Homomorphisms and anti-homomorphisms of neutrosophic INK-algebras

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Abstract

This article presents the concepts of neutrosophic INK-subalgebras and INK-ideals of INK-algebras. We also studied neutrosophic INK-subalgebras and INK-ideals that depend on homomorphisms and anti-homomorphisms.

Keywords: INK-algebra; neutrosophic set; neutrosophic INK-subalgebra; neutrosophic INK-ideal; homomorphism; anti-homomorphism.

1 Introduction

In 1965, Zadeh[10] took the fuzzy set (FS) theory, then so few investigators laid in FSs in BCI/BCK-algebras. Also, Atanassov[1] began intuitionistic fuzzy sets (IFSs) as a simplification of FSs in 1986, and it was requested in BCI/BCK-algebras, begun by Imai in the 1980s. Iséki[2] anticipated the idea of BCI-algebras, and Tanaka[3] has presented the notion of BCK-algebras. After, Neggers and Kim[7] have anticipated the Q-algebras. Ensuing these, copious canvassers put out articles using IFSs observation.

Later, the neutrosophic set (NS) theory was principally anticipated in 1999 by Smarandache.[9] NS has been plagiaristic from a fresh division of philosophy, that is, neutrosophy. NS is trained for allocating with vagueness, inconclusive, and unpredictable reports. NS methodologies are apt to work on demonstrating problems where human knowledge is essential and human calculation is required. Smarandache, who also recognized the idea of a single-valued NS, focused on real-world technical and engineering significance. NS is described by truth association (T), inconclusive association (I) and falsehood association functions (F). This thought is very trivial in many presentation areas when inconclusive is computed clearly and the truth association, inconclusive association, and falsehood association functions are independent.

In 2017, Kaviyarasu et al.[6] presented an idea named INK-algebra, which is the generality of TM/Q/BCK/BCI/BCH-algebras, and studied some properties. At first, Indhira and Kaviyarasu[5] worked on fuzzy subalgebras and K-ideals in INK-algebras. Subsequently, they deliberated fuzzy p-ideals in INK-algebras in.[4] Rajakumari and Balasubramanian[8] executed anti-fuzzy translation, subalgebras, and K-ideals of INK-algebras in 2019. Afterwards, they worked on NSs in INK-algebras in 2020.

In this paper, we present the concepts of neutrosophic INK-subalgebras and INK-ideals of INK-algebras. We also studied neutrosophic INK-subalgebras and INK-ideals that depend on homomorphisms and anti-homomorphisms.

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2 Preliminaries

In this segment, we have used some of the definitions that are being utilized for this work.

Definition 2.1. An algebra $(\ddot{I}, \bullet, 0)$ is said to be an *INK-algebra* if it mollifies the ensuing situations for any $\tilde{\ae}_1, \tilde{\ae}_2, \tilde{\ae}_3 \in \ddot{I}$,

INK-1: $((\tilde{\ae}_1 \bullet \tilde{\ae}_2) \bullet (\tilde{\ae}_1 \bullet \tilde{\ae}_3)) \bullet (\tilde{\ae}_3 \bullet \tilde{\ae}_2) = 0$,

INK-2: $((\tilde{\ae}_1 \bullet \tilde{\ae}_3) \bullet (\tilde{\ae}_2 \bullet \tilde{\ae}_3)) \bullet (\tilde{\ae}_1 \bullet \tilde{\ae}_2) = 0$,

INK-3: $\tilde{\ae}_1 \bullet 0 = \tilde{\ae}_1$,

INK-4: $\tilde{\ae}_1 \bullet \tilde{\ae}_2 = 0, \tilde{\ae}_2 \bullet \tilde{\ae}_1 = 0 \Rightarrow \tilde{\ae}_1 = \tilde{\ae}_2$,

where $\bullet$ is a binary and 0 is a constant of $\ddot{I}$.

Definition 2.2. An INK-algebra $\ddot{I}$ be composed of a non-empty subset $\breve{I}$ is said to be an *INK-subalgebra* of $\ddot{I}$ if $\tilde{\ae}_1 \bullet \tilde{\ae}_2 \in \breve{I}$ for all $\tilde{\ae}_1, \tilde{\ae}_2 \in \breve{I}$.

