Possibility neutrosophic bipolar fuzzy soft sets and their applications

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Abstract

In this work, the possibility neutrosophic bipolar soft sets interact with the possibility bipolar fuzzy soft sets, as well as complementation, union, intersection, AND, and OR. This paper extends the concept of bipolar neutrosophic soft sets to the possibility neutrosophic bipolar soft sets. Our main goal was to demonstrate De Morgan's law, associate law and distributive law, which are all the laws related to the possibility neutrosophic bipolar soft sets. Also, we present an algorithm that uses a soft set model to solve the decision-making problem primarily in order to simplify the process.

Keywords: neutrosophic bipolar soft set; possibility neutrosophic bipolar soft set; decision-making problem.

1 Introduction

Real-world systems are becoming increasingly complex, and decision-makers are finding it increasingly difficult to identify the optimal way of solving these problems. It is difficult to choose between alternative options, but you can find the best option if you decide to look for it. In fact, a number of firms find creating opportunities, objectives, and viewpoint constraints quite challenging due to their different viewpoints. In order to achieve multiple objectives simultaneously, individuals or groups should consider multiple priorities when decision-making (DM). In our daily work, we deal with a wide range of MADM-related problems. Thus, it is necessary to improve DM abilities. Multi-criteria decision-making (MCDM) involves aggregation operators. Decision contexts that involve cycles or seasons, or decision criteria that depend on time, are especially adept at utilizing this utility. In addition, symmetric properties of sine functions can be utilized in scenarios in which both negative and positive influences are equally weighted. Using symmetric aggregation operators, favourable and unfavourable outcomes can be assessed appropriately. MCDM allows effective aggregation of multiple criteria using sine operational laws-derived aggregation operators. By capturing the inherent characteristics and relationships between the criteria, we will be able to generate DM with greater accuracy and insight.

A fuzzy set (FS) has been proposed by Zadeh[22] to deal with uncertainty, an intuitionistic fuzzy set (IFS) has been proposed by Atanassov,[7] a Pythagorean fuzzy set (PFS) by Yager[20] and a neutrosophic set (NSS) by Smarandache.[19] FS have membership values (MVs) that correspond to what level of belongingness each element possesses; grades correspond to these levels. According to Atanassov,[7] the sum of membership value (MV) and non-membership value (NMV) should not exceed one. If the sum of MV and NMV exceeds one, we

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

might experience trouble DM. Yager[20] proposes PFS as a generalization of IFS, where MV and NMV cannot have a square sum greater than one. Several applications of PFS are discussed by Akram et al.[3] The notion of a picture fuzzy set (PicFS) was developed by Cuong and Kreinovich.[10] It was observed that MV ξ, neutral τ and NMV δ with $0 \leq \xi + \tau + \delta \leq 1$; for $\xi, \tau, \delta \in [0, 1]$. Assuring consistency between the data acquisition process and the actual decision environment by using PicFS definitions such as "yes", "abstain", "no", and "refusal". The PicFS concept has been studied in a limited manner, but its applications are numerous. Shahzaib et al.[6] defined the spherical FS (SFS) using MADM. In SFS by $0 \leq \xi^2 + \tau^2 + \delta^2 \leq 1$ rather than $0 \leq \xi + \tau + \delta \leq 1$.

Neutrosophic sets (NSSs) were developed by Smarandache.[19] FS and IFS differ in this neutrality, referred to as neutrosophy. There is a truth value (TV), an indeterminacy value (IV) and a false value (FV). Each component of the universe has levels of TV, IV, and FV between $[0, 1]$. An NSS is a generalization of classical sets, functional sets, integral sets, etc. There are many algebraic structures and aggregation operators discussed by Palanikumar et al.[16,17] Broumi et al. have extended the idea to the idea of interval-valued Fermatean neutrosophic sets and applied it to graphs, as seen in.[8,9] The concept of soft set (SS) was developed by Molodtsov.[15] In terms of objectivity and complexity, SS more accurately models real-world DM problems. There are two types of soft sets such as fuzzy soft sets (FSSs)[12] and intuitionistic fuzzy soft sets (IFSSs).[13] These two theories can be used to address a variety of DM problems. Adeel et al. deal with the fuzzy linguistic TOPSIS based on m-polar attributes. TOPSIS using GDMs was discussed by Eraslan et al.[11] A soft approach to possibility FS is described by Alkhazaleh.[4] A FSS has recently been extended to Pythagorean FSS by Peng et al.[18] Many researchers[21]-[5] discussed the concept of Pythagorean with its extension based on DM. Here, the possibility neutrosophic soft sets are extended to parameterizing possibility neutrosophic bipolar sets. Using a soft model, we will establish a similarity measure.

Using bipolar possibility sets as the basis for this research extends the concept of possibility neutrosophic bipolar soft sets. The paper is divided into six sections. A summary of the introduction can be found in section 1. Section 2 represents a neutrosophic set and some relevant definitions. A possibility neutrosophic bipolar soft sets and their basic operations are provided in Section 3. Section 4 discusses the similarity between two possibility neutrosophic bipolar soft sets. Section 5 discusses the application of the possibility neutrosophic bipolar soft set model. Section 6 presents a comparison between the possibility neutrosophic bipolar soft set and neutrosophic bipolar soft set. Section 7 provides a conclusion.

2 Preliminaries

Definition 2.1. [20,21] Let U be a universal, the PFS $\Lambda = \{\langle \varsigma, \varpi_\Lambda(\varsigma), \upsilon_\Lambda(\varsigma)\rangle : \varsigma \in U\}$, where $\varpi_\Lambda(\varsigma)$ and $\upsilon_\Lambda(\varsigma)$ represent the MV and NMV of Λ respectively. The mapping $\varpi_\Lambda : U \to [0,1]$, $\upsilon_\Lambda : U \to [0,1]$ and $0 \leq (\varpi_\Lambda(\varsigma))^2 + (\upsilon_\Lambda(\varsigma))^2 \leq 1$. The IV is $\pi_\Lambda(\varsigma) = \left[\sqrt{1 - (\varpi_\Lambda(\varsigma))^2 - (\upsilon_\Lambda(\varsigma))^2}\right]$. $\Lambda = \langle \varpi_\Lambda, \upsilon_\Lambda\rangle$ is called a Pythagorean fuzzy number (PFN).

Definition 2.2. [14] The Pythagorean bipolar fuzzy set (PBFS) $\Lambda = \{\langle \varsigma, \varpi^+_\Lambda(\varsigma), \upsilon^+_\Lambda(\varsigma), \varpi^-_\Lambda(\varsigma), \upsilon^-_\Lambda(\varsigma)\rangle : \varsigma \in U\}$, where $\varpi^+_\Lambda(\varsigma), \upsilon^+_\Lambda(\varsigma)$, $\varpi^-_\Lambda(\varsigma), \upsilon^-_\Lambda(\varsigma)$ represent the positive MV, positive NMV, negative MV and negative NMV of Λ respectively. The mapping $\varpi^+_\Lambda, \upsilon^+_\Lambda : U \to [0,1]$, $\varpi^-_\Lambda, \upsilon^-_\Lambda : U \to [-1,0]$ such that $0 \leq (\varpi^+_\Lambda(\varsigma))^2 + (\upsilon^+_\Lambda(\varsigma))^2 \leq 1$ and $-1 \leq -\left[(\varpi^-_\Lambda(\varsigma))^2 + (\upsilon^-_\Lambda(\varsigma))^2\right] \leq 0$. The indeterminacy value (IV) is $\pi^+_\Lambda(\varsigma) = \left[\sqrt{1 - (\varpi^+_\Lambda(\varsigma))^2 - (\upsilon^+_\Lambda(\varsigma))^2}\right]$ and $\pi^-_\Lambda(\varsigma) = -\left[\sqrt{1 - (\varpi^-_\Lambda(\varsigma))^2 - (\upsilon^-_\Lambda(\varsigma))^2}\right]$. Since $\Lambda = \langle \varpi^+_\Lambda, \upsilon^+_\Lambda, \varpi^-_\Lambda, \upsilon^-_\Lambda\rangle$ is called a Pythagorean bipolar fuzzy number (PBFN).

Definition 2.3. [1,2] (i) Let U and E be the universal and set of parameters, respectively. Then $(\mathcal{X}, \Lambda)$ is said to be bipolar FSS (BFSS) on U if $\Lambda \subseteq E$ and $\mathcal{X} : \Lambda \to \mathcal{BF}^U$.
(ii) The $(\mathcal{X}, \Lambda)$ is called a Pythagorean BFSS (PBFSS) on U if $\Lambda \subseteq E$ and $\mathcal{X} : \Lambda \to P\mathcal{BF}^U$. Here $\mathcal{BF}^U$ and $P\mathcal{BF}^U$ are called the set of all bipolar fuzzy subsets and Pythagorean bipolar fuzzy subsets of U, respectively.

Definition 2.4. [14] Given that $\xi_1 = \langle \varpi^+_{\xi_1}, \upsilon^+_{\xi_1}, \varpi^-_{\xi_1}, \upsilon^-_{\xi_1}\rangle$, $\xi_2 = \langle \varpi^+_{\xi_2}, \upsilon^+_{\xi_2}, \varpi^-_{\xi_2}, \upsilon^-_{\xi_2}\rangle$ and $\xi_3 = \langle \varpi^+_{\xi_3}, \upsilon^+_{\xi_3}, \varpi^-_{\xi_3}, \upsilon^-_{\xi_3}\rangle$ are any three PBFNs. Then
(1) $\xi_1^c = \langle \upsilon^+_{\xi_1}, \varpi^+_{\xi_1}, \upsilon^-_{\xi_1}, \varpi^-_{\xi_1}\rangle$
(2) $\xi_2 \cup \xi_3 = \left\langle \max(\varpi^+_{\xi_2}, \varpi^+_{\xi_3}), \max(\upsilon^+_{\xi_2}, \upsilon^+_{\xi_3}), \min(\upsilon^-_{\xi_2}, , \upsilon^-_{\xi_3}) \min(\varpi^-_{\xi_2}, \varpi^-_{\xi_3}) \right\rangle$
(3) $\xi_2 \cap \xi_3 = \left\langle \min(\varpi^+_{\xi_2}, \varpi^+_{\xi_3}), \min(\upsilon^+_{\xi_2}, \upsilon^+_{\xi_3}), \max(\varpi^-_{\xi_2}, \varpi^-_{\xi_3}), \max(\upsilon^-_{\xi_2}, \upsilon^-_{\xi_3}) \right\rangle$

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

(4) $\xi_2 \geq \xi_3$ if and only if $\varpi^+_{\xi_2} \geq \varpi^+_{\xi_3}$ and $\upsilon^+_{\xi_2} \geq \upsilon^+_{\xi_3}$ and $\varpi^-_{\xi_2} \leq \varpi^-_{\xi_3}$ and $\upsilon^-_{\xi_2} \leq \upsilon^-_{\xi_3}$
(5) $\xi_2 = \xi_3$ if and only if $\varpi^+_{\xi_2} = \varpi^+_{\xi_3}$ and $\upsilon^+_{\xi_2} = \upsilon^+_{\xi_3}$ and $\varpi^-_{\xi_2} = \varpi^-_{\xi_3}$ and $\upsilon^-_{\xi_2} = \upsilon^-_{\xi_3}$.

Definition 2.5. [4] Let U and E be the universal and set of parameters respectively. The (U, E) is a soft universe. The function $\mathcal{X} : E \to \mathcal{X}(U)$ and τ be a fuzzy subset of E. That is, $\tau : E \to \mathcal{X}(U)$. Let $\mathcal{X}_\tau : E \to \mathcal{X}(U) \times \mathcal{X}(\mathbb{U})$ and $\mathcal{X}_\tau(\varepsilon) = (\mathcal{X}(\varepsilon)(\varsigma), \tau(\varepsilon)(\varsigma)), \forall \varsigma \in U$. Then $\mathcal{X}_\tau$ is called a possibility FSS on (U, E).

