Enhancing neutrosophic fuzzy goal programming approach for solving multi-objective probabilistic linear programming problems

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Abstract

This article considers a stochastic multi-objective linear programming problem in a neutrosophic framework. The right-hand sides of the constraints are random variables associated with their means and variances, and the single-valued trapezoidal neutrosophic numbers are included in the coefficients of the objective functions to investigate the problem. The problem is successfully converted into the related deterministic programming model based on the formulation of the scoring function and several distributions, such as the Uniform, Exponential, and Gama distributions. Applying fuzzy goal programming, the deterministic problem is solved. An example is then solved to demonstrate the solution methodology.

Keywords: Optimization; Optimization problems; Multi-objective stochastic linear programming; Neutrosophic numbers; Exponential distribution; Uniform distribution; Gama Distribution; Fuzzy programming; Optimal compromise solution; Optimization Problems; Decision Making; Goal programming; Lingo computer package.

1. Introduction

The fact that the coefficients in the formulation are not constants but instead fluctuate and are unpredictable is one of the challenges that arise in the applications of mathematical programming. Several writers [1–3] have explored this problem. Charnes et al. [3] first presented the idea of probabilistic constraints for handling uncertain situations. In [4] non-linear conditions equivalent to probabilistic constraints were introduced.

Stochastic programming is the theory and methods for adding stochastic variations into a mathematical programming problem [5]. Risk programming in linear programming (LP) models with chance constraints is one of the three major stochastic programming approaches [3]. Methods for interactive stochastic programming have been created by Teghem et al. [7] and Leclercq et al. [6]. A review article on stochastic linear programming with a single objective function is introduced in [8]. Dantzig [9] suggested a two-step method for translating stochastic programming models into their deterministic counterparts. In his article on single-objective chance constrained programming with joint constraints, Jagannathan [10] addressed this problem. Biswal et al. [11] used an exponential distribution to calculate the deterministic

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equivalent of the goal function and the left-side constraint coefficients. Using a linear optimization setting, the authors of [12] provide a modern overview of how concepts from optimization, probability theory, and multicriteria decision analysis are intertwined to deal with situations involving the presence of multiple objective functions and the stochastic nature of data. Fuzzy random coefficients for multiobjective linear programming were the subject of research by Li and coworkers [13]. Fuzzy random simulation was utilized for this purpose. Zimmermann [14] tackled the first LP problem with competing goals by using appropriate membership functions. Leberling [15] used a special set of non-linear membership functions to solve the vector maximum LP models. It was his use of the fuzzy min-operator in conjunction with both linear and non-linear membership functions that demonstrated that the best answers to issues with multiple objectives usually include some kind of compromise. Many authors investigated various optimization techniques for goal programming problems [16-22].

The earliest description of fuzzy multiobjective linear programming (MOLP) problems appears in [21]. Different methods for addressing MOLP problems under uncertainty are presented in [22- 26]. In their analysis of multiple multiobjective linear programming problems, Othman and Zainuddin [25] included fully fuzzy stochastic problems as well as a comprehensive discussion of fuzziness and/or randomness in the goals and/or constraints. The probabilistic fuzzy goal multi-objective for supply chain networks is stated in [26] and solved using three distinct approaches. Karakutuk and Ornek [27] suggested a goal-programming model that was solved using many goal-programming models and the normalization method. A stochastic multi-objective mixed-integer optimization model [28] has been designed to guarantee production efficiency under uncertainty. In [29] Vidhya et al. resolved the neutrosophic MOLP. In [30] Pramanik and Banerjee proposed goal programming strategy to multi-objective linear programming problem with neutrosophic numbers, where the coefficients of objective functions and the constraints are considered as neutrosophic numbers. In [31] Maiti et al. presented an algorithm for solving the neutrosophic bi-level decentralised MOLP problem.

The remainder of the article is laid out as follows: In Section 2, essential preliminary information is presented. Using random variables as constraints and neutrosophic numbers as objective function coefficients, Section 3 describes a MOLP problem. The problem is handled using fuzzy goal programming in Section 4. Using the method described in Section 5, the neutrosophic optimum compromise solution is determined. A numerical example is presented for explanation in Section 6. Section 7 concludes with some findings and ideas for further work.

2. PRLIMINARIES

In order to identify our problem in a more effective manner, we recollect fundamental concepts and conclusions regarding intuitionistic trapezoidal fuzzy numbers (ITFN), trapezoidal fuzzy numbers (TFN), and the neutrosophic set (NS).

Definition 1. (TFNs, [32]). A fuzzy number $\tilde{F} = (r_1, r_2, r_3, r_4)$. Is a TFN where $r_1, r_2, r_3, r_4 \in R$ and has membership function (MF) defined as:

$$\mu_{\tilde{F}}(\chi) = \begin{cases} \frac{\chi - r_1}{r_2 - r_1}, & r_1 \le \chi \le r_2, \\ 1, & r_2 \le \chi \le r_3, \\ \frac{r_4 - \chi}{r_4 - r_3}, & r_3 \le \chi \le r_4, \\ 0, & \text{otherwise}, \end{cases}$$

Definition 2. (Intuitionistic fuzzy set, [33]). A fuzzy set $\tilde{F}$ is an intuitionistic fuzzy set $\tilde{F}^{IN}$of a non-empty set Z if $\tilde{F}^{IN} = \{\langle \chi, \mu_{\tilde{F}^{IN}}, \rho_{\tilde{F}^{IN}} \rangle : \chi \in Z\}$, where $\mu_{\tilde{F}^{IN}}$, and $\rho_{\tilde{F}^{IN}}$ are the MF and the non-MF functions where $\mu_{\tilde{F}^{IN}}$, $\rho_{\tilde{F}^{IN}}: Z \to [0, 1]$ and $0 \le \mu_{\tilde{F}^{IN}} + \rho_{\tilde{F}^{IN}} \le 1, \ \forall \chi \in Z$.

