



Designing a Computer Simulation Model to Study the Failure Mechanism in Rotating Shaft

Warshine Barry

University of Debrecen, Department of Mathematical and Computational Science, Debrecen, Hungary

Email: warshinabarrykurd@gmail.com

Abstract

The phenomenon of the failure of rotating mechanical structures is considered one of the dangerous phenomena that researchers are trying to study to find effective ways to avoid it. Many mathematical models describing the mechanism of crack formation in structures and the occurrence of failure in them have been adopted. In this paper, a computer simulation model of the failure mechanism in the intermediate conduction shaft of the ship's propeller (oil shuttle tanker) was designed based on the mathematical model that describes the stages of failure according to the number of revolutions that the shaft bears and the dimensions of fatigue cracks in it. In the beginning, a detailed mathematical model of the crack propagation mechanism was deduced as an initial stage, through which the differential equations that express the failure of the shaft were described. In the second stage, numerical simulations were implemented using the Euler and Rung-Kutta numerical methods used in solving ordinary differential equations. Also, for both methods using the Matlab language to reach the results and accurate graphs and analyze them. In this way it is possible to predict the number of cycles that the column will bear before the collapse, and this will reduce the occurrence of sudden collapses and save time, effort, and costs.

Keywords: Simulation; Matlab; Rotating Shaft; Mathematical model

1. Introduction

The occurrence of breakdown in rotating axles is of particular importance in many structures such as the shuttle oil tanker used in the maritime transportation of flammable petroleum materials, in which any accident may cause serious disasters, and it is important to conduct a comprehensive study of one of the most important components of the oil tanker, which is the intermediate connecting shaft of the ship's propeller, which connects the Blades of the propeller and the engine, and is one of the components most exposed to breakdown after the Blades, as its breakdown entails enormous costs, as well as exorbitant maintenance cost [1].

Figure (1) shows the stages of crack formation in the column from the stabilization stage to the final collapse

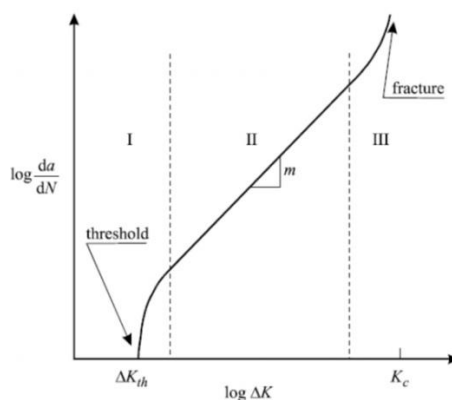


Figure (1) a: diagram representing the relationship between the logarithm of the crack propagation rate (da/dN) and the logarithm of the stress intensity gradient Δk

It determines the relationship between the crack propagation rate relative to the number of cycles da/dN and the stress intensity gradient ΔK and is divided into three zones determined by stress intensity factors. In the first zone, the cracks are micro-small and stable and characterized by the fact that the maximum stress intensity factor is smaller than the threshold of the stress intensity gradient ΔK_{th} i.e. $\Delta K_{th} > K_{max}$. If its value is greater than the threshold, the microcracks grow and reach the second zone in the diagram, which is the most important zone in describing the crack situation where the cracks are somewhat stable and grow linearly according to the Paris Model [2].

When the value of the maximum stress intensity factor becomes greater than the fracture toughness, i.e., $\Delta K_{th} > K_{max}$, the cracks become unstable, and their growth speed increases very significantly until a sudden collapse of the structure occurs [3].

2. References studies:

The researcher Nicolò Bachschmid confirmed what the scientist Irwin stated in 1957 that the stress intensity factor at the crack tip describes the fatigue state of the studied structure. If the Stress Intensity Factor reaches a value greater than the fracture toughness value, the crack becomes unstable, and its propagation speed increases until the final collapse of the structure [3].

The researcher (Herrmann) adopted the stress intensity factor as a criterion to determine the condition of crack propagation in the main axis (low-speed axis) of the wind turbine wind turbine with a capacity of 2.1 MW, where the crack propagation was simulated for the turbine axis and the fatigue life was estimated by changing the type of metal and reaching the most suitable metal for this axis based on the Paris model, which was modeled using the Block Diagram method and using the computer environment (ANSYS) [4]. The researcher Rajesh M. Metkar models the crack propagation according to the Paris model of a crankshaft in an engine. (Internal compression), where the criterion in knowing the number of cycles necessary for the crack to progress until the collapse occurred was the arrival of the value of the greatest stress intensity factor to a value greater than the fracture toughness value ($K_{max} > K_C$) [5].

