Neutrosophic Integrals by Reduction Formula and Partial Fraction Methods for Indefinite Integrals

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Abstract

Neutrosophic mathematics is a branch of mathematics that deals with ambiguity, indeterminacy, and incompleteness in mathematical objects and procedures. To account for Neutrosophic uncertainty, several mathematical concepts—including the reduction formula, partial fractions, and area finding—are extended in this field. The Neutrosophic reduction formula is a technique for summarising simpler words from a complex mathematical expression when the coefficientss a nd/or values may be ambiguous or unknown. By taking the potential of insufficient information into account, expands the traditional reduction formula. A rational function can be broken down using the Neutrosophic partial fraction into several simpler expressions, where the coefficients and/or values may be ambiguous or unknown. By considering, this expands the traditional partial fraction. The potential for inaccurate information. A method for calculating the area under a curve where the curve's form or position may be unknown or ambiguous is area finding via neutrosophic integration. By considering the potential of having insufficient information, this expands the traditional area of searching. These ideas can be used in fields like decision-making, expert systems, and artificial intelligence and are crucial for handling problems in the real world that entail uncertainty, indeterminacy, and incompleteness.

Keywords: Definite neutrosophic integral; Area of neutrosophic curves; length of neutrosophic volumes of neutrosophic revolution; Indeterminacy in integrals.

1. Introduction

A brand-new mathematical framework called neutrosophic mathematics allows for the existence of ambiguity, imprecision, and incompleteness in mathematical objects and processes. Due to its numerous applications in areas including decision-making, expert systems, and artificial intelligence, it has attracted

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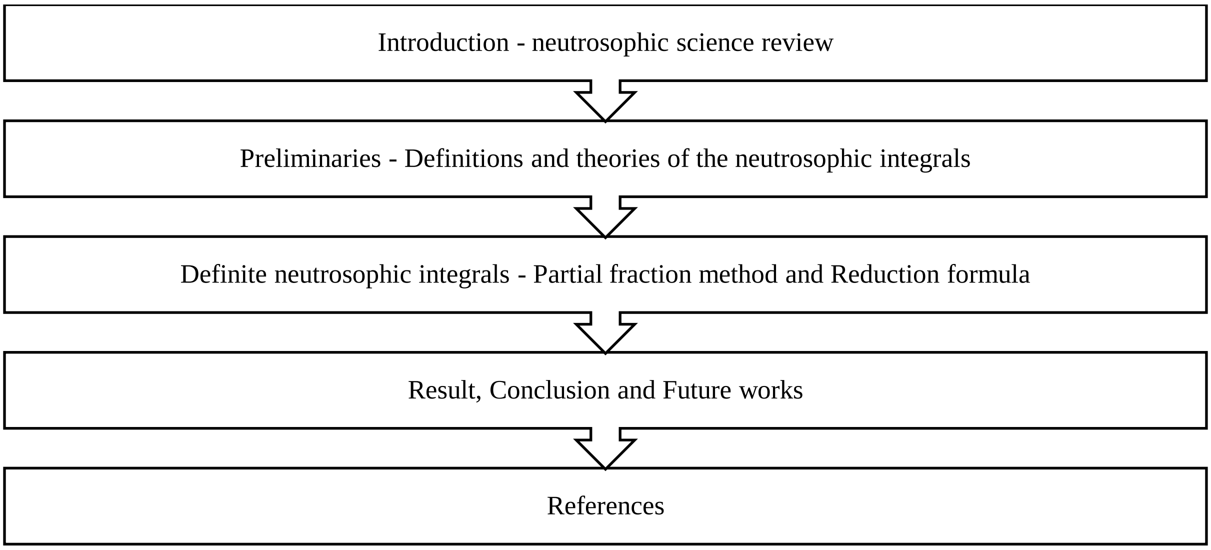
more attention in recent years. Three fundamental ideas in mathematics—the partial fraction, the neutrososphic reduction formula, and area determination via neutrosophic integration—are the focus of this study. To deal with the existence of neutrosophic uncertainty, these notions are expansions of their classical counterparts.
The neutrosophic set was first developed by Smarandache [6] in 1998, and Yaser Ahmad Alhasan introduced the integration in the neutrosophic environment with indeterminacy [14-16]. Researchers have also applied the extended versions of Neutrosophic Fermatean sets and graphs in decision making. Broumi et al have constructed the theory of classical Fermatean and complex Fermatean neutrosophic graphs and their applications in decision making [17-19]. Interval –valued Fermatean neutrosophic representations are also applied in decision making [20]. The neutrosophic soft sets of varied dimensions are also well discoursed in decision making together with different optimizations algorithms [21,22].

