Enhancing neutrosophic fuzzy compromise approach for solving stochastic bi- level linear programming problems with right- hand sides of constraints follow normal distribution

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Abstract

In this paper, a bi-level chance constrained programming problem is considered when the coefficients of the objective function is presented as neutrosophic numbers and the right- hand side of the constraints is normal variables and the constraints have a joint probability distribution. While the probability problem and applying the score and accurate functions the problem is converted into an equivalent deterministic non- linear programming problem, a fuzzy programming approach is applied by defining membership function. A linear membership function is used for obtaining optimal compromise solution. A numerical example is given to illustrate the proposed methodology.

Keywords: Optimization; neutrosophic set; single valued neutrosophic numbers Chance- constrained programming; Bi-level linear programming; Decision Making; Normal distribution; Joint constraints; Incomplete Gamma function; Membership function; Fuzzy programming approach; Optimal compromise solution

1. Introduction

Decision making in many planning problem involves hierarchical administrative structure, which involves decisions of several individual with independent and conflicting objectives. Bi- level mathematical programming (BLMP) is identified as mathematical programming that solves decentralized planning problems with two decision makers (DMs) in a two levels or hierarchical organization. The basic concept of the BLMP technique is that the decision maker at the upper level and the lower level is termed as the followers. When the leader attempts to optimize his objective, the follower tries to find an optimal solution according to each of possible decisions may by the leader (Dempe, [1-2]; and Colson et al., [3]). BLP has researched from theoretical to computational and successfully applied to variety of fields as transport network design (Gzara,[4]; Fontainne, and Minner, [5]), Principle- agent problems (Cecchini et al., [6]), and electricity markets(

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

Zhang et al., [7]). A numerous methods for solving BLP problems are introduced. Most of these methods are based on the concepts of vertex enumeration and transformation methods. Bialas and Karwan [8-9] pioneers of linear BLP problem proposed a vertex enumeration method, namely Kth- best solution. These are solved by the simplex method, although it is developed to find a global optimal solution, computational and storage requirements of the algorithm increase vastly with the number of variables. Bard and Falk [10] developed the grid search algorithm. They transformed the product terms into a series of equalities. Anandalingam, [11] and Shih et al., [12] applied mathematical programming and fuzzy programming approaches for multi- level programming problems, respectively. Ye and Zhu [13] studied the linear BLP problem where the constraint region has determined implicitly by the solution set of a lower level mathematical programming problem. Chen and Florian, [14] studied the regularity and optimality conditions for the linear BLP problem using a formulation involving the marginal value function of the parametric lower level problem. Falk and Liu, [15] investigated the linear BLP suing an approach which employs strong stability on local optimizers of the lower level problem to convert it locally into an equivalent non- differentiable optimization problem. Gendreau et al., [16] argued target the linear BLP is an instance of a linear hierarchical decision process where the lower level constraint set is independent on decisions taken at the upper level. They proposed approach is applied to solve the NP- hard problem using an adaptive search method related to the Tabu search metaheuristic. Dempe and Schmidt, [17] introduced an algorithm for solving BLP. This algorithm combined a direction finding problem with regularization for the lower level problem, where the upper level objective function was included in the regularization to yield uniqueness of the follower's solution set. Safaei and Saraj [18] introduced a method to solve a FFBLP problem, where the problem involving triangular fuzzy numbers in all the parameters and decision variables. They decomposed the considered problem into three deterministic linear programming problems with bounded variables constraints. Ren, [19] proposed a new approach based on deviation degree measures and a ranking function to solve a class of fully fuzzy bilevel linear programming problems in which all coefficients and decision variables are fuzzy numbers. Ren, [20] presented interactive programming method to find the fuzzy optimal solution of FFBLP problem. Hammad, [21] solved the evacuation location assignment problem using a bilvel multiobjective optimization model, where the model incorporates two decision makers' spaces, namely**,** urban planners and evacuees. Tahernejad et al., [22] used a generalized branch- and- cut approach for solving mixed integer bi-level linear optimization problems through a comprehensive algorithm framework. Rey, [23] introduced the literature on exact methodologies for the discrete network design probe and attempts to categorize the main approaches have employed.

Stochastic programming deals with the methods and theory of stochastic variations into a mathematical programming (Segupta, [24]). Three main approaches to stochastic programming (Goicoechea et al. [25]) are recognized as:

i. Chance constrained programming, and
ii. Two- stage programming.

The chance- constrained programming solves problem with chance constraints (i. e., constraints having finite probability violated). Leclercq [26] and Teghem et al. [27] introduced interactive methods in probabilistic programming. Sinha et al. [28] studied multiobjective probabilistic linear programming with right- hand -side of the constraints distributed randomly with known means and variances and applied fuzzy programming approach for obtaining the optimal compromise solution for the corresponding deterministic problem.

In many scientific areas, such as system analysis and operators research, a model has to be setup using data which is only approximately known. Fuzzy sets theory, introduced by Zadeh [29] makes this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade [30] extended the use of algebraic operations on real numbers to fuzzy numbers by use of a fuzzification principle. Fuzzy Set Theory has been widely used to solve many practical problems, including financial risk management, since it allows us to describe and treat imprecise and uncertain elements present in a decision problem. Then the imperfect knowledge of the returns on the assets and the uncertainty involved in the behavior of financial markets may also be introduced by means of fuzzy quantities and/ or fuzzy constraints. Decision making in a fuzzy environment was developed by Bellman and Zadeh [31]. Zimmermann [32] is the first one discussed the nature in a goal programming problems and later followed by Narasimhan [33] and a lot of other authors working in that field. Al-Sharqi et al. [34-36] introduced several algebraic contributions supported by algorithms that employed these algebraic structures in solving decision-making real-life problems. Leberling [37] applied the min- operator for solving multi-criteria optimization problem. Fuzzy linear programming with multiple objective functions gas studied by Zimmermann [38].

A bi- level organization has the following common features (Sinha and Biswal, [39])

- Interactive decision making units within a predominantly hierarchical structure,
- Execution of decision is sequential , from upper to lower level,
- Each unit independently maximizes or minimizes its own benefits, but is affected by the actions of other units through externalities,
- External effect on a decision maker's problem can be reflected in both the objective function and the set of feasible decision space.