Definition 3. (Intuitionistic fuzzy number, [34]). An intuitionistic fuzzy set $\tilde{F}^{IN}$ on R is called an Intuitionistic fuzzy number if each of the following conditions valid:

1. $\exists u \in R: \mu_{\tilde{F}^{IN}}(u) = 1$, and $\rho_{\tilde{F}^{IN}}(u) = 0$,
2. $\mu_{\tilde{F}^{IN}}: R \to [0, 1]$ is continuous and $0 \le \mu_{\tilde{F}^{IN}} + \rho_{\tilde{F}^{IN}} \le 1, \forall \chi \in Z$,
3. $\mu_{\tilde{F}^{IN}}$, and $\rho_{\tilde{F}^{IN}}$ are

$$\mu_{\tilde{F}^{IN}}(\chi) = \begin{cases} 0, & \chi < r_1 \\ H(\chi), & r_1 \le \chi \le r_2 \\ 1, & \chi = r_2 \\ I(\chi), & r_2 \le \chi \le r_3 \\ 0, & r_4 \le \chi. \end{cases} \text{ and } \rho_{\tilde{F}^{IN}}(\chi) = \begin{cases} 0, & \chi < r_1 \\ F(\chi), & r_1 \le \chi \le r_2 \\ 1, & \chi = r_2 \\ G(\chi), & r_2 \le \chi \le r_3 \\ 0, & r_4 \le \chi. \end{cases}$$

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Where $H, I, F, G: R \rightarrow [0,1]$, H and G are monotonic increasing functions, I and F are monotonic decreasing functions and satisfy $0 \leq H(\chi) + F(\chi) \leq 1$, and $0 \leq I(\chi) + G(\chi) \leq 1$.

Definition 4. (Trapezoidal intuitionistic fuzzy number, [35]).

A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{F}^{IN} = (r_1, r_2, r_3, r_4), (r'_1, r_2, r_3, r'_4)$, where $r'_1 \leq r_1 \leq r_2 \leq r_3 \leq r_4 \leq r'_4$ with defined as

$$\mu_{\tilde{F}^{INT}}(\chi) = \begin{cases} \frac{\chi - r_1}{r_2 - r_1}, & r_1 \leq \chi \leq r_2, \\ 1, & r_2 \leq \chi \leq r_3, \\ \frac{r_4 - \chi}{r_4 - r_3}, & r_3 \leq \chi \leq r_4, \\ 0, & \text{otherwise}, \end{cases} \text{and} \quad \rho_{\tilde{F}^{INT}}(\chi) = \begin{cases} \frac{\chi - r'_1}{r_2 - r'_1}, & r'_1 \leq \chi \leq r_2, \\ 1, & r_2 \leq \chi \leq r_3, \\ \frac{r'_4 - \chi}{r'_4 - r_3}, & r_3 \leq \chi \leq r'_4, \\ 0, & \text{otherwise}, \end{cases}$$

Definition 5. (Neutrosophic set, [36-38]). A NS $\bar{F}^N$ of non-empty set Z is described as $\bar{F}^N = \{\langle z, I_{\bar{F}^N}(z), J_{\bar{F}^N}(z), V_{\bar{F}^N}(z)\rangle : z \in Z, I_{\bar{F}^N}(z), J_{\bar{F}^N}(z), V_{\bar{F}^N}(z) \in (0_-, 1^+)\}$,where $I_{\bar{F}^N}(z), J_{\bar{F}^N}(z)$,and $V_{\bar{F}^N}(z)$ are the truth, the indeterminacy, and the falsity membership functions, $\nexists$ any restrictions on, $0^- \leq I_{F^N}(z) + J_{\bar{F}^N}(z) + V_{F^N}(z) \leq 3^+$, $(0_-, 1^+)$ is a non-standard unit interval.

Definition 6. (Single-valued neutrosophic set, [39]). A Single-valued neutrosophic set $\bar{F}^{SVN}$ of a set $Z, Z \neq \varphi$ is defined as

$\bar{F}^{SVN} = \{\langle z, I_{\bar{F}^N}(z), J_{\bar{F}^N}(z), V_{\bar{F}^N}(z)\rangle : z \in Z\}$, where $I_{\bar{F}^N}(z), J_{\bar{F}^N}(z)$, and $V_{\bar{F}^N}(z) \in [0,1]$ $\forall z \in Z$ and $0 \leq I_{\bar{F}^N}(z) + J_{\bar{F}^N}(z) + V_{\bar{F}^N}(z) \leq 3$.

Definition 7. (Single-valued neutrosophic number, [40]). Let $\tau_{\tilde{f}}, \varphi_{\tilde{f}}, \omega_{\tilde{f}} \in [0,1]$ and $r_1, r_2, r_3, r_4 \in R$ and has (MF) defined as and. Then $\tilde{f}^N = \langle (r_1, r_2, r_3, r_4) : \tau_{\tilde{f}}, \varphi_{\tilde{f}}, \omega_{\tilde{f}} \rangle$ is a specific NS on R, whose truth, indeterminacy, and falsity MFs are

