Some Important Theories about Duality and the Economic Interpretation of Neutrosophic Linear Models and Their Dual Models

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Abstract

The essence of operations research is focused on creating and using models. The first step is to create the model, which requires a set of data that is determined from the aspects of the real system that the model must represent. The model is an acceptable model if it achieves the purpose for which it was formulated, and since programming issues linearity is concerned with allocating scarce resources, including labor, machinery, and capital, and using them in the best possible way, such that costs are reduced to a minimum or profits are maximized, by choosing the optimal decision from several available options. Since linear models are used in many fields, it was necessary to prepare studies that meet the needs of decision makers who have made solutions to linear programming problems a safe haven for them. The duality theory is considered one of the most important linear programming theories because it is used in many fields and is relied upon in the economic interpretation of the content of linear models. It provides a comprehensive study of the system represented by the linear model and its dual model. In this research, we present a study of the theory of neutrosophic dual and its economic interpretation by presenting a set of theorems that can be relied upon in explaining the results of solving both the original neutrosophic models and their dual.

Keywords: Neutrosophic science; Linear models; Neutrosophic Duality theory; Economic interpretation of the dual model; Applications of duality theory.

1. Introduction:

When looking at the concepts of neutrosophic science, the research and studies that have been presented by researchers and scholars in this field, and the stages of its development that can be viewed through the links available in this research [1], we find that the science of operations research, with its various methods, was in dire need of such concepts because it is the science that it is considered the applied side of mathematics and touches on the details of our lives. It is a science whose methods depend on data that is provided by experts. These data are classical values specific to the time period in which they were collected, and thus will provide specific solutions that require decision makers to prepare and prepare for any sudden change that may occur in the environment. Work, and this is what prompted us to say that the science of operations research is in dire need of the concepts of neutrosophic science because when using neutrosophic values while collecting data on the case under study, we find that we take into account the best and worst conditions and thus we obtain solutions that are neutrosophic values, values that are not completely defined and have a margin. Freedom puts us in a safe work environment. This is what we have achieved through research and studies that were presented to reformulate some operations research methods using the concepts of neutrosophic science, see [2-15].

Most companies and institutions rely on studies provided by experts and researchers using operations research methods in order to ensure a safe work environment away from danger and danger and to give decision makers a margin of freedom, after the studies and research presented using the concepts of neutrosophic science in most fields of science have proven their ability to provide more accurate results. From the results that we were obtaining using classical data-driven studies, we reformulated many operations research topics using neutrosophic concepts [1-20], and to complement what we have done, we present in this research a study of dual linear models and the binary simplex algorithm that gives us a solution for the original and dual models at the same time so that we can provide a clear study that helps decision makers in companies and institutions develop plans and programs through which the highest profit and lowest cost are achieved.

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2. Discussion:

In this research, we present a study of some important theories related to linear neutrosophic models and their dual models, through whose optimal solutions we can provide an economic explanation that helps in making ideal decisions for managing systems that operate according to these models and achieve the maximum profit, based on what is stated in the following references: [16 -19]

We begin this study by presenting the matrix form of the dual models:

Find the dual model using matrices:

Here we distinguish two cases:

The first case: The original model is symmetrical and maximization type:

original model

Find

$$NZ = NC\,\mathrm{X} \longrightarrow Max$$

Constraints

$$NA\,\mathrm{X} \leq NB$$
$$\mathrm{X} \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{X} = [x_1 \;\; x_2 \dots x_n]$$

Dual model

Find

$$NL = NB\,\mathrm{Y} \longrightarrow Min$$

Constraints

$$NA^T\mathrm{Y} \geq NC$$
$$\mathrm{Y} \geq 0$$

Where:

$$NA^T = \begin{bmatrix} Na_{11} & Na_{21} \dots Na_{m1} \\ Na_{12} & Na_{22} \dots Na_{m2} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{1n} & Na_{2n} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{Y} = [y_1 \;\; y_2 \dots y_m]$$