Recently, many researchers have introduced their studied about bilevel optimization in different environments (see, Liu et al. [38] and Camacho- Vallejo et al. [39]).

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

The rest of the paper is organized as: Section 2, introduced some preliminaries related to the neutrosophic set and trapezoidal neutrosophic numbers. Section 3 formulates the probabilistic BLP problem. Section 4 applies the fuzzy programming approach for obtaining the optimal compromise solution of the BLP problem. Section 5 gives a numerical example to illustrate the proposed solution procedure. Finally, some concluding remarks are reported in section 6.

2. Preliminaries

In this section, some basic notions of fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy numbers and neutrosophic sets are recalled (see, Atanassov, [40]; Smarandache, [41]; Wang et la., [42]; and Wang et al., [43])

***Definition 1(*Neutrosophic set)** A neutrosophic set $\widetilde{\mathcal{B}}$ of non-empty set $\mathcal{X}$ is defined as:
$\widetilde{\mathcal{B}}^{\mathrm{N}} = \{\langle x;\ \mathrm{I}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{J}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{V}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x)\rangle : x \in \mathcal{X}, \mathrm{I}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{J}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{V}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x) \in]0^-, 1^+[\}$
where $\mathrm{I}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{J}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x)$, and $\mathrm{V}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x)$ are functions that fulfill truth-membership, indeterminacy-membership, and falsity-membership: $0^- \leq \mathrm{Sup}\{\mathrm{I}_{\widetilde{\mathrm{B}}^{\mathrm{N}}}(x)\} + \mathrm{Sup}\{\mathrm{J}_{\widetilde{\mathrm{B}}^{\mathrm{N}}}(x)\} + \mathrm{Sup}\ \{\mathrm{V}_{\widetilde{\mathrm{B}}^{\mathrm{N}}}(x)\} \leq 3^+$, where $]0^-, 1^+[$ is a nonstandard unit interval
***Definition 2* (Single-valued neutrosophic set)**A Single-valued neutrosophic set $\widetilde{\mathcal{B}}^{\mathcal{SVN}}$ of a non-empty set $\mathcal{X}$ is described as $\widetilde{\mathcal{B}}^{\mathcal{SVN}} = \{\langle x;\ \mathrm{I}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{J}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{V}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x)\rangle : x \in \mathcal{X}\}$ where $\mathrm{I}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x), \mathrm{J}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x)$ and $\mathrm{V}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x) \in [0, 1]$ for each $x \in \mathcal{X}$ and $0 \leq \mathrm{I}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x) + \mathrm{J}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x) + \mathrm{V}_{\widetilde{\mathcal{B}}^{\mathrm{N}}}(x) \leq 3$.
***Definition 3(*Single-valued pentagonal fuzzy neutrosophic number (SVPFN))**. Let $\zeta_{\tilde{\rho}}, \sigma_{\tilde{\rho}}, \psi_{\tilde{\rho}} \in [0, 1]$ and $\mathrm{r, s, t, u, v} \in \mathcal{R}$ where, $\mathrm{r} \leq \mathrm{s} \leq \mathrm{t} \leq \mathrm{u} \leq \mathrm{v}$. Then a SVPFN, $\tilde{\rho}^{\mathrm{PN}} = \langle(\mathrm{r, s, t, u, v});\ \zeta_{\tilde{\rho}}, \sigma_{\tilde{\rho}}, \psi_{\tilde{\rho}}\rangle$ is a specific neutrosophic set on $\mathcal{R}$, with truth-membership, hesitant-membership, and falsity-membership functions are

$$\zeta_{\tilde{\rho}^{\mathrm{PN}}}(x) = \begin{cases} 0, & x < \mathrm{r}; \\ \zeta_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{s-r})^2}(x-\mathrm{r})^2\right), & \mathrm{r} \leq x \leq \mathrm{s}; \\ \zeta_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{t-s})^2}(x-\mathrm{t})^2+1\right), & \mathrm{s} \leq x \leq \mathrm{t}; \\ \zeta_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{u-t})^2}(x-\mathrm{t})^2+1\right), & \mathrm{t} \leq x \leq \mathrm{u}; \\ \zeta_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{v-u})^2}(x-\mathrm{v})^2\right), & \mathrm{u} \leq x \leq \mathrm{v}; \\ 0, & x > \mathrm{v}. \end{cases}$$

$$\sigma_{\tilde{\mathrm{p}}^{\mathrm{PN}}}(x) = \begin{cases} 0, & x < \mathrm{r}; \\ \sigma_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{s-r})^2}(x-\mathrm{r})^2\right), & \mathrm{r} \leq x \leq \mathrm{s}; \\ \sigma_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{t-s})^2}(x-\mathrm{t})^2+1\right), & \mathrm{s} \leq x \leq \mathrm{t}; \\ \sigma_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{u-t})^2}(x-\mathrm{t})^2+1\right), & \mathrm{t} \leq x \leq \mathrm{u}; \\ \sigma_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{v-u})^2}(x-\mathrm{v})^2\right), & \mathrm{u} \leq x \leq \mathrm{v}; \\ 0, & x > \mathrm{v}. \end{cases}$$

$$\psi_{\tilde{\mathrm{p}}^{\mathrm{PN}}}(x) = \begin{cases} 0, & x < \mathrm{r}; \\ \psi_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{s-r})^2}(x-\mathrm{r})^2\right), & \mathrm{r} \leq x \leq \mathrm{s}; \\ \psi_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{t-s})^2}(x-\mathrm{t})^2+1\right), & \mathrm{s} \leq x \leq \mathrm{t}; \\ \psi_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{u-t})^2}(x-\mathrm{t})^2+1\right), & \mathrm{t} \leq x \leq \mathrm{u}; \\ \psi_{\tilde{\rho}^{\mathrm{PN}}}\left(\frac{1}{2}\frac{1}{(\mathrm{v-u})^2}(x-\mathrm{v})^2\right), & \mathrm{u} \leq x \leq \mathrm{v}; \\ 0, & x > \mathrm{v}. \end{cases}$$

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

Where $\zeta_{\tilde{\rho}^{PN}}, \sigma_{\tilde{\rho}^{PN}}$, and$\psi_{\tilde{\rho}^{PN}}$ symbolise the max-truth, min-hesitant, and min-falsity membership degrees, respectively. SVPFN, $\tilde{\rho}^{PN} = \langle(r, s, t, u, v);\ \zeta_{\tilde{\rho}}, \sigma_{\tilde{\rho}}, \psi_{\tilde{\rho}}\ \rangle$ may described in an ill-defined quantity about ρ, that is approximately equal to $[s, u]$.