3 Possibility neutrosophic bipolar soft sets

Definition 3.1. Let U and E be the universal and set of parameters, respectively. The $(\mathcal{X}, A)$ is a neutrosophic bipolar soft set (NBSS) on U if $\mathcal{X} : A \to \mathcal{NB}^U$. Here $\mathcal{NB}^U$ is called the set of all neutrosophic bipolar subsets of U.

Example 3.2. A set of three bikes $U = \{u_1, u_2, u_3\}$ under consideration and parameters $E = \{\varepsilon_1 =$ design, $\varepsilon_2 =$ price, $\varepsilon_3 =$ mileage, $\varepsilon_4 =$ durable$\}$. Suppose that $\mathcal{X} : E \to \mathcal{NB}^U$ is given by

$$\mathcal{X}^{\mathcal{B}}(\varepsilon_1) = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.75, 0.85, 0.65, -0.35, -0.45, -0.55\rangle} \\ \frac{u_2}{\langle 0.35, 0.65, 0.85, -0.85, -0.65, -0.55\rangle} \\ \frac{u_3}{\langle 0.85, 0.65, 0.55, -0.45, -0.65, -0.75\rangle} \end{array} \right\}; \quad \mathcal{X}^{\mathcal{B}}(\varepsilon_2) = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.45\ 0.75, 0.85, -0.75, -0.35, -0.25\rangle} \\ \frac{u_2}{\langle 0.65, 0.75\ 0.75, -0.85, -0.45, -0.35\rangle} \\ \frac{u_3}{\langle 0.95, 0.55, 0.25, -0.75, -0.55, -0.55\rangle} \end{array} \right\};$$

$$\mathcal{X}^{\mathcal{B}}(\varepsilon_3) = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.95, 0.55, 0.45, -0.25, -0.45, -0.85\rangle} \\ \frac{u_2}{\langle 0.65, 0.85, 0.55, -0.45, -0.55, -0.85\rangle} \\ \frac{u_3}{\langle 0.55, 0.85, 0.75, -0.95, -0.35, -0.25\rangle} \end{array} \right\}; \quad \mathcal{X}^{\mathcal{B}}(\varepsilon_4) = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.75, 0.65, 0.65, -0.45, -0.65, -0.75\rangle} \\ \frac{u_2}{\langle 0.85, 0.65, 0.55, -0.65, -0.75, -0.85\rangle} \\ \frac{u_3}{\langle 0.65, 0.55, 0.85, -0.55, -0.55, -0.65\rangle} \end{array} \right\}.$$

Matrix form:

$$\begin{pmatrix} \langle 0.75, 0.85, 0.65, -0.35, -0.45, -0.55\rangle & \langle 0.35, 0.65, 0.85, -0.85, -0.65, -0.55\rangle & \langle 0.85, 0.65, 0.55, -0.45, -0.65, -0.75\rangle \\ \langle 0.45\ 0.75, 0.85, -0.75, -0.35, -0.25\rangle & \langle 0.65, 0.75\ 0.75, -0.85, -0.45, -0.35\rangle & \langle 0.95, 0.55, 0.25, -0.75, -0.55, -0.55\rangle \\ \langle 0.95, 0.55, 0.45, -0.25, -0.45, -0.85\rangle & \langle 0.65, 0.85, 0.55, -0.45, -0.55, -0.85\rangle & \langle 0.55, 0.85, 0.75, -0.95, -0.35, -0.25\rangle \\ \langle 0.75, 0.65, 0.65, -0.45, -0.65, -0.75\rangle & \langle 0.85, 0.65, 0.55, -0.65, -0.75, -0.85\rangle & \langle 0.65, 0.55, 0.85, -0.55, -0.55, -0.65\rangle \end{pmatrix}$$

Definition 3.3. Let (U, E) represent a soft universe. Suppose that $\mathcal{X} : E \to \mathcal{PNB}^U$, and p is a neutrosophic bipolar subset of E and $p : E \to \mathcal{PNB}^U$, $\mathcal{PNB}^U$ represents the collection of all neutrosophic bipolar subsets of U. If $\mathcal{X}^{\mathcal{NB}}_p : E \to \mathcal{PNB}^U \times \mathcal{PNB}^U$, where $\mathcal{X}^{\mathcal{NB}}_p(\varepsilon) = \left\{ \left\langle \mathcal{B}(\varepsilon)(\varsigma), p(\varepsilon)(\varsigma) \right\rangle : \varsigma \in U \right\}$. Then $\mathcal{X}^{\mathcal{NB}}_p$ is a possibility NBSSs (PNBSS) on (U, E).

$$\mathcal{X}^{\mathcal{NB}}_p(\varepsilon) = \left\{ \begin{array}{c} \Big\langle \varsigma, (\varpi^+_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^+_{\mathcal{X}(\varepsilon)}(\varsigma), \chi^+_{\mathcal{X}(\varepsilon)}(\varsigma), \varpi^-_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^-_{\mathcal{X}(\varepsilon)}(\varsigma), \chi^-_{\mathcal{X}(\varepsilon)}(\varsigma)), \\ (\varpi^+_{p(\varepsilon)}(\varsigma), \upsilon^+_{p(\varepsilon)}(\varsigma), \chi^+_{p(\varepsilon)}(\varsigma), \varpi^-_{p(\varepsilon)}(\varsigma), \upsilon^-_{p(\varepsilon)}(\varsigma), \chi^-_{p(\varepsilon)}(\varsigma))\Big\rangle : \varsigma \in U \end{array} \right\}$$

Example 3.4. Let $U = \{u_1, u_2, u_3\}$ and $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ is a set of parameters. Suppose that $\mathcal{X}^{\mathcal{NB}}_p : E \to \mathcal{PNB}^U \times \mathcal{PNB}^U$ is defined by

$$\mathcal{X}^{\mathcal{B}}_p(\varepsilon_1) = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.55,0.65,0.65,-0.25,-0.75,-0.75),(0.55,0.85,0.45,-0.75,-0.65,-0.25)\rangle} \\ \frac{u_2}{\langle (0.85,0.75,0.35,-0.65,-0.65-0.45),(0.75,0.65,0.25,-0.55,-0.55,-0.45)\rangle} \\ \frac{u_3}{\langle (0.75,0.85,0.45,-0.55,-0.15,-0.85),(0.65,0.75,0.35,-0.75,-0.75,-0.55)\rangle} \end{array} \right\};$$

$$\mathcal{X}^{\mathcal{B}}_p(\varepsilon_2) = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.65,0.65,0.35,-0.15,-0.75,-0.75),(0.85,0.65,0.15,-0.65,-0.55,-0.35)\rangle} \\ \frac{u_2}{\langle (0.25,0.55,0.85,-0.65,-0.55,-0.35),(0.55,0.55,0.35,-0.55,-0.75,-0.45)\rangle} \\ \frac{u_3}{\langle (0.45,0.45,0.55,-0.15,-0.35,-0.85),(0.75,0.65,0.25,-0.65,-0.55,-0.55)\rangle} \end{array} \right\};$$

$$\mathcal{X}^{\mathcal{B}}_p(\varepsilon_3) = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.25,0.65,0.65,-0.75,-0.75,-0.35),(0.55,0.55,0.45,-0.65,-0.65,-0.25)\rangle} \\ \frac{u_2}{\langle (0.75,0.55,0.35,-0.65,-0.65,-0.25),(0.65,0.65,0.35,-0.55,-0.55,-0.35)\rangle} \\ \frac{u_3}{\langle (0.85,0.45,0.15,-0.45,-0.55,-0.55),(0.75,0.75,0.45,-0.85,-0.35,-0.15)\rangle} \end{array} \right\}.$$

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Definition 3.5. Suppose that $\mathcal{X}_p^{\mathcal{NB}}$ and $\mathcal{Y}_q^{\mathcal{NB}}$ are two PNBSSs on (U,E). Now $\mathcal{X}_p^{\mathcal{NB}} \subseteq \mathcal{Y}_q^{\mathcal{NB}}$ if and only if
(1) $\mathcal{X}(\varepsilon)(\varsigma) \subseteq \mathcal{Y}(\varepsilon)(\varsigma)$ if $\forall e \in E$,

$$\left\{\begin{array}{l} \varpi^+_{\mathcal{X}(\varepsilon)}(\varsigma) \leq \varpi^+_{\mathcal{Y}(\varepsilon)}(\varsigma),\ \upsilon^+_{\mathcal{X}(\varepsilon)}(\varsigma) \leq \upsilon^+_{\mathcal{Y}(\varepsilon)}(\varsigma)\ \chi^+_{\mathcal{X}(\varepsilon)}(\varsigma) \geq \chi^+_{\mathcal{Y}(\varepsilon)}(\varsigma), \\ \varpi^-_{\mathcal{X}(\varepsilon)}(\varsigma) \geq \varpi^-_{\mathcal{Y}(\varepsilon)}(\varsigma),\ \upsilon^-_{\mathcal{X}(\varepsilon)}(\varsigma) \geq \upsilon^-_{\mathcal{Y}(\varepsilon)}(\varsigma)\ \chi^-_{\mathcal{X}(\varepsilon)}(\varsigma) \leq \chi^-_{\mathcal{Y}(\varepsilon)}(\varsigma) \end{array}\right\}$$

(2) $p(\varepsilon)(\varsigma) \subseteq q(\varepsilon)(\varsigma)$ if $\forall e \in E$,

$$\left\{\begin{array}{l} \varpi^+_{p(\varepsilon)}(\varsigma) \leq \varpi^+_{q(\varepsilon)}(\varsigma),\ \upsilon^+_{p(\varepsilon)}(\varsigma) \leq \upsilon^+_{q(\varepsilon)}(\varsigma)\ \chi^+_{p(\varepsilon)}(\varsigma) \geq \chi^+_{q(\varepsilon)}(\varsigma), \\ \varpi^-_{p(\varepsilon)}(\varsigma) \geq \varpi^-_{q(\varepsilon)}(\varsigma),\ \upsilon^-_{p(\varepsilon)}(\varsigma) \geq \upsilon^-_{q(\varepsilon)}(\varsigma)\ \chi^-_{p(\varepsilon)}(\varsigma) \leq \chi^-_{q(\varepsilon)}(\varsigma) \end{array}\right\}.$$

Example 3.6. Consider the PNBSS $\mathcal{X}_p^{\mathcal{NB}}$ in the Example 3.4. A PNBSS might be represented as follows:

$$\mathcal{Y}_q^{\mathcal{NB}}(\varepsilon_1) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.65,0.7,0.45,-0.55,-0.8,-0.65),(0.55,0.9,0.35,-0.85,-0.75,-0.15)\rangle} \\ \frac{u_2}{\langle(0.85,0.8,0.15,-0.75,-0.7,-0.35),(0.85,0.7,0.15,-0.65,-0.6,-0.35)\rangle} \\ \frac{u_3}{\langle(0.85,0.9,0.05,-0.65,-0.6,-0.75),(0.75,0.8,0.25,-0.85,-0.8,-0.25)\rangle} \end{array}\right\};$$

$$\mathcal{Y}_q^{\mathcal{NB}}(\varepsilon_2) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.75,0.7,0.25,-0.35,-0.8,-0.55),(0.85,0.7,0.05,-0.75,-0.6,-0.25)\rangle} \\ \frac{u_2}{\langle(0.55,0.6,0.65,-0.7,-0.75,-0.25),(0.65,0.6,0.35,-0.65,-0.8,-0.35)\rangle} \\ \frac{u_3}{\langle(0.65,0.5,0.35,-0.25,-0.4,-0.65),(0.85,0.7,0.15,-0.75,-0.6,-0.45)\rangle} \end{array}\right\};$$

$$\mathcal{Y}_q^{\mathcal{NB}}(\varepsilon_3) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.45,0.7,0.55,-0.85,-0.8,-0.25),(0.65,0.6,0.35,-0.85,-0.7,-0.05)\rangle} \\ \frac{u_2}{\langle(0.75,0.6,0.25,-0.75,-0.7,-0.15),(0.75,0.7,0.25,-0.65,-0.6,-0.25)\rangle} \\ \frac{u_3}{\langle(0.85,0.5,0.05,-0.6,-0.65,-0.45),(0.85,0.8,0.35,-0.85,-0.4,-0.15)\rangle} \end{array}\right\}.$$

Definition 3.7. The complement of $\mathcal{X}_p^{\mathcal{NB}}$ is $\mathcal{X}_{p^c}^{\mathcal{NB}} = \left\langle \mathcal{BF}^c(\varepsilon)(\varsigma), p^c(\varepsilon)(\varsigma)\right\rangle$, where

$$\begin{aligned} \mathcal{BF}^c(\varepsilon)(\varsigma) &= (\chi^+_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^+_{\mathcal{X}(\varepsilon)}(\varsigma), \varpi^+_{\mathcal{X}(\varepsilon)}(\varsigma), \chi^-_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^-_{\mathcal{X}(\varepsilon)}(\varsigma), \varpi^-_{\mathcal{X}(\varepsilon)}(\varsigma)), \\ p^c(\varepsilon)(\varsigma) &= (\chi^+_{p(\varepsilon)}(\varsigma), \upsilon^+_{p(\varepsilon)}(\varsigma), \varpi^+_{p(\varepsilon)}(\varsigma), \chi^-_{p(\varepsilon)}(\varsigma), \upsilon^-_{p(\varepsilon)}(\varsigma), \varpi^-_{p(\varepsilon)}(\varsigma)). \end{aligned}$$

Here $\mathcal{X}_{(p^c)^c}^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}}$.

