i*Y)} \rangle$,then

$T_{\bigcup_{i\in I} X_i*Y} \supseteq T_{\bigcup_{i\in I}(X_i*Y)}$, $C_{\bigcup_{i\in I} X_i*Y} \supseteq C_{\bigcup_{i\in I}(X_i*Y)}$, $U_{\bigcup_{i\in I} X_i*Y} \subseteq U_{\bigcap_{i\in I}(X_i*Y)}$ *and* $F_{\bigcup_{i\in I} X_i*Y} \subseteq F_{\bigcap_{i\in I}(X_i*Y)}$

The proof is similar to the procedure used in propositions 8 and 9.

11.The Quadri Partitioned Neutrosophic opening satisfies

The following properties are satisfies by the Quadri Partitioned Neutrosophic opening.

Proposition 11.1: The Quadri Partitioned Neutrosophic opening satisfies :

Monotonicity , for all X, Y,Z $\in \mathbb{Q}(Z^2)$

i) $X \subseteq Y \Longrightarrow \langle T_{X\circ Z}, C_{X\circ Z}, U_{X\circ Z}, F_{X\circ Z} \rangle \subseteq \langle T_{Y\circ Z}, C_{Y\circ Z}, U_{Y\circ Z}, F_{Y\circ Z} \rangle$

$T_{X\circ Z} \subseteq T_{Y\circ Z}$, $C_{X\circ Z} \subseteq C_{Y\circ Z}$, $U_{X\circ Z} \supseteq U_{Y\circ Z}$ *and* $F_{X\circ Z} \supseteq F_{Y\circ Z}$

Proposition 11. 2: For any family $(X_i|i \in I)$ in $\mathbb{Q}(Z^2)$ and Y$\in \mathbb{Q}(Z^2)$

(i)if $\langle T_{\bigcap_{i\in I} X_i\circ Y}, C_{\bigcap_{i\in I} X_i\circ Y}, U_{\bigcap_{i\in I} X_i\circ Y}, F_{\bigcap_{i\in I} X_i\circ Y} \rangle \subseteq \langle T_{\bigcap_{i\in I}(X_i\circ Y)}, C_{\bigcap_{i\in I}(X_i\circ Y)}, U_{\bigcap_{i\in I}(X_i\circ Y)}, F_{\bigcap_{i\in I}(X_i\circ Y)} \rangle$,then

$T_{\bigcap_{i\in I} X_i\circ Y} \subseteq T_{\bigcap_{i\in I}(X_i\circ Y)}$, $C_{\bigcap_{i\in I} X_i\circ Y} \subseteq C_{\bigcap_{i\in I}(X_i\circ Y)}$, $U_{\bigcap_{i\in I} X_i\circ Y} \supseteq U_{\bigcup_{i\in I}(X_i\circ Y)}$ *and* $F_{\bigcap_{i\in I} X_i\circ Y} \supseteq F_{\bigcup_{i\in I}(X_i\circ Y)}$

Proposition 11.3: For any family $(x_i|i \in I)$ in $\mathbb{Q}(Z^2)$ and Y$\in \mathbb{Q}(Z^2)$

if $\langle T_{\bigcup_{i\in I} X_i\circ Y}, C_{\bigcup_{i\in I} X_i\circ Y}, U_{\bigcup_{i\in I} X_i\circ Y}, F_{\bigcup_{i\in I} X_i\circ Y} \rangle \supseteq \langle T_{\bigcup_{i\in I}(X_i\circ Y)}, C_{\bigcup_{i\in I}(X_i\circ Y)}, U_{\bigcup_{i\in I}(X_i\circ Y)}, F_{\bigcup_{i\in I}(X_i\circ Y)} \rangle$,then

$T_{\bigcup_{i\in I} X_i\circ Y} \supseteq T_{\bigcup_{i\in I}(X_i\circ Y)}$, $C_{\bigcup_{i\in I} X_i\circ Y} \supseteq C_{\bigcup_{i\in I}(X_i\circ Y)}$, $U_{\bigcup_{i\in I} X_i\circ Y} \subseteq U_{\bigcap_{i\in I}(X_i\circ Y)}$ and $F_{\bigcup_{i\in I} X_i\circ Y} \subseteq F_{\bigcap_{i\in I}(X_i\circ Y)}$

The proof is similar to the procedure used in propositions 8 and 9.

11. Conclusion

The concept of the quadripartitioned neutrosophic set's mathematical morphology is introduced in this study. The set theory for mathematical morphology serves as the foundation for its introduction. There were numerous fundamental definitions for the quadripartitioned neutrosophic set in mathematical morphological operations as well as explanations of its algebraic features. The concept of quadripartitioned neutrosophic sets' mathematical morphology will be used in image processing in the future. Medical images, for instance, use image enhancement, retrieval, and smoothing.

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