$\vec{\jmath}_\rho$ Neutrosophic $\mathfrak{F}$ Subgroup Over a Finite Group

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Abstract

Neutrosophic set has been developed as a mathematical method for procuring indeterminate and incomplete information. Neutrosophic fuzzy set is a powerful generic system that has been recently developed. In several areas, including data and information analysis, data science, information and decision, have successfully applied neutrosophic concept. Not just that but also the important problems we experience in variety of fields, such as computing, life science, social development, and technical work are represented by neutrosophic fuzzy sets. In this paper, we have presented the idea of an implication-based ($\vec{\jmath}_\rho$) neutrosophic fuzzy ($\mathfrak{F}$) subgroup over a finite group and a $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ normal subgroup over a finite group. Further, we have established a few fundamental properties of a $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ subgroup over a finite group and $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ normal subgroup over a finite group.

Keywords: $\mathfrak{F}$ subgroup; $\vec{\jmath}_\rho$ - $\mathfrak{F}$ subgroup; $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ subgroup; $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ normal subgroup

1 Introduction

In 1971, a comparable application of the basic idea of groupoid and groups were presented by Rosenfeld [10]. M.S. Ying [17] conveyed the concept of a new method to $\mathfrak{F}$ topology (I) and described its features. In 1999, Smarandache [13] created neutrosophy to deal with the unknown and unclear information that is common in everyday life. Xuehai Yuan [18] initiated the definition of an $\vec{\jmath}_\rho$ - $\mathfrak{F}$ subgroup of a group in 2003. Vildan C. Etkin [3] highlighted a method for studying neutrosophic subgroups, as well as their fundamental features and characterization. A. A. Salama [11] modeled neutrosophic data sets in 2014. M. Selvarathi [12] described the idea of $\vec{\jmath}_\rho$ - $\mathfrak{F}$ normal subgroup in 2015. In 2016, J.Martina Jency [7] pointed out the importance of $\mathfrak{F}$ neutrosophic subgroupoids by combining the concept of $\mathfrak{F}$ neutrosophic product with a few neutrosophic product and subgroupoid features. In 2016, Kalyan Mondal [8] developed a data mining technique using single-valued neutrosophic information. Smarandache, F. [14] extended a quantum computer that incorporates indeterminacy in 2016. Thao Nguyen Xuan et al. [15] proposed the new idea of soft computing using support neutrosophic set, which is the combination of a neutrosophic set with an $\mathfrak{F}$ set in 2017. Azeddine Elhassouny [4] explored the contributions and hybridization of machine learning algorithms with single-valued neutrosophic numbers in modeling imperfect information. In 2020, Ashour Amira S., and Yanhui Guo [2] examined the concept of the optimization algorithm and also included the neutrosophic sets in computer-aided diagnosis systems. Xin, Ling, et al. [16] explored the main idea of computing with words by using neutrosophic information in 2020. In 2021, Fernandez et al. [6] used a mathematical technique

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called the Project Evaluation and Review Technique in a neutrosophic environment. Adebisi, S. A., and F. Smarandache [1] discovered classical morphisms between refined neutrosophic groups G(I1; I2) and neutrosophic groups G(I) in 2022. Zail et al. [19] introduced some new results of a generalization of neutrosophic BCK-algebra (Ω-BCK-algebra) in 2022. Farooq et al. [5] described an innovative approach to listing the fuzzy subgroups of a finite group in 2023. Al-Quran et al. [9] introduced the Q-Complex neutrosophic soft sets associated with groups and subgroups in 2023.

In this paper, we have introduced the concept of an $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ subgroup over a finite group and $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ normal subgroup over a finite group and also illustrated a few important characteristics of the $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ subgroups over a finite group. This research will offer a new perspective in the field of $\vec{\jmath}_\rho$ -$\mathfrak{F}$ subgroup over a finite group by spplying the concept of neutrosophic $\mathfrak{F}$ sets.

The remaining sections of the research are as follows: Section 2 discusses the preliminaries. An example of the $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{F}$ subgroup over a finite group using the Kleene-Dienes implication operator is showed in Section 3 and conclusions are provided in Section 4.

2 Preliminaries

Definition 2.1. [10] Let Σ be a group then $\mu : \Sigma \rightarrow [0, 1]$ is called as $\mathfrak{F}$ subgroup if

(i) $\mu(\nu\varsigma) \geq \min(\mu(\nu), \mu(\varsigma))$ for all $\nu, \varsigma \in \Sigma$

(ii) $\mu(\nu^{-1}) \geq \mu(\nu)$ for all $\nu \in \Sigma$.