Definition 3.8. The union and intersection of $\mathcal{X}_p^{\mathcal{NB}}$ and $\mathcal{Y}_q^{\mathcal{NB}}$ are defined as $T_v : E \to \mathcal{PNB}^U \times \mathcal{PNB}^U$, $S_w : E \to \mathcal{PNB}^U \times \mathcal{PNB}^U$ such that $T_v(\varepsilon)(\varsigma) = (T(\varepsilon)(\varsigma), v(\varepsilon)(\varsigma))$, $S_w(\varepsilon)(\varsigma) = (S(\varepsilon)(\varsigma), w(\varepsilon)(\varsigma))$, where $T(\varepsilon)(\varsigma) = \mathcal{X}(\varepsilon)(\varsigma) \cup \mathcal{Y}(\varepsilon)(\varsigma)$, $v(\varepsilon)(\varsigma) = p(\varepsilon)(\varsigma) \cup q(\varepsilon)(\varsigma)$, $S(\varepsilon)(\varsigma) = \mathcal{X}(\varepsilon)(\varsigma) \cap \mathcal{Y}(\varepsilon)(\varsigma)$ and $w(\varepsilon)(\varsigma) = p(\varepsilon)(\varsigma) \cap q(\varepsilon)(\varsigma), \forall \varsigma \in U$.

Example 3.9. Let $\mathcal{X}_p^{\mathcal{NB}}$ and $\mathcal{Y}_q^{\mathcal{NB}}$ be the two PNBSSs. Let $\mathcal{X}_q^{\mathcal{NB}}$ and $\mathcal{Y}_q^{\mathcal{NB}}$ is defined as follows:

$$\mathcal{X}_q^{\mathcal{NB}}(\varepsilon_1) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.65,0.7,0.7,-0.55,-0.8,-0.65),(0.55,0.9,0.35,-0.85,-0.75,-0.15)\rangle} \\ \frac{u_2}{\langle(0.9,0.7,0.4,-0.75,-0.7,-0.5),(0.8,0.7,0.3,-0.65,-0.6,-0.5)\rangle} \\ \frac{u_3}{\langle(0.8,0.7,0.5,-0.2,-0.8,-0.9),(0.7,0.6,0.4,-0.8,-0.35,-0.6)\rangle} \end{array}\right\};$$

$$\mathcal{X}_q^{\mathcal{NB}}(\varepsilon_2) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.7,0.85,0.4,-0.2,-0.7,-0.8),(0.9,0.7,0.2,-0.7,-0.6,-0.5)\rangle} \\ \frac{u_2}{\langle(0.3,0.6,0.4,-0.4,-0.6,-0.8),(0.6,0.6,0.35,-0.6,-0.8,-0.5)\rangle} \\ \frac{u_3}{\langle(0.65,0.6,0.3,-0.2,-0.7,-0.9),(0.8,0.7,0.3,-0.7,-0.6,-0.6)\rangle} \end{array}\right\};$$

$$\mathcal{Y}_q^{\mathcal{NB}}(\varepsilon_1) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.3,0.35,0.35,-0.15,-0.75,-0.25),(0.6,0.85,0.35,-0.25,-0.5,-0.5)\rangle} \\ \frac{u_2}{\langle(0.4,0.75,0.5,-0.7,-0.25,-0.2),(0.6,0.25,0.2,-0.35,-0.6,-0.2)\rangle} \\ \frac{u_3}{\langle(0.6,0.65,0.2,-0.1,-0.25,-0.4),(0.4,0.6,0.3,-0.5,-0.7,-0.6)\rangle} \end{array}\right\};$$

$$\mathcal{Y}_q^{\mathcal{NB}}(\varepsilon_2) = \left\{\begin{array}{c} \frac{u_1}{\langle(0.8,0.85,0.7,-0.4,-0.25,-0.3),(0.2,0.75,0.1,-0.3,-0.6,-0.4)\rangle} \\ \frac{u_2}{\langle(0.6,0.65,0.9,-0.3,-0.65,-0.4),(0.3,0.45,0.35,-0.2,-0.75,-0.8)\rangle} \\ \frac{u_3}{\langle(0.45,0.7,0.6,-0.5,-0.35,-0.4),(0.4,0.7,0.3,-0.6,-0.55,-0.9)\rangle} \end{array}\right\}.$$

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Then

$$\left(\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}}\right)(\varepsilon_1) = \left\{\begin{array}{l} \langle(0.65, 0.7, 0.35, -0.55, -0.8, -0.25), (0.6, 0.9, 0.35, -0.85, -0.5, -0.15)\rangle \\ \langle(0.9, 0.75, 0.4, -0.75, -0.7, -0.2), (0.8, 0.7, 0.2, -0.65, -0.6, -0.2)\rangle \\ \langle(0.8, 0.7, 0.2, -0.2, -0.8, -0.4), (0.7, 0.6, 0.3, -0.8, -0.7, -0.6)\rangle \end{array}\right\},$$

$$\left(\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}}\right)(\varepsilon_2) = \left\{\begin{array}{l} \langle(0.8, 0.85, 0.4, -0.4, -0.7, -0.3), (0.9, 0.75, 0.1, -0.7, -0.6, -0.4)\rangle \\ \langle(0.6, 0.65, 0.4, -0.4, -0.65, -0.4), (0.6, 0.6, 0.35, -0.6, -0.8, -0.5)\rangle \\ \langle(0.65, 0.7, 0.3, -0.5, -0.7, -0.4), (0.8, 0.7, 0.3, -0.7, -0.6, -0.6)\rangle \end{array}\right\}$$

and

$$\left(\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}}\right)(\varepsilon_1) = \left\{\begin{array}{l} \langle(0.3, 0.35, 0.7, -0.5, -0.75, -0.65), (0.55, 0.85, 0.35, -0.25, -0.75, -0.5)\rangle \\ \langle(0.4, 0.7, 0.5, -0.7, -0.25, -0.5), (0.6, 0.25, 0.3, -0.35, -0.6, -0.5)\rangle \\ \langle(0.6, 0.650.5, -0.1, -0.25, -0.9), (0.4, 0.6, 0.4, -0.5, -0.35, -0.6)\rangle \end{array}\right\},$$

$$\left(\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}}\right)(\varepsilon_2) = \left\{\begin{array}{l} \langle(0.7, 0.85, 0.7, -0.2, -0.25, -0.8), (0.2, 0.7, 0.2, -0.3, -0.6, -0.5)\rangle \\ \langle(0.3, 0.6, 0.9, -0.3, -0.6, -0.8), (0.3, 0.45, 0.35, -0.2, -0.75, -0.8)\rangle \\ \langle(0.45, 0.6, 0.6, -0.2, -0.35, -0.9), (0.4, 0.7, 0.3, -0.6, -0.55, -0.9)\rangle \end{array}\right\}.$$

Definition 3.10. A PNBSS $\Delta_\delta^{\mathcal{NB}}(\varepsilon)(\varsigma) = \left\langle \Delta(\varepsilon)(\varsigma), \delta(\varepsilon)(\varsigma) \right\rangle$ is called a possibility null NBSS $\Delta_\delta^{\mathcal{NB}} : E \to \mathcal{PNB}^U \times \mathcal{PNB}^U$, where $\Delta^+(\varepsilon)(\varsigma) = (0, 0, 1)$, $\delta^+(\varepsilon)(\varsigma) = (0, 0, 1)$ and $\Delta^-(\varepsilon)(\varsigma) = (-1, -1, 0)$ and $\delta^-(\varepsilon)(\varsigma) = (-1, -1, 0),\ \forall \varsigma \in U$.

Definition 3.11. A PNBSS $\Phi_\omega^{\mathcal{NB}}(\varepsilon)(\varsigma) = \left\langle \Phi(\varepsilon)(\varsigma), \omega(\varepsilon)(\varsigma) \right\rangle$ is called a possibility absolute NBSS $\Phi_\omega^{\mathcal{NB}} : E \to \mathcal{PNB}^U \times \mathcal{PNB}^U$, where $\Phi^+(\varepsilon)(\varsigma) = (1, 1, 0)$, $\omega^+(\varepsilon)(\varsigma) = (1, 1, 0)$ and $\Phi^-(\varepsilon)(\varsigma) = (0, 0, -1)$ and $\omega^-(\varepsilon)(\varsigma) = (0, 0, -1), \forall \varsigma \in U$.

Theorem 3.12. *Let $\mathcal{X}_p^{\mathcal{NB}}$ be a PNBSS. Then the following properties are holds:*
(1) $\mathcal{X}_p^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{X}_p^{\mathcal{NB}},\ \mathcal{X}_p^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{X}_p^{\mathcal{NB}}$
(2) $\mathcal{X}_p^{\mathcal{NB}} \subseteq \mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{X}_p^{\mathcal{NB}},\ \mathcal{X}_p^{\mathcal{NB}} \subseteq \mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{X}_p^{\mathcal{NB}}$
(3) $\mathcal{X}_p^{\mathcal{NB}} \cup \Delta_\delta^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}},\ \mathcal{X}_p^{\mathcal{NB}} \cap \Delta_\delta^{\mathcal{NB}} = \Delta_\delta^{\mathcal{NB}}$
(4) $\mathcal{X}_p^{\mathcal{NB}} \cup \Phi_\omega^{\mathcal{NB}} = \Phi_\omega^{\mathcal{NB}},\ \mathcal{X}_p^{\mathcal{NB}} \cap \Phi_\omega^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}}$.

Proof. Based on Definitions 3.7, 3.10 and 3.11, the proof is given. □

Remark 3.13. Let $\mathcal{X}_p^{\mathcal{NB}}$ be a PNBSS on (U, E). If $\mathcal{X}_p^{\mathcal{NB}} \neq \Phi_\omega^{\mathcal{NB}}$ or $\mathcal{X}_p^{\mathcal{NB}} \neq \Delta_\delta^{\mathcal{NB}}$, then $\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{X}_{p^c}^{\mathcal{NB}} \neq \Phi_\omega^{\mathcal{NB}}$ and $\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{X}_{p^c}^{\mathcal{NB}} \neq \Delta_\delta^{\mathcal{NB}}$.