Definition 4: Suppose, $\tilde{\rho}^{PN} = \langle(r, s, t, u, v);\ \zeta_{\tilde{\rho}}, \sigma_{\tilde{\rho}}, \psi_{\tilde{\rho}}\ \rangle$, and $\tilde{q}^{PN} = \langle(r', s', t', u', v');\ \zeta_{\tilde{q}}, \sigma_{\tilde{q}}, \psi_{\tilde{q}}\ \rangle$ be two pentagonal neutrosophic numbers. Then we have

1. $\tilde{\rho}^{PN} \oplus \tilde{q}^{PN} = \langle(r + r', s + s', t + t', u + u', v + v');\ \zeta_{\tilde{\rho}^{PN}} \wedge \zeta_{\tilde{q}^{PN}}, \sigma_{\tilde{\rho}^{PN}} \vee\ \sigma_{\tilde{q}^{PN}}, \psi_{\tilde{\rho}^{PN}} \vee \psi_{\tilde{q}^{PN}}\rangle$,
2. $\tilde{\rho}^{PN} \ominus \tilde{q}^{PN} = \langle(r - v', s - u', t - t', u - s', v - r');\ \zeta_{\tilde{\rho}^{PN}} \wedge \zeta_{\tilde{q}^{PN}}, \sigma_{\tilde{\rho}^{PN}} \vee\ \sigma_{\tilde{q}^{PN}}, \psi_{\tilde{\rho}^{PN}} \vee \psi_{\tilde{q}^{PN}}\rangle$,
3. $\tilde{\rho}^{PN} \otimes \tilde{q}^{PN} = \langle(rr', ss', tt', uu', vv');\ \zeta_{\tilde{\rho}^{PN}}\zeta_{\tilde{q}^{PN}}, \sigma_{\tilde{\rho}^{PN}} +\ \sigma_{\tilde{q}^{PN}} - \sigma_{\tilde{\rho}^{PN}}\sigma_{\tilde{q}^{PN}}, \psi_{\tilde{\rho}^{PN}}+\psi_{\tilde{q}^{PN}} - \psi_{\tilde{\rho}^{PN}}\psi_{\tilde{q}^{PN}}\rangle$,
4. $m\ \tilde{\rho}^{PN} = \begin{cases} \langle(mr, ms, mt, mu, mv);\ \zeta_{\tilde{\rho}^{PN}}, \sigma_{\tilde{\rho}^{PN}}, \psi_{\tilde{\rho}^{PN}}\ \rangle, m > 0, \\ \langle(mv, mu, mt, ms, mr);\ \zeta_{\tilde{\rho}^{PN}}, \sigma_{\tilde{\rho}^{PN}}, \psi_{\tilde{\rho}^{PN}}\ \rangle, m < 0, \end{cases}$
5. $\tilde{\rho}^{PN^{-1}} = \langle\left(\frac{1}{v}, \frac{1}{u}, \frac{1}{t}, \frac{1}{s}, \frac{1}{r}\right);\ \zeta_{\tilde{\rho}^{PN}}, \sigma_{\tilde{\rho}^{PN}}, \psi_{\tilde{\rho}^{PN}}\ \rangle, \tilde{\rho}^{PN} \neq 0.$

***Definition 5*: (Accuracy and Score function**). Let $\tilde{\rho}^{PN} = \langle(r, s, t, u, v);\ \zeta_{\tilde{\rho}^{PN}}, \sigma_{\tilde{\rho}^{PN}}, \psi_{\tilde{\rho}^{PN}}\ \rangle$ be a SVPFN number, then

1. Accuracy $AC(\tilde{p}^{PN}) = \left(\frac{1}{15}\right)(r + s + t + u + v) * [2 + \zeta_{\tilde{\rho}^{PN}} - \sigma_{\tilde{\rho}^{PN}}]$.
2. Score $SC(\tilde{p}^{PN}) = \left(\frac{1}{15}\right)(r + s + t + u + v) * \left[2 + \zeta_{\tilde{\rho}^{PN}} - \sigma_{\tilde{\rho}^{PN}} - \psi_{\tilde{\rho}^{PN}}\right]$.

Definition 6: [42]. The order relations between $\tilde{\rho}^{PN}$and $\tilde{q}^{PN}$based on $SC(\tilde{\rho}^{PN})$ and $AC(\tilde{q}^{PN})$ are defined as:

1. If $SC(\tilde{\rho}^{PN}) < SC(\tilde{q}^{PN})$, then $\tilde{\rho}^{PN} < \tilde{q}^{PN}$,
2. If $SC(\tilde{\rho}^{PN}) = SC(\tilde{q}^{PN})$, then $\tilde{\rho}^{PN} = \tilde{q}^{PN}$,
3. If $AC(\tilde{\rho}^{PN}) < AC(\tilde{q}^{PN})$, then $\tilde{\rho}^{PN} < \tilde{q}^{PN}$,
4. If $AC(\tilde{\rho}^{PN}) > AC(\tilde{q}^{PN})$, then $\tilde{\rho}^{PN} > \tilde{q}^{PN}$,
5. If $AC(\tilde{\rho}^{PN}) = AC(\tilde{q}^{PN})$, then $\tilde{\rho}^{PN} = \tilde{q}^{PN}$.

3.Problem statement and solution concepts

A bi- level linear chance constrained programming problem with a joint probability constraints and trapezoidal neutrosophic numbers in the objective functions coefficients can be formulated as follows

$$\max_{x_1} Z_1(x) = \tilde{c}_{11}^{N}x_1 + \tilde{c}_{12}^{N}x_2 \tag{1}$$

Where x_2 solves

$$\max_{x_2} Z_2(x) = \tilde{c}_{21}^{N}x_1 + \tilde{c}_{22}^{N}x_2 \tag{2}$$

Subject to

$$Pr[A_{i1}x_1 + A_{i2}x_2 \leq b_i] \geq 1 - \alpha, i = 1, 2, \dots, m; \tag{3}$$

$$x_1, x_2 \geq 0. \tag{4}$$

Where $Z = (Z_1, Z_2)$, $\tilde{C}^{\ N}$ is the single valued trapezoidal neutrosophic cost coefficient matrix, x is the decision vector, A is the coefficient matrix, and $b_i's$ are independent normal random variables with known means and variances. Let n be the total number of decision variables in the system, and m be the total number of constraints of the problem. Let us define:

$x_1 = \{x_1^1, x_1^2, \dots, x_1^{n_1}\}$: Decision variables under the control of the center,

$x_2 = \{x_2^1, x_2^2, \dots, x_2^{n_2}\}$: Decision variables under the control of division1.

