Neutrosophic Soft n-Topological Spaces: A Framework for Decision-Making Problems

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Abstract

This article introduces the concept of neutrosophic soft n-topological spaces and their application in decision-making problems. Neutrosophic soft sets are used to define the open sets in these spaces, which allow for greater flexibility and uncertainty in the decision-making process. The concept of neutrosophic soft n-homeomorphism is also introduced, which describes the topological equivalence between two neutrosophic soft n-topological spaces. The article provides examples of how neutrosophic soft n-topological spaces can be used in decision-making problems, such as medical diagnosis and stock market analysis. The theory and applications presented in this article provide a valuable tool for dealing with uncertainty in decision-making problems.

Keywords: Neutrosophic Soft n-Topological Space; Neutrosophic Soft Continuous Map; Neutrosophic Soft n-Homeomorphism; Neutrosophic Soft Filters; Decision-Making Problems

1. Introduction

In this article, we introduce the concept of neutrosophic soft n-topological spaces and their applications in decision-making problems. Neutrosophic soft sets are a generalization of fuzzy sets, which allow for the representation of uncertain, indeterminate, and inconsistent information. Similarly, neutrosophic soft n-topological spaces extend the idea of fuzzy n-topological spaces to incorporate neutrosophic soft sets **[8]**. The paper begins with a brief introduction to neutrosophic soft sets and fuzzy n-topological spaces. We then define neutrosophic soft n-topological spaces and explore their properties, including neutrosophic soft continuity, neutrosophic soft separation axioms, and neutrosophic soft connectedness. We also define the concept of neutrosophic soft n-homeomorphism and examine its properties.

Finally, we discuss the applications of neutrosophic soft n-topological spaces in decision-making problems, including decision-making with incomplete and inconsistent information, decision-making under risk and uncertainty, and decision-making in social networks.

Overall, this article provides a comprehensive introduction to neutrosophic soft n-topological spaces and their applications in decision-making problems. The results presented in this paper can be useful in various fields such as computer science, engineering, and economics.

2. Preliminary

This section provides fundamental definitions and theorems related to neutrosophic set theory and neutrosophic soft set theory.

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2.1. Neutrosophic sets [1]

Consider a space X consisting of points or objects, where a generic element in X is denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T, an indeterminacy-membership function I, and a falsity-membership function F [1]. In other words, T, I, and F are functions from X to the real standard or non-standard subsets of the interval [-0, 1+]. There is generally no restriction on the sum of T(x), I(x), and F(x), so -0 ≤ T(x) + I(x) + F(x) ≤ 3+. T, I, and F are referred to as the neutrosophic components, and the set of all neutrosophic sets in X is denoted by N(X).

2.1 Definition [1]: Let $S, M \in N(X)$.

1. Subset: $M \subset S$ if $T_M(b) \leq T_S(b), I_M(b) \leq I_S(b), F_M(b) \geq F_S(b)$ for all $b \in X$.

2. Equality: $M = S$ if $M \subset S$ and $S \subset M$.

3. Union:

$$M \cup S = \{< b, max\{T_M(b), T_S(b)\}, max\{I_M(b), I_S(b)\}, min\{F_M(b), F_S(b)\} >: b \in X\}.$$

4. Intersection:

$$M \cap S = \{< b, min\{T_M(b), T_S(b)\}, min\{I_M(b), I_S(b)\}, max\{F_M(b), F_S(b)\} >: b \in X\}.$$

More generally, the intersection and the union of a collection of neutrosophic sets $\{M_i\} \in N(X)$ are defined by:

The intersection and union of a collection of neutrosophic sets $\{\{M_i\} \in N(X)$ are defined more generally as follows:

$$\cap_{i \in I} M_i = \left\{< b, min\left\{T_{M_i}(b)\right\}, min\left\{I_{M_i}(b)\right\}, max\left\{F_{M_i}(b)\right\} >: b \in X\right\},$$

$$\cup_{i \in I} M_i = \left\{< b, max\left\{T_{M_i}(b)\right\}, max\left\{I_{M_i}(b)\right\}, min\left\{F_{M_i}(b)\right\} >: b \in X\right\}.$$

5. The neutrosophic set defined as $T_M(b) = 1, I_M(b) = 1$ and $F_M(b) = 0$ for all $b \in X$ is called the universal NS denoted by 1_X. Also, the neutrosophic set defined as $T_M(b) = 0, I_M(b) = 0$ and $F_M(b) = 1$ for all $b \in X$ is called the empty NS denoted by 0_X.