$$\tau_{\tilde{f}^N}(z) = \begin{cases} \tau_{\tilde{f}^N}\left(\frac{z - r_1}{r_2 - r_1}\right), & r_1 \leq z < r_2 \\ \tau_{\tilde{f}^N}, & r_2 \leq z \leq r_3 \\ \tau_{\tilde{f}^N}\left(\frac{r_4 - z}{z - r_3}\right), & r_3 \leq z \leq r_4 \\ 0, & \text{otherwise}, \end{cases}, \varphi_{\tilde{f}}^N(z) = \begin{cases} \frac{r_2 - z + \varphi_{\tilde{f}^N}(z - r_1)}{r_2 - r_1}, & r_1 \leq z < r_2 \\ \varphi_{\tilde{f}^N}, & r_2 \leq z \leq r_3 \\ \frac{z - r_3 + \varphi_{\tilde{f}^N}(r_4 - z)}{r_4 - r_3}, & r_3 \leq z \leq r_4 \\ 1, & \text{otherwise}, \end{cases}, \text{and}$$

$$\omega_{\tilde{f}}^N(z) = \begin{cases} \dfrac{r_2 - z + \omega_{\tilde{f}}^N(z - r_1)}{r_2 - r_1}, & r_1 \leq z < r_2 \\ \omega_{\tilde{f}}^N, & r_2 \leq z \leq r_3 \\ \dfrac{z - r_3 + \omega_{\tilde{f}}^N(r_4 - z)}{r_4 - r_3}, & r_3 \leq z \leq r_4 \\ 1, & \text{otherwise}. \end{cases}$$

A single- valued trapezoidal neutrosophic number (SVTNN) $\tilde{f}^N = \langle (r_1, r_2, r_3, r_4) : \tau_{\tilde{f}^N}, \varphi_{\tilde{f}^N}, \omega_{\tilde{f}^N} \rangle$ may represented in ill- defined quantity about f, which is nearly equal to $[r_2, r_3]$.

Definition 8. Let $\tilde{f}^N = \langle (r_1, r_2, r_3, r_4) : \tau_{\tilde{f}^N}, \varphi_{\tilde{f}^N}, \omega_{\tilde{f}^N} \rangle$, and $\tilde{g}^N = \langle (r'_1, r'_2, r'_3, r'_4) : \tau_{\tilde{g}^N}, \varphi_{\tilde{g}^N}, \omega_{\tilde{g}^N} \rangle$ be two SVTNNs, the arithmetic operations on $\tilde{f}^N$, and $g^N$are:

1. $\tilde{f}^N \oplus \tilde{g}^N = \langle (r_1 + r'_1, r_2 + r'_2, r_3 + r'_3, r_4 + r'_4);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle$,
2. $\tilde{f}^N \ominus \tilde{g}^N = \langle (r_1 - r'_4, r_2 - r'_3, r_3 - r'_2, r_4 - r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle$,
3. $\tilde{f}^N \otimes \tilde{g}^N = \begin{cases} \langle (r_1 r'_1, r_2 r'_2, r_3 r'_3, r_4 r'_4);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 > 0,\ r'_4 > 0 \\ \langle (r_1 r'_4, r_2 r'_3, r_3 r'_2, r_4 r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 > 0 \\ \langle \left(r_4 r'_4, r_3 r'_3, r_2 r'_2, r_1 r'_1{}'\right);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 < 0, \end{cases}$
4. $\tilde{f}^N \oslash \tilde{g}^N = \begin{cases} \langle (r_1/r'_4, r_2/r'_3, r_3/r'_2, r_4/r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 > 0,\ r'_4 > 0 \\ \langle (r_4/r'_4, r_3/r'_3, r_2/r'_2, r_1/r'_1);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 > 0 \\ \langle (r_4/r'_1, r_3/r'_2, r_2/r'_3, r_1/r'_4);\ \tau_{\tilde{f}^N} \wedge \tau_{\tilde{g}^N}, \varphi_{\tilde{f}^N} \vee \varphi_{\tilde{g}^N}, \omega_{\tilde{f}^N} \vee \omega_{\tilde{g}^N} \rangle, r_4 < 0,\ r'_4 < 0, \end{cases}$
5. $\alpha \tilde{f}^N = \begin{cases} \langle (\alpha r_1, \alpha r_2, \alpha r_3, \alpha r_4);\ \tau_{\tau_{\tilde{f}^N}},\ \varphi_{\tau_{\tilde{f}^N}}, \omega_{\tau_{\tilde{f}^N}} \rangle, \alpha > 0, \\ \langle (\alpha r_4, \alpha r_3, \alpha r_2, \alpha r_1);\ \tau_{\tau_{\tilde{f}^N}},\ \varphi_{\tau_{\tilde{f}^N}}, \omega_{\tau_{\tilde{f}^N}} \rangle, \alpha < 0, \end{cases}$
6. $\tilde{f}^{N^{-1}} = \langle (1/r_4, 1/r_3, 1/r_2, 1/r_1);\ \tau_{\tau_{\tilde{f}^N}},\ \varphi_{\tau_{\tilde{f}^N}}, \omega_{\tau_{\tilde{f}^N}} \rangle, \tilde{f}^N \neq 0.$

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Definition 9. (Score and Accuracy functions of SVTNN). Any two SVTNNs $\tilde{f}$, and $\tilde{g}$ can be ordered according to their score and accuracy functions as:

1. Accuracy function $AC(\tilde{f}^N) = \left(\frac{1}{16}\right)[r_1 + r_2 + r_3 + r_4] * \left[\tau_{\tau_{\tilde{f}N}} + \left(1 - \varphi_{\tau_{\tilde{f}N}}(z)\right) + \left(1 + \omega_{\tau_{\tilde{f}N}}(z)\right)\right]$,
2. Score function $SC(\tilde{f}^N) = \left(\frac{1}{16}\right)[r_1 + r_2 + r_3 + r_4] * \left[\tau_{\tau_{\tilde{f}N}} + \left(1 - \varphi_{\tau_{\tilde{f}N}}(z)\right) + \left(1 - \omega_{\tau_{\tilde{f}N}}(z)\right)\right]$.