Theorem 3.14. *Let $\mathcal{X}_p^{\mathcal{NB}}$, $\mathcal{Y}_q^{\mathcal{NB}}$ and $\mathcal{Z}_r^{\mathcal{NB}}$ be any three PNBSSs over (U, E). Then*
(1) $\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}} = \mathcal{Y}_q^{\mathcal{NB}} \cup \mathcal{X}_p^{\mathcal{NB}}$
(2) $\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}} = \mathcal{Y}_q^{\mathcal{NB}} \cap \mathcal{X}_p^{\mathcal{NB}}$
(3) $\mathcal{X}_p^{\mathcal{NB}} \cup (\mathcal{Y}_q^{\mathcal{NB}} \cup \mathcal{Z}_r^{\mathcal{NB}}) = (\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}}) \cup \mathcal{Z}_r^{\mathcal{NB}}$
(4) $\mathcal{X}_p^{\mathcal{NB}} \cap (\mathcal{Y}_q^{\mathcal{NB}} \cap \mathcal{Z}_r^{\mathcal{NB}}) = (\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}}) \cap \mathcal{Z}_r^{\mathcal{NB}}$
(5) $(\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}})^c = \mathcal{X}_{p^c}^{\mathcal{NB}} \cap \mathcal{Y}_{q^c}^{\mathcal{NB}}$
(6) $(\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}})^c = \mathcal{X}_{p^c}^{\mathcal{NB}} \cup \mathcal{Y}_{q^c}^{\mathcal{NB}}$
(7) $(\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}}) \cap \mathcal{X}_p^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}}$
(8) $(\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}}) \cup \mathcal{X}_p^{\mathcal{NB}} = \mathcal{X}_p^{\mathcal{NB}}$
(9) $\mathcal{X}_p^{\mathcal{NB}} \cup (\mathcal{Y}_q^{\mathcal{NB}} \cap \mathcal{Z}_r^{\mathcal{NB}}) = (\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Y}_q^{\mathcal{NB}}) \cap (\mathcal{X}_p^{\mathcal{NB}} \cup \mathcal{Z}_r^{\mathcal{NB}})$
(10) $\mathcal{X}_p^{\mathcal{NB}} \cap (\mathcal{Y}_q^{\mathcal{NB}} \cup \mathcal{Z}_r^{\mathcal{NB}}) = (\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Y}_q^{\mathcal{NB}}) \cup (\mathcal{X}_p^{\mathcal{NB}} \cap \mathcal{Z}_r^{\mathcal{NB}})$.

Proof. The proof follows from Definition 3.7 and 3.8. (9) Now,

$$\begin{aligned} \mathcal{X}_p(\varepsilon)(\varsigma) \cup (\mathcal{Y}_q(\varepsilon)(\varsigma) \cap \mathcal{Z}_r(\varepsilon)(\varsigma)) &= \mathcal{X}_p(\varepsilon)(\varsigma) \cup \mathcal{Z}_h(\varepsilon)(\varsigma) \\ &= \mathcal{M}_m(\varepsilon)(\varsigma) \\ &= (\mathcal{M}(\varepsilon)(\varsigma), m(\varepsilon)(\varsigma)) \\ &= (\mathcal{X}(\varepsilon)(\varsigma) \cup \mathcal{Z}(\varepsilon)(\varsigma)), (p(\varepsilon)(\varsigma) \cup h(\varepsilon)(\varsigma)) \end{aligned}$$

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

$$
\begin{aligned}
(\mathcal{X}(\varepsilon)(\varsigma) \cup \mathcal{Z}(\varepsilon)(\varsigma)) &= \max\left\{\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}, \max\left\{\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}, \min\left\{\chi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \chi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\} \\
&= \max\left\{\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \min\left\{\varpi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma), \varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \max\left\{\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \min\left\{\upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma), \upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \min\left\{\chi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \max\left\{\chi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma), \chi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\} \\
&= \min\left\{\max\left\{\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \varpi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma)\right\}, \max\left\{\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \min\left\{\max\left\{\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma)\right\}, \max\left\{\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \max\left\{\min\left\{\chi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \chi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma)\right\}, \min\left\{\chi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma), \chi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\} \\
&= \min\left\{\left\{\varpi^{+}_{\mathcal{X}(\varepsilon)\cup\mathcal{Y}(\varepsilon)}(\varsigma)\right\}, \left\{\varpi^{+}_{\mathcal{X}(\varepsilon)\cup\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \min\left\{\left\{\upsilon^{+}_{\mathcal{X}(\varepsilon)\cup\mathcal{Y}(\varepsilon)}(\varsigma)\right\}, \left\{\upsilon^{+}_{\mathcal{X}(\varepsilon)\cup\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \max\left\{\left\{\chi^{+}_{\mathcal{X}(\varepsilon)\cap\mathcal{Y}(\varepsilon)}(\varsigma)\right\}, \left\{\chi^{+}_{\mathcal{X}(\varepsilon)\cap\mathcal{Z}(\varepsilon)}(\varsigma)\right\}\right\} \\
&= \left\{\varpi^{+}_{(\mathcal{X}(\varepsilon)\cup\mathcal{Y}(\varepsilon))\cap(\mathcal{X}(\varepsilon)\cup\mathcal{Z}(\varepsilon))}(\varsigma)\right\}, \left\{\chi^{+}_{(\mathcal{X}(\varepsilon)\cap\mathcal{Y}(\varepsilon))\cup(\mathcal{X}(\varepsilon)\cap\mathcal{Z}(\varepsilon))}(\varsigma)\right\}.
\end{aligned}
$$

Similarly, to prove that $\left\{\varpi^{-}_{(\mathcal{X}(\varepsilon)\cup\mathcal{Y}(\varepsilon))\cap(\mathcal{X}(\varepsilon)\cup\mathcal{Z}(\varepsilon))}(\varsigma)\right\}, \left\{\chi^{-}_{(\mathcal{X}(\varepsilon)\cap\mathcal{Y}(\varepsilon))\cup(\mathcal{X}(\varepsilon)\cap\mathcal{Z}(\varepsilon))}(\varsigma)\right\}$ and

$$
\begin{aligned}
(p(\varepsilon)(\varsigma) \cup h(\varepsilon)(\varsigma)) &= \max\left\{\varpi^{+}_{p(\varepsilon)}(\varsigma), \varpi^{+}_{h(\varepsilon)}(\varsigma)\right\}, \max\left\{\upsilon^{+}_{p(\varepsilon)}(\varsigma), \upsilon^{+}_{h(\varepsilon)}(\varsigma)\right\}, \min\left\{\chi^{+}_{p(\varepsilon)}(\varsigma), \chi^{+}_{h(\varepsilon)}(\varsigma)\right\} \\
&= \max\left\{\varpi^{+}_{p(\varepsilon)}(\varsigma), \min\left\{\varpi^{+}_{q(\varepsilon)}(\varsigma), \varpi^{+}_{r(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \max\left\{\upsilon^{+}_{p(\varepsilon)}(\varsigma), \min\left\{\upsilon^{+}_{q(\varepsilon)}(\varsigma), \upsilon^{+}_{r(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \min\left\{\chi^{+}_{p(\varepsilon)}(\varsigma), \max\left\{\chi^{+}_{q(\varepsilon)}(\varsigma), \chi^{+}_{r(\varepsilon)}(\varsigma)\right\}\right\} \\
&= \min\left\{\max\left\{\varpi^{+}_{p(\varepsilon)}(\varsigma), \varpi^{+}_{q(\varepsilon)}(\varsigma)\right\}, \max\left\{\varpi^{+}_{p(\varepsilon)}(\varsigma), \varpi^{+}_{r(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \min\left\{\max\left\{\upsilon^{+}_{p(\varepsilon)}(\varsigma), \upsilon^{+}_{q(\varepsilon)}(\varsigma)\right\}, \max\left\{\upsilon^{+}_{p(\varepsilon)}(\varsigma), \upsilon^{+}_{r(\varepsilon)}(\varsigma)\right\}\right\}, \\
&\quad \max\left\{\min\left\{\chi^{+}_{p(\varepsilon)}(\varsigma), \chi^{+}_{q(\varepsilon)}(\varsigma)\right\}, \min\left\{\chi^{+}_{p(\varepsilon)}(\varsigma), \chi^{+}_{r(\varepsilon)}(\varsigma)\right\}\right\}
\end{aligned}
$$

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

$$= \min\left\{\left\{\varpi^{+}_{p(\varepsilon)\cup q(\varepsilon)}(\varsigma)\right\},\left\{\varpi^{+}_{p(\varepsilon)\cup r(\varepsilon)}(\varsigma)\right\}\right\},$$
$$\min\left\{\left\{\upsilon^{+}_{p(\varepsilon)\cup q(\varepsilon)}(\varsigma)\right\},\left\{\upsilon^{+}_{p(\varepsilon)\cup r(\varepsilon)}(\varsigma)\right\}\right\},$$
$$\max\left\{\left\{\chi^{+}_{p(\varepsilon)\cap q(\varepsilon)}(\varsigma)\right\},\left\{\chi^{+}_{p(\varepsilon)\cap r(\varepsilon)}(\varsigma)\right\}\right\}$$
$$= \left\{\varpi^{+}_{(p(\varepsilon)\cup q(\varepsilon))\cap(p(\varepsilon)\cup r(\varepsilon))}(\varsigma)\right\},\left\{\chi^{+}_{(p(\varepsilon)\cap q(\varepsilon))\cup(p(\varepsilon)\cap r(\varepsilon))}(\varsigma)\right\}.$$

Similarly, to prove that $\left\{\varpi^{-}_{(p(\varepsilon)\cup q(\varepsilon))\cap(p(\varepsilon)\cup r(\varepsilon))}(\varsigma)\right\},\left\{\chi^{-}_{(p(\varepsilon)\cap q(\varepsilon))\cup(p(\varepsilon)\cap r(\varepsilon))}(\varsigma)\right\}$.

Hence, $\mathcal{X}\cup(\mathcal{Y}\cap\mathcal{Z})=(\mathcal{X}\cup\mathcal{Y})\cap(\mathcal{X}\cup\mathcal{Z})$. □

Definition 3.15. Let $(\mathcal{X}_p^{\mathcal{NB}},\Lambda)$ and $(\mathcal{Y}_q^{\mathcal{NB}},\Xi)$ be any two PNBSSs. Then $(\mathcal{X}_p^{\mathcal{NB}},\Lambda)\wedge(\mathcal{Y}_q^{\mathcal{NB}},\Xi)=(\mathcal{Z}_r^{\mathcal{NB}},\Lambda\times\Xi)$, where $\mathcal{Z}_r^{\mathcal{NB}}(\tau,\sigma)=(\mathcal{Z}(\tau,\sigma)(\varsigma),r(\tau,\sigma)(\varsigma))$ such that $\mathcal{Z}(\tau,\sigma)=\mathcal{X}(\tau)\cap\mathcal{Y}(\sigma)$ and $r(\tau,\sigma)=p(\tau)\cap q(\sigma),\forall(\tau,\sigma)\in\Lambda\times\Xi$.

Definition 3.16. Let $(\mathcal{X}_p^{\mathcal{NB}},\Lambda)$ and $(\mathcal{Y}_q^{\mathcal{NB}},\Xi)$ be two PNBSSs. Then $(\mathcal{X}_p^{\mathcal{NB}},\Lambda)\vee(\mathcal{Y}_q^{\mathcal{NB}},\Xi)=(\mathcal{Z}_r^{\mathcal{NB}},\Lambda\times\Xi)$, where $\mathcal{Z}_r^{\mathcal{NB}}(\tau,\sigma)=(\mathcal{Z}(\tau,\sigma)(\varsigma),r(\tau,\sigma)(\varsigma))$ such that $\mathcal{Z}(\tau,\sigma)=\mathcal{X}(\tau)\cup\mathcal{Y}(\sigma)$ and $r(\tau,\sigma)=p(\tau)\cup q(\sigma),\forall(\tau,\sigma)\in\Lambda\times\Xi$.