Let $x = (x_1, x_2)$, and $n = n_1 + n_2$. Inequality (3) is a joint probabilistic constraint and

$0 < \alpha < 1$ is specified probability. Here, we assume that the decision variables are deterministic.

Now, let the mean and standard deviation of $b_i's$ denoted by $E(b_i)$ and $\sigma(b_i)$, respectively. Then inequality (3) can be rewritten as follows:

$$\prod_{i=1}^{m} Pr[A_{i1}x_1 + A_{i2}x_2 \leq b_i] \geq 1 - \alpha \tag{5}$$

Inequality (5) can be simplified as

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

$$\prod_{i=1}^{m} \Pr\left[\frac{\mathrm{b_i}-\mathrm{E}(\mathrm{b_i})}{\sigma(\mathrm{b_i})} \geq \frac{\mathrm{A_{i1}x_1+A_{i2}x_2}-\mathrm{E}(\mathrm{b_i})}{\sigma(\mathrm{b_i})}\right] \geq 1-\alpha \tag{6}$$

Where $\frac{\mathrm{b_i}-\mathrm{E}(\mathrm{b_i})}{\sigma(\mathrm{b_i})}, i = 1, 2, \dots, m$ is standard normal variable with $\mathrm{E}(\mathrm{b_i}) = 0$ and $\sigma^2(\mathrm{b_i}) = 1$.

Therefore,

$$\prod_{i=1}^{m} \phi\left[\frac{\mathrm{A_{i1}x_1+A_{i2}x_2}-\mathrm{E}(\mathrm{b_i})}{\sigma(\mathrm{b_i})}\right] \geq 1-\alpha \tag{7}$$

Where $\phi(.)$ characterized the cumulative distribution function of the standard normal variables. Let

$$\varphi = \frac{\mathrm{A_{i1}x_1+A_{i2}x_2}-\mathrm{E}(\mathrm{b_i})}{\sigma(\mathrm{b_i})}, i = 1, 2, \dots, m \tag{8}$$

And,

$$\phi(\varphi_i) = u_i, i = 1, 2, \dots, m \tag{9}$$

Then,

$$\prod_{i=1}^{m} u_i \geq 1-\alpha. \tag{10}$$

It is known that

$$\phi(\varphi_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\varphi_i} e^{\left(-\frac{z^2}{2}\right)dz} \tag{11}$$

Let us assume that $\frac{z^2}{2} = v$, hence (11) can be simplified as

$$\phi(\varphi_i) = \frac{1}{2\sqrt{\pi}} \int_{\frac{\varphi_i^2}{2}}^{\infty} e^{(-v)v^{(v^2-1)}dv} \tag{12}$$

$$(13) = \frac{1}{2\sqrt{\pi}}\left[\int_0^{\infty} e^{(-v)v^{(v^2-1)}dv} - \int_0^{\varphi_i^2/2} e^{(-v)v^{(v^2-1)}dv}\right]$$

$$= \frac{1}{2}\left(\frac{1}{\Gamma\left(\frac{1}{2}\right)}\gamma\left(\frac{1}{2}, \frac{\varphi_i^2}{2}\right) - 1\right)$$

$$= \frac{1}{2}\left(P\left(\frac{1}{2}, \frac{\varphi_i^2}{2}\right) - 1\right), \tag{14}$$

$$P(a, y) = \frac{\gamma(a,y)}{\Gamma(a)}$$

$$(15) \qquad = \frac{1}{\Gamma(a)}\int_0^{y} e^{(-v)v^{(a-1)}dv}, a > 0$$

Equation (15) is an Incomplete Gamma Function (Sinha et al., [46]), where

$$\gamma(a, y) = e^{(-y)y^a} \sum_{s=0}^{\infty} \frac{\Gamma(a)}{\Gamma(a+1+s)} y^s \tag{16}$$

Referring to equation (9), we have

$$e^{\left(\frac{\varphi_i^2}{2}\right)} \sum_{s=0}^{\infty} \frac{\varphi_i^{(2s-1)}}{2^s\Gamma\left(\frac{3}{2}+s\right)} = \sqrt{2}(2u_i+1) \tag{17}$$

Simply, equation (17) can rewritten as

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

$$\sum_{s=0}^{\infty} \frac{\varphi_i^{(2s-1)}}{\prod_{m=0}^{s}(2m+1)} = \sqrt{\frac{\pi}{2}}\ (2u_i+1)e^{\varphi_i^2/2} \tag{18}$$

Clearly, the series $\sum_{s=0}^{\infty} \frac{\varphi_i^{(2s-1)}}{\prod_{m=0}^{s}(2m+1)}$ is convergent for any φ_i . So, it can be expanded as follows

$$\sum_{s=0}^{\infty} \frac{\delta_i^{(2s-1)}}{\prod_{m=0}^{s}(2m+1)} = \varphi_i\left(1+\frac{1}{3}\varphi_i^2+\frac{1}{15}\varphi_i^4+\cdots\right)$$

$$\leq \varphi_i\left(1+\frac{1}{3}\varphi_i^2+\frac{1}{15}\varphi_i^4+\cdots\right)$$

$$= \varphi_i\left(\frac{3}{3-\varphi_i^2}\right), \varphi_i^2 < 3. \tag{19}$$

Equation (19) can be simplified as

$$\frac{3}{3-\varphi_i^2}\left(-\frac{\varphi_i^2}{2}\right) \leq \sqrt{\frac{\pi}{2}}\ (2u_i+1) \tag{20}$$