Definition 10. Based on the accuracy and the score functions the order relations between $\tilde{f}^N$ and $\tilde{g}^N$ are:

1. If $SC(\tilde{f}^N) < SC(\tilde{g}^N), \Rightarrow \tilde{f}^N < \tilde{g}^N$
2. If $SC(\tilde{f}^N) = SC(\tilde{g}^N), \Rightarrow \tilde{f}^N = \tilde{g}^N$,
3. If $AC(\tilde{f}^N) < AC(\tilde{g}^N), \Rightarrow \tilde{f}^N < \tilde{g}^N$,
4. If $AC(\tilde{f}^N) > AC(\tilde{g}^N), \Rightarrow \tilde{f}^N < \tilde{g}^N$,
5. If $AC(\tilde{f}^N) = AC(\tilde{g}^N), \Rightarrow \tilde{f}^N = \tilde{g}^N$.

To illustrate the basic properties, let $\tilde{f}^N = \langle(4, 8, 10, 16): \ .5,\ .3\ , .6\ \rangle$ and
$\tilde{g}^N = \langle(3, 7, 11, 14): \ .4,\ .5\ , .6\ \rangle$ be two single valued trapezoidal neutrosophic numbers, then

1. $\tilde{f}^N \oplus \tilde{g}^N = \langle(7, 15, 21, 30): \ .4,\ .5\ , .6\ \rangle$,
2. $\tilde{f}^N \ominus \tilde{g}^N = \langle(-10, -3, 3, 13): \ .4,\ .5\ , .6\ \rangle$,
3. $\tilde{f}^N \otimes \tilde{g}^N = \langle(12, 56, 110, 224): \ .4,\ .5\ , .6\ \rangle$,
4. $\tilde{f}^N \oslash \tilde{g}^N = \langle\left(\frac{4}{14}, \frac{8}{11}, \frac{10}{7}, \frac{16}{3}\right): \ .4,\ .5\ , .6\ \rangle$,
5. $4\tilde{f}^N = \langle(16, 32, 40, 64): \ .4,\ .5\ , .6\ \rangle$,
6. $\tilde{f}^{N^{-1}} = \langle\left(\frac{1}{16}, \frac{1}{10}, \frac{1}{8}, \frac{1}{4}\right): \ .4,\ .5\ , .6\ \rangle$,
7. $SC(\tilde{f}^N) = \left(\frac{1}{16}\right)(4 + 8 + 10 + 16) \times (.5 + (1 - .3) + (1 - .6)) = 3.8$,
8. $AC(\tilde{f}^N) = \left(\frac{1}{16}\right)(4 + 8 + 10 + 16) \times (.5 + (1 - .3) + (1 + .6)) = 6.65$.

3. Problem formulation and solution concepts

A stochastic MOLP problem in a neutrosophic setting is characterized as follows in chance-constrained programming.

$$\max\ \tilde{\mathcal{Z}}^{N^{(k)}} = \sum_{j=1}^{J} \tilde{c}_j^{N^{(k)}} x_j, \qquad k = 1, \dots, K \tag{1}$$

Subject to

$$P\left[\sum_{j=1}^{J} h_{ij}\, x_j \le p_i\right] \ge 1 - \gamma_i, i = 1, \dots, I$$

$$x_j \ge 0, j = 1 : J$$

Where, $\tilde{c}_j^{N^{(k)}}(\ j = 1, \dots\ J; k = 1, \dots, K)$ are SVTNNs, $h_{ij} \in R^{\mathcal{I} \times \mathcal{J}}$, and $p_i, i = 1, 2, \dots, I$ are random variables, $\gamma_i \in (0, 1)$ are specified probabilities, and $x_j, j = 1, 2, \dots, J$ are deterministic decision variables.

Definition 11. A point x is referred to be a stochastic neutrosophic feasible point if it satisfies the requirements in the problem (1).

Definition 12. A stochastic neutrosophic feasible point x° is called stochastic single-valued trapezoidal neutrosophic efficient solution to problem (1) iff there does not exists an another x such that:
$\tilde{\mathcal{Z}}^{N^{(k)}}\left(x, \tilde{c}_j^{N^{(k)}}\right) \ge \tilde{\mathcal{Z}}^{N^{(k)}}\left(x^\circ, \tilde{c}_j^{N^{(k)}}\right)$ and $\tilde{\mathcal{Z}}^{N^{(k)}}(x, \tilde{c}_j^{N^{(k)}}) \ne \tilde{\mathcal{Z}}^{N^{(k)}}(x^\circ, \tilde{c}_j^{N^{(k)}})$.

Based on the score function in definition 9, problem (1) is transformed to the following stochastic problem

$$\max\ \mathcal{Z}^{(k)} = \sum_{j=1}^{J} c_j^{(k)} x_j, \qquad k = 1, \dots, K \tag{2}$$

Subject to

$$P\left[\sum_{j=1}^{J} h_{ij}\, x_j \le p_i\right] \ge 1 - \gamma_i, \qquad i = 1, \dots, I$$

$$x_j \ge 0, j = 1, \dots, J.$$

The right-hand side values, p_i's are now considered random variables with the following known distributions:

(i) Uniform distribution,
(ii) Exponential distribution, and
(iii) Gamma distribution.