Theorem 3.17. *Let $(\mathcal{X}_p^{\mathcal{NB}},\Lambda)$ and $(\mathcal{Y}_q^{\mathcal{NB}},\Xi)$ be any two PNBSSs. Then*
(1) $((\mathcal{X}_p^{\mathcal{NB}},\Lambda)\wedge(\mathcal{Y}_q^{\mathcal{NB}},\Xi))^c=(\mathcal{X}_{p^c}^{\mathcal{NB}},\Lambda)\vee(\mathcal{Y}_{q^c}^{\mathcal{NB}},\Xi)$
(2) $((\mathcal{X}_p^{\mathcal{NB}},\Lambda)\vee(\mathcal{Y}_q^{\mathcal{NB}},\Xi))^c=(\mathcal{X}_{p^c}^{\mathcal{NB}},\Lambda)\wedge(\mathcal{Y}_{q^c}^{\mathcal{NB}},\Xi)$.

Proof. (1) Suppose that $(\mathcal{X}_p^{\mathcal{NB}},\Lambda)\wedge(\mathcal{Y}_q^{\mathcal{NB}},\Xi)=(\mathcal{Z}_r^{\mathcal{NB}},\Lambda\times\Xi)$. Now,

$$\mathcal{Z}_{r^c}^{\mathcal{NB}}(\tau,\sigma)=(\mathcal{Z}^c(\tau,\sigma)(\varsigma),r^c(\tau,\sigma)(\varsigma)),\forall(\tau,\sigma)\in\Lambda\times\Xi.$$

By Theorem 3.14 and Definition 3.15, $\mathcal{Z}^c(\tau,\sigma)=(\mathcal{X}(\tau)\cap\mathcal{Y}(\sigma))^c=\mathcal{X}^c(\tau)\cup\mathcal{Y}^c(\sigma)$ and $r^c(\tau,\sigma)=(p(\tau)\cap q(\sigma))^c=p^c(\tau)\cup q^c(\sigma)$. On the other side, $(\mathcal{X}_{p^c}^{\mathcal{NB}},\Lambda)\vee(\mathcal{Y}_{q^c}^{\mathcal{NB}},\Xi)=(\omega_o,\Lambda\times\Xi)$, where $\omega_o(\tau,\sigma)=(\omega(\tau,\sigma)(\varsigma),o(\tau,\sigma)(\varsigma))$ such that $\omega(\tau,\sigma)=\mathcal{X}^c(\tau)\cup\mathcal{Y}^c(\sigma)$ and $o(\tau,\sigma)=p^c(\tau)\cup q^c(\sigma),\forall(\tau,\sigma)\in\Lambda\times\Xi$. Thus, $\mathcal{Z}_r^c=\omega_o$. Hence, $((\mathcal{X}_p^{\mathcal{NB}},\Lambda)\wedge(\mathcal{Y}_q^{\mathcal{NB}},\Xi))^c=(\mathcal{X}_{p^c}^{\mathcal{NB}},\Lambda)\vee(\mathcal{Y}_{q^c}^{\mathcal{NB}},\Xi)$. Similarly to prove other parts. □

4 Similarity measure between two PNBSSs

Formula: Let U be a non-empty set of the universe and E be a set of parameters. Suppose that $\mathcal{X}_p^{\mathcal{NB}}$ and $\mathcal{Y}_q^{\mathcal{NB}}$ are two PNBSSs on (U,E). The similarity measure between two PNBSSs $\mathcal{X}_p^{\mathcal{NB}}$ and $\mathcal{Y}_q^{\mathcal{NB}}$ is denoted by $\mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}})$ and is defined as $\mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}})=\left[\Phi^{\mathcal{NB}}(\mathcal{X},\mathcal{Y})\cdot\Psi^{\mathcal{NB}}(p,q)\right]$ such that

$\Phi^{\mathcal{NB}}(\mathcal{X},\mathcal{Y})=\frac{\mathbb{T}_1^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))+\mathbb{T}_2^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))+\mathbb{S}^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))}{3}$ and

$\Psi^{\mathcal{NB}}(p,q)=1-\frac{\sum|(\xi_{1i}+\xi_{2i})-(\tau_{1i}+\tau_{2i})|}{\sum|(\xi_{1i}+\xi_{2i})+(\tau_{1i}+\tau_{2i})|}$, where

$$\mathbb{T}_1^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))=\frac{\sum_{i=1}^{n}\left[\left[\varpi^{+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\varpi^{+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\varpi^{-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\right]\right]}{\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-\varpi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(1-\varpi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\right]}\right]+\left[1+\sqrt{\left[(-1+\varpi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(-1+\varpi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\right]}\right]\right]},$$

$$\mathbb{T}_2^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))=\frac{\sum_{i=1}^{n}\left[\left[\upsilon^{+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\upsilon^{+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\right]+\left[\upsilon^{-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\upsilon^{-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\right]\right]}{\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-\upsilon^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(1-\upsilon^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\right]}\right]+\left[1+\sqrt{\left[(-1+\upsilon^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(-1+\upsilon^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\right]}\right]\right]},$$

$$\mathbb{S}^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))=\sqrt{1-\frac{\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\right|\right]}{\sum_{i=1}^{n}\left[\left[1+\left[(\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\right]\right]+\left[1+\left[(\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\right]\right]\right]}},$$

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

$$\xi_{1i}=\frac{\varpi^{2+}_{p(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{p(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{p(\varepsilon_i)}(\varsigma)+\chi^{2+}_{p(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{p(\varepsilon_i)}(\varsigma)+\chi^{2-}_{p(\varepsilon_i)}(\varsigma)\right]},\quad \tau_{1i}=\frac{\varpi^{2+}_{q(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{q(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{q(\varepsilon_i)}(\varsigma)+\chi^{2+}_{q(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{q(\varepsilon_i)}(\varsigma)+\chi^{2-}_{q(\varepsilon_i)}(\varsigma)\right]},$$
$$\xi_{2i}=\frac{\varpi^{2+}_{p(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{p(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{p(\varepsilon_i)}(\varsigma)+\upsilon^{2+}_{p(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{p(\varepsilon_i)}(\varsigma)+\upsilon^{2-}_{p(\varepsilon_i)}(\varsigma)\right]},\quad \tau_{2i}=\frac{\varpi^{2+}_{q(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{q(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{q(\varepsilon_i)}(\varsigma)+\upsilon^{2+}_{q(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{q(\varepsilon_i)}(\varsigma)+\upsilon^{2-}_{q(\varepsilon_i)}(\varsigma)\right]}.$$

Theorem 4.1. *Let $\mathcal{X}_p^{\mathcal{NB}}$, $\mathcal{Y}_q^{\mathcal{NB}}$ and $\mathcal{Z}_r^{\mathcal{NB}}$ be the any three PNBSSs. Then*
(1) $\mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}}) = \mathrm{Sim}(\mathcal{Y}_q^{\mathcal{NB}},\mathcal{X}_p^{\mathcal{NB}})$
(2) $0\leq \mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}})\leq 1$
(3) $\mathcal{X}_p^{\mathcal{NB}}=\mathcal{Y}_q^{\mathcal{NB}}\Rightarrow \mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}})=1$
(4) $\mathcal{X}_p^{\mathcal{NB}}\subseteq\mathcal{Y}_q^{\mathcal{NB}}\subseteq\mathcal{Z}_r^{\mathcal{NB}}\Rightarrow \mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Z}_r^{\mathcal{NB}})\leq \mathrm{Sim}(\mathcal{Y}_q^{\mathcal{NB}},\mathcal{Z}_r^{\mathcal{NB}})$
(5) $\mathcal{X}_p^{\mathcal{NB}}\cap\mathcal{Y}_q^{\mathcal{NB}}=\{\phi\}\Leftrightarrow \mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}})=0.$

Proof. A trivial proof can be found in (1), (2), and (5). It remains to prove that (3). Let $\mathcal{X}_p^{\mathcal{NB}}=\mathcal{Y}_q^{\mathcal{NB}}$. Now,

$$\begin{aligned}\mathbb{T}_1^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma)) &= \frac{\sum_{i=1}^n\left[\varpi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]}{\sum_{i=1}^n\left[\left[1-1+\varpi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]+\left[1-1+\varpi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]\right]}\\ &= \frac{\sum_{i=1}^n\left[\varpi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]}{\sum_{i=1}^n\left[\varpi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]}\\ &= 1,\end{aligned}$$

$$\begin{aligned}\mathbb{T}_2^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma)) &= \frac{\sum_{i=1}^n\left[\upsilon^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)+\upsilon^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]}{\sum_{i=1}^n\left[\left[1-1+\upsilon^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]+\left[1-1+\upsilon^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]\right]}\\ &= \frac{\sum_{i=1}^n\left[\upsilon^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)+\upsilon^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]}{\sum_{i=1}^n\left[\upsilon^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)+\upsilon^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\right]}\\ &= 1,\end{aligned}$$

and $\mathbb{S}^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma),\mathcal{Y}(\varepsilon)(\varsigma))=\sqrt{1}=1$. Thus, $\varPhi^{\mathcal{NB}}(\mathcal{X},\mathcal{Y})=\frac{1+1+1}{3}=1$ and $\varPsi^{\mathcal{NB}}(p,q)=1$. Hence, $\mathrm{Sim}(\mathcal{X}_p^{\mathcal{NB}},\mathcal{Y}_q^{\mathcal{NB}})=1$.

(4) Given that

$$\left\{\begin{array}{l}\mathcal{X}_p^{\mathcal{NB}}\subseteq\mathcal{Y}_q^{\mathcal{NB}}\Rightarrow\left\{\begin{array}{l}\varpi^+_{\mathcal{X}(\varepsilon)}(\varsigma)\leq\varpi^+_{\mathcal{Y}(\varepsilon)}(\varsigma),\ \upsilon^+_{\mathcal{X}(\varepsilon)}(\varsigma)\leq\upsilon^+_{\mathcal{Y}(\varepsilon)}(\varsigma),\ \chi^+_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\chi^+_{\mathcal{Y}(\varepsilon)}(\varsigma),\\ \varpi^-_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\varpi^-_{\mathcal{Y}(\varepsilon)}(\varsigma),\ \upsilon^-_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\upsilon^-_{\mathcal{Y}(\varepsilon)}(\varsigma),\ \chi^-_{\mathcal{X}(\varepsilon)}(\varsigma)\leq\chi^-_{\mathcal{Y}(\varepsilon)}(\varsigma),\\ \varpi^+_{p(\varepsilon)}(\varsigma)\leq\varpi^+_{q(\varepsilon)}(\varsigma),\ \upsilon^+_{p(\varepsilon)}(\varsigma)\leq\upsilon^+_{q(\varepsilon)}(\varsigma),\ \chi^+_{p(\varepsilon)}(\varsigma)\geq\chi^+_{q(\varepsilon)}(\varsigma),\\ \varpi^-_{p(\varepsilon)}(\varsigma)\geq\varpi^-_{q(\varepsilon)}(\varsigma),\ \upsilon^-_{p(\varepsilon)}(\varsigma)\geq\upsilon^-_{q(\varepsilon)}(\varsigma),\ \chi^-_{p(\varepsilon)}(\varsigma)\leq\chi^-_{q(\varepsilon)}(\varsigma),\end{array}\right\}\\ \mathcal{X}_p^{\mathcal{NB}}\subseteq\mathcal{Z}_r^{\mathcal{NB}}\Rightarrow\left\{\begin{array}{l}\varpi^+_{\mathcal{X}(\varepsilon)}(\varsigma)\leq\varpi^+_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \upsilon^+_{\mathcal{X}(\varepsilon)}(\varsigma)\leq\upsilon^+_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \chi^+_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\chi^+_{\mathcal{Z}(\varepsilon)}(\varsigma),\\ \varpi^-_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\varpi^-_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \upsilon^-_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\upsilon^-_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \chi^-_{\mathcal{X}(\varepsilon)}(\varsigma)\leq\chi^-_{\mathcal{Z}(\varepsilon)}(\varsigma),\\ \varpi^+_{p(\varepsilon)}(\varsigma)\leq\varpi^+_{r(\varepsilon)}(\varsigma),\ \upsilon^+_{p(\varepsilon)}(\varsigma)\leq\upsilon^+_{r(\varepsilon)}(\varsigma),\ \chi^+_{p(\varepsilon)}(\varsigma)\geq\chi^+_{r(\varepsilon)}(\varsigma),\\ \varpi^-_{p(\varepsilon)}(\varsigma)\geq\varpi^-_{r(\varepsilon)}(\varsigma),\ \upsilon^-_{p(\varepsilon)}(\varsigma)\geq\upsilon^-_{r(\varepsilon)}(\varsigma),\ \chi^-_{p(\varepsilon)}(\varsigma)\leq\chi^-_{r(\varepsilon)}(\varsigma),\end{array}\right\}\\ \mathcal{Y}_q^{\mathcal{NB}}\subseteq\mathcal{Z}_r^{\mathcal{NB}}\Rightarrow\left\{\begin{array}{l}\varpi^+_{\mathcal{Y}(\varepsilon)}(\varsigma)\leq\varpi^+_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \upsilon^+_{\mathcal{Y}(\varepsilon)}(\varsigma)\leq\upsilon^+_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \chi^+_{\mathcal{Y}(\varepsilon)}(\varsigma)\geq\chi^+_{\mathcal{Z}(\varepsilon)}(\varsigma),\\ \varpi^-_{\mathcal{Y}(\varepsilon)}(\varsigma)\geq\varpi^-_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \upsilon^-_{\mathcal{Y}(\varepsilon)}(\varsigma)\geq\upsilon^-_{\mathcal{Z}(\varepsilon)}(\varsigma),\ \chi^-_{\mathcal{Y}(\varepsilon)}(\varsigma)\leq\chi^-_{\mathcal{Z}(\varepsilon)}(\varsigma),\\ \varpi^+_{q(\varepsilon)}(\varsigma)\leq\varpi^+_{r(\varepsilon)}(\varsigma),\ \upsilon^+_{q(\varepsilon)}(\varsigma)\leq\upsilon^+_{r(\varepsilon)}(\varsigma),\ \chi^+_{q(\varepsilon)}(\varsigma)\geq\chi^+_{r(\varepsilon)}(\varsigma),\\ \varpi^-_{q(\varepsilon)}(\varsigma)\geq\varpi^-_{r(\varepsilon)}(\varsigma),\ \upsilon^-_{q(\varepsilon)}(\varsigma)\geq\upsilon^-_{r(\varepsilon)}(\varsigma),\ \chi^-_{q(\varepsilon)}(\varsigma)\leq\chi^-_{r(\varepsilon)}(\varsigma),\end{array}\right\}\end{array}\right\}\quad(*)$$