Definition 2.3. An INK-algebra $\ddot{I}$ be composed of a non-empty subset $\breve{I}$ is entitled as an *INK-ideal* of $\ddot{I}$ if it mollifies for all $\tilde{\ae}_1, \tilde{\ae}_2, \tilde{\ae}_3 \in \ddot{I}$,

(i) $0 \in \breve{I}$,

(ii) $(\tilde{\ae}_3 \bullet \tilde{\ae}_1) \bullet (\tilde{\ae}_3 \bullet \tilde{\ae}_2) \in \breve{I}, \tilde{\ae}_2 \in \breve{I} \Rightarrow \tilde{\ae}_1 \in \breve{I}$.

Definition 2.4. An NS $= (_T, _I, _F)$ in $\ddot{I}$ is termed a *neutrosophic INK-subalgebra* of $\ddot{I}$ if it mollifies the ensuing condition for all $\tilde{\ae}_1, \tilde{\ae}_2, \tilde{\ae}_3 \in \ddot{I}$,

(i) $_T(\tilde{\ae}_1 \bullet \tilde{\ae}_2) \geq \min\{_T(\tilde{\ae}_1), _T(\tilde{\ae}_2)\}$,

(ii) $_I(\tilde{\ae}_1 \bullet \tilde{\ae}_2) \leq \max\{_I(\tilde{\ae}_1), _I(\tilde{\ae}_2)\}$,

(iii) $_F(\tilde{\ae}_1 \bullet \tilde{\ae}_2) \leq \max\{_F(\tilde{\ae}_1), _F(\tilde{\ae}_2)\}$.

Definition 2.5. An NS $= (_T, _I, _F)$ in $\ddot{I}$ is termed a *neutrosophic INK-ideal* of $\ddot{I}$ if it mollifies the ensuing circumstances for all $\tilde{\ae}_1, \tilde{\ae}_2, \tilde{\ae}_3 \in \ddot{I}$,

(i) $_T(0) \geq _T(\tilde{\ae}_1), _I(0) \leq _I(\tilde{\ae}_1)$, and $_F(0) \leq _F(\tilde{\ae}_1)$,

(ii) $_T(\tilde{\ae}_1) \geq \min\{_T((\tilde{\ae}_3 \bullet \tilde{\ae}_1) \bullet (\tilde{\ae}_3 \bullet \tilde{\ae}_2)), _T(\tilde{\ae}_2)\}$,

(iii) $_I(\tilde{\ae}_1) \leq \max\{_I((\tilde{\ae}_3 \bullet \tilde{\ae}_1) \bullet (\tilde{\ae}_3 \bullet \tilde{\ae}_2)), _I(\tilde{\ae}_2)\}$,

(iv) $_F(\tilde{\ae}_1) \leq \max\{_F((\tilde{\ae}_3 \bullet \tilde{\ae}_1) \bullet (\tilde{\ae}_3 \bullet \tilde{\ae}_2)), _F(\tilde{\ae}_2)\}$.

Example 2.6. Consider an INK-algebra $\ddot{I} = \{0, o, q\}$ with the binary operation $\bullet$ as the following Cayley table:

$\bullet$	0	o	q
0	0	q	o
o	o	0	q
q	q	o	0

Now, we define the NS $= (_T, _I, _F)$ of $\ddot{I}$ by the ensuing values as follows:

	0	o	q
$_T$	0.2	0.5	0.6
$_I$	0.9	0.8	0.8
$_F$	0.4	0.4	0.3

Then $= (_T, _I, _F)$ is a neutrosophic INK-subalgebra of $\ddot{I}$.

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Definition 2.7. A mapping $¢ : \ddot{I} \to \breve{I}$ of INK-algebras is said to be a *homomorphism* (anti-homomorphism) if $¢(\tilde{æ}_1 \bullet \tilde{æ}_2) = ¢(\tilde{æ}_1) \bullet ¢(\tilde{æ}_2)$ $(¢(\tilde{æ}_1 \bullet \tilde{æ}_2) = ¢(\tilde{æ}_2) \bullet ¢(\tilde{æ}_1))$ for all $\tilde{æ}_1, \tilde{æ}_2 \in \ddot{I}$. If $¢ : \ddot{I} \to \breve{I}$ is a homomorphism (anti-homomorphism), then $¢(0) = 0$.

3 Main results

In this area, we have talked about homomorphisms and anti-homomorphisms of neutrosophic INK-subalgebras and neutrosophic INK-ideals of INK-algebras.