Firstly, let us consider p_i's are uniformly distributed. Then,

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$$g(p_i) = \begin{cases} \dfrac{1}{\alpha_i - \beta_i}, & \beta_i < p_i < \alpha_i \\ 0, & otherwise \end{cases}$$

With mean equal to $E(p_i) = \frac{\alpha_i + \beta_i}{2}$ and variance equal to $V(p_i) = \frac{{\alpha_i}^2 - {\beta_i}^2}{12}$. So, the constrain (2) of the stochastic problem becomes

$$\int_{\sum_{,j=1}^{J} h_{ij}\, x_j}^{\alpha_i} \frac{1}{\alpha_i - \beta_i}\, dp_i \geq 1 - \gamma_i = \left(\frac{p_i}{\alpha_i - \beta_i}\right)_{\sum_{,j=1}^{J} h_{ij}\, x_j}^{\alpha_i} \geq 1 - \gamma_i,$$

$$\text{or} \sum_{j=1}^{J} h_{ij}\, x_j \leq \beta_i + \gamma_i(\alpha_i - \beta_i).$$

Hence, the stochastic problem (2) is converted into the corresponding deterministic problem as

$$\max\ \mathcal{Z}^{(k)} = \textstyle\sum_{j=1}^{J} {c_j}^{(k)}\, x_j, \qquad k = 1{:}\,K \tag{3}$$

Subject to

$$\sum_{,j=1}^{J} h_{ij}\, x_j \leq \beta_i + \gamma_i(\alpha_i - \beta_i), \qquad i = 1, \dots, I$$

$$x_j \geq 0,\ j = 1, \dots, J.$$

Secondly, let p_i's be exponential distributed random variables. Then, $g(p_i) = \varepsilon_i \exp(-\varepsilon_i\, p_i)$, with mean equal to $(\frac{1}{\varepsilon_i})$ and variance equal to $(\frac{1}{\varepsilon_i})^2$. So, the constraint of the stochastic problem (2) becomes

$$\int_{\sum_{j=1}^{J} h_{ij}\, x_j}^{\infty} \varepsilon_i \exp(-\varepsilon_i\, p_i)\, dp_i \geq 1 - \gamma_i = \exp\left(\varepsilon_i \sum_{,j=1}^{J} h_{ij}\, x_j\right), i = 1{:}\,I,$$

$$\sum_{j=1}^{J} h_{ij}\, x_j \leq -\frac{\ln(1 - \gamma_i)}{\varepsilon_i}.$$

Hence, the stochastic problem (2) represented in the deterministic form become

$$\max\ \mathcal{Z}^{(k)} = \textstyle\sum_{j=1}^{J} {c_j}^{(k)}\, x_j, \qquad k = 1, \dots, K \tag{4}$$

Subject to

$$\textstyle\sum_{j=1}^{J} h_{ij}\, x_j \leq -\frac{\ln(1-\gamma_i)}{\varepsilon_i}, \qquad i = 1, \dots, I$$

$$x_j \geq 0,\ j = 1, \dots, J.$$

Finally, let p_i's be Gamma distributed random variables. Then,

$$g(p_i) = \begin{cases} \frac{\varepsilon_i^{\eth} \exp(-\varepsilon_i p_i) p_i^{\eth - 1}}{\mathrm{Gamma}(\eth)}, & 0 < p_i < \infty \\ 0, & otherwise \end{cases}$$

With mean is $\eth/\varepsilon$ and variance is $\eth/\varepsilon^2$. Then, the constraint of problem (1) becomes

$$\int_{\sum_{j=1}^{J} h_{ij}\, x_j}^{\infty} \frac{\varepsilon_i^{\eth} \exp(-\varepsilon_i p_i)\, p_i^{\eth - 1}}{\mathrm{Gamma}(\eth\,)}\, dp_i \geq 1 - \gamma_i, i = 1, \dots, I.$$

So,

$$\int_{\sum_{j=1}^{J} h_{ij}\, x_j}^{\infty} \frac{\varepsilon_i^{\eth} \exp(-\varepsilon_i p_i)\, p_i^{\eth - 1}}{\mathrm{Gamma}(\eth\,)}\, dp_i = (-1)^{\eth} \exp\left(-\varepsilon_i \sum_{j=1}^{J} h_{ij}\, x_j\right) \begin{bmatrix} \left(-\varepsilon_i \sum_{j=1}^{J} h_{ij}\, x_j\right)^{\eth - 1} - (\eth - 1)\left(-\varepsilon_i \sum_{j=1}^{J} h_{ij}\, x_j\right)^{\eth - 2} \\ + \cdots + (-1)^{\eth - 1}(\eth - 1)(\eth - 2) \dots 2.1 \end{bmatrix},$$

$$i = 1, \dots, I$$

Thus, the stochastic problem (2) is converted into the corresponding crisp form

$$\max\ \mathcal{Z}^{(k)} = \textstyle\sum_{j=1}^{J} {c_j}^{(k)}\, x_j, \qquad k = 1, \dots, K \tag{5}$$

Subject to

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$$\frac{(-1)^{\mathfrak{d}}}{\mathrm{Gamma}(\delta)}\exp\left(-\varepsilon_i\sum_{j=1}^{J}h_{ij}\,x_j\right)\left[\left(-\varepsilon_i\sum_{j=1}^{J}h_{ij}\,x_j\right)^{\mathfrak{d}-1}-(\mathfrak{d}-1)\left(-\varepsilon_i\sum_{j=1}^{J}h_{ij}\,x_j\right)^{\mathfrak{d}-2}+\cdots\right.$$
$$\left.+(-1)^{\mathfrak{d}-1}(\mathfrak{d}-1)(\mathfrak{d}-2)\ldots 2.1\right], i=1,\ldots,I.$$
$$x_j \geq 0, j=1,\ldots,J.$$