Thus problem (1)- (4) can be converted into the following deterministic problem as

$$\max_{x_1} Z_1(x) = c_{11}x_1 + c_{12}x_2 \tag{21}$$

Where x_2 solves

$$\max_{x_2} Z_2(x) = c_{21}x_1 + c_{22}x_2 \tag{22}$$

Subject to

$$\frac{3}{3-\varphi_i^2}\left(-\frac{\varphi_i^2}{2}\right) \leq \sqrt{\frac{\pi}{2}}\ (2u_i+1), i = 1,2,\dots,m; \tag{23}$$

$$\prod_{i=1}^{m} u_i \geq 1-\alpha; \tag{24}$$

$$A_{i1}x_1 + A_{i2}x_2 - \varphi_i\sigma(b_i) = E(b_i), i = 1,2,\dots,m; \tag{25}$$

$$0 \leq u_i \leq 1, i = 1,2,\dots,m; \tag{26}$$

$$x_1, x_2 \geq 0; \varphi_i, i = 1,2,\dots,m, \text{ are unrestricted in sign} \tag{27}$$

4. Fuzzy compromise approach to bi- level programming

In this section, a fuzzy compromise approach is applied to solve the stochastic bi- level linear programming problem under neutrosophic environment as:

The decision maker firstly solves the upper level problem

$$\max_{x_1} Z_1(x) = c_{11}x_1 + c_{12}x_2$$

Subject to (28)

The constraints (23)- (27).

Let $\left(x_1^U, x_2^U, \varphi_i^U, u_i^U; Z_1^U\right)$ be the solution for the upper problem where $Z_1^U = Z_1\left(x_1^U, x_2^U, \varphi_i^U, u_i^U\right)$.

. Then, the decision maker solves the lower level problem

$$\max_{x_2} Z_2(x) = c_{21}x_1 + c_{22}x_2$$

Subject to (29)

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

The constraints (23)- (27).

Let $\left(x_1^L, x_2^L, \varphi_i^L, u_i^L;\ Z_1^L\ \right)$ be the solution for the upper problem where $Z_2^L = Z_2(x_1^L, x_2^L, \varphi_i^L, u_i^L)$.

The decision maker (DM) at the upper level gives some tolerance to the decision vector x_1^U which enables the DM at lower level to search for his/ her optimum in a wider feasible region. Also, the DM gives some tolerance to the objective function Z_1 to direct the DM at the lower level to search for his/ her solutions in the right direction. Let $\pm q_1$ be the tolerance on x_1^U, and the membership function (Zimmermann, [36]) for x_1 is defined as

$$\mu_{x_1}(x_1) = \begin{cases} \frac{(x_1-(x_1^U-q_1))}{q_1}, (x_1^U - q_1) \le x_1 \le x_1^U; \\ \frac{((x_1^U+q_1)-x_1)}{q_1}, x_1^U \le x_1 \le x_1^U + q_1 \\ 0, \text{Otherwise} \end{cases} \tag{30}$$

Assume the membership function (Zimmermann, [36]) for the objective function Z_1 is

$$\mu_{Z_1}(Z_1(x)) = \begin{cases} 0,\ Z_1(x) < Z_1^\circ; \\ \frac{Z_1(x)-Z_1^\circ}{Z_1^U - Z_1^\circ}, Z_1^\circ \le Z_1(x) \le Z_1^U; \\ 1,\ Z_1(x) > Z_1^U. \end{cases} \tag{31}$$

The above information introduced in equations (30) and (31) is sent to the DM at lower level as additional constraints. Hence, the lower level problem becomes

$$\max_{x_2} Z_2(x) = c_{21}x_1 + c_{22}x_2$$

Subject to (32)

The constraints (23)- (27),

$\mu_{x_1}(x_1) \ge \vartheta I;\ \mu_{Z_1}\left(Z_1(x)\right) \ge \theta; \vartheta, \theta \in [0,1].$

Where, ϑ and θ are the minimum acceptable degree of satisfaction for the decision variable x_1 and the objective functionZ_1, respectively. I is the unit column vector which has the same dimension as x_1. In order to rate the satisfaction of each of his/ her potential solution, the DM at lower level establishes the membership function for the objective function Z_2 as

$$\mu_{Z_2}(Z_2(x)) = \begin{cases} 0,\ Z_2(x) < Z_2^\circ; \\ \frac{Z_2(x)-Z_2^\circ}{Z_2^L - Z_2^\circ}, Z_2^\circ \le Z_2(x) \le Z_2^L; \\ 1,\ Z_2(x) > Z_2^L. \end{cases} \tag{33}$$

The auxiliary model for the DM at the lower level is

$$\max \omega = \mu_{Z_2}\left(Z_2(x)\right) = \frac{Z_2(x)-Z_2^\circ}{Z_2^L - Z_2^\circ}$$

Subject to

The constraints (23)- (27),

$$\mu_{x_1}(x_1) \ge \vartheta I;\ \mu_{Z_1}\left(Z_1(x)\right) \ge \theta; \vartheta, \theta, \omega \in [0,1]. \tag{34}$$

Where ω is the minimum acceptable degree of satisfaction for objective functionZ_2. To resolve the conflict between the two DMs and also for avoiding the solution rejection by the DM at the upper level, The DM at lower level try to maximize all of $\vartheta, \theta, \omega$ simultaneously. Let us assume that $\chi = \min(\vartheta, \theta, \omega)$. Then a linear programming for the lower level problem (32) can be stated as

$\max \chi$

Subject to

$\frac{3}{3-\varphi_i^2}\left(-\frac{\varphi_i^2}{2}\right) \le \sqrt{\frac{\pi}{2}}\ (2u_i + 1), i = 1, 2, \dots, m;$

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

$\prod_{i=1}^{m} u_i \geq 1-\alpha;$

$A_{i1}x_1 + A_{i2}x_2 - \varphi_i\sigma(b_i) = E(b_i), i = 1, 2, \dots, m;$

$0 \leq u_i \leq 1, i = 1, 2, \dots, m;$

$$\frac{\left(x_1-(x_1^U-q_1)\right)}{q_1} \geq \chi I; \tag{35}$$

$\frac{\left((x_1^U+q_1)-x_1\right)}{q_1} \geq \chi I;$

$\mu_{Z_1}\left(Z_1(x)\right) = \frac{Z_1(x)-Z_1^{\circ}}{Z_1^U - Z_1^{\circ}} \geq \chi;$

$\mu_{Z_2}\left(Z_2(x)\right) = \frac{Z_2(x)-Z_2^{\circ}}{Z_2^L - Z_2^{\circ}} \geq \chi;$

$x_1, x_2 \geq 0; \varphi_i, i = 1, 2, \dots, m,$ are unrestricted in sign; $\chi \in [0,1]$.