6. Complement: $M^c = 1_X \backslash M$

2.2 Definition [2]:A subset Γ of N(Y) is called a neutrosophic topology on Y if it satisfies the following conditions:

1. The empty neutrosophic set $0_X$and the universal neutrosophic set 1_X belong to Γ.
2. The union of any number of neutrosophic sets in Γ belongs to Γ.
3. The intersection of a finite number of neutrosophic sets in Γ belongs to Γ.

The pair (Y,Γ) is then called a neutrosophic topology on Y.

1.2 Neutrosophic soft set

2.3. Definition [4]: Let P be an initial universe set, and let E be a set of parameters. The pair (L,E) is defined as a neutrosophic soft set (NSS) over P, where L is a mapping from E to N(P).

The set of all neutrosophic soft sets over P is denoted by NSS(P). A neutrosophic soft set (L,E) can be expressed as:$(L, E) = \{(e, \{< x, T_{L(e)}(x), I_{L(e)}(x), F_{L(e)}(x) >\}: x \in P, e \in E\}$.

2.4 Definition [4]:

Let X be an initial universe set, and let E be a set of parameters. Then, the neutrosophic soft set ${x^e}_{(\alpha,\beta,\gamma)}$ is defined as:

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${x^e}_{(\alpha,\beta,\gamma)}$ = {(x, T(x,α), I(x,β), F(x,γ)) | x ∈ X},

where T(x,α), I(x,β), and F(x,γ) are the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively, defined on X. The values of these functions are given by:

T(x,α) = {0, 1, α}, I(x,β) = {0, 1, β}, and F(x,γ) = {0, 1, γ},

where 0 ≤ α,β,γ ≤ 1.

Thus, the neutrosophic soft set ${x^e}_{(\alpha,\beta,\gamma)}$ assigns to each element x in X a truth-membership value α, an indeterminacy-membership value β, and a falsity-membership value γ.

2.5 Definition [3]: Let $(w,E),(M,E) \in NSS(P)$. Then for all $x \in P$

1. Subset: $(w,E) \subset (M,E)$ if $T_{w(e)}(x) \leq T_{M(e)}(x), I_{w(e)}(x) \leq I_{M(e)}(x)$ and $F_{w(e)}(x) \geq F_{M(e)}(x)$ for all $e \in E,$

2. Equality: $(w,E) = (M,E)$ if $(w,E) \subset (M,E)$ and $(M,E) \subset (w,E),$

3. Intersection:

$$(w,E) \cap (z,E) = \left\{(e,\{< x, min\{T_{w(e)}(x), T_{z(e)}(x)\}, max\{I_{w(e)}(x), I_{z(e)}(x)\}, max\{F_{w(e)}(x), F_{z(e)}(x)\} >\}: e \in E\right\},$$

4. Union:

$$(w,E) \cup (z,E) = \left\{(e,\{< x, max\{T_{w(e)}(x), T_{z(e)}(x)\}, min\{I_{w(e)}(x), I_{z(e)}(x)\}, min\{F_{w(e)}(x), F_{z(e)}(x)\} >\}: e \in E\right\}.$$

More generally, the intersection and the union of a collection of $\{(w_i,E)\} \subset NSS(P)$ are defined by:

$$\cup_{i\in I}(w_i,E) = \left\{(e,\left\{< x, max\left\{T_{w_{i(e)}}(x)\right\}, max\left\{I_{w_{i(e)}}(x)\right\}, min\left\{F_{w_{i(e)}}(x)\right\} >\right\}: e \in E\right\}$$

$$\cap_{i\in I}(w_i,E) = \left\{(e,\left\{< x, min\left\{T_{w_{i(e)}}(x)\right\}, min\left\{I_{w_i(e)}(x)\right\}, max\left\{F_{w_i(e)}(x)\right\} >\right\}: e \in E\right\}$$

5. The NSS defined as and $F_{w(e)}(x) = 0$, for all $e \in E$ and $x \in P$ is called the universal NSS denoted by $1_{(P,E)}$. Also, the neutrosophic set defined as $T_{w(e)}(x) = 0, I_{w(e)}(x) = 0$ and $F_{w(e)}(x) = 1$ for all $e \in E$ and $x \in P$ is called the empty NSS denoted by $0_{(P,E)}$.
The NSS defined as $T_{w(e)}(x) = 1, I_{w(e)}(x) = 1,$ and $F_{w(e)}(x) = 0$ for all e∈E and x∈P is called the universal NSS and is denoted by $1_{(P,E)}$. Similarly, the NSS defined as as $T_{w(e)}(x) = 0, I_{w(e)}(x) = 0,$ and$F_{w(e)}(x) = 1$ for all e∈E and x∈P is called the empty NSS and is denoted by $0_{(P,E)}$.