Definition 3.1. Let $¢ : \ddot{I} \to \breve{I}$ be a homomorphism of INK-algebras. Intended any NS $= (_T, _I, _F)$ in $\breve{I}$, we explain a fresh NS $[¢] = (_T[¢], _I[¢], _F[¢])$ such that for all $\tilde{æ}_1 \in \ddot{I}$,

$$
\begin{aligned}
_T[¢] : \ddot{I} \to [0,1], {}_T[¢](\tilde{æ}_1) &= {}_T(¢(\tilde{æ}_1)),\\
_I[¢] : \ddot{I} \to [0,1], {}_I[¢](\tilde{æ}_1) &= {}_I(¢(\tilde{æ}_1)),\\
_F[¢] : \ddot{I} \to [0,1], {}_F[¢](\tilde{æ}_1) &= {}_F(¢(\tilde{æ}_1)).
\end{aligned}
$$

Theorem 3.2. *Let $¢ : \ddot{I} \to \breve{I}$ be a homomorphism of INK-algebras. If $= (_T, _I, _F)$ is a neutrosophic INK-subalgebra of $\breve{I}$, then the NS $[¢] = (_T[¢], _I[¢], _F[¢])$ is a neutrosophic INK-subalgebra of $\ddot{I}$.*

Proof. Let $\tilde{æ}_1, \tilde{æ}_2 \in \ddot{I}$. Then

$$
\begin{aligned}
_T[¢](\tilde{æ}_1 \bullet \tilde{æ}_2) &= {}_T(¢(\tilde{æ}_1 \bullet \tilde{æ}_2))\\
&= {}_T(¢(\tilde{æ}_1) \bullet ¢(\tilde{æ}_2))\\
&\geq \min\{_T(¢(\tilde{æ}_1)), {}_T(¢(\tilde{æ}_2))\}\\
&= \min\{_T[¢](\tilde{æ}_1), {}_T[¢](\tilde{æ}_2)\},
\end{aligned}
$$

$$
\begin{aligned}
_I[¢](\tilde{æ}_1 \bullet \tilde{æ}_2) &= {}_I(¢(\tilde{æ}_1 \bullet \tilde{æ}_2))\\
&= {}_I(¢(\tilde{æ}_1) \bullet ¢(\tilde{æ}_2))\\
&\leq \max\{_I(¢(\tilde{æ}_1)), {}_I(¢(\tilde{æ}_2))\}\\
&= \max\{_I[¢](\tilde{æ}_1), {}_I[¢](\tilde{æ}_2)\},
\end{aligned}
$$

$$
\begin{aligned}
_F[¢](\tilde{æ}_1 \bullet \tilde{æ}_2) &= {}_F(¢(\tilde{æ}_1 \bullet \tilde{æ}_2))\\
&= {}_F(¢(\tilde{æ}_1) \bullet ¢(\tilde{æ}_2))\\
&\leq \max\{_F(¢(\tilde{æ}_1)), {}_F(¢(\tilde{æ}_2))\}\\
&= \max\{_F[¢](\tilde{æ}_1), {}_F[¢](\tilde{æ}_2)\}.
\end{aligned}
$$

Hence, $[¢]$ is a neutrosophic INK-subalgebra of $\ddot{I}$. □

Theorem 3.3. *Let $¢ : \ddot{I} \to \breve{I}$ be an epimorphism (onto homomorphism) of INK-algebras and $= (_T, _I, _F)$ be an NS in $\breve{I}$. If $[¢] = (_T[¢], _I[¢], _F[¢])$ is a neutrosophic INK-subalgebra of $\ddot{I}$, then is a neutrosophic INK-subalgebra of $\breve{I}$.*

Proof. Let $æ_1, æ_2 \in \breve{I}$. Then there exist $\tilde{æ}_1, \tilde{æ}_2 \in \ddot{I}$ such that $¢(\tilde{æ}_1) = æ_1$ and $¢(\tilde{æ}_2) = æ_2$. Thus

$$
\begin{aligned}
_T(æ_1 \bullet æ_2) &= {}_T(¢(\tilde{æ}_1) \bullet ¢(\tilde{æ}_2))\\
&= {}_T(¢(\tilde{æ}_1 \bullet \tilde{æ}_2))\\
&= {}_T[¢](\tilde{æ}_1 \bullet \tilde{æ}_2)\\
&\geq \min\{_T[¢](\tilde{æ}_1), {}_T[¢](\tilde{æ}_2)\}\\
&= \min\{_T(¢(\tilde{æ}_1)), {}_T(¢(\tilde{æ}_2))\}\\
&= \min\{_T(æ_1), {}_T(æ_2)\},
\end{aligned}
$$

