Innovative Theoretical Approach: Bipolar Pythagorean Neutrosophic Sets (BPNSs) in Decision-Making

Noraini Ahmad*[1], Zahari Rodzi*[2,3], Faisal Al-Sharqi[4,5], Ashraf Al-Quran[6], Abdalwali Lutfi[7], Zanariah Mohd Yusof[8], Nor Aini Hassanuddin[9]

[1] Pusat Asasi UiTM, UiTM Cawangan Selangor, Kampus Dengkil, 43800 Selangor, Malaysia

[2] College of Computing, Informatics and Mathematics, UiTM Cawangan Negeri Sembilan, Kampus Seremban, 73000 Negeri Sembilan, Malaysia

[3] Fakulti Sains dan Teknologi, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor Malaysia

[4] Department of Mathematics, Faculty of Education for Pure Sciences, University Of Anbar, Ramadi, Anbar, Iraq

[5] College of Engineering, National University of Science and Technology, Dhi Qar, Iraq

[6] Department of Mathematics and Statistics, College of Science, King Faisal University, Al-Ahsa 31982, Saudi Arabia

[7] Department of Accounting, College of Business, King Faisal University, Al-Ahsa 31982, Saudi Arabia

[8,9] College of Computing, Informatics and Mathematics, UiTM Cawangan Terengganu, Kampus Kuala Terengganu, 21080 Terengganu, Malaysia

Emails: norainiahmad@uitm.edu.my; zahari@uitm.edu.my; faisal.ghazi@uoanbar.edu.iq; aalquran@kfu.edu.sa; aalkhassawneh@kfu.edu.sa; zanariah297@uitm.edu.my; norai548@uitm.edu.my

Abstract

In this paper, the concept of bipolar Pythagorean fuzzy neutrosophic sets (BPNSs) will be introduced which represents an innovative synthesis, incorporating bipolarity, Pythagorean fuzzy sets, and neutrosophic sets to provide a comprehensive solution to the multifaceted issues encountered in decision-making challenges. BPNS is proposed to enhance the expressiveness and flexibility of the model by incorporating both positive and negative preferences that can lead to more accurate and realistic modeling of complex decision scenarios. The essential idea of BPNSs is defined and some definitions for instance complement, intersection, and union are described. The basic operation, the derivation of its properties, and the numerical example of the method are integrated to execute the proposed model.

Keywords: Pythagorean fuzzy set; Neutrosophic set; Bipolar fuzzy set; Decision-making problem

1. Introduction

When making judgments, several factors, such as insufficient knowledge, complexity, volatility, and dynamic changes in the external environment, can contribute to the uncertain environment and the fuzziness of the information. Numerous theories have been established to address the issue of uncertainty and ambiguity in information due to the significance of that information and the quickly growing volume of uncertain data that has been gathered and amassed. The fuzzy set theory was first presented by Zadeh [1] and provides a mathematical framework for dealing with situations when category boundaries are not clear, and information is inaccurate. This is especially important for domains like linguistics, artificial intelligence, control systems, and decision-making. The

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generalization of fuzzy sets involves exploring various extensions and modifications of the traditional concept of fuzzy sets introduced by Zadeh. These generalizations expand the scope and applicability of fuzzy set theory by addressing more complex and nuanced aspects of uncertainty, imprecision, and vagueness. Examples of generalization of fuzzy sets include Type-2 Fuzzy Sets [2], Interval-Valued Fuzzy Sets (IVFS) [3], Generalized Fuzzy Sets [4], Rough Fuzzy Sets [5], Hybrid Fuzzy Sets [6] and Hesitant Fuzzy Set [7].

Atanassov [8] extends the concepts of fuzzy sets by introducing intuitionistic, fuzzy sets, IFS which is a generalization of fuzzy sets whose elements have a degree of membership and non-membership function. This method is widely spread use in numerous fields such as education, management, social science, medicine, and engineering because of the efficient instrument for dealing with information that is vague and imprecise [9,10,11,12]. The researcher's exploration is expanded upon with the introduction of IFS principles [13,14]. A new set, the neutrosophic fuzzy set, was created by Smarandache [15] to incorporate the trisected components of membership, non-membership, and indeterminacy degrees, offering a more sophisticated and subtle means of describing uncertainty. The goal of this expansion is to represent the intricacy of fuzzy, neutral, and uncertain real-world information. These components capture the truth, uncertainty, and falsehood associated with each element's membership in the set. IFS includes a degree of hesitation (θ), which represents neutrality or uncertainty about membership. NFS, on the other hand, includes a degree of indeterminacy (I), which accounts for uncertainty and ambiguity. The idea of neutrosophic with a vague set was investigated by Hashim [16] to offer an interval-based membership structure to manage ambiguous neutrosophic data.