Clearly, $\varpi^+_{\mathcal{X}(\varepsilon)}(\varsigma)\cdot\varpi^+_{\mathcal{Z}(\varepsilon)}(\varsigma)\leq\varpi^+_{\mathcal{Y}(\varepsilon)}(\varsigma)\cdot\varpi^+_{\mathcal{Z}(\varepsilon)}(\varsigma)$ and $\varpi^-_{\mathcal{X}(\varepsilon)}(\varsigma)\cdot\varpi^-_{\mathcal{Z}(\varepsilon)}(\varsigma)\leq\varpi^-_{\mathcal{Y}(\varepsilon)}(\varsigma)\cdot\varpi^-_{\mathcal{Z}(\varepsilon)}(\varsigma)$ imply that

$$\begin{aligned}&\sum_{i=1}^n\left[\left[\varpi^+_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\varpi^+_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^-_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\varpi^-_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]\\ &\leq\sum_{i=1}^n\left[\left[\varpi^+_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\varpi^+_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^-_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\varpi^-_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right].\end{aligned}\quad(1)$$

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Clearly, $(\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2 \leq (\varpi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2$ and $(\varpi^{-}_{\mathcal{X}(\varepsilon)}(\varsigma))^2 \leq (\varpi^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2$. Thus, $\left[(1-(\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right] \geq \left[(1-(\varpi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]$ and

$$\left[1-\sqrt{\left[(1-(\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right] \leq \left[1-\sqrt{\left[(1-(\varpi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]. \quad (2)$$

Similarly,

$$\left[1-\sqrt{\left[(1-(\varpi^{-}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right] \leq \left[1-\sqrt{\left[(1-(\varpi^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]. \quad (3)$$

Equations (2) and (3), we get

$$\begin{aligned}&\textstyle\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-(\varpi^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]+\left[1-\sqrt{\left[(1-(\varpi^{-}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]\right]\\ &\leq\textstyle\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-(\varpi^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]+\left[1-\sqrt{\left[(1-(\varpi^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\varpi^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]\right]. \quad (4)\end{aligned}$$

Equations (1) and (4), we get

$$\begin{aligned}&\frac{\sum_{i=1}^{n}\left[\left[\varpi^{+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\varpi^{+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\varpi^{-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]}{\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-\varpi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(1-\varpi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]+\left[1-\sqrt{\left[(1-\varpi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(1-\varpi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]\right]}\\ &\leq\frac{\sum_{i=1}^{n}\left[\left[\varpi^{+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\varpi^{+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\varpi^{-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]}{\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-\varpi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\cdot(1-\varpi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]+\left[1-\sqrt{\left[(1-\varpi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\cdot(1-\varpi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]\right]}. \quad (5)\end{aligned}$$

Clearly, $\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma)\cdot\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma) \leq \upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma)\cdot\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma)$ and $\upsilon^{-}_{\mathcal{X}(\varepsilon)}(\varsigma)\cdot\upsilon^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma) \leq \upsilon^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma)\cdot\upsilon^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma)$ imply that

$$\begin{aligned}&\sum_{i=1}^{n}\left[\left[\upsilon^{+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\upsilon^{+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[\upsilon^{-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\upsilon^{-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]\\ &\leq\sum_{i=1}^{n}\left[\left[\upsilon^{+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\upsilon^{+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[\upsilon^{-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\upsilon^{-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]. \quad (6)\end{aligned}$$

Clearly, $(\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2 \leq (\upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2$ and $(\upsilon^{-}_{\mathcal{X}(\varepsilon)}(\varsigma))^2 \leq (\upsilon^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2$. Thus, $\left[(1-(\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right] \geq \left[(1-(\upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]$ and

$$\left[1-\sqrt{\left[(1-(\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right] \leq \left[1-\sqrt{\left[(1-(\upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]. \quad (7)$$

Similarly,

$$\left[1-\sqrt{\left[(1-(\upsilon^{-}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right] \leq \left[1-\sqrt{\left[(1-(\upsilon^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]. \quad (8)$$

Equations (7) and (8), we get

$$\begin{aligned}&\textstyle\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-(\upsilon^{+}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]+\left[1-\sqrt{\left[(1-(\upsilon^{-}_{\mathcal{X}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]\right]\\ &\leq\textstyle\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-(\upsilon^{+}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{+}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]+\left[1-\sqrt{\left[(1-(\upsilon^{-}_{\mathcal{Y}(\varepsilon)}(\varsigma))^2)\cdot(1-(\upsilon^{-}_{\mathcal{Z}(\varepsilon)}(\varsigma))^2)\right]}\;\right]\right]. \quad (9)\end{aligned}$$

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Equations (6) and (9), we get

$$\frac{\sum_{i=1}^{n}\left[\left[v^{+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot v^{+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[v^{-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot v^{-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]}{\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-v^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(1-v^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]+\left[1-\sqrt{\left[(1-v^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma))\cdot(1-v^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]\right]}$$
$$\leq\frac{\sum_{i=1}^{n}\left[\left[v^{+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot v^{+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]+\left[v^{-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot v^{-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right]\right]}{\sum_{i=1}^{n}\left[\left[1-\sqrt{\left[(1-v^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\cdot(1-v^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]+\left[1-\sqrt{\left[(1-v^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma))\cdot(1-v^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma))\right]}\right]\right]}. \quad (10)$$

Clearly, $\chi^{2+}_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\chi^{2+}_{\mathcal{Y}(\varepsilon)}(\varsigma)\geq\chi^{2+}_{\mathcal{Z}(\varepsilon)}(\varsigma)$ and $\chi^{2-}_{\mathcal{X}(\varepsilon)}(\varsigma)\geq\chi^{2-}_{\mathcal{Y}(\varepsilon)}(\varsigma)\geq\chi^{2-}_{\mathcal{Z}(\varepsilon)}(\varsigma)$.
Thus, $\left[\chi^{2+}_{\mathcal{X}(\varepsilon)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]\geq\left[\chi^{2+}_{\mathcal{Y}(\varepsilon)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]$ and $\left[\chi^{2-}_{\mathcal{X}(\varepsilon)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]\geq\left[\chi^{2-}_{\mathcal{Y}(\varepsilon)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]$.
Hence,

$$\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|\right]$$
$$\geq\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|\right]. \quad (11)$$

Also, $\left[\chi^{2+}_{\mathcal{X}(\varepsilon)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]\geq\left[\chi^{2+}_{\mathcal{Y}(\varepsilon)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]$and $\left[\chi^{2-}_{\mathcal{X}(\varepsilon)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]\geq\left[\chi^{2-}_{\mathcal{Y}(\varepsilon)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon)}(\varsigma)\right]$.
Hence,

$$\sum_{i=1}^{n}\left[\left[1+[\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]+\left[1+[\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]\right]$$
$$\geq\sum_{i=1}^{n}\left[\left[1+[\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]+\left[1+[\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]\right]. \quad (12)$$

Equations (11) and (12), we get

$$\frac{\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|\right]}{\sum_{i=1}^{n}\left[\left[1+[\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]+\left[1+[\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]\right]}$$
$$\geq\frac{\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|\right]}{\sum_{i=1}^{n}\left[\left[1+[\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]+\left[1+[\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]\right]}$$

and hence,

$$\sqrt{1-\frac{\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|\right]}{\sum_{i=1}^{n}\left[\left[1+[\chi^{2+}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]+\left[1+[\chi^{2-}_{\mathcal{X}(\varepsilon_i)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]\right]}}$$
$$\leq\sqrt{1-\frac{\sum_{i=1}^{n}\left[\left|\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)-\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|+\left|\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)-\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)\right|\right]}{\sum_{i=1}^{n}\left[\left[1+[\chi^{2+}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\chi^{2+}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]+\left[1+[\chi^{2-}_{\mathcal{Y}(\varepsilon_i)}(\varsigma)\cdot\chi^{2-}_{\mathcal{Z}(\varepsilon_i)}(\varsigma)]\right]\right]}}. \quad (13)$$

https://doi.org/10.54216/IJNS.230116
Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Equations (10) and (13), we get

$$\Phi^{\mathcal{NB}}(\mathcal{X},\mathcal{Z}) \leq \Phi^{\mathcal{NB}}(\mathcal{Y},\mathcal{Z}). \tag{14}$$

By Equation (*), we have $\xi_{1i} \leq \tau_{1i} \leq \delta_{1i}$ and $\xi_{2i} \leq \tau_{2i} \leq \delta_{2i}$, where

$\xi_{1i} = \frac{\varpi^{2+}_{p(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{p(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{p(\varepsilon_i)}(\varsigma) + \chi^{2+}_{p(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{p(\varepsilon_i)}(\varsigma) + \chi^{2-}_{p(\varepsilon_i)}(\varsigma)\right]}$, $\tau_{1i} = \frac{\varpi^{2+}_{q(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{q(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{q(\varepsilon_i)}(\varsigma) + \chi^{2+}_{q(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{q(\varepsilon_i)}(\varsigma) + \chi^{2-}_{q(\varepsilon_i)}(\varsigma)\right]}$,

$\xi_{2i} = \frac{\varpi^{2+}_{p(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{p(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{p(\varepsilon_i)}(\varsigma) + \upsilon^{2+}_{p(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{p(\varepsilon_i)}(\varsigma) + \upsilon^{2-}_{p(\varepsilon_i)}(\varsigma)\right]}$, $\tau_{2i} = \frac{\varpi^{2+}_{q(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{q(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{q(\varepsilon_i)}(\varsigma) + \upsilon^{2+}_{q(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{q(\varepsilon_i)}(\varsigma) + \upsilon^{2-}_{q(\varepsilon_i)}(\varsigma)\right]}$,

$\delta_{1i} = \frac{\varpi^{2+}_{r(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{r(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{r(\varepsilon_i)}(\varsigma) + \chi^{2+}_{r(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{r(\varepsilon_i)}(\varsigma) + \chi^{2-}_{r(\varepsilon_i)}(\varsigma)\right]}$, $\delta_{2i} = \frac{\varpi^{2+}_{r(\varepsilon_i)}(\varsigma)+\varpi^{2-}_{r(\varepsilon_i)}(\varsigma)}{\left[\varpi^{2+}_{r(\varepsilon_i)}(\varsigma) + \upsilon^{2+}_{r(\varepsilon_i)}(\varsigma)\right]+\left[\varpi^{2-}_{r(\varepsilon_i)}(\varsigma) + \upsilon^{2-}_{r(\varepsilon_i)}(\varsigma)\right]}$.