Researcher Tom Lassen conducted a study on the collapse of the intermediate propeller shaft in a shuttle oil tanker, in which the crack length factor in comparison with the shaft diameter was used as a criterion to find out the state of crack propagation in the shaft so that if the crack length reaches a value greater than the shaft radius, its growth speed will increase until the final collapse occurs [1].

The researcher Lucas Gimenes de Moura adopted the Euler numerical method to simulate the dynamic behavior of cracks as a function of position and depth and the coefficient of stress intensity at its introduction [6]. This justifies the need for reliable and accurate models to represent cracks in rotating columns and simulate their dynamic behavior to predict the formation of cracks and detect them at an early stage, which was the subject of some studies, as in the Study [7].

Most of the previous studies relied on the stress intensity factor as a criterion to find out the length of the crack at which the collapse will occur by comparing it with the fracture toughness of the axis material, and also relied on comparing the length of the crack with the radius of the studied axis, which will be adopted in the current study in addition to studying the mechanism of crack propagation in the studied axis using the Paris model and simulation using the Euler and Rung-Kutta numerical methods and programming them using the MATLAB environment to arrive at computer results to be compared with the applied results of practical experiments in reference studies.

3. Mathematical model of crack propagation in the intermediate connecting shaft of an oil tanker propeller:

The mechanism of breakdown in the intermediate connecting shaft of the oil tanker propeller shown schematically in Figure (2) was studied, the theory of linear elastic mechanics fracture was applied to deduce the fatigue life consisting of two stages-the crack initiation crack initiation stage and the crack propagation propagation period crack progression stage until the breakdown occurs.

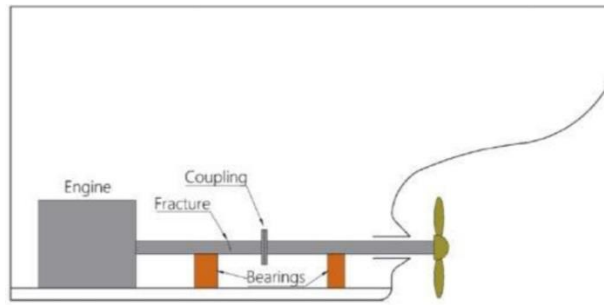


Figure 2: diagram of the shaft and the location of the fracture

The equation expressing the fatigue life of the axis is given as follows:

$$N = N_i + N_p \tag{1}$$

N: the total number of cycles that represents the fatigue life. (Cycle)

N_i: the number of cycles needed for cracks to form. (Cycle)

N_p: the number of cycles required for the cracks to progress until collapse. (Cycle)

The number of cycles required for N_p crack progression is calculated using the Paris model, which gives the relationship between the crack propagation rate relative to the number of cycles and the stress intensity factor at the crack introduction and is given by the following relationship [8]:

$$\frac{da}{dN} = C(\Delta K)^m \tag{2}$$

a: slit length (mm)

N: number of cycles (cycle)

Δk: gradation of the stress intensity factor. (MPa√mm) (the stress intensity factor is proportional to the square root of the crack length).

C, m: constants related to matter. And it has the following values in this study m=3, C=1.83 E-13 [1].

To determine when a column according to the Paris model becomes collapsible under the smallest load cycle, there are two criteria, at least one of which must be met:

The maximum stress intensity factor K_{max} at each increase in the crack propagation rate is compared with the fracture toughness K_C and when it exceeds it (K_{max} > K_C) the crack growth accelerates abruptly until collapse occurs.

The length of the crack a is compared with the radius of the rotating shaft d and when the length of the crack exceeds the radius (a > d) the shaft collapses.