4. Fuzzy goal programming approach

Based on the three terms fuzzy goals (G), fuzzy constraints (C), and fuzzy decision (D) established by Bellman and Zadeh [32], the fuzzy decision is described as

$$D = C \cap G \tag{6}$$

The following membership function is characterized the problem:

$$\mu_D(x) = \min\big(\mu_C(x), \mu_G(x)\big) \tag{7}$$

With the following membership function, let us describe the problem's fuzzy gools. The linear membership function (MP) [14] is given by

$$\mu^k\big(\mathcal{Z}^{(k)}\big)\begin{cases} 0, & \mathcal{Z}^{(k)} \leq m_k, \\ \dfrac{\mathcal{Z}^{(k)}-m_k}{M_k-m_k}, & m_k < \mathcal{Z}^{(k)} < M_k, \\ 1, & \mathcal{Z}^{(k)} \geq m_{\mathscr{k}} \end{cases} \tag{8}$$

Where, m_k , and M_k represents the lower and upper bounds of $\mathcal{Z}^{(k)}$, $m_k \neq M_k$, and can be calculated as

$$m_k = \min_x \mathcal{Z}^{(k)}, \text{and } M_k = \max_x \mathcal{Z}^{(k)}, \quad k=1,\ldots,K \tag{9}$$

Using the MP defined in (8), problems (3), (4), and (5), respectively are rewritten as

$$\max \ \min_k\big(\ \mu^k\big(\mathcal{Z}^{(k)}\big)\big),\ k=1,\ldots,K$$

Subject to

$$\sum_{,j=1}^{J}h_{ij}\,x_j \leq \beta_i+\gamma_i(\alpha_i-\beta_i), \qquad i=1,\ldots,I.$$
$$x_j \geq 0, j=1,..,J,$$

$$\max \ \min_k\big(\ \mu^k\big(\mathcal{Z}^{(k)}\big)\big),\ k=1,\ldots,K \tag{11}$$

Subject to

$$\textstyle\sum_{,j=1}^{J}h_{ij}\,x_j \leq -\frac{\ln(1-\gamma_i)}{\varepsilon_i}, \qquad i=1,\ldots,I.$$
$$x_j \geq 0, j=1,\ldots,J, \text{ and}$$

$$\max \ \min_k\big(\ \mu^k(\mathcal{Z})\big),\ k=1,\ldots,K \tag{12}$$

Subject to

$$\frac{(-1)^{\mathfrak{d}}}{\mathrm{Gamma}(\delta)}\exp(-\varepsilon_i)\left[\left(-\varepsilon_i\sum_{j=1}^{J}h_{ij}\,x_j\right)^{\mathfrak{d}-1}-(\mathfrak{d}-1)\left(-\varepsilon_i\sum_{j=1}^{J}h_{ij}\,x_j\right)^{\mathfrak{d}-2}+\cdots+(-1)^{\mathfrak{d}-1}(\mathfrak{d}-1)(\mathfrak{d}-2)\ldots 2.1\right], i$$
$$=1,\ldots,I,$$
$$x_j \geq 0, j=1,\ldots,J.$$

By introducing the auxiliary variables ϑ, θ and $\mathcal{E}$ problems (10), (11) and (12) can be formed the following problems

$$\max \vartheta \tag{13}$$

Subject to

$$\vartheta \leq \frac{\mathcal{Z}^{(k)}-m_k}{M_k-m_k},$$
$$\textstyle\sum_{j=1}^{J}h_{ij}\,x_j \leq \beta_i+\gamma_i(\alpha_i-\beta_i),\ i=1,\ldots,I.$$
$$x_j \geq 0, j=1,\ldots,J; 0 \leq \ \vartheta \leq 1,$$

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$$\max \theta \tag{14}$$

Subject to

$$\theta \leq \frac{\mathcal{Z}^{(k)} - m_k}{M_k - m_k},$$

$$\sum_{j=1}^{J} h_{ij}\, x_j \leq -\frac{\ln(1-\gamma_i)}{\varepsilon_i}, \quad i = 1, \dots, I.$$

$$x_j \geq 0, j = 1, \dots, J;\ 0 \leq \theta \leq 1, \text{ and}$$

$$\max \epsilon \tag{15}$$

Subject to

$$\varepsilon \leq \frac{\mathcal{Z}^{(k)} - m_k}{M_k - m_k},$$

$$\frac{(-1)^{\mathfrak{d}}}{\mathrm{Gamma}(\mathfrak{d})} \exp\left(-\varepsilon_i \sum_{j=1}^{J} h_{ij}\, x_j\right)\left[\left(-\varepsilon_i \sum_{j=1}^{J} h_{ij}\, x_j\right)^{\mathfrak{d}-1} - (\mathfrak{d}-1)\left(-\varepsilon_i \sum_{j=1}^{J} h_{ij}\, x_j\right)^{\mathfrak{d}-2} + \cdots \right.$$
$$\left. + (-1)^{\mathfrak{d}-1}(\mathfrak{d}-1)(\mathfrak{d}-2)\dots 2.1\right], i = 1, \dots, I,$$

$$x_j \geq 0, j = 1, \dots, J, ;\ 0 \leq \epsilon \leq 1.$$

As a goal programming, problems (13), (14) and (15) can be reformulated. So, let us consider the negative and positive deviational variables:

$$\mathcal{Z}^{(k)} - d_-^k + d_+^k = \mathcal{S}^k, k = 1, \dots, K.$$

Where, $\mathcal{S}^k$ is the aspiration level of the k^{th} objective function.

Hence, problem (13) may be transformed into

$$\max \vartheta \tag{16}$$

Subject to

$$\vartheta \leq \frac{\mathcal{Z}^{(k)} - m_k}{M_k - m_k},$$

$$\sum_{j=1}^{J} h_{ij}\, x_j \leq \beta_i + \gamma_i(\alpha_i - \beta_i), i = 1, \dots, I,$$

$$\mathcal{Z}^{(k)} - d_-^k + d_+^k = \mathcal{S}^k, k = 1, \dots, K$$

$$x_j, d_-^k, d_+^k \geq 0, j = 1, \dots, J; k = 1, \dots, K; 0 \leq \vartheta \leq 1.$$

Similarly, Problems (14), and (15) can be rewritten as (16).