The second case: The model is symmetrical and miniaturized

original model:

Find

$$NZ = NC\,\mathrm{X} \longrightarrow Min$$

constraints

$$NA\mathrm{X} \geq NB$$
$$\mathrm{X} \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{X} = [x_1 \;\; x_2 \dots x_n]$$

Dual model:

Find

$$NL = NB\,\mathrm{Y} \longrightarrow Max$$

Constraints

$$NA^T\mathrm{Y} \leq NC$$
$$\mathrm{Y} \geq 0$$

Where:

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$$A^T = \begin{bmatrix} a_{11} & a_{21} \dots a_{m1} \\ a_{12} & a_{22} \dots a_{m2} \\ \dots & \dots \dots \dots \\ a_{1n} & a_{2n} \dots a_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad Y = [y_1 \ y_2 \dots y_m]$$

We summarize the process of finding neutrosophic conjugate models using matrices in the following steps:

1. We define a new non-negative variable for each constraint of the original model
2. We make the wind (cost) vector in the original model a column vector of constants in the companion model
3. We make the constants column vector in the original model the cost (profit) vector in the dual model
4. We transform a matrix of the parsimony of the variables of the constraints in the original model into the parsimony of the variables in the dual model
5. We reverse the direction of the constraint inequalities
6. We reverse the direction of the examples, that is, we change the increase to the maximum limit to a decrease to the minimum limit, and vice versa.

Duality theorems:

1- Weak duality theorem:

Let the following linear model:

Find

$$Z = NC\,X \longrightarrow Max$$

Constraints

$$NA\,X \le NB$$
$$X \ge 0$$

Dual model:

$$L = NB\,Y \longrightarrow Min$$

Constraints

$$NA^T Y \ge NC$$
$$Y \ge 0$$

The value of the objective function of the dual model for any acceptable solution is always highest than or equal to the value of the objective function of the original model at the same solution, i.e.

$$NB\,Y' \ge NCX'$$

Where X' is an acceptable solution for the original model and Y' is an acceptable solution for the dual model.

Results from the theory:

- The value of the objective function of the original model of the maximization type for any acceptable solution represents the minimum value of the objective function in the dual model.
- The minimum value of the objective function in the dual model for any acceptable solution represents the upper limit of the maximum value of the objective function in the original model.
- If the original model and its solution are infinite (that is, the maximum value of NZ seeks infinity, $MaxNZ \longrightarrow \infty$), then the dual model does not have an acceptable solution.
- If the dual problem has an indefinite solution (that is, the minimum value of NL seeks infinity $MinNL \longrightarrow \infty$), then the original problem is infinite.

To illustrate the Weak duality theorem, we present the following example:

Original model

Find

$$Z = [1,2]x_1 + [0,1]x_2 + [3,4]x_3 + [2,3]x_4 \longrightarrow Max$$

Constraints

$$x_1 + 2x_2 + 2x_3 + 3x_4 \le 20$$
$$2x_1 + x_2 + 3x_3 + 2x_4 \le 20$$
$$x_1, x_2, \dots, x_4 \ge 0$$

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Dual model

Find

$$L = 20y_1 + 20y_2 \longrightarrow Min$$

Constraints

$$y_1 + 2y_2 \geq [1,2]$$
$$2y_1 + y_2 \geq [0,1]$$
$$2y_1 + 3y_2 \geq [3,4]$$
$$3y_1 + 2y_2 \geq [2,3]$$
$$y_1, y_2 \geq 0$$

$x_1' = x_2' = x_3' = x_4' = [0,1]$ is an acceptable solution for the original model and $y_1' = y_2' = [0.5,1]$ is an acceptable solution for the dual model, (the solution is Acceptable if it satisfies all constraints). We determine the value of the objective functions corresponding to the acceptable solution. We find:

The value of the objective function in the original form

$$NZ = NCX' \in [1,2]x_1' + [0,1]x_2' + [3,4]x_3' + [2,3]x_4' = [6,10].[0,1] = [0,10]$$

And the value of the objective function in the dual model

$$NL = NBY' \in 20y_1' + 20y_2' = 20[0.5,1] + 20[0.5,1] = [20,40]$$

We note that $CX' < BY'$ where ($X' = [x_1' \ x_2' \ x_3' \ x_4'], Y' = [y_1' \ y_2']$) and this fulfills the weak duality theorem Likewise, according to result (2), the maximum value of the objective function in the original model cannot exceed [20,40], and the inverse of results (3) and (4) is also true.