According to the stress intensity factor according to the Irwin method with the following relationship: [9]

$$K_I = \sigma\beta\sqrt{\pi a} \tag{3}$$

$$\Delta K = K_{max} - K_{min} = (\sigma_{max} - \sigma_{min})\beta\sqrt{\pi a} = \Delta\sigma\beta\sqrt{\pi a} \tag{4}$$

By compensating for the relationship (4) in the relationship (2):

$$\frac{da}{dN} = C(\Delta\sigma\beta\sqrt{\pi a})^m \tag{5}$$

To obtain the number of cycles necessary for the progression of the N_p notch, the relation (5) between the two terms a_c, a_{th} is integrated as follows:

$$\int_0^{N_p} dN = \frac{1}{C} \cdot \int_{a_{th}}^{a_c} \frac{da}{[\Delta K(a)]^m} \tag{6}$$

$$\int_0^{N_p} dN = \frac{1}{C} \cdot \int_{a_{th}}^{a_c} \frac{da}{[\Delta\sigma\beta\sqrt{\pi a}]^m} \tag{7}$$

(a_{th}) and (a_c) are calculated as follows:

$$a_{th} = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{s_f} \right)^2 \tag{8}$$

The threshold value of the stress intensity gradient is calculated in terms of the Crystal diameter according to the relation:

$$\Delta K_{th} = s_f \sqrt{\pi \frac{D}{2}} \tag{9}$$

$$a_c = \frac{1}{\pi} \left(\frac{K_{IC}}{\sigma_{max}} \right)^2 \tag{10}$$

Where:

K_I: is the stress intensity factor [Mpa√mm]

K_{max}: maximum stress intensity factor [Mpa√mm]

K_{min}: microstress intensity factor [Mpa√mm]

σ_{max}: applied maximum stress [MPa]

σ_{min}: applied microstress [MPa]

a_{th} : the length of the primary slit (at the threshold δk_{th}) [mm]

a_c : critical slit length [mm]

β : an dimensionless factor related to the geometry of the slit and takes in this study the value $\beta=0.95$). [1]

D: the value of the grain diameter of the material in the crystal lattice.

s_f : fatigue limit.

The value of the fatigue limit is taken from the scheme (S-N), which expresses the relationship between the number of load cycles N

(Number of cycle) for the applied stress of the axle metal 42CrMO4 according to DIN EN 10083.

$\Delta K_{th} = 63$ Mpa

Compensating for the threshold value of the stress intensity gradient in relation (8) is the value of the initial crack at which the simulation will begin:

$$K_{th} = \frac{1}{\pi} \left(\frac{63}{50.26} \right)^2 = 0.5mm$$

4. Results and discussion:

For the metal 42CrMo4 steel, numerical simulation of crack growth was carried out according to the paris model expressed by relation (5), which proposes an initial value problem, through a linear Ordinary Differential Equation of the first rank using the Euler numerical method, which calculates the value of the solution Y_{i+1} at the moment X_{i+1}

Based on the solution value Y_i at the previous moment X_i and the time step value [10] has follows:

$$y(x_{i+1}) = y(x_i) + f(x_i, y_i)h \quad (11)$$

$$x_{i+1} = x_i + h \quad (12)$$

And $f(x_i, y_i)$ is given from the initial conditions by the relation:

$$f(x, y) = \frac{dy}{dx}, y(x_0) = y_0 \quad (13)$$

Table 1: parameters adopted in the study [1] (Tom Lassen, 2021)

N_p	B	M	C
6.68E+07	0.95	3	1.83E-13

This method is programmed in the Matlab language as follows:

```

Clc
Clear
syms a
m=3;
c=1.83E-13;
B=0.95;
a_f= 180 %mm ;
sigma_max=36;
sigma_min=0;
h=10000
f=c*((sigma_max - sigma_min)*B*sqrt(pi)*sqrt(a))^m;
N(1)=0;
A(1)=0.5;
Nf=1000000000000;
nsim=(Nf-N(1))/h
for i=1:nsim
N(i+1)=N(i)+h ;
n=N(i);
a=A(i);
A(i+1)=A(i)+h*subs(f);
if a > a_f
disp(sigma_max)
disp(N(i))
disp(A(i))
break
end
end
end
plot(N,A)
grid

```

xlabel('N [cycle]')
ylabel('a [mm]')

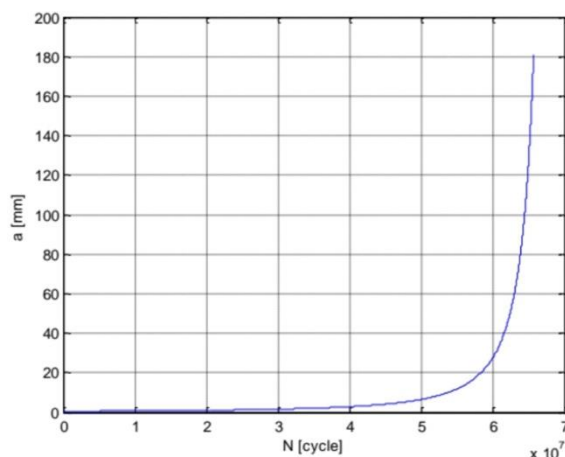


Figure 3: simulation results according to Euler

For greater accuracy, the mathematical model was simulated according to the Runge-Kutta numerical method, which solves linear and nonlinear differential equations and gives a dependent value at a certain moment in terms of the parameters of the previous moment and derivatives then, and the Runge-Kutta equations are deduced from the Taylor Prism, which is given by the following relation [11]:

$$y(x_0 + h) = y(x_0) + \frac{hy'_n}{1!} + \frac{h^2y''_n}{2!} + \frac{h^3y'''_n}{3!} + \frac{h^4y''''_n}{4!} + \dots + \frac{h^N y^n}{N!} \quad (14)$$

Thus, the general formula of the Runge-Kutta method is given at $n=4$ according to the following equations [12]:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (15)$$

$$k_1 = h f(x_n, y_n) \quad (16)$$

$$k_2 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \quad (17)$$

$$k_3 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \quad (18)$$

$$k_4 = h f(x_n + h, y_n + k_3) \quad (19)$$

The code for the Runge-Kutta method is designed according to the following:

```
Clc
Clear
syms a
m=3;
c=1.83E-13;
B=0.95;
a_f=180 % mm ;
sigma_max=36;
sigma_min=0;
h=10000;
f=c*((sigma_max - sigma_min)*B*sqrt(pi)*sqrt(a))^m;
N(1)=0;
A(1)=0.5;
Nf=1000000000;
nsim=(Nf-N(1))/h
for i=1:nsim
a=A(i);
n=N(i);
k1=subs(f);
a=A(i)+0.5*k1*h;
n=N(i)+0.5*h;
k2=subs(f);
a=A(i)+0.5*k2*h;
n=N(i)+0.5*h;
k3=subs(f);
a=A(i)+k3*h;
```

```

n=N(i)+h;
k4=subs(f);
A(i+1)=A(i)+(1/6)*(k1+2*k2+2*k3+k4)*h;
N(i+1)=N(i)+h;
if a > a_f
disp(N(i))
disp(A(i))
break
end
end
end
figure
plot(N,A)
grid
xlabel('N [cycle]')
ylabel('a [mm]')

```

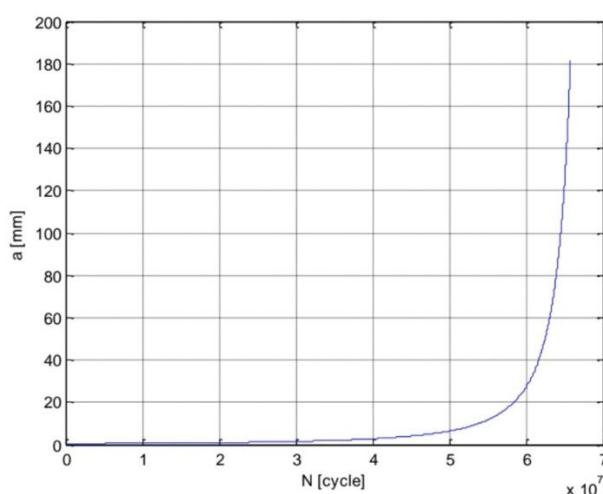


Figure 4: simulation results according to Rung-Kutta

It is noted from the results in Figures (3) and (4) that the column collapses at the number of cycles according to Euler estimated.

$N_p = 65730000$ cycle and the number of cycles necessary for the progress of the crack until collapse according to Rung-Kutta, $N_p = 65780000$ cycle are estimated, which are values that accurately correspond to the results in the reference Study[1], which confirms the correctness of the designed models and can be used for any other column depending on its parameters.

5. Conclusions

- 1- it was reached to write a characterization of the equations representing the slit extension as a function of the number of cycles
- 2- these equations were solved using numerical methods for solving differential equations (Rung-Kutta, Euler)
- 3- the behavior of the system was simulated under certain conditions approved for the study [1].
- 4- by comparing the results with the study it was concluded that the computer model is reliable to predict the behavior of the notch with the number of cycles.
- 5- a mathematical model was reached that simulates the extension of cracks until the collapse occurred in the intermediate connecting column of the oil tanker fan, which predicted the number of cycles that the column endured before the collapse.

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