Problem (35) is a fuzzy linear programming problem, which can be solved by the DM at lower level.

5. Numerical example

$$\max_{x_1} Z_1(x) = \tilde{c}_{11}^N x_1 + \tilde{c}_{12}^N x_2 \tag{36}$$

Where x_2 solves

$$\max_{x_2} Z_2(x) = \tilde{c}_{21}^N x_1 + \tilde{c}_{22}^N x_2 \tag{37}$$

Subject to

$$\Pr\begin{pmatrix} 3x_1 - 5x_2 \leq b_1; 3x_1 - x_2 \leq b_2; \\ 3x_1 + x_2 \leq b_3;\ 3x_1 + 4x_2 \leq b_4; \\ x_1 + 3x_2 \leq b_5 \end{pmatrix} \geq 0.85; \tag{38}$$

$$x_1, x_2 \geq 0. \tag{39}$$

The mean and standard deviation of b_1, b_2, b_3, b_4, and b_5 are given as

$E\ (b_1) = 15, \sigma(b_1) = 8;\ \ E(b_2) = 21,\ \sigma(b_2) = 11; E(b_3) = 27, \sigma(b_3) = 14;$

$E(b_4) = 45, \sigma(b_4) = 23;\ E(b_5) = 30, \sigma(b_5) = 15;\ \tilde{c}_{11}^N = \langle(3, 5, 6, 8); 0.6, 0.5, 0.4\rangle,$
$\tilde{c}_{12}^N = \langle(0, 1, 3, 6); 0.7, 0.5, 0.3\rangle = \tilde{c}_{21}^N, \tilde{c}_{22}^N = \langle(1, 3, 4, 6); 0.6, 0.3, 0.5\rangle$

The equivalent deterministic problem corresponding to (32)- (35) is

$$\max_{x_1} Z_1(x) = 2x_1 - x_2 \tag{40}$$

Where x_2 solves

$$\max_{x_2} Z_2(x) = x_1 + 2x_2 \tag{41}$$

Subject to

$$x_1 - 5x_2 - 8\varphi_1 = 15; \tag{42}$$

$$3x_1 - x_2 - 11\varphi_2 = 21; \tag{43}$$

$$3x_1 + x_2 - 14\varphi_3 = 27; \tag{44}$$

$$3x_1 + 4x_2 - 23\varphi_4 = 45; \tag{45}$$

$$x_1 + 3x_2 - 15\varphi_5 = 30; \tag{46}$$

$$3\varphi_1 e^{(-\frac{\varphi_1^2}{2})} \leq 1.2533141(1-2u_1)(3-\varphi_1^2); \tag{47}$$

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

$$3\varphi_2 e^{(-\frac{\varphi_2^2}{2})} \leq 1.2533141(1-2u_2)(3-\varphi_2^2); \tag{48}$$

$$3\varphi_3 e^{(-\frac{\varphi_3^2}{2})} \leq 1.2533141(1-2u_3)(3-\varphi_3^2); \tag{49}$$

$$3\varphi_4 e^{(-\frac{\varphi_4^2}{2})} \leq 1.2533141(1-2u_4)(3-\varphi_4^2); \tag{50}$$

$$3\varphi_5 e^{(-\frac{\varphi_5^2}{2})} \leq 1.2533141(1-2u_5)(3-\varphi_5^2); \tag{51}$$

$$u_1u_2u_3u_4u_5 \geq 0.85; \tag{52}$$

$$u_1,\ u_2,\ u_3, u_4, u_5 \leq 1; \tag{53}$$

$$x_1, x_2, u_1,\ u_2,\ u_3, u_4, u_5 \geq 0; \tag{54}$$

$$\varphi_1, \varphi_2, \varphi_3, \varphi_4 \text{ and } \varphi_5 \text{ are unrestricted in sign} \tag{55}$$

The solution of Z_1 is

$$x = \begin{pmatrix} 14.28260 \\ 0 \end{pmatrix}, \varphi = \begin{pmatrix} 1.234568 \\ 1.986165 \\ 1.131986 \\ 1.234568 \\ 1.234568 \end{pmatrix}, u = \begin{pmatrix} 1 \\ 0.85 \\ 1 \\ 1 \\ 1 \end{pmatrix}, Z_1 = 28.56521.$$

The solution of Z_2 is

$$x = \begin{pmatrix} 5.778806 \\ 18.00455 \end{pmatrix}, \varphi = \begin{pmatrix} 1.234568 \\ 1.234568 \\ 1.234568 \\ 1.928462 \\ 1.986165 \end{pmatrix}, u = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0.85 \end{pmatrix}, Z_2 = 41.78791.$$

Since the two solutions are not equivalent, then there is no satisfactory solution. We find that the upper- bound $Z_1^U = 28.56521$, and lower- bound $Z_1^{\circ} = -6.446938$. Since Z_1° is negative in sign, we set $Z_1^{\circ} = 0$. Also, the upper- bound $Z_2^U = 41.78791$, and lower- bound $Z_2^{\circ} = 14.28260$. Let the DM at upper level specifies x_1 to be around 14.28260 with 8.503794 (negative) and 0.5 tolerance. The membership functions for the decision variable x_1 (i.e., $\mu_{x_1}(x_1)$) and objective function Z_1 (i.e., $\mu_{Z_1}(Z_1(x)$) of the DM at upper level are calculated using equations (31) and (31)m respectively. Next, the membership function for the objective function Z_2 (i.e., $\mu_{Z_2}(Z_2(x))$) of the DM at the lower level is formulated by applying the equation (33). Then, the auxiliary model for the lower- level DM is formulated as

$$\max \chi$$

Subject to

$$x_1 - 5x_2 - 8\varphi_1 = 15;$$

$$3x_1 - x_2 - 11\varphi_2 = 21;$$

$$3x_1 + x_2 - 14\varphi_3 = 27;$$

$$3x_1 + 4x_2 - 23\varphi_4 = 45;$$

$$x_1 + 3x_2 - 15\varphi_5 = 30;$$

$$3\varphi_1 e^{(-\frac{\varphi_1^2}{2})} \leq 1.2533141(1-2u_1)(3-\varphi_1^2);$$

$$3\varphi_2 e^{(-\frac{\varphi_2^2}{2})} \leq 1.2533141(1-2u_2)(3-\varphi_2^2);$$

$$3\varphi_3 e^{(-\frac{\varphi_3^2}{2})} \leq 1.2533141(1-2u_3)(3-\varphi_3^2);$$

$$3\varphi_4 e^{(-\frac{\varphi_4^2}{2})} \leq 1.2533141(1-2u_4)(3-\varphi_4^2);$$

$$3\varphi_5 e^{(-\frac{\varphi_5^2}{2})} \leq 1.2533141(1-2u_5)(3-\varphi_5^2);$$