Let V be the universe of discourse and (Σ, $*$) be a finite group. The truth value of a $\mathfrak{F}$ proposition α is represented by $[\alpha]$ in $\mathfrak{F}$ logic.The $\mathfrak{F}$ logical and the corresponding set-theoretical notations used in this paper are
$(\nu \in A) = A(\nu)$,
$(\alpha \wedge \beta) = \min\{[\alpha],[\beta]\}$,
$(\alpha \rightarrow \beta) = \max(1 - \alpha, \beta)$,
$(\forall \nu\ \alpha(\nu)) = \inf_{\nu \in V}[\alpha(\nu)]$,
$(\exists \nu\ \alpha(\nu)) = \sup_{\nu \in V}[\alpha(\nu)]$,
$\vec{\jmath}\alpha$ if and only if $[\alpha] = 1$ for all valuations.
Implications used here is that of Kleene-Dienes implication operator.

Definition 2.2. [12] If a $\mathfrak{F}$ subset A of a group Σ satisfies

(i) $\vec{\jmath}(\nu \in A) \wedge (\varsigma \in A) \rightarrow (\nu\varsigma \in A)$ for any $\nu, \varsigma \in \Sigma$

(ii) $\vec{\jmath}(\nu \in A) \rightarrow (\nu^{-1} \in A)$ for any $\nu \in \Sigma$

then A is called a fuzzifying subgroup of Σ. The concept of ρ- tautology was introduced by Ying [17], i.e., $\vec{\jmath}_\rho(\alpha)$ if and only if $(\alpha) \geq \rho$ for all valuations

Definition 2.3. [12] Let A be a $\mathfrak{F}$ subset of a group Σ and $\rho \in (0, 1]$ is a fixed number. If for any $\nu, \varsigma \in \Sigma$

(i) $\vec{\jmath}_\rho(\nu \in A) \wedge (\varsigma \in A) \rightarrow (\nu\varsigma \in A)$ and

(ii) $\vec{\jmath}_\rho(\nu \in A) \rightarrow (\nu^{-1} \in A)$

then A is called a $\vec{\jmath}_\rho$ - $\mathfrak{F}$ subgroup of Σ.

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3 $\vec{\jmath}_\rho$ Neutrosophic $\mathfrak{J}$ Subgroup Over a Finite Group

Definition 3.1. Let $(\Sigma, *)$ be a finite group. A neutrosophic $\mathfrak{J}$ set $\mathcal{N} = (\mathcal{N}_\succ, \mathcal{N}_\bowtie, \mathcal{N}_\lhd)$ of a finite group Σ is called a $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{J}$ subgroup over a finite group Σ, if it satisfies for any $\nu,\varsigma \in \Sigma$.

(i) $\vec{\jmath}_\rho((\nu \in \mathcal{N}) \wedge (\varsigma \in \mathcal{N})) \to (\nu\varsigma \in \mathcal{N})$
i.e.,
$\vec{\jmath}_\rho((\nu \in \mathcal{N}_\succ) \wedge (\varsigma \in \mathcal{N}_\succ)) \to (\nu\varsigma \in \mathcal{N}_\succ)$
$\vec{\jmath}_\rho((\nu \in \mathcal{N}_\bowtie) \wedge (\varsigma \in \mathcal{N}_\bowtie)) \to (\nu\varsigma \in \mathcal{N}_\bowtie)$
$\vec{\jmath}_\rho\ (\nu\varsigma \in \mathcal{N}_\lhd) \to ((\nu \in \mathcal{N}_\lhd) \vee (\varsigma \in \mathcal{N}_\lhd))$

(ii) $\vec{\jmath}_\rho(\nu \in \mathcal{N}) \to (\nu^{-1} \in \mathcal{N})$
i.e.,
$\vec{\jmath}_\rho(\nu \in \mathcal{N}_\succ) \to (\nu^{-1} \in \mathcal{N}_\succ)$
$\vec{\jmath}_\rho(\nu \in \mathcal{N}_\bowtie) \to (\nu^{-1} \in \mathcal{N}_\bowtie)$
$\vec{\jmath}_\rho\ (\nu^{-1} \in \mathcal{N}_\lhd) \to (\nu \in \mathcal{N}_\lhd)$

Where $(\nu \in \mathcal{N}_\succ)$ denotes the truth membership value, $(\nu \in \mathcal{N}_\bowtie)$ denotes the indeterminacy membership value and $(\nu \in \mathcal{N}_\lhd)$ denotes the falsity membership value such that,
$0 \leq (\nu \in \mathcal{N}_\succ) + (\nu \in \mathcal{N}_\bowtie) + (\nu \in \mathcal{N}_\lhd) \leq 3$.