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$$\begin{aligned}
{}_I(\text{æ}_1 \bullet \text{æ}_2) &= {}_I(\text{¢}(\tilde{\text{æ}}_1) \bullet \text{¢}(\tilde{\text{æ}}_2)) \\
&= {}_I(\text{¢}(\tilde{\text{æ}}_1 \bullet \tilde{\text{æ}}_2)) \\
&= {}_I[\text{¢}](\tilde{\text{æ}}_1 \bullet \tilde{\text{æ}}_2) \\
&\leq \max\{{}_I[\text{¢}](\tilde{\text{æ}}_1), {}_I[\text{¢}](\tilde{\text{æ}}_2)\} \\
&= \max\{{}_I(\text{¢}(\tilde{\text{æ}}_1)), {}_I(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_I(\text{æ}_1), {}_I(\text{æ}_2)\},
\end{aligned}$$

$$\begin{aligned}
{}_F(\text{æ}_1 \bullet \text{æ}_2) &= {}_F(\text{¢}(\tilde{\text{æ}}_1) \bullet \text{¢}(\tilde{\text{æ}}_2)) \\
&= {}_F(\text{¢}(\tilde{\text{æ}}_1 \bullet \tilde{\text{æ}}_2)) \\
&= {}_F[\text{¢}](\tilde{\text{æ}}_1 \bullet \tilde{\text{æ}}_2) \\
&\leq \max\{{}_F[\text{¢}](\tilde{\text{æ}}_1), {}_F[\text{¢}](\tilde{\text{æ}}_2)\} \\
&= \max\{{}_F(\text{¢}(\tilde{\text{æ}}_1)), {}_F(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_F(\text{æ}_1), {}_F(\text{æ}_2)\}.
\end{aligned}$$

Therefore, is a neutrosophic INK-subalgebra of $\breve{I}$. □

Theorem 3.4. *Let* $\text{¢} : \ddot{I} \to \breve{I}$ *be an anti-homomorphism of INK-algebras. If* $= ({}_T, {}_I, {}_F)$ *is a neutrosophic INK-subalgebra of* $\breve{I}$*, then the NS* $[\text{¢}] = ({}_T[\text{¢}], {}_I[\text{¢}], {}_F[\text{¢}])$ *is a neutrosophic INK-subalgebra of* $\ddot{I}$.

Proof. The proof is similar to the proof of Theorem 3.2. □

Theorem 3.5. *Let* $\text{¢} : \ddot{I} \to \breve{I}$ *be a homomorphism of INK-algebras. If* $= ({}_T, {}_I, {}_F)$ *is a neutrosophic INK-ideal of* $\breve{I}$*, then the NS* $[\text{¢}] = ({}_T[\text{¢}], {}_I[\text{¢}], {}_F[\text{¢}])$ *is a neutrosophic INK-ideal of* $\ddot{I}$.

Proof. Let $\tilde{\text{æ}}_1 \in \ddot{I}$. Then

$$\begin{aligned}
{}_T[\text{¢}](0) = {}_T(\text{¢}(0)) = {}_T(0) \geq {}_T(\text{¢}(\tilde{\text{æ}}_1)) = {}_T[\text{¢}](\tilde{\text{æ}}_1), \\
{}_I[\text{¢}](0) = {}_I(\text{¢}(0)) = {}_I(0) \leq {}_I(\text{¢}(\tilde{\text{æ}}_1)) = {}_I[\text{¢}](\tilde{\text{æ}}_1), \\
{}_F[\text{¢}](0) = {}_F(\text{¢}(0)) = {}_F(0) \leq {}_F(\text{¢}(\tilde{\text{æ}}_1)) = {}_F[\text{¢}](\tilde{\text{æ}}_1).
\end{aligned}$$