5. SOLUTION PROCEDURE

This approach for solving problem (1) may be structured as follows:

Step 1: Convert the neutrosophic stochastic problem (1) into the related stochastic problem (2) based on the score function in definition 9, and thus into the deterministic problem by using the chance restricted programming approach previously stated,

Step 2: Determine the solution to the first objective function of the deterministic problem obtained from step1. Continue this process k times. If all solutions are identical, choose one of them as the optimal compromise solution, continue to Step 7. Otherwise, go to step3,

Step 3: Determine the optimal lower limit m_k, the worst possible upper bound M_k, and the objective function's value at the k solution**,**

Step 4: Construct the membership function of each objective function based on the relationship (8) and the initial goal level,

Step 5: Develop problem (16) and its the similar equations,

Step 7: Stop.

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6. NUMERICAL EXAMPLES

Example [1]

Take into consideration the following problem with p_i's are exponential random variables

$$\max\ \tilde{Z}^{N^{(1)}} = \tilde{c}_1^{N^{(1)}} x_1 + \tilde{c}_2^{N^{(1)}} x_2$$
$$\max\ \tilde{Z}^{N^{(2)}} = \tilde{c}_1^{N^{(2)}} x_1 + \tilde{c}_2^{N^{(2)}} x_2$$

Subject to

$$P(x_1 + x_2 \le p_1) \ge 0.94; \quad P(4x_1 + 3x_2 \le p_2) \ge 0.93;$$
$$P(2x_1 + 5x_2 \le p_3) \ge 0.91; \quad x_j \ge 0, j = 1, 2.$$

Where,

$$\tilde{c}_1^{N^{(2)}} = \langle (5, 8, 10, 14); 0.3, 0.6, 0.6 \rangle, \tilde{c}_2^{N^{(1)}} = \langle (4, 8, 11, 15); 0.6, 0.3, 0.2 \rangle,$$
$$\tilde{c}_1^{N^{(1)}} = \langle (14, 17, 21, 28); 0.8, 0.2, 0.6 \rangle, \tilde{c}_2^{N^{(2)}} = \langle (12, 15, 19, 22); 0.6, 0.4, 0.5 \rangle,$$
$$E(p_1) = 7, E(p_2) = 9, E(p_3) = 8, \gamma_1 = 0.06, \gamma_2 = 0.07, \gamma_3 = 0.09.$$

Step 1: Using the score function of the SVTNN, and the chance constrained discussed as before, the above problem become

$$\max Z^{(1)} = 10x_1 + 5x_2$$
$$\max Z^{(2)} = 3x_1 + 7x_2$$

Subject to

$$x_1 + x_2 \le 0.433,$$
$$4x_1 + 3x_2 \le 0.653,$$
$$2x_1 + 5x_2 \le 0.755,$$
$$x_j \ge 0, j = 1, 2.$$

Step 2: The solution of each single objective function with respect to the constraints is
$\dot{X} = (x_1, x_2) = (0.16325,\ 0)$, and $\dot{Y} = (x_1, x_2) = (.071536,\ 0.122286)$.

Step 3: The objective values corresponding to $\dot{X}$, and $\dot{Y}$.

$$Z^{(1)}(\dot{X}) = 1.6325, Z^{(1)}(\dot{Y}) = 1.3268, Z^{(2)}(\dot{X}) = 0.4898, Z^{(2)}(\dot{Y}) = 1.07061.$$

The lower and upper bounds of each objective function are:

$$1.32679 \le Z^{(1)} \le 1.6325, \text{ and } 0.48975 \le Z^{(2)} \le 1.070607.$$

Step 4: The MFs of the first and second objective functions are:

$$7.53699\, x_1 + 3.76849\, x_2 - 0.2304130\theta \ge 1,$$
$$5.16478\, x_1 + 12.051159 x_2 - 0.8431510\theta \ge 1.$$

Step 5: Formulate the above problem corresponding to the problem (16) as

$$\max \theta \quad \text{Subject to}$$
$$7.53699\, x_1 + 3.76849\, x_2 - 0.2304130\theta \ge 1,$$
$$5.16478\, x_1 + 12.051159 x_2 - 0.8431510\theta \ge 1,$$
$$x_1 + x_2 \le 0.433,$$
$$4x_1 + 3x_2 \le 0.653,$$
$$2x_1 + 5x_2 \le 0.755,$$
$$10x_1 + 5x_2 - d_-^1 + d_+^1 = 1.6325,$$
$$3x_1 + 7x_2 - d_-^2 + d_+^2 = 1.070607,$$
$$0 \le \theta \le 1, d_-^1, d_+^1, d_-^2, d_+^2, x_1, x_2 \ge 0.$$

The problem is solved using the Lingo package, and the solution is

Table 1: The optimal compromise solution of example1

Variables	Objective
$x_1^* = 0.1135,\ x_2^* = 0.0663$	$\theta^* = 0.45, Z^{(1)^*} = 1.4665,$
$d_-^1 = 0, d_+^1 = 0.3649, \quad d_-^2 = 0, d_+^2 = 0.2657$	$Z^{(2)^*} = 0.8046$

Table 2: The optimal compromise neutrosophic solution example1

Variables	Objective
$x_1^* = 0.1135$	$\tilde{Z}^{N^{(1)}} = \langle (1.855, 0.72495, 3.115, 4.1755); 0.6, 0.3, 0.6 \rangle$
$x_2^* = 0.0663$	$\tilde{Z}^{N^{(2)}} = \langle (1.3655, 1.9055, 2.3985, 3.052); 0.3, 0.6, 0.6 \rangle$