6. Complement: $(w,E)^c = 1_{(P,E)} \backslash (w,E) = \{e,\{x, F_{w(e)}(x), 1 - I_{w(e)}(x), T_{w(e)}(x) >\}: e \in E\}$
Clearly, the complements of $1_{(X,E)}$ and $0_{(X,E)}$ are defined:

$$(1_{(P,E)})^c = 1_{(P,E)} \backslash 1_{(P,E)} = \{(e,\{< x, 0,0,1 >\} : e \in E\} = 0_{(P,E)}$$

$$(0_{(P,E)})^c = 1_{(P,E)} \backslash 0_{(P,E)} = \{(e,\{< x, 1,0,0 >\}: e \in E\} = 1_{(P,E)}$$

2.6 Definition [4]: The neutrosophic soft topology on a set Y is defined as follows: Let Γ be a subset of NSS(Y). Γ is called a neutrosophic soft topology on Y if the following conditions hold:

NST1) The empty NSS)$0_{(P;E)}$and the universal NSS $1_{(P,E)}$ belong to Γ.

NST2) The union of any number of NSSs in Γ again belongs to Γ.

NST3) The intersection of a finite number of NSSs in Γ belongs to Γ.

The pair (Y,Γ) is called a neutrosophic soft topological space. The elements of Γ are called neutrosophic soft open sets. An NSS whose complement is neutrosophic soft open is called a neutrosophic soft closed set.

2.7 Definition [5]: A neutrosophic soft supra topology on Y is denoted by Γ ⊂ NSS(Y) that satisfies the following conditions: $0_{(Y;E)}$ and $1_{(Y,E)}$ belong to Γ, and the union of any number of NSSs in Γ is again a member of Γ.

2.8 Definition [6]: A neutrosophic soft bitopological space is denoted by (X, τ_1, τ_2, E), where(X, τ_1, E) and (X, τ_2, E) are two neutrosophic soft topological spaces. The sets belonging to τ_i are called neutrosophic soft i-open sets for i=1,2.

2.9 Definition [7]: If $(X, \tau_1, E), (X, \tau_2, E)$ and (X, τ_3, E) are three neutrosophic soft topological spaces, then $(X, \tau_1, \tau_2, \tau_3, E)$ is named as neutrosophic soft tri-topological space. The sets belonging to τ_i are called neutrosophic soft i −open sets for $i = 1,2,3$. **[8]**

3. Neutrosophic Soft n-Topological Spaces

Neutrosophic soft set theory is an extension of neutrosophic set theory, which allows the assignment of a degree of membership, indeterminacy, and non-membership to the elements of a set. In this article, we introduce the concept of neutrosophic soft n-topological spaces, which is a generalization of neutrosophic soft set theory and n-topological spaces.

Let X be a set, and let T be a neutrosophic soft set on X, which is a collection of triplets (x, μ(x), λ(x)) where μ(x) and λ(x) are the degrees of membership and non-membership of x, respectively, and $\mu(x) + \lambda(x) \leq 1$ for all x ∈ X. We denote T as (X, μ, λ).

Now, let n be a positive integer, and let $P = \{P_1, P_2, ..., P_n\}$ be a partition of X, where each P_i is a subset of X.

A neutrosophic soft n-topology on X is a collection $T = \{T_i \mid i = 1, 2, ..., n\}$ of neutrosophic soft sets on X such that:

- T_i is a neutrosophic soft set on X for each i = 1, 2, ..., n
- T_i is a proper neutrosophic soft subset of T_j whenever $i \neq j$
- T_i contains all elements of P_i
- T_i is closed under arbitrary union of neutrosophic soft sets.

We call the pair (X, T) a neutrosophic soft n-topological space. In other words, a neutrosophic soft n-topology on X is a collection of n neutrosophic soft sets that covers the set X and is closed under arbitrary union.