Let $\tilde{\text{æ}}_1, \tilde{\text{æ}}_2, \tilde{\text{æ}}_3 \in \ddot{I}$. Then

$$\begin{aligned}
{}_T[\text{¢}](\tilde{\text{æ}}_1) &= {}_T(\text{¢}(\tilde{\text{æ}}_1)) \\
&\geq \min\{{}_T((\text{¢}(\tilde{\text{æ}}_3) \bullet \text{¢}(\tilde{\text{æ}}_1)) \bullet (\text{¢}(\tilde{\text{æ}}_3) \bullet \text{¢}(\tilde{\text{æ}}_2))), {}_T(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \min\{{}_T(\text{¢}(\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet \text{¢}(\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2)), {}_T(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \min\{{}_T(\text{¢}((\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet (\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2))), {}_T(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \min\{{}_T[\text{¢}]((\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet (\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2)), {}_T[\text{¢}](\tilde{\text{æ}}_2)\},
\end{aligned}$$

$$\begin{aligned}
{}_I[\text{¢}](\tilde{\text{æ}}_1) &= {}_I(\text{¢}(\tilde{\text{æ}}_1)) \\
&\leq \max\{{}_I((\text{¢}(\tilde{\text{æ}}_3) \bullet \text{¢}(\tilde{\text{æ}}_1)) \bullet (\text{¢}(\tilde{\text{æ}}_3) \bullet \text{¢}(\tilde{\text{æ}}_2))), {}_I(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_I(\text{¢}(\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet \text{¢}(\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2)), {}_I(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_I(\text{¢}((\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet (\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2))), {}_I(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_I[\text{¢}]((\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet (\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2)), {}_I[\text{¢}](\tilde{\text{æ}}_2)\},
\end{aligned}$$

$$\begin{aligned}
{}_F[\text{¢}](\tilde{\text{æ}}_1) &= {}_F(\text{¢}(\tilde{\text{æ}}_1)) \\
&\leq \max\{{}_F((\text{¢}(\tilde{\text{æ}}_3) \bullet \text{¢}(\tilde{\text{æ}}_1)) \bullet (\text{¢}(\tilde{\text{æ}}_3) \bullet \text{¢}(\tilde{\text{æ}}_2))), {}_F(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_F(\text{¢}(\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet \text{¢}(\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2)), {}_F(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_F(\text{¢}((\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet (\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2))), {}_F(\text{¢}(\tilde{\text{æ}}_2))\} \\
&= \max\{{}_F[\text{¢}]((\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_1) \bullet (\tilde{\text{æ}}_3 \bullet \tilde{\text{æ}}_2)), {}_F[\text{¢}](\tilde{\text{æ}}_2)\}.
\end{aligned}$$

Hence, $[\text{¢}]$ is a neutrosophic INK-ideal of $\ddot{I}$. □

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Theorem 3.6. *Let* $¢ : \ddot{I} \to \breve{I}$ *be an epimorphism of INK-algebras and* $= ({}_T, {}_I, {}_F)$ *be an NS in* $\breve{I}$. *If* $[¢] = ({}_T[¢], {}_I[¢], {}_F[¢])$ *is a neutrosophic INK-ideal of* $\ddot{I}$, *then is a neutrosophic INK-ideal of* $\breve{I}$.

Proof. Let $æ_1 \in \breve{I}$. Then there exists $\tilde{æ_1} \in \ddot{I}$ such that $¢(\tilde{æ_1}) = æ_1$. Thus

$$\begin{aligned} {}_T(0) &= {}_T(¢(0)) = {}_T[¢](0) \geq {}_T[¢](\tilde{æ_1}) = {}_T(¢(\tilde{æ_1})) = {}_T(æ_1), \\ {}_I(0) &= {}_I(¢(0)) = {}_I[¢](0) \leq {}_I[¢](\tilde{æ_1}) = {}_I(¢(\tilde{æ_1})) = {}_I(æ_1), \\ {}_F(0) &= {}_F(¢(0)) = {}_F[¢](0) \leq {}_F[¢](\tilde{æ_1}) = {}_F(¢(\tilde{æ_1})) = {}_F(æ_1). \end{aligned}$$

Let $æ_1, æ_2, æ_3 \in \breve{I}$. Then there exist $\tilde{æ_1}, \tilde{æ_2}, \tilde{æ_3} \in \ddot{I}$ such that $¢(\tilde{æ_1}) = æ_1, ¢(\tilde{æ_2}) = æ_2$, and $¢(\tilde{æ_3}) = æ_3$. Thus