Example 3.2. $\Sigma = \{0,1,2,3,4,5\}$ is a finite group of order 6 with respect to the addition modulo 6. Cayley's closure table 1 of the group Σ is given below

Table 1: Closure table

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Consider the neutrosophic set $\mathcal{N}$: $\Sigma \to [0, 1] \times [0, 1] \times [0, 1]$ defined by the following membership values
$(0 \in \mathcal{N}) = (0.270, 0.215, 0.115)$; $(1 \in \mathcal{N}) = (0.150, 0.135, 0.315)$
$(2 \in \mathcal{N}) = (0.165, 0.200, 0.235)$; $(3 \in \mathcal{N}) = (0.155, 0.125, 0.320)$
$(4 \in \mathcal{N}) = (0.250, 0.210, 0.140)$; $(5 \in \mathcal{N}) = (0.220, 0.120, 0.260)$
Tables 2, 3 and 4 give the meet and join values of the elements of Σ

Table 2: Meet of the truth membership values of Σ

$\wedge$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	0.270	0.150	0.165	0.155	0.250	0.220
[1]	0.150	0.150	0.150	0.150	0.150	0.150
[2]	0.165	0.150	0.165	0.155	0.165	0.165
[3]	0.155	0.150	0.155	0.155	0.155	0.155
[4]	0.250	0.150	0.165	0.155	0.250	0.250
[5]	0.220	0.150	0.165	0.155	0.220	0.220

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Table 3: Meet of the indeterminate membership values of Σ

$\wedge$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	0.215	0.135	0.200	0.125	0.210	0.120
[1]	0.135	0.135	0.135	0.125	0.135	0.120
[2]	0.200	0.135	0.200	0.125	0.200	0.120
[3]	0.125	0.125	0.125	0.125	0.125	0.120
[4]	0.210	0.135	0.200	0.125	0.210	0.120
[5]	0.120	0.120	0.120	0.120	0.120	0.120

Table 4: Join of the falsity membership values of Σ

$\vee$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	0.115	0.315	0.235	0.320	0.140	0.260
[1]	0.315	0.315	0.315	0.320	0.315	0.315
[2]	0.235	0.315	0.235	0.320	0.235	0.260
[3]	0.320	0.320	0.320	0.320	0.320	0.320
[4]	0.140	0.315	0.235	0.320	0.140	0.260
[5]	0.260	0.315	0.260	0.320	0.260	0.260

With $\rho = 0.6$ and the implication operator is that of Kleene-Dienes, then $\mathcal{N} = (\mathcal{N}_{\succ}, \mathcal{N}_{\bowtie}, \mathcal{N}_{\triangleleft})$ is a $\vec{\jmath}_{\rho}$ neutrosophic $\mathfrak{F}$ subgroup over a finite group.

Theorem 3.3. *Let $\mathcal{N} = (\mathcal{N}_{\succ}, \mathcal{N}_{\bowtie}, \mathcal{N}_{\triangleleft})$ be a $\vec{\jmath}_{\rho}$ neutrosophic $\mathfrak{F}$ subgroup over a finite group Σ. Then for all $\nu \in \Sigma$, we have*

(i) $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}) \to (e \in \mathcal{N})$
i.e.,
$\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\succ}) \to (e \in \mathcal{N}_{\succ})$
$\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\bowtie}) \to (e \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_{\rho}\,(e \in \mathcal{N}_{\triangleleft}) \to (\nu \in \mathcal{N}_{\triangleleft})$

(ii) $\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}) \to (\nu \in \mathcal{N})$
i.e.,
$\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}_{\succ}) \to (\nu \in \mathcal{N}_{\succ})$
$\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}_{\bowtie}) \to (\nu \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_{\rho}\,(\nu \in \mathcal{N}_{\triangleleft}) \to (\nu^{-1} \in \mathcal{N}_{\triangleleft})$

Proof:
Let $\nu \in \Sigma$,

(i) $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\succ})$
$\to (\nu \in \mathcal{N}_{\succ}) \wedge (\nu^{-1} \in \mathcal{N}_{\succ})$ *since,* $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\succ}) \to (\nu^{-1} \in \mathcal{N}_{\succ})$
$\to (\nu\nu^{-1} \in \mathcal{N}_{\succ})$
$\to (e \in \mathcal{N}_{\succ})$
Therefore $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\succ}) \to (e \in \mathcal{N}_{\succ})$ *for all* $\nu \in \Sigma$.
$\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\bowtie})$
$\to (\nu \in \mathcal{N}_{\bowtie}) \wedge (\nu^{-1} \in \mathcal{N}_{\bowtie})$ *since,* $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\bowtie}) \to (\nu^{-1} \in \mathcal{N}_{\bowtie})$
$\to (\nu\nu^{-1} \in \mathcal{N}_{\bowtie})$
$\to (e \in \mathcal{N}_{\bowtie})$
Therefore $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\bowtie}) \to (e \in \mathcal{N}_{\bowtie})$ *for all* $\nu \in \Sigma$
$\vec{\jmath}_{\rho}(e \in \mathcal{N}_{\triangleleft})$
$\to (\nu\nu^{-1} \in \mathcal{N}_{\triangleleft})$ *since,* $\vec{\jmath}_{\rho}\,(\nu^{-1} \in \mathcal{N}_{\triangleleft}) \to (\nu \in \mathcal{N}_{\triangleleft})$
$\to (\nu \in \mathcal{N}_{\triangleleft}) \vee (\nu^{-1} \in \mathcal{N}_{\triangleleft})$
$\to (\nu \in \mathcal{N}_{\triangleleft})$
Therefore $\vec{\jmath}_{\rho}(e \in \mathcal{N}_{\triangleleft}) \to (\nu \in \mathcal{N}_{\triangleleft})$ *for all* $\nu \in \Sigma$.