Now, let's define some basic concepts related to neutrosophic soft n-topological spaces:

- Neutrosophic Soft n-Interior: The neutrosophic soft n-interior of a neutrosophic soft set A is the largest neutrosophic soft set that is contained in A and is an element of T_i for each i = 1, 2, ..., n. We denote the neutrosophic soft n-interior of A as $\text{int}_n(A)$.
- Neutrosophic Soft n-Closure: The neutrosophic soft n-closure of a neutrosophic soft set A is the smallest neutrosophic soft set that contains A and is an element of T_i for each i = 1, 2, ..., n. We denote the neutrosophic soft n-closure of A as $\text{cl}_n(A)$.

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- Neutrosophic Soft n-Exterior: The neutrosophic soft n-exterior of a neutrosophic soft set A is the largest neutrosophic soft set that is disjoint from A and is an element of T_i for each i = 1, 2, ..., n. We denote the neutrosophic soft n-exterior of A as $\text{ext}_n(A)$.

- Neutrosophic Soft n-Frontier: The neutrosophic soft n-frontier of a neutrosophic soft set A is the set $\text{int}_n(A) \cup \text{ext}_n(A)^c$. We denote the neutrosophic soft n-frontier of A as $\text{fr}_n(A)$.

Now, let's define some more advanced concepts:

- Neutrosophic Soft n-Continuity: Let (X, T) and

3. Neutrosophic Soft n-Topological Spaces

In this section, we introduce the concept of neutrosophic soft n-topological spaces, which is an extension of neutrosophic soft topological spaces and neutrosophic n-topological spaces.

3.1 Definition: A neutrosophic soft n-topological space (NSnTS) on a non-empty set X is a neutrosophic soft set (NSS) (T,I,F) of n-approximation spaces $(X_i, t_i)(i = 1,2, \dots, n)$ where t_i is a fuzzy n-topology on X_i, and the following conditions are satisfied:

1. For any $x_i \in X_i$, $T((x_1, x_2,..., x_n))>0$ if and only if $(x_1, x_2,..., x_n) \in T(t_1, t_2, \dots, t_n)$ where $T(t_1, t_2, \dots, t_n) = (x_1, x_2, \dots, x_n) \in X_1 \times X_2 \times \dots\dots \times X_n : (x_i \in U_i)$ for some $U_i \in t_i : i = 1,2, \dots, n$
2. For any $x_i \in X_i$, $F((x_1, x_2,..., x_n))>0$ if $(x_1, x_2, \dots, x_n) \notin F(t_1, t_2, \dots, t_n)$ where $F(t_1, t_2, \dots, t_n) = (x_1, x_2, \dots, x_n) \in X_1 \times X_2 \times \dots\dots \times X_n : (x_i \in F_i)$ for some $F_i \in t_i : i = 1,2, \dots, n$
3. For any $x_i \in X_i$, $I((x_1, x_2,..., x_n))>0$ if and only if
$$(x_1, x_2, \dots, x_n) \notin T(t_1, t_2, \dots, t_n) \cup F(t_1, t_2, \dots, t_n).$$

4. The neutrosophic soft set (T, I, F) is closed under arbitrary neutrosophic soft set intersection and finite neutrosophic soft set union.

5. Note that in the definition above, we use the Cartesian product $X_1 \times X_2 \times \dots\dots \times X_n$ to represent the set of all n-tuples $(x_1, x_2, \dots, x_n)$, where $x_i \in X_i : i = 1,2, \dots, n$

3.2 Examples:

Let $X = \{a, b, c\}$ X_1 = {a,b}, X_2 = {b,c}, and X_3 = {a,c}. Let t_1= {{a},{a,b}}, t_2= {{b},{b,c}}, and t_3= {{a},{a,c}}. Then the NSS (T,I,F) defined by $T((t_1, t_2, t_3)) = ((x_1, x_2, x_3))= 1/2$ for all $x_1, x_2, x_3) \in X_1 \times X_2 \times X_3$

4. Neutrosophic Soft n-Open Sets and Neutrosophic Soft n-Continuity

In this section, we define the concepts of neutrosophic soft n-open sets and neutrosophic soft n-continuity.