In 2013, the Pythagorean fuzzy sets (PFSs) concepts were invented by Yager [17], which offer a unique method for modeling uncertainty and vagueness with a high level of accuracy, and precision unlike intuitionistic fuzzy sets (IFSs). Akram [18] suggested using the Pythagorean fuzzy rough CRITIC-REGIME (PFR-CRITIC-REGIME) technique to address the issue of an electric ferry's sustainable supply chain in public transportation. Ismail [19] merged the theory of the Pythagorean fuzzy set and neutrosophic fuzzy set to model complex systems where uncertainty, indeterminacy, and ambiguity coexist. Zahari et al [21- 23] employed some based SVNS techniques to solve real-life issues. Al-Sharqi et al. [24-27] introduced several algebraic contributions supported by algorithms that employed these algebraic structures in solving decision-making real-life problems. The bipolar fuzzy set which is a new variant of the fuzzy set has been explored recently. The bipolar fuzzy set theory gives a theoretical foundation for bipolar clustering, decision analysis, and coordination by fusing polarity and fuzziness into a single model. The bipolar judgmental uses double-sided or negative and positive aspects of human perception and cognition [28]. Deli [29], combined the theory of bipolar and neutrosophic sets by considering both positive/negative degrees from bipolar fuzzy sets and truth/indeterminacy/falsity degrees from neutrosophic fuzzy sets in the same modeling framework. The new idea from Mandal [30] which is the combination of bipolar fuzzy set and Pythagorean fuzzy set gives another direction to the decision-making phenomenon. Combining these two frameworks might involve integrating the positive and negative degrees from the bipolar fuzzy set with the membership and non-membership degrees from the Pythagorean fuzzy set.

Consequently, this paper proposed a new extension Bipolar Pythagorean Fuzzy Neutrosophic (BPNSs) which is a development of the bipolarity, neutrosophic logic, and Pythagorean fuzzy sets to enhance usefulness in the decision-making environment. These sets were introduced to address multi-attribute decision-making problems where decision-makers need to consider both the degree of desirability and the degree of undesirability of different alternatives with respect to multiple attributes. These generalizations of fuzzy sets expand the toolbox of mathematical tools available to model and analyse uncertainty in diverse fields, including artificial intelligence, decision analysis, control systems, and pattern recognition. The outline of this paper is arranged as follows. Some basic mathematical concepts to enhance the understanding of BPNSs are presented in section 2. The definitions of BPNSs and their operational properties are discussed in section 3. The numerical examples of BPNSs are shown in section 4. Finally, the last section contains the conclusion of the paper.

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2. Preliminaries

This section explains the fundamental ideas allied to bipolar neutrosophic sets, Bipolar Pythagorean fuzzy sets, and neutrosophic sets.

Definition 2.1. [16] Let z be a nonvoid set. BN represent bipolar neutrosophic set in z is defined as:

$$BN = \left\{\left\langle z, \chi_B^+(z), \gamma_B^+(z), \kappa_B^+(z), \chi_B^-(z), \gamma_B^-(z), \kappa_B^-(z)\right\rangle : z \in Z\right\}$$

Where $\chi_B^+(z), \gamma_B^+(z), \kappa_B^+(z) : Z \to [1,0]$ and $\chi_B^-(z), \gamma_B^-(z), \kappa_B^-(z) : Z \to [-1,0]$

Definition 2.2. [11] Let z be a nonvoid set. P represent Pythagorean fuzzy set in z is given by:

$$P = \left\{\left\langle z, \alpha_P(z), \beta_P(z) \mid z \in Z\right\rangle\right\}$$

Where $\alpha_P(z), \beta_P(z) : Z \to [0,1]$ denote the degree of membership and non-membership of the element $z \in Z$ respectively. Follow by the condition below:

$0 \le \alpha_P(z)^2 + \beta_P(z)^2 \le 1$ and the degree of indeterminacy, $I_P(z) = \sqrt{1 - \alpha_P(z)^2 - \beta_P(z)^2}$.