2- Optimality criterion theorem:

If acceptable solutions x' , y' are found for the symmetrical dual models such that the values of the objective functions corresponding to these solutions are equal, then these acceptable solutions are optimal solutions for these models.

Proof:

We assume that X'' is another acceptable solution to the original model according to the previous theory, then:

$$NCX'' < NBY'$$

But according to the assumption

$$NCX' = NBY'$$

Therefore:

$$NCX' \leq NBX'$$

For all acceptable solutions of the original models, therefore, by definition, X' is an optimal solution of the original model. In the same way, we prove that Y' is an optimal solution of the companion model.

3- Main duality theorem:

If each of the original and companion models has an acceptable solution, then they both have optimal solutions that make the optimal values of the objective functions equal.

Proof:

When both the original and dual models have an acceptable solution, according to results 1 and 2 of Theorem 1, we have a minimum for NL and an upper limit for ZN. In other words, the original or dual model cannot have an infinite solution. Therefore, both problems must have optimal solutions.

Economic interpretation of linear models and their dual models:

Linear programming problems are concerned with allocating the available resources of labor, materials, machinery, and capital and using them in the best possible way so that costs are reduced to their minimum and profits are maximized. What is meant by the term best is choosing the best solution from among the set of acceptable solutions.

What is meant by the economic interpretation of the linear models and their dual models is the interpretation of the results that we obtain when we find the optimal solution for both the original and dual models. We illustrate the economic interpretation of the original and dual models through the following example:

Example:

A factory wants to transfer its products from two warehouses to three retail centers at the lowest possible cost. The following table shows the data provided by the factory official:

Sales centers / Stores	B_1	B_2	B_3	Available quantities
A_1	[1,3]	[2,4]	[0,3]	300
A_2	[4,6]	[1,4]	[1,5]	600
Quantities required	200	300	400	900 / 900

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The plant manager's request was for a transportation plan with a minimum cost so that the distribution centres' orders could be met from the available quantities
The previous issue is a balanced transfer issue because

$$\sum_{i=1}^{2} a_i = \sum_{j=1}^{3} b_j = 900$$

Formulating the neutrosophic mathematical model,[15].
We assume x_{ij} the quantity transported from store i where $i = 1,2$, to distribution center j, where $j = 1,2,3$. Thus, we obtain the following linear model:
Find

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constans:

$$x_{11} + x_{12} + x_{13} \le 300$$
$$x_{11} + x_{21} + x_{23} \le 600$$
$$x_{11} + x_{21} \ge 200$$
$$x_{12} + x_{22} \ge 300$$
$$x_{13} + x_{23} \ge 400$$
$$x_{\text{ij}} \ge 0 \;;\; \text{i} = 1,2 \;, \text{j} = 1,2,3$$

We write the model in the following symmetrical form:
Since the objective function is a minimization function, all constraints must be of type greater than or equal to, so the model is written in the following symmetric form:
Find

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constans:

$$-x_{11} - x_{12} - x_{13} \ge -300$$
$$-x_{11} - x_{21} - x_{23} \ge -600$$
$$x_{11} + x_{21} \ge 200$$
$$x_{12} + x_{22} \ge 300$$
$$x_{13} + x_{23} \ge 400$$
$$x_{\text{ij}} \ge 0 \;;\; \text{i} = 1,2 \;, \text{j} = 1,2,3$$

Forming the dual model, we obtain the following linear model:
Find

$$Z = -300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5 \longrightarrow Max$$