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Example [2]

Consider the following problem when b_i's are uniformly distributed

$$\max Z^{(1)} = \tilde{q}_1^{N^{(1)}} x_1 + \tilde{q}_2^{N^{(1)}} x_2$$

$$\max Z^{(2)} = \tilde{q}_1^{N^{(2)}} x_1 + \tilde{q}_2^{N^{(2)}} x_2$$

Subject to

$$P(2x_1 + x_2 \le 7.6) \ge 1 - \gamma_1,$$

$$P(5x_1 + 3x_2 \le 7.8) \ge 1 - \gamma_2,$$

$$P(x_1 + 4x_2 \le 7.2) \ge 1 - \gamma_2,$$

$$x_j \ge 0, j = 1, 2.$$

Where, $\tilde{q}_1^{N^{(2)}} = \langle (0, 1, 3, 6); 0.7, 0.5, 0.3 \rangle, \tilde{q}_2^{N^{(1)}} = \langle (5, 7, 9, 11); 0.9, 0.7, 0.5 \rangle,$

$\tilde{q}_1^{N^{(1)}} = \langle (0, 1, 3, 6); 0.7, 0.5, 0.3 \rangle, \tilde{q}_2^{N^{(2)}} = \langle (4, 8, 11, 15); 0.6, 0.3, 0.2 \rangle,$

$E(b_i) = 6, V(b_i) = 4, \gamma_1 = 0.9, \gamma_2 = 0.95, \gamma_3 = 0.8.$

Step 1: Using the score function and the chance constrained programming, the problem can be stated as

$$\max Z^{(1)} = x_1 + 3x_2$$

$$\max Z^{(2)} = x_1 + 5x_2$$

Subject to

$$2x_1 + x_2 \le 0.76,$$

$$5x_1 + 3x_2 \le 0.78,$$

$$x_1 + 4x_2 \le 0.72,$$

$$x_j \ge 0, j = 1, 2.$$

Step 2: The solution of each single objective function is

$\ddot{X} = (0.564706,\ 1.658823)$, and $\ddot{Y} = (x_1, x_2) = (0,\ 1.8)$.

Step 3: The objective values corresponding to $\ddot{X}$, and $\ddot{Y}$.

$$Z^{(1)}(\ddot{X}) = 5.542276, Z^{(1)}(\ddot{Y}) = 9, Z^{(2)}(\ddot{X}) = 8.858821, Z^{(2)}(\ddot{Y}) = 9\,.$$

$$5.542276 \le Z^{(1)} \le 9, \text{ and } 8.858821 \le Z^{(2)} \le 9.$$

Step 4: The MFs of the first and second objective functions are

$$7.53699\, x_1 + 3.76849\, x_2 - 0.230413\vartheta \ge 1,$$

$$5.16478\, x_1 + 12.051159 x_2 - 0.843151\vartheta \ge 1.$$

Step 5: Formulate the above problem corresponding to the problem (16) as

$$\max \vartheta$$

Subject to

$$7.53699\, x_1 + 3.76849\, x_2 - 0.230413\vartheta \ge 1,$$

$$5.16478\, x_1 + 12.051159 x_2 - 0.843151\vartheta \ge 1$$

$$2x_1 + x_2 \le 0.76,$$

$$5x_1 + 3x_2 \le 0.78,$$

$$x_1 + 4x_2 \le 0.72,$$

$$x_1 + 3x_2 - d_-^1 + d_+^1 = 9,$$

$$x_1 + 5x_2 - d_-^2 + d_+^2 = 9,$$

$$0 \le \vartheta \le 1\,, d_-^1, d_+^1, d_-^2, d_+^2, x_1, x_2 \ge 0.$$

In this example, the Lingo computer package is used to provide a solution that is

Table 3: The optimal compromise solution of example 2

Variables	Objective
$x_1^* = 0.1131, \quad x_2^* = 0.0715$ $d_-^1 = 0, d_+^1 = 8.6724, \ \ d_-^2 = 0, d_+^2 = 8.5293$	$\vartheta^* = 0.5289, Z^{(1)^*} = 0.3276,$ $Z^{(2)^*} = 0.4706$

Table 4: The optimal compromise neutrosophic solution example 2

Variables	Objective
$x_1^* = 0.1131$ $x_2^* = 0.0715$	$\tilde{Z}^{N^{(1)}} = \langle (0.3575, 0.6136, 0.9828, 1.465\ ); 0.7, 0.7, 0.5\ \rangle$ $\tilde{Z}^{N^{(2)}} = \langle (0.286, 0.6851, 1.1258, 1.7511); 0.6, 0.5, 0.3 \rangle$

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7. Concluding Remarks And Future Works

In this study, a fuzzy goal programming technique for addressing a probabilistic MOLP problem in a neutrosophic environment has developed. The methodology is implemented by establishing the membership function and aspirational level. The benefit of this technique is that it is more flexible, and the resulting solution presents a preferable compromise solution that is more realistic than the standpoint for the decision maker. The proposed technique can be extended to multi-objective nonlinear programming problems, priority based-goal programming involving neutrosophic parameters or other uncertainty settings. In the Future work might contain the additional extension of this study to other fuzzy-like structure (i. e., Neutrosophic set, interval-valued fuzzy set, Spherical fuzzy set, Pythagorean fuzzy set etc. In addition, one can consider new fuzzy systems such as interval type-2, interval type-3, Possibility Interval-valued Intuitionistic fuzzy set, Possibility Neutrosophic set, Possibility Interval-valued Neutrosophic set, Possibility Interval-valued fuzzy set, Possibility fuzzy expert set etc., with applications in decision-making.

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