Definition 2.3. [17] Let be a nonvoid set. P_B represent a bipolar Pythagorean fuzzy set in z is given by:

$$P_B = \left\{\left\langle z, \alpha_P^+(z), \beta_P^+(z), \alpha_P^-(z), \beta_P^-(z)\right\rangle \mid z \in Z\right\}$$

Where $\alpha_P^+(z), \beta_P^+(z) : Z \to [0,1]$, $\alpha_P^-(z), \beta_P^-(z) : Z \to [-1,0]$ and $0 \le \alpha_P^+(z)^2 + \beta_P^+(z)^2 \le 1$ and $0 \le \alpha_P^-(z)^2 + \beta_P^-(z)^2 \le 1$.

Definition 2.4. [13] Let z be a nonvoid set. P_N represent Pythagorean neutrosophic set, in z is defined as:

$$P_N = \left\{\left\langle z, \alpha_P(z), I_P(z), \beta_P(z)\right\rangle \mid z \in Z\right\}$$

Where $\alpha_P(z)$ is the truth $I_P(z)$ is the indeterminacy and $\beta_P(z)$ is the false membership function and $z, \alpha_P(z), I_P(z), \beta_P(z) : Z \to [0,1]$.

3. Bipolar Pythagorean Neutrosophic Set (BPNS)

This section introduces the fundamental idea of the bipolar Pythagorean fuzzy neutrosophic (BPNS) with its formal definition and basic operations of union, intersection, and complement.

Definition 3.1. A bipolar Pythagorean neutrosophic set (BPNS) with components D in Z is defined as:

$$D = \left\{\left\langle z, \alpha_D^+(z), I_D^+(z), \beta_D^+(z), \alpha_D^-(z), I_D^-(z), \beta_D^-(z)\right\rangle : z \in Z\right\}$$

Where $\alpha_D^+(z), I_D^+(z), \beta_D^+(z) : Z \to [0,1]$ and $\alpha_D^-(z), I_D^-(z), \beta_D^-(z) : Z \to [-1,0]$. The condition for all z in Z are satisfied as shown below:

$$0 \le \alpha_D^+(z)^2 + \beta_D^+(z)^2 \le 1$$

$$0 \le \alpha_D^-(z)^2 + \beta_D^-(z)^2 \le 1$$

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$$0 \le \alpha_D^+(z)^2 + I_D^+(z)^2 + \beta_D^+(z)^2 \le 2$$

$$0 \le \alpha_D^-(z)^2 + I_D^-(z)^2 + \beta_D^-(z)^2 \le 2$$

Let $E = \left\{\left\langle z, \alpha_E^+(z), I_E^+(z), \beta_E^+(z), \alpha_E^-(z), I_E^-(z), \beta_E^-(z)\right\rangle : z \in Z\right\}$ Then, $D \subseteq E$ if and only if

$\alpha_D^+(z) \le \alpha_E^+(z), I_D^+(z) \le I_E^+(z), \beta_D^+(z) \le \beta_E^+(z)$ and

$\alpha_D^-(z) \ge \alpha_E^-(z), I_D^-(z) \ge I_E^-(z), \beta_D^-(z) \ge \beta_E^-(z)$ for all z in Z

Also, $D = E$ if and only if

$\alpha_D^+(z) = \alpha_E^+(z), I_D^+(z) = I_E^+(z), \beta_D^+(z) = \beta_E^+(z)$ and

$\alpha_D^-(z) = \alpha_E^-(z), I_D^-(z) = I_E^-(z), \beta_D^-(z) = \beta_E^-(z)$ for all z in Z

The complement of a BPNSs, D is

$$D^C = \left\{\left\langle z, \beta_D^+(z), I_D^+(z), \alpha_D^+(z), \beta_D^-(z), I_D^-(z), \alpha_D^-(z)\right\rangle\right\}$$

The union D or E are

$$D \cup E = \left\{\max(\alpha_D^+, \alpha_E^+), \min(I_D^+, I_E^+), \min(\beta_D^+, \beta_E^+), \min(\alpha_D^-, \alpha_E^-), \min(I_D^-, I_E^-), \max(\beta_D^-, \beta_E^-)\right\} \tag{1}$$