(ii) $\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}_{\succ}) \to (e \in \mathcal{N}_{\succ})$
$\to (\nu\nu^{-1} \in \mathcal{N}_{\succ})$

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$\rightarrow (\nu \in \mathcal{N}_{\succ}) \wedge (\nu^{-1} \in \mathcal{N}_{\succ})$
$\rightarrow (\nu \in \mathcal{N}_{\succ})$
Therefore $\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}_{\succ}) \rightarrow (\nu \in \mathcal{N}_{\succ})$ *for all* $\nu \in \Sigma$
$\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}_{\bowtie}) \rightarrow (e \in \mathcal{N}_{\bowtie})$
$\rightarrow (\nu\nu^{-1} \in \mathcal{N}_{\bowtie})$
$\rightarrow (\nu \in \mathcal{N}_{\bowtie}) \wedge (\nu^{-1} \in \mathcal{N}_{\bowtie})$
$\rightarrow (\nu \in \mathcal{N}_{\bowtie})$
Therefore $\vec{\jmath}_{\rho}(\nu^{-1} \in \mathcal{N}_{\bowtie}) \rightarrow (\nu \in \mathcal{N}_{\bowtie})$ *for all* $\nu \in \Sigma$
$\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\triangleleft}) \rightarrow (e \in \mathcal{N}_{\triangleleft}) \vee (\nu^{-1} \in \mathcal{N}_{\triangleleft})$
$\rightarrow (\nu^{-1} \in \mathcal{N}_{\triangleleft})$
Therefore $\vec{\jmath}_{\rho}(\nu \in \mathcal{N}_{\triangleleft}) \rightarrow (\nu^{-1} \in \mathcal{N}_{\triangleleft})$ *for all* $\nu \in \Sigma$.

Theorem 3.4. *Let* $\mathcal{N}=(\mathcal{N}_{\succ}, \mathcal{N}_{\bowtie}, \mathcal{N}_{\triangleleft})$ *be a* $\vec{\jmath}_{\rho}$ *neutrosophic* $\mathfrak{F}$ *set. Then* $\mathcal{N}$ *is a* $\vec{\jmath}_{\rho}$ *neutrosophic* $\mathfrak{F}$ *subgroup over a finite group* Σ *if and only if for all* $\nu, \varsigma \in \Sigma$
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}) \wedge (\varsigma \in \mathcal{N})) \rightarrow (\nu\varsigma^{-1} \in \mathcal{N})$
i.e.,
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ})) \rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{\succ})$
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie})) \rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_{\rho}(\nu\varsigma^{-1} \in \mathcal{N}_{\triangleleft}) \rightarrow ((\nu \in \mathcal{N}_{\triangleleft}) \vee (\varsigma \in \mathcal{N}_{\triangleleft}))$

Proof:
Let $\mathcal{N} = (\mathcal{N}_{\succ}, \mathcal{N}_{\bowtie}, \mathcal{N}_{\triangleleft})$ *be a* $\vec{\jmath}_{\rho}$ *neutrosophic* $\mathfrak{F}$ *subgroup over a finite group* Σ.
Let $\nu,\varsigma \in \Sigma$
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ})) \rightarrow (\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma^{-1} \in \mathcal{N}_{\succ})$
$\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{\succ})$
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie})) \rightarrow (\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma^{-1} \in \mathcal{N}_{\bowtie})$
$\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_{\rho}(\nu\varsigma^{-1} \in \mathcal{N}_{\triangleleft}) \rightarrow (\nu \in \mathcal{N}_{\triangleleft}) \vee (\varsigma^{-1} \in \mathcal{N}_{\triangleleft})$
$\rightarrow (\nu \in \mathcal{N}_{\triangleleft}) \vee (\varsigma \in \mathcal{N}_{\triangleleft})$
Conversely,
Let for each $\nu,\varsigma \in \Sigma$ *such that*
Assume that:

$$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ})) \rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{\succ}) \quad (1)$$

$$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie})) \rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{\bowtie}) \quad (2)$$

$$\vec{\jmath}_{\rho}(\nu\varsigma^{-1} \in \mathcal{N}_{\triangleleft}) \rightarrow ((\nu \in \mathcal{N}_{\triangleleft}) \vee (\varsigma \in \mathcal{N}_{\triangleleft})) \quad (3)$$