$$\begin{aligned} {}_T(æ_1) &= {}_T(¢(\tilde{æ_1})) \\ &= {}_T[¢](\tilde{æ_1}) \\ &\geq \min\{{}_T[¢]((\tilde{æ_3} \bullet \tilde{æ_1}) \bullet (\tilde{æ_3} \bullet \tilde{æ_2})), {}_T[¢](\tilde{æ_2})\} \\ &= \min\{{}_T(¢((\tilde{æ_3} \bullet \tilde{æ_1}) \bullet (\tilde{æ_3} \bullet \tilde{æ_2}))), {}_T(¢(\tilde{æ_2}))\} \\ &= \min\{{}_T(¢(\tilde{æ_3} \bullet \tilde{æ_1}) \bullet ¢(\tilde{æ_3} \bullet \tilde{æ_2})), {}_T(¢(\tilde{æ_2}))\} \\ &= \min\{{}_T((¢(\tilde{æ_3}) \bullet ¢(\tilde{æ_1})) \bullet (¢(\tilde{æ_3}) \bullet ¢(\tilde{æ_2}))), {}_T(¢(\tilde{æ_2}))\} \\ &= \min\{{}_T((æ_3 \bullet æ_1) \bullet (æ_3 \bullet æ_2)), {}_T(æ_2)\}, \end{aligned}$$

$$\begin{aligned} {}_I(æ_1) &= {}_I(¢(\tilde{æ_1})) \\ &= {}_I[¢](\tilde{æ_1}) \\ &\leq \max\{{}_I[¢]((\tilde{æ_3} \bullet \tilde{æ_1}) \bullet (\tilde{æ_3} \bullet \tilde{æ_2})), {}_I[¢](\tilde{æ_2})\} \\ &= \max\{{}_I(¢((\tilde{æ_3} \bullet \tilde{æ_1}) \bullet (\tilde{æ_3} \bullet \tilde{æ_2}))), {}_I(¢(\tilde{æ_2}))\} \\ &= \max\{{}_I(¢(\tilde{æ_3} \bullet \tilde{æ_1}) \bullet ¢(\tilde{æ_3} \bullet \tilde{æ_2})), {}_I(¢(\tilde{æ_2}))\} \\ &= \max\{{}_I((¢(\tilde{æ_3}) \bullet ¢(\tilde{æ_1})) \bullet (¢(\tilde{æ_3}) \bullet ¢(\tilde{æ_2}))), {}_I(¢(\tilde{æ_2}))\} \\ &= \max\{{}_I((æ_3 \bullet æ_1) \bullet (æ_3 \bullet æ_2)), {}_I(æ_2)\}, \end{aligned}$$

$$\begin{aligned} {}_F(æ_1) &= {}_F(¢(\tilde{æ_1})) \\ &= {}_F[¢](\tilde{æ_1}) \\ &\leq \max\{{}_F[¢]((\tilde{æ_3} \bullet \tilde{æ_1}) \bullet (\tilde{æ_3} \bullet \tilde{æ_2})), {}_F[¢](\tilde{æ_2})\} \\ &= \max\{{}_F(¢((\tilde{æ_3} \bullet \tilde{æ_1}) \bullet (\tilde{æ_3} \bullet \tilde{æ_2}))), {}_F(¢(\tilde{æ_2}))\} \\ &= \max\{{}_F(¢(\tilde{æ_3} \bullet \tilde{æ_1}) \bullet ¢(\tilde{æ_3} \bullet \tilde{æ_2})), {}_F(¢(\tilde{æ_2}))\} \\ &= \max\{{}_F((¢(\tilde{æ_3}) \bullet ¢(\tilde{æ_1})) \bullet (¢(\tilde{æ_3}) \bullet ¢(\tilde{æ_2}))), {}_F(¢(\tilde{æ_2}))\} \\ &= \max\{{}_F((æ_3 \bullet æ_1) \bullet (æ_3 \bullet æ_2)), {}_F(æ_2)\}. \end{aligned}$$

Hence, is a neutrosophic INK-ideal of $\breve{I}$. □

Theorem 3.7. *Let* $¢ : \ddot{I} \to \breve{I}$ *be an anti-homomorphism of INK-algebras. If* $= ({}_T, {}_I, {}_F)$ *is a neutrosophic INK-ideal of* $\breve{I}$, *then the NS* $[¢] = ({}_T[¢], {}_I[¢], {}_F[¢])$ *is a neutrosophic INK-ideal of* $\ddot{I}$.

Proof. The proof is similar to the proof of Theorem 3.5. □

4 Conclusion

In this paper, we have introduced and studied the concepts of neutrosophic INK-subalgebras and INK-ideals of INK-algebras. In addition, we have studied neutrosophic INK-subalgebras and INK-ideals that depend on homomorphisms and anti-homomorphisms.

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