4.1. Neutrosophic Soft n-Open Sets

Definition 4.1: Let X be a neutrosophic soft n-topological space and let A ⊆ X. A neutrosophic soft n-open set of X is a neutrosophic soft set O ⊆ X such that for every neutrosophic soft set B ⊆ X, if A ⊆ B and B is neutrosophic soft n-compact, then O ⊆ B.

Remark 4.1: We denote by $N_s^n(X)$ the set of all neutrosophic soft n-open sets of X. It is easy to see that the empty set ∅ and the whole space X are neutrosophic soft n-open sets.

Example 4.1: Let X = {a, b, c} and T, I, and F be truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively, defined as follows:

T(a) = 1, T(b) = 0.8, T(c) = 0.5

I(a) = 0.1, I(b) = 0.5, I(c) = 0.9

F(a) = 0, F(b) = 0.2, F(c) = 0.5

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Let A = {a, b}. Then the neutrosophic soft set O = {<a, 0.5, 0, 0.8>, <b, 0.5, 0.1, 0>} is a neutrosophic soft n-open set of X. Indeed, let B = {<a, 1, 0, 0>, <b, 0.8, 0, 0.2>, <c, 0.5, 0.9, 0>} be a neutrosophic soft n-compact set containing A. Then, we have O ⊆ B.

5. Neutrosophic Soft n-Continuity

Definition 4.2: Let X and Y be neutrosophic soft n-topological spaces and let f : X → Y be a neutrosophic soft map. We say that f is neutrosophic soft n-continuous at x ∈ X if for every neutrosophic soft n-open set V ⊆ Y containing f(x), there exists a neutrosophic soft n-open set U ⊆ X containing x such that f(U) ⊆ V.

Remark 4.2: We say that f is neutrosophic soft n-continuous if f is neutrosophic soft n-continuous at every point x ∈ X. We denote by $C_s^n(X,Y)$ the set of all neutrosophic soft n-continuous maps from X to Y.

Example 4.2: Let X and Y be neutrosophic soft n-topological spaces defined as in Example 3.1 and

let f : X → Y be a neutrosophic soft map defined by f(a) = p, f(b) = q, and f(c) = r, where p, q, and r are neutrosophic soft sets in Y defined as follows:

p = {<A, 0.9, 0, 0.2>, <B, 0.5, 0.1, 0.8>, <C, 0.3, 0.7, 0.4>},

q = {<A, 0.4, 0.6, 0.1>, <B, 0.8, 0.3, 0.7>, <C, 0.2, 0.5, 0.9>}, and

r = {<A, 0.2, 0.8, 0.3>, <B, 0.6, 0.2, 0.5>, <C, 0.9, 0.4, 0.6>}. Then, f is a neutrosophic soft continuous map, since for every neutrosophic soft open set V in Y, $f^{-1}(V)$ is a neutrosophic soft open set in X.

To see this, let V = {<A, 0.1, 0.2, 0.3>, <B, 0.4, 0.5, 0.6>, <C, 0.7, 0.8, 0.9>} be a neutrosophic soft open set in Y. Then, since p, q, and r are neutrosophic soft sets in Y, we have:

- p ∩ V = {<A, 0.1, 0, 0.2>, <B, 0.4, 0.1, 0.8>, <C, 0.3, 0, 0.4>}, which is a neutrosophic soft set in X;
- q ∩ V = {<A, 0.4, 0.6, 0.1>, <B, 0.8, 0.3, 0.6>, <C, 0.2, 0, 0.9>}, which is a neutrosophic soft set in X;
- r ∩ V = {<A, 0.2, 0.8, 0.3>, <B, 0.6, 0.2, 0.5>, <C, 0.7, 0, 0.6>}, which is a neutrosophic soft set in X.

Therefore, $f^{-1}(V)$= (p ∩ V) ∪ (q ∩ V) ∪ (r ∩ V)

= {<A, 0.1, 0, 0.2>, <A, 0.4, 0.6, 0.1>, <A, 0.2, 0.8, 0.3>, <B, 0.4, 0.1, 0.8>, <B, 0.8, 0.3, 0.6>, <B, 0.6, 0.2, 0.5>, <C, 0.3, 0, 0.4>, <C, 0.2, 0, 0.9>, <C, 0.7, 0, 0.6>}, which is a neutrosophic soft set in X. Therefore, f is a neutrosophic soft continuous map.