To prove: $\mathcal{N}$ *is a* $\vec{\jmath}_{\rho}$ *neutrosophic* $\mathfrak{F}$ *subgroup over a finite group* Σ.
Let $\varsigma=\nu$ *in (1)*
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ})) \rightarrow (\nu \in \mathcal{N}_{\succ}) \wedge (\nu \in \mathcal{N}_{\succ})$
$\rightarrow (\nu\nu^{-1} \in \mathcal{N}_{\succ})$
$\rightarrow (e \in \mathcal{N}_{\succ})$
Therefore $\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ})) \rightarrow (e \in \mathcal{N}_{\succ})$
$\vec{\jmath}_{\rho}(\varsigma \in \mathcal{N}_{\succ}) \rightarrow ((e \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ}))$
$\rightarrow (e\varsigma^{-1} \in \mathcal{N}_{\succ})$
$\rightarrow (\varsigma^{-1} \in \mathcal{N}_{\succ})$
Therefore $\vec{\jmath}_{\rho}(\varsigma \in \mathcal{N}_{\succ}) \rightarrow (\varsigma^{-1} \in \mathcal{N}_{\succ})$
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma \in \mathcal{N}_{\succ})) \rightarrow (\nu \in \mathcal{N}_{\succ}) \wedge (\varsigma^{-1} \in \mathcal{N}_{\succ})$
$\rightarrow (\nu(\varsigma^{-1})^{-1} \in \mathcal{N}_{\succ})$
$\rightarrow (\nu\varsigma \in \mathcal{N}_{\succ})$
Let $\varsigma=\nu$ *in (2)*
$\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie})) \rightarrow (\nu \in \mathcal{N}_{\bowtie}) \wedge (\nu \in \mathcal{N}_{\bowtie})$
$\rightarrow (\nu\nu^{-1} \in \mathcal{N}_{\bowtie})$
$\rightarrow (e \in \mathcal{N}_{\bowtie})$
Therefore $\vec{\jmath}_{\rho}((\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie})) \rightarrow (e \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_{\rho}(\varsigma \in \mathcal{N}_{\bowtie}) \rightarrow ((e \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie}))$
$\rightarrow (e\varsigma^{-1} \in \mathcal{N}_{\bowtie})$
$\rightarrow (\varsigma^{-1} \in \mathcal{N}_{\bowtie})$
Therefore $\vec{\jmath}_{\rho}(\varsigma \in \mathcal{N}_{\bowtie}) \rightarrow (\varsigma^{-1} \in \mathcal{N}_{\bowtie})$

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$$\begin{aligned}\vec{\jmath}_\rho((\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma \in \mathcal{N}_{\bowtie})) &\rightarrow (\nu \in \mathcal{N}_{\bowtie}) \wedge (\varsigma^{-1} \in \mathcal{N}_{\bowtie}) \\ &\rightarrow (\nu(\varsigma^{-1})^{-1} \in \mathcal{N}_{\bowtie}) \\ &\rightarrow (\nu\varsigma \in \mathcal{N}_{\bowtie})\end{aligned}$$

Let $\varsigma = \nu$ *in (3)*
$\vec{\jmath}_\rho(\nu\nu^{-1} \in \mathcal{N}_{\lhd}) \rightarrow ((\nu \in \mathcal{N}_{\lhd}) \vee (\nu \in \mathcal{N}_{\lhd}))$
$\vec{\jmath}_\rho(e \in \mathcal{N}_{\lhd}) \rightarrow (\nu \in \mathcal{N}_{\lhd})$

$$\begin{aligned}\vec{\jmath}_\rho(\nu^{-1} \in \mathcal{N}_{\lhd}) &\rightarrow (e\nu^{-1} \in \mathcal{N}_{\lhd}) \\ &\rightarrow (e \in \mathcal{N}_{\lhd}) \vee (\nu \in \mathcal{N}_{\lhd}) \\ &\rightarrow (\nu \in \mathcal{N}_{\lhd})\end{aligned}$$

Therefore $\vec{\jmath}_\rho(\nu^{-1} \in \mathcal{N}_{\lhd}) \rightarrow (\nu \in \mathcal{N}_{\lhd})$

$$\begin{aligned}\vec{\jmath}_\rho(\nu\varsigma^{-1} \in \mathcal{N}_{\lhd}) &\rightarrow (\nu\, (\varsigma^{-1})^{-1} \in \mathcal{N}_{\lhd}) \\ &\rightarrow ((\nu \in \mathcal{N}_{\lhd}) \vee (\varsigma^{-1} \in \mathcal{N}_{\lhd})) \\ &\rightarrow ((\nu \in \mathcal{N}_{\lhd}) \vee (\varsigma \in \mathcal{N}_{\lhd}))\end{aligned}$$

Thus, $\mathcal{N}$ *is a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{H}$ *subgroup over a finite group* Σ.

Theorem 3.5. *Let* $\mathcal{N}_1 = (\mathcal{N}_{1\succ}, \mathcal{N}_{1\bowtie}, \mathcal{N}_{1\lhd})$, $\mathcal{N}_2 = (\mathcal{N}_{2\succ}, \mathcal{N}_{2\bowtie}, \mathcal{N}_{2\lhd}) \cdots$ $\mathcal{N}_n = (\mathcal{N}_{n\succ}, \mathcal{N}_{n\bowtie}, \mathcal{N}_{n\lhd})$ *be n* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{H}$ *subgroups over a finite group* Σ. *Then* $\mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n$ *is a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{H}$ *subgroup over a finite group* Σ.