Different integration techniques including integration by part, and definite integral in neutrosophic environments were explained by Yaser Ahmad Alhasan [14-16]. In continuation with these methods, the reduction formulae and partial fraction methods for neutrosophic indefinite integrals are discussed. The neutrosophic reduction formula is a way, to sum up simpler words from a complex mathematical expression where the coefficients and/or values may be ambiguous or unknown. This expansion of the traditional reduction formula gives a more thorough strategy for resolving real-world issues where there is a lack of information. By breaking down a rational function into a collection of simpler terms using the neutrosophic partial fraction, indeterminate or unknown coefficients and/or values can be introduced into the equation. This expansion of the traditional partial fraction provides a more reliable method for resolving issues in the real world where there is a lack of information.
This article comprises four sections which are shown below



2. Preliminaries

Integration is important in human life and one of its most important applications is the calculation of area, size, and arc length. In our reality, we find things that cannot be precisely defined and that contain an indeterminacy part. This is the reason for studying neutrosophic integration and methods of its integration in a Neutrosophic environment.

Definition 2.1: Neutrosophic Real Number [12]
Suppose that w is a neutrosophic number, then it takes the following standard form: $w=a+bI$ where a, b are real coefficients, and I represent indeterminacy, such 0.I=0 and In=I, for all positive integers n.

Definition 2.2: Neutrosophic Indefinite Integral [14]
We just extend the classical definition of anti-derivative. The neutrosophic antiderivative of neutrosophic function $f(x)$ is the neutrosophic function $F(x)$ such that $F'(x)=f(x)$.

Definition2.3 [14] Let $f:Df\subseteq R\rightarrow Rf\cup\{I\}$, to evaluate $\int f(x)dx$
Put: $x=g(u) \Rightarrow dx=\acute{g}(u)du$

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By substitution, we get: $\int f(x)dx=\int f(u)\acute{g}(u)du$
then we can directly integral it.

Propositions:2.4[14] If $\int f(x,I)dx=\varphi(x,I)$ $then$, $\int f((a+bI)x+c+dI))dx=(1a-ba(a+b)I)\varphi((a+bI)x+c+dI))+C$
where C is an indeterminate real constant, $a\neq 0,a\neq -b$ and b,c,d are real numbers, while I = indeterminacy.

Propositions:2.5[14] Let f:$Df\subseteq R\rightarrow Rf\cup\{I\}$ then:
$\int \acute{f}(x,I)f(x,I)dx=ln|f(x,I)|+C$
where C is an indeterminate real constant (i.e. constant of the form $a+bI$, where a, b are real numbers, while I = indeterminacy).

Propositions:2.6[14] Let f: $Df\subseteq R\rightarrow Rf\cup\{I\}$, then:
$\int \acute{f}(x,I)\sqrt{}f(x,I)dx=2\sqrt{}f(x,I)+C$
where $C$is an indeterminate real constant (i.e. constant of the form $a+bI$, where a, b are real numbers, while I = indeterminacy).

Propositions:2.7[14] If f:$Df\subseteq R\rightarrow Rf\cup\{I\}$, then:
$\int[f(x,I)]n\acute{f}(x)\ dx=[f(x,I)]n+1n+1+C$
Where n is any rational number. C is an indeterminate real constant (i.e. constant of the form $a+bI$, where a, b are real numbers, while I = indeterminacy).
Result: 2.8[14] Integration of products of neutrosophic trigonometric function:
I. $\int\sin(a+bI)x\cos n(a+bI)x\ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following two cases:

- Case n is odd:

• Split of $\cos(a+bI)x$
• Apply $\cos 2(a+bI)x=1-\sin 2(a+bI)x$
• We substitution $u=\sin(a+bI)x$

- Case m is odd:

• Split of $\sin(a+bI)x$
• Apply $\sin 2(a+bI)x=1-\cos 2(a+bI)x$
• We substitution $u=\cos(a+bI)x$.

3. The Indefinite Integrals

3.1 Neutrosophic Integration by Partial Fraction Methods:

When the integration's bounds or the integrand itself exhibits imprecision, ambiguity, or inconsistency, indeterminacy occurs in neutrosophic calculus. Neutrosophic indefinite integrals provide a framework that manages erroneous or contradictory information throughout the integration process in an effort to reflect this inherent uncertainty. We can determine the values of complex integrals in classical integration by using the partial fraction approach. Partial fraction methods are crucial in neutrosophic integrals as well for converting the quadratic terms into linear neutrosophic integral elements. Three different partial fraction approaches are covered in this section along with pertinent examples.