Clearly, $(\xi_{1i} + \xi_{2i}) - (\delta_{1i} + \delta_{2i}) \leq (\tau_{1i} + \tau_{2i}) - (\delta_{1i} + \delta_{2i})$.
Thus, $\left|(\tau_{1i} + \tau_{2i}) - (\delta_{1i} + \delta_{2i})\right| \leq \left|(\xi_{1i} + \xi_{2i}) - (\delta_{1i} + \delta_{2i})\right|$. Hence,

$$-\left|(\xi_{1i} + \xi_{2i}) - (\delta_{1i} + \delta_{2i})\right| \leq -\left|(\tau_{1i} + \tau_{2i}) - (\delta_{1i} + \delta_{2i})\right|. \tag{15}$$

Thus,

$$\left|(\xi_{1i} + \xi_{2i}) + \delta_i\right| \leq \left|(\tau_{1i} + \tau_{2i}) + \delta_i\right|. \tag{16}$$

Equations (15) and (16), we get

$\frac{-\left|(\xi_{1i}+\xi_{2i})-(\delta_{1i}+\delta_{2i})\right|}{\left|(\xi_{1i}+\xi_{2i})+(\delta_{1i}+\delta_{2i})\right|} \leq \frac{-\left|(\tau_{1i}+\tau_{2i})-(\delta_{1i}+\delta_{2i})\right|}{\left|(\tau_{1i}+\tau_{2i})+(\delta_{1i}+\delta_{2i})\right|} \Rightarrow 1 - \frac{\left|(\xi_{1i}+\xi_{2i})-(\delta_{1i}+\delta_{2i})\right|}{\left|(\xi_{1i}+\xi_{2i})+(\delta_{1i}+\delta_{2i})\right|} \leq 1 - \frac{\left|(\tau_{1i}+\tau_{2i})-(\delta_{1i}+\delta_{2i})\right|}{\left|(\tau_{1i}+\tau_{2i})+(\delta_{1i}+\delta_{2i})\right|}$.

Hence,

$$\Psi^{\mathcal{NB}}(p,r) \leq \Psi^{\mathcal{NB}}(q,r). \tag{17}$$

Equations (14) and (17), we have $\Phi^{\mathcal{NB}}(\mathcal{X},\mathcal{Z}) \cdot \Psi^{\mathcal{NB}}(p,r) \leq \Phi^{\mathcal{NB}}(\mathcal{Y},\mathcal{Z}) \cdot \Psi^{\mathcal{NB}}(q,r)$.

Hence, $\mathrm{Sim}(\mathcal{X}^{\mathcal{NB}}_p, \mathcal{Z}^{\mathcal{NB}}_r) \leq \mathrm{Sim}(\mathcal{Y}^{\mathcal{NB}}_q, \mathcal{Z}^{\mathcal{NB}}_r)$. □

Example 4.2. Calculate the similarity between the two PNBSSs such as $\mathcal{X}^{\mathcal{NB}}_p$ and $\mathcal{Y}^{\mathcal{NB}}_q$. We choose the first sample of PNBSS and parameter $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$.

$\mathcal{X}^{\mathcal{NB}}_p(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{X}(\varepsilon)$	$(0.65, 0.7, 0.65, -0.4, -0.9, -0.65)$	$(0.85, 0.75, 0.55, -0.8, -0.8, -0.6)$	$(0.75, 0.85, 0.65, -0.6, -0.9, -0.7)$
$p(\varepsilon)$	$(0.65, 0.6, 0.6, -0.75, -0.7, -0.45)$	$(0.85, 0.7, 0.45, -0.65, 0.75, -0.55)$	$(0.75, 0.8, 0.6, -0.75, -0.65 - 0.75)$

$\mathcal{Y}^{\mathcal{NB}}_q(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{Y}(\varepsilon)$	$(0.35, 0.5, 0.45, -0.4, -0.7, -0.4)$	$(0.45, 0.6, 0.65, -0.7, -0.8 - 0.3)$	$(0.7, 0.65, 0.35, -0.2, -0.6, -0.6)$
$q(\varepsilon)$	$(0.6, 0.45, 0.65, -0.45, -0.45, -0.35)$	$(0.7, 0.6, 0.35, -0.5, -0.5, -0.25)$	$(0.55, 0.7, 0.45, -0.65, -0.6, -0.75)$

Now, $\mathbb{T}^{\mathcal{NB}}_1(\mathcal{X}(\varepsilon)(\varsigma), \mathcal{Y}(\varepsilon)(\varsigma)) = \frac{X_1}{Y_1}$, where

$$X_1 = \left\{ \begin{array}{l} (0.65 \times 0.35) + (0.85 \times 0.45) + (0.75 \times 0.7) + \\ (-0.35 \times -0.4) + (-0.8 \times -0.65) + (-0.6 \times -0.15) \end{array} \right\} = 1.885$$

and

$$Y_1 = \left\{ \begin{array}{l} (1 - \sqrt{(1-0.65^2) \times (1-0.35^2)}) + (1 - \sqrt{(1-0.85^2) \times (1-0.45^2)}) \\ +(1 - \sqrt{(1-0.75^2) \times (1-0.7^2)}) + (1 - \sqrt{(1-(-0.35)^2) \times (1-(-0.4)^2)}) \\ +(1 - \sqrt{(1-(-0.8)^2) \times (1-(-0.65)^2)}) + (1 - \sqrt{(1-(-0.6)^2) \times (1-(-0.15)^2)}) \end{array} \right\} = 2.239884.$$

https://doi.org/10.54216/IJNS.230116
Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Hence, $\frac{X_1}{Y_1} = \frac{1.885}{2.239884} = 0.841561$. Now, $\mathbb{T}_2^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma), \mathcal{Y}(\varepsilon)(\varsigma)) = \frac{X_2}{Y_2}$, where

$$X_2 = \left\{ \begin{array}{l} (0.7 \times 0.5) + (0.75 \times 0.6) + (0.85 \times 0.65) + \\ (-0.55 \times -0.65) + (-0.75 \times -0.75) + (-0.85 \times -0.6) \end{array} \right\} = 2.7825$$

and

$$Y_2 = \left\{ \begin{array}{l} (1 - \sqrt{(1-0.7^2) \times (1-0.5^2)}) + (1 - \sqrt{(1-0.75^2) \times (1-0.6^2)}) \\ +(1 - \sqrt{(1-0.85^2) \times (1-0.65^2)}) + (1 - \sqrt{(1-(-0.55)^2) \times (1-(-0.65)^2)}) \\ +(1 - \sqrt{(1-(-0.75)^2) \times (1-(-0.75)^2)}) + (1 - \sqrt{(1-(-0.85)^2) \times (1-(-0.6)^2)}) \end{array} \right\} = 2.958467.$$

Hence, $\frac{X_2}{Y_2} = \frac{2.7825}{2.958467} = 0.940521$. Therefore,

$$\begin{aligned} \mathbb{S}^{\mathcal{NB}}(\mathcal{X}(\varepsilon)(\varsigma), \mathcal{Y}(\varepsilon)(\varsigma)) &= \sqrt{1 - \frac{1.63}{6.54785}} = 0.866639, \\ \Phi^{\mathcal{NB}}(\mathcal{X}, \mathcal{Y}) &= \frac{0.841561 + 0.940521 + 0.866639}{3} = 0.882907, \\ \Psi^{\mathcal{NB}}(p, q) &= 1 - \frac{0.334003}{6.836356} = 0.951143, \\ \text{Sim}(\mathcal{X}_p^{\mathcal{NB}}, \mathcal{Y}_q^{\mathcal{NB}}) &= 0.882907 \times 0.951143 = 0.839771. \end{aligned}$$

5 Application of PNBSS using soft model

5.1 Algorithm

In order to select the best choice, the following algorithm is used:

1. Enter the PPBFSS $\mathcal{X}_p^{\mathcal{NB}}$ in tabular form.
2. Choose parameters $A \subseteq E$.
3. Find $\mathbb{T}_1^{\mathcal{NB}}, \mathbb{T}_2^{\mathcal{NB}}$ and $\mathbb{S}^{\mathcal{NB}}$.
4. Find $\Phi^{\mathcal{NB}} = \frac{\mathbb{T}_1^{\mathcal{NB}} + \mathbb{T}_2^{\mathcal{NB}} + \mathbb{S}^{\mathcal{NB}}}{3}$.
5. $\Psi^{\mathcal{NB}} = 1 - \frac{\sum |\xi_i - \tau_i|}{\sum |\xi_i + \tau_i|}$ and $1 \leq i \leq 5$.
6. Calculate similarity using $\Phi^{\mathcal{NB}}$ and $\Psi^{\mathcal{NB}}$.
7. Determine maximum similarity $= \text{Max}\{\text{similarity}^i\}$, $1 \leq i \leq 5$.
8. The final decision involves choosing the best solution for the problem.

5.2 Survey Study

1. Luxury hotel : An exceptional guest experience is what distinguishes luxury hotels from other high-end accommodations. They offer top-notch guest service, top-of-the-line amenities, and luxurious amenities. Guests staying in these hotels are provided with an opulent, comfortable, and indulgent experience.
2. Business hotel : In business hotels, also known as corporate hotels or business traveller's hotels, accommodations are specifically tailored to meet the needs and preferences of travellers. In addition to providing a comfortable and productive environment, these hotels cater to individuals or groups travelling for business purposes.

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

3. Resort hotel : In its simplest form, a resort hotel offers guests a wide spectrum of leisure, recreational, and entertainment activities, all conveniently located on-site. In addition to offering an immersive vacation experience, resorts are usually located in scenic or exotic locations.
4. Elegant Hotel : Boutique hotels are small lodging establishments that are often owned or operated independently. Most travellers prefer small hotels because they offer a more intimate, authentic, and personalized experience. Small hotels accommodate different preferences and interests with a variety of options.
5. Budget hotel : Budget hotels are less expensive accommodation options for travellers seeking to save money on their accommodation expenses and are sometimes referred to as economy hotels. The goal of budget hotels is to provide clean, comfortable rooms with essential amenities without frills and luxuries commonly associated with upscale hotels.

Parameters :

1. Customer Service(ε_1): It is important for guests to feel welcome and comfortable during their stay. Any hotel can make a positive first impression by having a smiling, courteous staff member address guests by name.
2. Cleanliness and Hygiene(ε_2): Cleaning protocols for guest rooms should be rigorously followed by housekeeping staff. All surfaces should be thoroughly cleaned, bed linens changed, and the bathroom sanitized. Keeping guest rooms clean, functional, and free from damaged or worn fixtures, furniture, and equipment.
3. Food and Beverage(ε_3): There are also family-friendly restaurants that offer a diverse range of dishes in a relaxed and family-friendly atmosphere. Hotel restaurants usually offer upscale fine dining as well as casual breakfast buffets.
4. Location(ε_4): Hotel resorts located in city centres are located within easy access to a city's main attractions and business districts. Public transportation, corporate offices, shopping centres, restaurants, and cultural attractions are easily accessible. Beachfront resorts, mountain resorts, and tropical islands are examples of resorts in scenic locations. A complete vacation experience is provided by these resorts, which offer recreational and leisure activities.
5. Pricing and safety(ε_5): The price of a hotel depends on a variety of factors such as location, hotel type (e.g., budget, midrange, luxury), time of year, and time of booking. Additionally, special events, holidays, and peak seasons can affect pricing. The rates for hotel rooms can often be lowered if you book well in advance. Reservations made early often result in discounts.