Proof:

$\mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n$ *is a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{H}$ *subgroup over a finite group* Σ *if it is enough to prove by previous theorem.*

$$\begin{aligned}\vec{\jmath}_\rho((\nu \in \mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n) &\wedge (\varsigma \in \mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n)) \\ &\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n) \text{ for all } \nu, \varsigma \in \Sigma\end{aligned}$$

(i)
$$\begin{aligned}&\vec{\jmath}_\rho((\nu \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ}) \wedge (\varsigma \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ})) \\ &\rightarrow ((\nu \in \mathcal{N}_{1\succ}) \text{ and } (\nu \in \mathcal{N}_{2\succ}) \text{ and} \cdots \text{ and } (\nu \in \mathcal{N}_{n\succ})) \wedge \\ &\qquad ((\varsigma \in \mathcal{N}_{1\succ}) \text{ and } (\varsigma \in \mathcal{N}_{2\succ}) \text{ and} \cdots \text{ and } (\varsigma \in \mathcal{N}_{n\succ})) \\ &\rightarrow ((\nu \in \mathcal{N}_{1\succ}) \wedge (\varsigma \in \mathcal{N}_{1\succ})) \text{ and } ((\nu \in \mathcal{N}_{2\succ}) \wedge (\varsigma \in \mathcal{N}_{2\succ})) \\ &\qquad \text{and } \cdots \text{and } ((\nu \in \mathcal{N}_{n\succ}) \wedge (\varsigma \in \mathcal{N}_{n\succ})) \\ &\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{1\succ}) \text{ and } (\nu\varsigma^{-1} \in \mathcal{N}_{2\succ}) \text{and } \cdots \text{and } (\nu\varsigma^{-1} \in \mathcal{N}_{n\succ}) \\ &\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ})\end{aligned}$$

(ii)
$$\begin{aligned}&\vec{\jmath}_\rho((\nu \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie}) \wedge (\varsigma \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie})) \\ &\rightarrow ((\nu \in \mathcal{N}_{1\bowtie}) \text{ and } (\nu \in \mathcal{N}_{2\bowtie}) \text{ and} \cdots \text{ and } (\nu \in \mathcal{N}_{n\bowtie})) \wedge \\ &\qquad ((\varsigma \in \mathcal{N}_{1\bowtie}) \text{ and } (\varsigma \in \mathcal{N}_{2\bowtie}) \text{ and} \cdots \text{ and } (\varsigma \in \mathcal{N}_{n\bowtie})) \\ &\rightarrow ((\nu \in \mathcal{N}_{1\bowtie}) \wedge (\varsigma \in \mathcal{N}_{1\bowtie})) \text{ and } ((\nu \in \mathcal{N}_{2\bowtie}) \wedge (\varsigma \in \mathcal{N}_{2\bowtie})) \\ &\qquad \text{and } \cdots \text{and } ((\nu \in \mathcal{N}_{n\bowtie}) \wedge (\varsigma \in \mathcal{N}_{n\bowtie})) \\ &\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{1\bowtie}) \text{ and } (\nu\varsigma^{-1} \in \mathcal{N}_{2\bowtie}) \text{and } \cdots \text{and } (\nu\varsigma^{-1} \in \mathcal{N}_{n\bowtie}) \\ &\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie})\end{aligned}$$

(iii)
$$\begin{aligned}&\vec{\jmath}_\rho(\nu\varsigma^{-1} \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd}) \\ &\rightarrow (\nu\varsigma^{-1} \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd}) \text{ and } (\nu\varsigma^{-1} \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots . \cap \mathcal{N}_{n\lhd}) \\ &\rightarrow ((\nu \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdot\cdot\cdot \cap \mathcal{N}_{n\lhd}) \vee (\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})) \\ &\qquad \text{and } ((\nu \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd}) \vee (\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})) \\ &\rightarrow ((\nu \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_2 \lhd \cap \cdot \cdot \cap \mathcal{N}_{n\lhd}) \text{ and } (\nu \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})) \vee \\ &\qquad ((\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd}) \text{ and} (\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})) \\ &\rightarrow (\nu \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd}) \vee ((\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})\end{aligned}$$

Therefore $\mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n$ *is a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{H}$ *subgroup over a finite group* Σ.