Constans:

$$-y_1 + y_3 \le [1,3]$$
$$-y_1 + y_4 \le [2,4]$$
$$-y_1 + y_5 \le [0,3]$$
$$-y_2 + y_3 \le [4,6]$$
$$-y_2 + y_4 \le [1,4]$$
$$-y_2 + y_5 \le [1,5]$$
$$y_1, y_2, y_3, y_4, y_5 \ge 0$$

We formulate an appropriate text for the accompanying model based on the text of the original problem:
It is clear from the original model that the factory's goal is to transport all of its products at the lowest possible cost:
Text of the issue dual to the attached form:
A transport company submitted to a factory an offer that it would transport the entire quantity in the first warehouse, i.e., 300 units, at a price of y_1 monetary unit per unit, and transfer the entire quantity available in the second warehouse, 600, at a price of y_2 monetary units per unit. The company pledged that it would deliver $(200\,,300\,,400)$ units. To the three retail centers, respectively. These units are sold in these centers at a price of (y_3, y_4, y_5) monetary units, respectively. So that you can convince the official in the factory that if he accepts her offer, the cost of transportation in his factory will be less than the cost if he carries out the transportation process, so that he carries out the transportation process.
She used the constraints in the dual model and conducted the following discussion:
You pay the cost of transporting one unit from the first factory to the first sales center, an amount whose value belongs to the range $[1,3]$, but if you use the transport company, the cost is $(y_3 - y_1)$, and we have from the first entry in the accompanying model

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$$y_3 - y_1 \leq [1,3]$$

Here the official in the laboratory will notice that the transportation company's offer is an appropriate offer.
In the same way we discuss all the limitations of the dual model, the conclusion that the factory official will reach is that the cost of transportation on any route if the transportation company's offer is accepted is less than or equal to the cost that he would pay if he himself carried out the transportation process.
The transport company will adopt the values $(y_1, y_2, y_3, y_4, y_5)$,because it will achieve maximum profit through them, as the transport company's profit is calculated from the relationship:

$$-300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5$$

It is the same as the objective function of the dual model, meaning that the dual model represents the transportation company that is trying to maximize its profits
The basic theorem of association states that the optimal values of the model and the dual model are always equal. The manufacturer does not save any money because he will pay the transportation company the minimum cost of transportation, but it saves the trouble of solving the original model to determine the minimum cost of transportation, and for the transportation company, it has guaranteed the deal to transport the goods with the maximum profit.
From the above, we can say that the optimal solution for the accompanying model from an economic perspective is the price that is paid for the available capabilities (resources). We know that these capabilities are limited. According to the main association theory, the value of the objective function in the original model and the dual model are equal, so if X^0 and Y^0 for two optimal solutions,

$$NZ_0 = NCX^0 = NBY^0 = NL_0.$$

In other words, the optimal value of the original or dual model is given by the following relation:

$$NL_0 = NB\,Y^0$$

Where NB is the matrix, whose elements are the limited quantities of resources, and Y^0 are the optimal values of the variables in the dual model. If any change occurs in the level of these resources, it will affect the optimal value of the objective function, that is, NL_0. Accordingly, the optimal value of the variable in the dual model (for each constraint in the original model, it gives the net change in the optimal value of the objective function. These values are called shadow prices for constrained resources. Shadow prices can be used to determine whether obtaining additional quantities of resources at prices that encourage them is economically feasible.

3. Conclusion and Results:

From the previous study, we find that it is sufficient to find the optimal solution for the original and companion models, to find the optimal solution for one of them, and using the theories found in the research to deduce the optimal solution for the other. We can also use the results to provide an economic explanation that helps in making ideal decisions, especially when we take the data as neutrosophic values, because the optimal solution for the original model gives us the best production plan makes the value of that production as large as possible within the available capabilities. As for the optimal solution for the accompanying model, it gives us the best values for the prices of raw materials, which if used without waste will also give us the best production plan, and the result is the greatest profit.

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