The intersection D and E are

$$D \cap E = \left\{\min(\alpha_D^+, \alpha_E^+), \max(I_D^+, I_E^+), \max(\beta_D^+, \beta_E^+), \max(\alpha_D^-, \alpha_E^-), \max(I_D^-, I_E^-), \min(\beta_D^-, \beta_E^-)\right\} \tag{2}$$

Then, the operations for BPNSs are defined as below:

Definition 3.2. Let $r = \left\langle \alpha_r^+, I_r^+, \beta_r^+ \alpha_r^-, I_r^-, \beta_r^- \right\rangle$ and $s = \left\langle \alpha_s^+, I_s^+, \beta_s^+, \alpha_s^-, I_s^-, \beta_s^- \right\rangle$ be two BPNSs and t is a scalar. The following is the definition of the operational rules, which include addition, multiplication, scalar multiplication, and power operations.

$$r \oplus s = \left\langle \begin{array}{l} \sqrt{(\alpha_r^+)^2 + (\alpha_s^+)^2 - (\alpha_r^+)^2 \cdot (\alpha_s^+)^2}, I_r^+ I_s^+, \beta_r^+ \beta_s^+, -(\alpha_r^- \cdot \alpha_s^-), \\ -(-I_r^- - I_s^- - I_r^- I_s^-, -\sqrt{(\beta_r^-)^2 + (\beta_s^-)^2 - (\beta_r^-)^2 \cdot (\beta_s^-)^2} \end{array} \right\rangle \tag{3}$$

$$r \otimes s = \left\langle \begin{array}{l} \alpha_r^+ \cdot \alpha_s^+, I_r^+ I_s^+ - I_r^+ I_s^+, \sqrt{\left(\beta_r^+\right)^2 + \left(\beta_s^+\right)^2 - \left(\beta_r^+\right)^2 \cdot \left(\beta_s^+\right)^2}, \\ -\sqrt{\left(\alpha_r^-\right)^2 + \left(\alpha_s^-\right)^2 - \left(\alpha_r^-\right)^2 \cdot \left(\alpha_s^-\right)^2}, -\left(I_r^- \cdot I_s^-\right), -\left(\beta_r^- \cdot \beta_s^-\right) \end{array} \right\rangle \tag{4}$$

$$t(r) = \left\langle \sqrt{1 - \left(1 - \left(\alpha_r^+\right)^2\right)^t}, \left(I_r^+\right)^t, \left(\beta_r^+\right)^t, -\left(-\alpha_r^-\right)^t, -\left(-I_r^-\right)^t, -\sqrt{1 - \left(1 - \left(-\beta_r^-\right)^2\right)^t} \right\rangle \tag{5}$$

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$$(r)^t = \left\langle \left(\alpha_r^+\right)^t, 1-\left(1-I_r^+\right)^t, \sqrt{1-\left(1-\left(\beta_r^+\right)^2\right)^t}, -\sqrt{1-\left(1-\left(-\alpha_r^-\right)^2\right)^t}, -(-I_r^-)^t, -(\beta_r^-)^t \right\rangle \tag{6}$$

Theorem 3.1. Let $r = \left\langle \alpha_r^+, I_r^+, \beta_r^+, \alpha_r^-, I_r^-, \beta_r^- \right\rangle$ and $s = \left\langle \alpha_s^+, I_s^+, \beta_s^+, \alpha_s^-, I_s^-, \beta_s^- \right\rangle$ be two BPNSs and t is a scalar and $t > 0, t_1 > 0, t_2 > 0$, then the following operations are valid:

$$r \oplus s = s \oplus r \tag{7}$$

$$r \otimes s = s \otimes r \tag{8}$$

$$t(r \oplus s) = t(r) \oplus t(s) \quad t > 0 \tag{9}$$

$$t_1(r) \oplus t_2(r) = (t_1 \oplus t_2) r \text{ where } t_1, t_2 > 0 \tag{10}$$

$$(r)^t \otimes (s)^t = (r \otimes s)^t \text{ where } t > 0 \tag{11}$$

$$(r)^{t_1} \otimes (r)^{t_2} = r^{(t_1+t_2)} \text{ where } t_1, t_2 > 0 \tag{12}$$

Proof.