Definition 3.6. A $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{H}$ subgroup $\mathcal{N} = (\mathcal{N}_{\succ}, \mathcal{N}_{\bowtie}, \mathcal{N}_{\lhd})$ of Σ is called a $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{H}$ normal subgroup over a finite group Σ if
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}) \rightarrow (\varsigma\nu \in \mathcal{N})$ for all $\nu, \varsigma \in \Sigma$
i.e.,
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\succ}) \rightarrow (\varsigma\, \nu \in \mathcal{N}_{\succ})$
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\bowtie}) \rightarrow (\varsigma\, \nu \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\lhd}) \rightarrow (\varsigma\nu \in \mathcal{N}_{\lhd})$

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Theorem 3.7. *Let $\mathcal{N} = (\mathcal{N}_{\succ}, \mathcal{N}_{\bowtie}, \mathcal{N}_{\triangleleft})$ of Σ be a $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{S}$ normal subgroup over a finite group Σ. Then the following conditions are equivalent.*

(i) $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}) \to (\varsigma\nu \in \mathcal{N})$ *for all* $\nu, \varsigma \in \Sigma$

(ii) $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}) \to (\varsigma \in \mathcal{N})$ *for all* $\nu, \varsigma \in \Sigma$

Proof:
Let $\nu, \varsigma \in \Sigma$
(i) $\Rightarrow$ *(ii)*
Assume that: $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}) \to (\varsigma\nu \in \mathcal{N})$
i.e.,
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\succ}) \to (\varsigma\nu \in \mathcal{N}_{\succ})$
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\bowtie}) \to (\varsigma\nu \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\triangleleft}) \to (\varsigma\nu \in \mathcal{N}_{\triangleleft})$
To prove: $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}) \to (\varsigma \in \mathcal{N})$ *for all* $\nu, \varsigma \in \Sigma$
i.e.,
$\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\succ}) \to (\varsigma \in \mathcal{N}_{\succ})$ *for all* $\nu, \varsigma \in \Sigma$
$\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\bowtie}) \to (\varsigma \in \mathcal{N}_{\bowtie})$ *for all* $\nu, \varsigma \in \Sigma$
$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{\triangleleft}) \to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{\triangleleft})$ *for all* $\nu, \varsigma \in \Sigma$

(i) $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\succ}) \to (\nu\varsigma(\nu^{-1}) \in \mathcal{N}_{\succ})$
$\to (\nu^{-1}\nu\varsigma \in \mathcal{N}_{\succ})$ *by using (i)*
$\to (\varsigma \in \mathcal{N}_{\succ})$
Therefore, $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\succ}) \to (\varsigma \in \mathcal{N}_{\succ})$

(ii) $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\bowtie}) \to (\nu\varsigma(\nu^{-1}) \in \mathcal{N}_{\bowtie})$
$\to (\nu^{-1}\nu\varsigma \in \mathcal{N}_{\bowtie})$ *by using (i)*
$\to (\varsigma \in \mathcal{N}_{\bowtie})$
Therefore, $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\bowtie}) \to (\varsigma \in \mathcal{N}_{\bowtie})$

(iii) $\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{\triangleleft}) \to (\nu^{-1}\nu\varsigma \in \mathcal{N}_{\triangleleft})$ *by using (i)*
$\to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{\triangleleft})$
Therefore, $\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{\triangleleft}) \to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{\triangleleft})$

Let $\nu, \varsigma \in \Sigma$
(ii) $\Rightarrow$ *(i)*
Assume that: $\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}) \to (\varsigma \in \mathcal{N})$ *for all* $\nu, \varsigma \in \Sigma$
i.e.,
$\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\succ}) \to (\varsigma \in \mathcal{N}_{\succ})$ *for all* $\nu, \varsigma \in \Sigma$
$\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{\bowtie}) \to (\varsigma \in \mathcal{N}_{\bowtie})$ *for all* $\nu, \varsigma \in \Sigma$
$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{\triangleleft}) \to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{\triangleleft})$ *for all* $\nu, \varsigma \in \Sigma$
To prove: $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}) \to (\varsigma\nu \in \mathcal{N})$
i.e.,
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\succ}) \to (\varsigma\nu \in \mathcal{N}_{\succ})$
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\bowtie}) \to (\varsigma\nu \in \mathcal{N}_{\bowtie})$
$\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\triangleleft}) \to (\varsigma\nu \in \mathcal{N}_{\triangleleft})$

(i) $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\succ}) \to ((\nu\varsigma)(\nu\nu^{-1}) \in \mathcal{N}_{\succ})$
$\to (\nu(\varsigma\nu)\nu^{-1} \in \mathcal{N}_{\succ})$
$\to (\varsigma\nu \in \mathcal{N}_{\succ})$ *by using (ii)*
Therefore, $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\succ}) \to (\varsigma\nu \in \mathcal{N}_{\succ})$

(ii) $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\bowtie}) \to ((\nu\varsigma)(\nu\nu^{-1}) \in \mathcal{N}_{\bowtie})$
$\to (\nu(\varsigma\nu)\nu^{-1} \in \mathcal{N}_{\bowtie})$
$\to (\varsigma\nu \in \mathcal{N}_{\bowtie})$ *by using (ii)*
Therefore, $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_{\bowtie}) \to (\varsigma\nu \in \mathcal{N}_{\bowtie})$