Doi: https://doi.org/10.54216/IJNS.230120
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$\frac{x_1-5.778806}{8.503794} \geq \chi;$

$\frac{14.7826-x_1}{0.5} \geq \chi;$

$\frac{2x_1-x_2}{28.56521} \geq \chi;$

$\frac{x_1+2x_2-14.28260}{14.28260} \geq \chi;$

$u_1u_2u_3u_4u_5 \geq 0.85;$

$u_1,\ u_2,u_3,u_4,u_5 \leq 1;$

$x_1,x_2,u_1,\ u_2,u_3,u_4,u_5 \geq 0;$

$\varphi_1,\varphi_2,\varphi_3,\varphi_4$ and φ_5 are unrestricted in sign; $\chi \in [0,1]$

The satisfactory solution is

$$x = \begin{pmatrix} 14.33240 \\ 2.94464 \end{pmatrix}, \varphi = \begin{pmatrix} 0.00000 \\ 1.73205 \\ 1.35299 \\ 1.000000 \\ 1.000000 \end{pmatrix}, u = \begin{pmatrix} 0.97000 \\ 0.97000 \\ 0.97000 \\ 0.97000 \\ 0.97000 \end{pmatrix}, Z = \begin{pmatrix} 25.72016 \\ 20.22168 \end{pmatrix}, and$$

$\chi = 0.90040162.$

The satisfaction degree is

$$\mu_{x_1}(x_1) = \frac{\left(x_1 - (x_1^U - q_1)\right)}{q_1} = 0.95, \mu_{x_1}(x_1) = \frac{\left((x_1^U + q_1) - x_1\right)}{q_1} = 1.5 > 1,$$

$$\mu_{Z_1}(Z_1(x) = \frac{Z_1(x)-Z_1^{\circ}}{Z_1^U - Z_1^{\circ}} = 0.90040, \mu_{Z_2}(Z_2(x) = \frac{Z_2(x)-Z_2^{\circ}}{Z_2^L - Z_2^{\circ}} = 0.90040.$$

6. Conclusion and future works

In this paper, bi- level linear programming problem with the right- hand side of the constraints randomly distributed has introduced. The purpose of this paper is to convert the stochastic bi- level linear programming based on the incomplete Gamma function into the corresponding deterministic model whose constraints presented in an infinite series, this series is not suitably for the numerical problem computation. Also, these series became convergent by means of a geometric series. And then the constraints turned to deterministic form. After obtaining a deterministic problem, fuzzy programming approach has applied for obtaining optimal compromise solution. Finally, an example is introduced to clarify the efficiency of the proposed approach and compare the results obtained by one of the most prominent evolutionary algorithms, the genetic algorithm (GA), to validate the accuracy and reliability of simulation results. In addition, the methodology has illustrated through a numerical example. Future work may include the further extension of this study to other fuzzy-like structures (i. e. interval-valued fuzzy set, Neutrosophic set, Pythagorean fuzzy set, Spherical fuzzy set, etc. with more discussion and suggestive comments.

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Acknowledgments: This work was supported by the Deanship of Scientific Research, Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia [Grant No. GRANT5246].

Funding: This support was instrumental in the realization of our research. This work was supported by the Deanship of Scientific Research, Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia [Grant No. GRANT5246].

References

[1] Dempe, S. Foundations of bilevel Programming, Kluwer Academic, Dordrecht, Netherlands, 2002.
[2] Dempe, S. (2003). Annotated bibliography on bilevel programming and mathematical programs with equilibrium constraints. Optimization, 52(3): 333- 359.

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

[3] Colson, B., Marcotte, P., and Savard, G. (2007). An overview of bilevel optimization. Annals of Operations Research 153(1): 235- 256.

[4] Gzara, F. (2013). A cutting plane approach for bilevel hazardous material transport network design. Operations Research Letters, 41(1): 40-46.

[5] Fontaine, P., and Minner, S. (2014). Benders decomposition for discrete- continuous linear bilevel problems with application to traffic network design. Transportation Research Part B: Methodological,(70): 163- 172.

[6] Cecchini, M., Ecker, J., Kupferschmid, M., and Leitch, R. (2013). Solving nonlinear principle- agent problems using bilevel programming. European Journal of Operational Research, 230(2): 364- 373.

[7] Zhang, G., Zhang, G., Gao, Y., and Lu, J. (2011). Competitive strategic bidding optimization in electricity markets using bilevel programming and swarm technique. IEEE Transactions on Industrial Electronics, 58(6): 2138-2146.

[8] Bialas, W.F., and Karwan, M. H. (1982). On two- level optimization, IEEE Transaction Automatic Control, AC-27, 211-214.

[9] Bialas, W.F., and Karwan, M. H. (1982). Two- level linear programming. Management Science, (30): 1004- 1020.

[10] Bard, J. F., and Falk, J.E. (1982). An explicit solution to the multi- level programming problem. Computers Operations Research, (9): 77- 100.Applications, Kluwer Academic Publisers, Dordrecht, The Netherlands, 1998.

[11] Anandalingam, G. (1988). A mathematical programming model of decentralized multi- level systems. Journal of Operational Research Society, 39(11): 1021- 1033.

[12] Shih, H- S., Lai, Y- J., and Lee, E. S.(1996). Fuzzy approach for multi- level programming problems. Computers Operations Research, 23(1): 73- 91.

[13] Ye, J. J., and Zhu, D. L. (1995). Optimality conditions for bi- level linear programming problems. Optimization, 33(1): 9- 27.