$$r \oplus s = \left\langle \begin{array}{l} \sqrt{(\alpha_r^+)^2 + (\alpha_r^+)^2 - (\alpha_s^+)^2 \cdot (\alpha_s^+)^2}, I_r^+ I_s^+, \beta_r^+ \beta_s^+, -(\alpha_r^- \cdot \alpha_s^-), -(-I_r^- - I_s^- - I_r^- I_s^-), \\ -\sqrt{(\beta_r^-)^2 + (\beta_s^-)^2 - (\beta_r^-)^2 \cdot (\beta_s^-)^2} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \sqrt{(\alpha_s^+)^2 + (\alpha_r^+)^2 - (\alpha_s^+)^2 \cdot (\alpha_r^+)^2}, I_s^+ I_r^+, \beta_s^+ \beta_r^+, -(\alpha_s^- \cdot \alpha_r^-), -(-I_s^- - I_r^- - I_s^- I_r^-), \\ -\sqrt{(\beta_s^-)^2 + (\beta_r^-)^2 - (\beta_s^-)^2 \cdot (\beta_r^-)^2} \end{array} \right\rangle$$

$$= s \oplus r$$

$$r \otimes s = \left\langle \begin{array}{l} \alpha_r^+ \cdot \alpha_s^+, I_r^+ + I_s^+ - I_r^+ I_s^+, \sqrt{(\beta_r^+)^2 + (\beta_s^+)^2 - (\beta_r^+)^2 \cdot (\beta_s^+)^2}, \\ -\left(\sqrt{(\alpha_r^-)^2 + (\alpha_s^-)^2 - (\alpha_r^-)^2 \cdot (\alpha_s^-)^2}, -\left(I_r^- \cdot I_s^-\right), -\left(\beta_r^- \cdot \beta_s^-\right) \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \alpha_s^+ \cdot \alpha_r^+, I_s^+ + I_r^+ - I_s^+ I_r^+, \sqrt{(\beta_s^+)^2 + (\beta_r^+)^2 - (\beta_s^+)^2 \cdot (\beta_r^+)^2}, \\ -\left(\sqrt{(\alpha_s^-)^2 + (\alpha_r^-)^2 - (\alpha_s^-)^2 \cdot (\alpha_r^-)^2}, -\left(I_s^- \cdot I_r^-\right), -\left(\beta_s^- \cdot \beta_r^-\right) \right) \end{array} \right\rangle$$

$$= s \otimes r$$

In the same way that (i) and (ii), (iii) and (iv) have simple proofs.

$$(r)^t \otimes (s)^t = \left\langle (\alpha_r^+)^t, 1-(1-I_r^+)^t, \sqrt{1-(1-(\beta_r^+)^2)^t}, -\sqrt{1-(1-(-\alpha_r^-)^2)^t}, -(-I_r^-), -(-\beta_r^-)^t \right\rangle \otimes$$

$$\left\langle (\alpha_s^+)^t, 1-(1-I_s^+)^t, \sqrt{1-(1-(\beta_s^+)^2)^t}, -\sqrt{1-(1-(-\alpha_s^-)^2)^t}, -(-I_s^-)^t, -(-\beta_s^-)^t \right\rangle$$

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$$= \left\langle \begin{array}{l} (\alpha_r^+ \cdot \alpha_s^+)^t, 1-(1-(I_r^+ + I_s^+ - I_r^+ \cdot I_s^+)^t), \sqrt{1-(1-(\beta_r^+)^2 + (\beta_s^+)^2 - (\beta_r^+)^2 \cdot (\beta_s^+)^2)^t}, \\ -\sqrt{1-(1+((\alpha_r^-)^2 + (\alpha_s^-)^2 - (\alpha_r^-)^2 \cdot (\alpha_s^-)^2)^t}, -(-(I_r^- \cdot I_s^-))^t, -(-(\beta_r^- \cdot \beta_s^-))^t \end{array} \right\rangle$$

$$= (r \otimes s)^t$$

The proofs of (vi) can easily as (v).

4. Numerical Examples

The numerical examples for BPNSs are defined bellows:

Let,

$A = \{\langle 0.5, 0.3, 0.2, -0.6, -0.4, -0.3\rangle \mid y \in Y\}$ and $B = \{\langle 0.9, 0.6, 0.2, -0.6, -0.5, -0.3\rangle \mid y \in Y\}$

are the bipolar Pythagorean fuzzy neutrosophic numbers.