Based on expert assessments against the criteria, we select the most appropriate option from a large number of alternatives.

Table 1
PNBSS for the ideal score

$\mathcal{L}_p^{\mathcal{NB}}(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{L}(\varepsilon)$	$(0.85, 0.9, 0.25, -0.85, -0.75, -0.35)$	$(0.9, 0.85, 0.35, -0.75, -0.8, -0.45)$	$(0.8, 0.8, 0.45, -0.85, -0.9, -0.35)$
$p(\varepsilon)$	$(1, 1, 0, -1, -1, 0)$	$(1, 1, 0, -1, -1, 0)$	$(1, 1, 0, -1, -1, 0)$

$\mathcal{L}_p^{\mathcal{NB}}(\varepsilon)$	ε_4	ε_5
$\mathcal{L}(\varepsilon)$	$(0.85, 0.85, 0.25, -0.85, -0.8, -0.5)$	$(0.75, 0.8, 0.35, -0.8, -0.95, -0.55)$
$p(\varepsilon)$	$(1, 1, 0, -1, -1, 0)$	$(1, 1, 0, -1, -1, 0)$

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

Table 2
PNBSS for the luxury hotel source

$\mathcal{A}^{\mathcal{NB}}_{p_1}(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{A}(\varepsilon)$	$(0.55, 0.8, 0.45, -0.65, -0.6, -0.85)$	$(0.85, 0.75, 0.65, -0.55, -0.75, -0.8)$	$(0.7, 0.65, 0.65, -0.65, -0.85, -0.65)$
$p_1(\varepsilon)$	$(0.85, 0.65, 0.25, -0.65, -0.75, -0.55)$	$(0.75, 0.7, 0.45, -0.75, -0.65, -0.7)$	$(0.65, 0.85, 0.65, -0.65, -0.6, -0.85)$

$\mathcal{A}^{\mathcal{NB}}_{p_1}(\varepsilon)$	ε_4	ε_5
$\mathcal{A}(\varepsilon)$	$(0.65, 0.75, 0.45, -0.75, -0.65, -0.8)$	$(0.6, 0.7, 0.55, -0.7, -0.8, -0.75)$
$p_1(\varepsilon)$	$(0.55, 0.65, 0.85, -0.75, -0.55, -0.65)$	$(0.45, 0.75, 0.65, -0.65, -0.45, -0.75)$

Table 3
PNBSS for the business hotel source

$\mathcal{NB}^{\mathcal{NB}}_{p_2}(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{NB}(\varepsilon)$	$(0.75, 0.85, 0.4, -0.65, -0.6, -0.55)$	$(0.85, 0.8, 0.5, -0.55, -0.5, -0.75)$	$(0.75, 0.75, 0.55, -0.65, -0.45, -0.65)$
$p_2(\varepsilon)$	$(0.85, 0.75, 0.45, -0.55, -0.75, -0.65)$	$(0.65, 0.8, 0.8, -0.85, -0.8, -0.5)$	$(0.65, 0.7, 0.65, -0.8, -0.65, -0.45)$

$\mathcal{NB}^{\mathcal{NB}}_{p_2}(\varepsilon)$	ε_4	ε_5
$\mathcal{NB}(\varepsilon)$	$(0.8, 0.65, 0.35, -0.55, -0.35, -0.75)$	$(0.65, 0.8, 0.55, -0.65, -0.85, -0.7)$
$p_2(\varepsilon)$	$(0.75, 0.75, 0.75, -0.85, -0.65, -0.75)$	$(0.85, 0.8, 0.65, -0.65, -0.5, -0.6)$

Table 4
PNBSS for the resort hotel source

$\mathcal{C}^{\mathcal{NB}}_{p_3}(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{C}(\varepsilon)$	$(0.6, 0.65, 0.4, -0.55, -0.45, -0.75)$	$(0.7, 0.6, 0.55, -0.45, -0.55, -0.7)$	$(0.55, 0.5, 0.5, -0.55, -0.75, -0.5)$
$p_3(\varepsilon)$	$(0.65, 0.7, 0.45, -0.6, -0.45, -0.65)$	$(0.55, 0.75, 0.55, -0.55, -0.55, -0.75)$	$(0.45, 0.9, 0.75, -0.6, -0.5, -0.8)$

$\mathcal{C}^{\mathcal{NB}}_{p_3}(\varepsilon)$	ε_4	ε_5
$\mathcal{C}(\varepsilon)$	$(0.6, 0.6, 0.4, -0.55, -0.55, -0.65)$	$(0.45, 0.55, 0.45, -0.65, -0.65, -0.55)$
$p_3(\varepsilon)$	$(0.45, 0.8, 0.65, -0.5, -0.65, -0.6)$	$(0.35, 0.85, 0.5, -0.55, -0.75, -0.7)$

Table 5
PNBSS for the elegant hotel source

$\mathcal{D}^{\mathcal{NB}}_{p_4}(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{D}(\varepsilon)$	$(0.8, 0.75, 0.7, -0.75, -0.5, -0.7)$	$(0.8, 0.7, 0.55, -0.65, -0.55, -0.85)$	$(0.65, 0.65, 0.75, -0.8, -0.85, -0.75)$
$p_4(\varepsilon)$	$(0.35, 0.5, 0.65, -0.75, -0.65, -0.35)$	$(0.45, 0.45, 0.85, -0.85, -0.5, -0.25)$	$(0.45, 0.45, 0.5, -0.8, -0.7, -0.5)$

$\mathcal{D}^{\mathcal{NB}}_{p_4}(\varepsilon)$	ε_4	ε_5
$\mathcal{D}(\varepsilon)$	$(0.75, 0.5, 0.8, -0.85, -0.75, -0.65)$	$(0.75, 0.7, 0.65, -0.75, -0.7, -0.7)$
$p_4(\varepsilon)$	$(0.65, 0.55, 0.75, -0.55, -0.8, -0.6)$	$(0.45, 0.6, 0.65, -0.65, -0.5, -0.55)$

Table 6
PNBSS for the budget hotel source

$\mathcal{E}_{p_5}^{\mathcal{NB}}(\varepsilon)$	ε_1	ε_2	ε_3
$\mathcal{E}(\varepsilon)$	$(0.6, 0.55, 0.9, -0.5, -0.6, -0.8)$	$(0.55, 0.65, 0.85, -0.65, -0.5, -0.6)$	$(0.45, 0.7, 0.5, -0.7, -0.65, -0.7)$
$p_5(\varepsilon)$	$(0.75, 0.8, 0.5, -0.65, -0.55, -0.6)$	$(0.6, 0.7, 0.6, -0.6, -0.5, -0.55)$	$(0.65, 0.75, 0.7, -0.7, -0.75, -0.7)$

$\mathcal{E}_{p_5}^{\mathcal{NB}}(\varepsilon)$	ε_4	ε_5
$\mathcal{E}(\varepsilon)$	$(0.5, 0.65, 0.75, -0.55, -0.55, -0.65)$	$(0.7, 0.75, 0.5, -0.45, -0.8, -0.7)$
$p_5(\varepsilon)$	$(0.7, 0.75, 0.5, -0.75, -0.6, -0.45)$	$(0.75, 0.8, 0.45, -0.65, -0.45, -0.65)$

Using Definition 4, we calculate the similarity between PNBSSs in Table 2 to Table 6 and its corresponding in Table 1. Similarity thresholds should be determined by the hotel source. Based on the table below, these five hotel sources have similarity measures.

	$\mathbb{T}_1^{\mathcal{NB}}$	$\mathbb{T}_2^{\mathcal{NB}}$	$\mathbb{S}^{\mathcal{NB}}$	$\Phi^{\mathcal{NB}}$	$\Psi^{\mathcal{NB}}$	Similarity
$(\mathcal{L}, \mathcal{A})$	0.943089	0.970399	0.851601	0.921696	0.813497	0.749797
$(\mathcal{L}, \mathcal{B})$	0.952596	0.909897	0.906051	0.922848	0.851133	0.785466
$(\mathcal{L}, \mathcal{C})$	0.870056	0.871331	0.924488	0.888625	0.678809	0.603207
$(\mathcal{L}, \mathcal{D})$	0.985568	0.925645	0.82036	0.910524	0.827599	0.753549
$(\mathcal{L}, \mathcal{E})$	0.854885	0.908018	0.824939	0.862614	0.842198	0.726492

As a result, the second hotel source has the highest similarity measure with $\mathbf{0.785466}$, which is closest to the ideal hotel source.

6 Comparison of PNBSS and NBSS

6.1 Algorithm

In order to select the best choice, the following algorithm is used:

1. Enter the NBSS $\mathcal{X}_p^{\mathcal{NB}}$ in tabular form
2. Choose parameters $A \subseteq E$.
3. Find $\mathbb{T}_1^{\mathcal{NB}}, \mathbb{T}_2^{\mathcal{NB}}$ and $\mathbb{S}^{\mathcal{NB}}$.
4. Calculate similarity using $\frac{\mathbb{T}_1^{\mathcal{NB}}+\mathbb{T}_2^{\mathcal{NB}}+\mathbb{S}^{\mathcal{NB}}}{3}$.
5. Determine maximum similarity $= \mathrm{Max}\{\text{similarity}^i\}$, $1 \leq i \leq 5$.
6. The final decision involves choosing the best solution for the problem.

The possibility parameter should be considered in order to determine its effects. We use the NBSS approach to analyze the survey study mentioned above. Based on the above 1-5 hotel sources, we calculated the similarity measure. We have

https://doi.org/10.54216/IJNS.230116

Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023

	$\mathbb{T}_1^{\mathcal{NB}}$	$\mathbb{T}_2^{\mathcal{NB}}$	$\mathbb{S}^{\mathcal{NB}}$	Similarity
$(\mathcal{L},\mathcal{A})$	0.943089	0.970399	0.851601	0.921696
$(\mathcal{L},\mathcal{B})$	0.952596	0.909897	0.906051	0.922848
$(\mathcal{L},\mathcal{C})$	0.870056	0.871331	0.924488	0.888625
$(\mathcal{L},\mathcal{D})$	0.985568	0.925645	0.82036	0.910524
$(\mathcal{L},\mathcal{E})$	0.854885	0.908018	0.824939	0.862614

PNBSS similarity measures have a significant impact on the parameters from the above results. By using a similarity measure, we observe that the first, third, fourth, and fifth hotel sources are far from the ideal source. A potential hotel should be chosen if the threshold $(0.75, 0.85, 0.4)$ is chosen in the (ε_1) by the second hotel source. By contrast, we cannot differentiate between the best hotel source and the PBFSS approach without the generalization parameter. Second hotel source similarity measurement is greatly influenced by the possibility parameter.

7 Conclusion

PBFSS without the generalization parameter is more scientific and reasonable in this regard. PPBFSS will be used to study decision-making phenomena in this work. We discussed some operational properties of PPBFSS as well as complements, unions, and intersections. As a result, we should consider the generalized possibility of spherical soft sets and neurosophic soft sets in the future.

Acknowledgments: This research project was supported by the Thailand Science Research and Innovation Fund and the University of Phayao (Grant No. FF67-UoE-Aiyared-Iampan).

Conflicts of Interest: The authors declare no conflict of interest.

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Received: May 19, 2023 Revised: August 14, 2023 Accepted: November 22, 2023