[14] Chen, Y., and Florian, M. (1995). The non- linear bi- level programming problem: Formulations, regularity and optimality conditions, Optimization, 32(3): 193- 209.

[15] Falk, J. E., and Liu, J. (1995). On bi- level programming I: General nonlinear cases. Mathematical Programming, 70(1): Ser A, 47- 72.

[16] Gendreau, M. Marcotte, P., and Savard, G. (1996). A hybrid Tabu- Ascent algorithm for the linear bilevel programming problem. Journal of Operational Research Society, (32): 783- 792.

[17] Dempe, S.., and Schemidt, H. (1996). On an algorithm solving 2- level programming problems with non- unique lower level solutions. Computers& Optimization Applications, 6(3): 227- 249.

[18] Safaei, N., and Saraj, M. (2014). A new method for solving fully fuzzy linear bi-level programming problems. International Journal of Applied Operational Research, 4(1): 39- 46.

[19] Ren, A. (2015). A novel method for solving the fully fuzzy bi-level linear programming problem. Mathematical Problems in Engineering, vol.2015, Article ID, 11 pages.

[20] Ren, A. (2016). Solving the fully fuzzy bilevel programming problem through deviations degree measures and a ranking function method. Mathematical Problems in Engineering, vol. 2016, Article ID 7069804, II pages. Doi.org/10.1155/2016/7069804.

[21] Hammad, A. W.A. (2019). A bilevel multiobjective optimization approach for solving the evacuation location assignment problem. Advanced in Civil Engineering, vol.2019, Article ID 6052931, 11 pages. Doi.org/10.1155/2019/6052931.

[22] Tahernejad, S., Ralphs, T.K., and DeNeger, S.T. (2020). A branch- and – cut algorithm for integer bilevel linear optimization problems and its implementation. Mathematical Programming Computation, doi.org/10.1007/s12532-020-00183-6.

[23] Rey, D. (2020). Computational benchmarking of exact method for the bilevel discrete network design problem. Transportation Research Procedia, (47): 11-18. Doi.org/10.1016/j.trpro.2020.03.67.

[24] Sengupta, J. K. Stochastic Programming: Methods and Applications, North- Holland Publishing Company, 1972.

[25] Goicoechea, A., Don, R. Hansen and Lucien Duckstein. Multiobjective Decision Analysis with Engineering and Business Applications, John Wiley& Sons, 1982.

[26] Leberling, H. (1981). On finding compromise solution in multi- criteria problems using the min- operator. Fuzzy Sets and Systems, (10): 33-41

[27] Teghem, J. Jr., Dufrance, D., Thauvoye, M., and Kunch, P. (1986). An interactive method for multiobjective linear programming under uncertainty. European Journal of Operations Research, (26): 65- 82.

[28] Sinha, S. B., Biswal, M.P., and Hulsurkar, S. (1998). Fuzzy programming to the multi- objective probabilistic linear programming problems when only $b_i's$ are probabilistic. The Journal of Fuzzy Mathematics, 6(1): 63- 73.

[29] Zadeh, L. A. (1965). Fuzzy sets. *Information Control*, 8(3): 338-353.

[30] Dubois, D., and Prade, H. Fuzzy sets and systems: theory and applications, Academic Press, New York, 1980.

[31] Bellman, R. E., and Zadeh, L.A. (1970). Decision making in a fuzzy environment. Management Science, (17): 141- 164.

[32] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and System*, 1(1): 45–55.

[33] Narasiman, R. (1980). Goal programming in a fuzzy environment. Decision Sciences, (11): 325- 336.

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023

[34] A. Al Quran, A. G. Ahmad, F. Al-Sharqi, A. Lutfi, Q-Complex Neutrosophic Set, International Journal of Neutrosophic Science, vol. 20(2), pp.08-19, 2023.
[35] F. Al-Sharqi, M. U. Romdhini, A. Al-Quran, Group decision-making based on aggregation operator and score function of Q-neutrosophic soft matrix, Journal of Intelligent and Fuzzy Systems, vol. 45, pp.305–321, 2023.
[36] Al-Quran, A., Al-Sharqi, F., Ullah, K., Romdhini, M. U., Balti, M. & Alomai, M., Bipolar fuzzy hypersoft set and its application in decision making. International Journal of Neutrosophic Science, 20, 65-77. 2023
[37] F. Al-Sharqi, A. Al-Quran, M. U. Romdhini, Decision-making techniques based on similarity measures of possibility interval fuzzy soft environment, Iraqi Journal for Computer Science and Mathematics, vol. 4, pp.18–29, 2023.
[38] Zimmermann, H-J. Fuzzy Set Theory and its Applications, Kluwer- Nijhoff Publishing, USA, 1985.
[39] Sinha, S., and Biswal, M.P. (2000). Fuzzy programming approach to bi- level linear programming problems. The Journal of Fuzzy Mathematics, 8(2): 337- 347.
[40] F. Al-Sharqi, Y. Al-Qudah and N. Alotaibi, Decision-making techniques based on similarity measures of possibility neutrosophic soft expert sets. Neutrosophic Sets and Systems, 55(1) (2023), 358-382.
[41] Camacho- Vallejo, Corpus, C., and Villegas, J. (2024). Metaheuristics for bilevel optimization: A comprehensive review. Computers& Operations Research, (161): 106410.
[42] Atanassov, K. T. (1986). Instuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1): 87- 96.
[43] Smarandache, F. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Lgics, American Research Press, Reoboth, NM, USA, 1998.
[44] Wang, H., Smarandache, F., Zhang, Y. Q., and Sunderraman, R. (2010). Single valued neutrosophic sets. Multistructure, (4): 410-413.
[45] Wang, J., Nie, R., Zhang, H., and Chen, X. (2013). New operators on triangular intuitionistic fuzzy numbers and their applications in system fault analysis. Information Sciences, (251): 69- 95.
[46] Sinha, S.B., Hulsurkar, S., Biswal, M. P. (2000). Fuzzy programming approach to multi- objective stochastic problems when $b_i's$ follow joint normal distribution. Fuzzy Sets and Systems, (109): 91- 96.

Doi: https://doi.org/10.54216/IJNS.230120
Received: May 27, 2023 Revised: August 11, 2023 Accepted: November 26, 2023