Example 4.1 the BPNS addition operation is:

$A \oplus B = \langle 0.5, 0.3, 0.2, -0.6, -0.4, -0.3\rangle \oplus \langle 0.9, 0.6, 0.2, -0.6, -0.5, -0.3\rangle$

$$= \left\langle \begin{array}{l} \sqrt{0.5^2 + 0.9^2 - (0.5^2)(0.9^2)}, (0.3)(0.6), (0.2)(0.2), -(-0.6)(-0.6), -(-(-0.4)-(-0.5) \\ -(-0.4)(-0.5)), -\sqrt{(-0.3)^2 + (-0.3)^2 - (-0.3)^2(-0.3)^2} \end{array} \right\rangle$$

$$= \langle 0.926, 0.18, 0.04, 0.36, -0.7, -0.4146\rangle$$

Where, $0 \le 0.8915 \le 2$ and $0 \le 0.4477 \le 2$ satisfying condition for definition 3.1

Example 4.2 the BPNS multiplication operation is:

$A \otimes B = \langle 0.5, 0.3, 0.2, -0.6, -0.4, -0.3\rangle \oplus \langle 0.9, 0.6, 0.2, -0.6, -0.5, -0.3\rangle$

$$= \left\langle \begin{array}{l} (0.5)(0.9), 0.3 + 0.6 - (0.3)(0.6), \sqrt{0.2^2 + 0.2^2 - (0.2)^2 \cdot (0.2)^2}, \\ -\sqrt{(-0.6)^2 + (-0.6)^2 - (-0.6)^2 \cdot (-0.6)^2}, -(-0.4)(-0.5), -(-0.3)(-0.3) \end{array} \right\rangle$$

$$= \langle 0.45, 0.72, 0.28, -0.7684, -0.2, -0.09\rangle$$

Where, $0 \le 0.7993 \le 2$ and $0 \le 0.6385 \le 2$ satisfying the condition for definition 3.1

Example 4.3 the BPNS scalar multiplication operation when, $t = 3$ is:

$3A = 3\langle 0.5, 0.3, 0.2, -0.6, -0.4, -0.3\rangle$

$$= \left\langle \sqrt{1-\left(1-(0.5)^2\right)^3}, 0.3^3, 0.2^3, -(0.6)^3, -(0.4)^3, -\sqrt{1-(1-(-(-0.3)^2)^3)} \right\rangle$$

$$= \langle 0.7603, 0.027, 0.008, -0.216, -0.064, -0.4964\rangle$$

Where, $0 \le 0.5788 \le 2$ and $0 \le 0.2972 \le 2$ satisfying the condition for definition 3.1

Example 4.4 the BPNS power operation when $t = 2$ is

$A^2 = (\langle 0.5, 0.3, 0.2, -0.6, -0.4, -0.3\rangle)^2$

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$$=\left\langle 0.5^2, 1-(1-0.3)^2, \sqrt{1-(1-0.2^2)^2}, -\sqrt{1-(1-(0.6)^2)^2}, -(0.4^2), -(0.3^2)\right\rangle$$

$$=\left\langle 0.25, 0.51, 0.28, -0.7684, -0.16, -0.09\right\rangle$$

Where, $0 \le 0.401 \le 2$ and $0 \le 0.3823 \le 2$ satisfying the condition for definition 3.1

5. Conclusions

In real-world scenarios, information may be conflicting or contradictory. The combination of bipolar Pythagorean fuzzy sets and neutrosophic sets allows for a more flexible representation of conflicting information, considering both positive and negative influences. This study provided an effective demonstration of the Bipolar Pythagorean Fuzzy Neutrosophic (BPNS) concept and its operation. The notion was expanded to include Pythagorean fuzzy sets, neutrosophic logic, and bipolarity. The new expansion greatly broadens the current theories for handling uncertainties and makes new areas for field research and pertinent applications possible. The intention would probably be to improve the depiction of complex and multilayered uncertainty by utilising the advantages of both frameworks. A more sophisticated approach to handling indeterminacy and enhanced differentiation between positive and negative uncertainties are possible benefits of such a combination. The hybrid model that is produced may provide a more thorough means of expressing several aspects of uncertainty at the same time.

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