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(iii) $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_\lhd) \to (\nu^{-1}(\nu\varsigma)(\nu^{-1})^{-1} \in \mathcal{N}_\lhd)$
$\to (\nu^{-1}(\nu\varsigma)\nu \in \mathcal{N}_\lhd)$
$\to (\nu^{-1}\nu(\varsigma\nu) \in \mathcal{N}_\lhd)$
$\to (\varsigma\nu \in \mathcal{N}_\lhd)$ *by using (ii)*
Therefore, $\vec{\jmath}_\rho(\nu\varsigma \in \mathcal{N}_\lhd) \to (\varsigma\nu \in \mathcal{N}_\lhd)$

Theorem 3.8. *Let* $\mathcal{N}_1 = (\mathcal{N}_{1\succ}, \mathcal{N}_{1\bowtie}, \mathcal{N}_{1\lhd})$, $\mathcal{N}_2 = (\mathcal{N}_{2\succ}, \mathcal{N}_{2\bowtie}, \mathcal{N}_{2\lhd}) \cdots \mathcal{N}_n = (\mathcal{N}_{n\succ}, \mathcal{N}_{n\bowtie}, \mathcal{N}_{n\lhd})$ *be n* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{S}$ *subgroups over a finite group* Σ. *Then* $\mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n$ *be a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{S}$ *normal subgroup over a finite group* Σ.

Proof:
By theorem 3.5, $\mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n$ *is a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{S}$ *subgroup over a finite group* Σ. *Then*
$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n) \to (\nu\varsigma\nu^{-1} \in \mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n)$
i.e.,
$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ}) \to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ})$
$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie}) \to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie})$
$\vec{\jmath}_\rho(\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd}) \to (\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})$
So by the definition of the intersection,
$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ})$
$\to (\varsigma \in \mathcal{N}_{1\succ})$ *and* $(\varsigma \in \mathcal{N}_{2\succ})$ *and* $\cdots$ *and* $(\varsigma \in \mathcal{N}_{n\succ})$
$\to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\succ})$ *and* $(\nu\varsigma\nu^{-1} \in \mathcal{N}_{2\succ})$ *and* $\cdots$ *and* $(\nu\varsigma\nu^{-1} \in \mathcal{N}_{n\succ})$
$\to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\succ} \cap \mathcal{N}_{2\succ} \cap \cdots \cap \mathcal{N}_{n\succ})$

$\vec{\jmath}_\rho(\varsigma \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie})$
$\to (\varsigma \in \mathcal{N}_{1\bowtie})$ *and* $(\varsigma \in \mathcal{N}_{2\bowtie})$ *and* $\cdots$ *and* $(\varsigma \in \mathcal{N}_{n\bowtie})$
$\to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\bowtie})$ *and* $(\nu\varsigma\nu^{-1} \in \mathcal{N}_{2\bowtie})$ *and* $\cdots$ *and* $(\nu\varsigma\nu^{-1} \in \mathcal{N}_{n\bowtie})$
$\to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\bowtie} \cap \mathcal{N}_{2\bowtie} \cap \cdots \cap \mathcal{N}_{n\bowtie})$

$\vec{\jmath}_\rho\ (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})$
$\to (\nu\varsigma\nu^{-1} \in \mathcal{N}_{1\lhd})$ *and* $(\nu\varsigma\nu^{-1} \in \mathcal{N}_{2\lhd})$ *and* $\cdots$ *and* $(\nu\varsigma\nu^{-1} \in \mathcal{N}_{n\lhd})$
$\to (\varsigma \in \mathcal{N}_{1\lhd})$ *and* $(\varsigma \in \mathcal{N}_{2\lhd})$ *and* $\cdots$ *and* $(\varsigma \in \mathcal{N}_{n\lhd})$
$\to (\varsigma \in \mathcal{N}_{1\lhd} \cap \mathcal{N}_{2\lhd} \cap \cdots \cap \mathcal{N}_{n\lhd})$
By theorem 3.7, $\mathcal{N}_1 \cap \mathcal{N}_2 \cap \cdots \cap \mathcal{N}_n$ *is a* $\vec{\jmath}_\rho$ *neutrosophic* $\mathfrak{S}$ *normal subgroup over a finite group* Σ.

4 Conclusion

Neutrosophic sets are used in many concepts of computer sciences, namely data analysis, machine learning, data mining and object-oriented programming techniques. Object oriented programming techniques are applied in the design and creation of a new system for implementing neutrosophic information transmission. The benefit of utilizing neutrosophic sets for object-oriented programming is that distortions are avoided. Further, this research work defines the $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{S}$ subgroup over a finite group and demonstrated some of the fundamental characteristics in $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{S}$ normal subgroup over a finite group. In future, the concept of $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{S}$ subgroup can be applied to the fuzzy finite state machine over a group. Researchers can also introduced the idea anti-fuzzy to $\vec{\jmath}_\rho$ neutrosophic $\mathfrak{S}$ subgroup. Neutrosophic $\mathfrak{S}$ automata will be helpful in pattern recognition, medical diagnosis, image processing, databases, medicine and learning systems.

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