Example 4.3: Let X and Y be neutrosophic soft n-topological spaces defined as in Example 3.1 and let f : X → Y be a neutrosophic soft map defined by f(a) = p, f(b) = q, and f(c) = r, where p, q, and r are neutrosophic soft sets in Y defined as follows:

p = {<A, 0.9, 0, 0.2>, <B, 0.5, 0.1, 0.3>, <C, 0.3, 0.3, 0.1>}, q = {<A, 0.7, 0.1, 0.1>, <B, 0.3, 0.3, 0.2>, <C, 0.4, 0.2, 0.1>}, and r = {<A, 0.6, 0.2, 0.2>, <B, 0.2, 0.2, 0.2>, <C, 0.1, 0.5, 0.1>}.

Suppose U = {<A, 0.8, 0, 0.1>, <B, 0.3, 0.2, 0.1>}, which is a neutrosophic soft open set in Y. Then, $f^{-1}(U)$ = {a, b}, which is a neutrosophic soft open set in X. Hence, f is a neutrosophic soft continuous map.

6. Neutrosophic Soft n-Topological Spaces In Decision-Making Problems

Neutrosophic soft n-topological spaces have been increasingly applied in decision-making problems due to their ability to handle uncertainty, indeterminacy, and vagueness. These spaces provide a flexible framework for modeling complex systems where the boundaries between open, closed, and neither open nor closed sets are blurred.

One example of their application is in the field of medical diagnosis. Neutrosophic soft n-topological spaces can be used to represent patient data, which often contains imprecise and uncertain information. By analyzing this data within the framework of neutrosophic soft n-topological spaces, medical professionals can make more informed decisions about patient care.

Another example is in financial investment decision-making. Here, neutrosophic soft n-topological spaces can be used to represent market data, such as stock prices, which are often subject to fluctuation and uncertainty. By analyzing this data within the framework of neutrosophic soft n-topological spaces, investors can make more informed decisions about when to buy, hold, or sell particular stocks.

Overall, the ability of neutrosophic soft n-topological spaces to handle uncertainty, indeterminacy, and vagueness makes them a powerful tool in decision-making problems across a variety of fields.

Example: Suppose we have a decision-making problem where a company wants to evaluate the performance of its employees. The evaluation is based on three criteria: productivity, punctuality, and teamwork. The performance of each employee is represented by a neutrosophic soft set, where the membership degrees represent the degree of satisfaction for each criterion.

Example 5.1: Let us consider two employees, John and Jane. John's performance is represented by {<Productivity, 0.7, 0.2, 0.1>, <Punctuality, 0.5, 0.3, 0.2>, <Teamwork, 0.6, 0.3, 0.1>}, while Jane's performance is represented by {<Productivity, 0.8, 0.1, 0.1>, <Punctuality, 0.4, 0.4, 0.2>, <Teamwork, 0.5, 0.4, 0.1>}.

To compare the performance of John and Jane, we can define a neutrosophic soft distance function between their performance sets. Let d be the neutrosophic soft distance function defined as

$d(X,Y) = \max\{d(A,B) : \langle A, \alpha, \beta, \gamma\rangle \in X \text{ and } \langle B, \alpha, \beta, \gamma\rangle \in Y\}$. We can use the Hamming distance as the distance measure, so we have $d(A,B) = 0$ if $A = B$ and $d(A,B) = 1$ otherwise.

Applying the neutrosophic soft distance function, we get d({<Productivity, 0.7, 0.2, 0.1>, <Punctuality, 0.5, 0.3, 0.2>, <Teamwork, 0.6, 0.3, 0.1>}, {<Productivity, 0.8, 0.1, 0.1>, <Punctuality, 0.4, 0.4, 0.2>, <Teamwork, 0.5, 0.4, 0.1>}) = 0.67.

Based on the neutrosophic soft distance, we can conclude that John's performance is closer to Jane's performance than we might initially think. However, this is just one example of how neutrosophic soft n-topological spaces can be used in decision-making problems. There are many other applications of this theory in various fields, such as finance, healthcare, and engineering.

In conclusion, neutrosophic soft n-topological spaces provide a useful framework for decision-making problems where uncertainty and indeterminacy are present. The concept of neutrosophic soft n-homeomorphism provides a way to compare and identify similar structures in these spaces, while neutrosophic soft filters allow for the exploration of subsets and their properties. The development of these theories and their application in decision-making problems can lead to more informed and accurate decision-making processes in real-world scenarios.

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