Numerous functions exist whose neutrosophic integrals are not directly attainable by using higher power integrals and whose neutrosophic integrals are not instantly reducible to one of the conventional forms. However, in some instances, such integrals may be linearly related to the neutrosophic integral of another expression using an algebraic formula, while the latter may be either instantly integrable or at any other easier to integrate than the original function.

	Denominator consisting of…	Expression	Form of Partial Fractions
Type I	Quadratic term which can be factored	$(f(x))/((x+aI)^2-(a+bI)^2)$	$\frac{A}{x+x_0I}+\frac{B}{a+a_0I}$
Type II	Repeated linear factor	$(f(x))/((x+aI)^3)$	$\frac{A}{x+aI}+\frac{B}{(x+aI)^2}$

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			$+\frac{C}{(x+aI)^3}$
Type III	Quadratic term which cannot factored and repeated linear factors	$\frac{f(x)}{(ax^2+bx+c)(gx+h)}$	$\frac{Ax+B}{ax^2+bx+c}+\frac{C}{gx+h}$

Type 1: Denominator consisting of quadratic term which can be factored:

Solve: $\int \frac{1}{(x+x_0I)^2-(a+a_0I)^2}dx$

Solution:

These forms should be thrown into partial fraction. Thus

$$\int \frac{1}{(x+x_0I)^2-(a+a_0I)^2}dx = \int \frac{A}{(x+x_0I)+(a+a_0I)}dx + \int \frac{B}{(x+x_0I)-(a+a_0I)}dx \qquad (3.1)$$

Consider $\frac{1}{(x+x_0I)^2-(a+a_0I)^2} = \frac{A}{(x+x_0I)+(a+a_0I)} + \frac{B}{(x+x_0I)-(a+a_0I)}$

$$\frac{1}{(x+x_0I)^2-(a+a_0I)^2} = \frac{A[(x+x_0I)-(a+a_0I)]+B[(x+x_0I)+(a+a_0I)]}{(x+x_0I)^2-(a+a_0I)^2}$$

$$1=A[(x+x_0I)-(a+a_0I)]+B[(x+x_0I)+(a+a_0I)]$$

Put $x=a \;\&x_0=a_0$

$$1=B[(a+a_0I)+(a+a_0I)]$$

$B=\frac{1}{2(a+a_0I)}$

Put $x=-a \;\&x_0=-a_0$

$1=A[(-a-a_0I)-(a+a_0I)]$

$A=\frac{-1}{2(a+a_0I)}$

$$\frac{1}{(x+x_0I)^2-(a+a_0I)^2} = \frac{1}{2(a+a_0I)}\left[\frac{1}{(x+x_0I)-(a+a_0I)} - \frac{1}{(x+x_0I)+(a+a_0I)}\right]$$

$$\int \frac{1}{(x+x_0I)^2-(a+a_0I)^2}dx = \frac{1}{2(a+a_0I)}\left[\int \frac{1}{(x+x_0I)-(a+a_0I)}dx - \int \frac{1}{(x+x_0I)+(a+a_0I)}dx\right]$$

$$=\frac{1}{2(a+a_0I)}\log\left[\frac{(x+x_0I)-(a+a_0I)}{(x+x_0I)+(a+a_0I)}\right].$$

Type 2:Denominator consisting of repeated linear factor:

Solve: $\int \frac{1}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2}dx$

Solution:

These forms should be thrown into partial fraction. Thus

Consider $\frac{1}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2} = \frac{A}{((x+x_0I)-(3+I)} + \frac{B}{((x+x_0I)-(2+3I))} + \frac{C}{((x+x_0I)-(2+3I))^2}$

$$\frac{1}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2} = \frac{A\left(((x+x_0I)-(2+3I))^2+B(x+x_0I)-(2+3I)\right)+C((x+x_0I)-(3+I))}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2}$$

$$1=A\left(((x+x_0I)-(2+3I))^2+(B(x+x_0I)-(2+3I)\right)+C((x+x_0I)-(3+I)$$

Put $x=2 \;\&x_0=3$

$1=C((x+x_0I)-(3+I)$

1=C(2+3I-3-I)

1=C(-1+2I)

$C=\frac{1}{-1+2I}$

Similarly we can find $A=\frac{1}{(2+3I)(1-2I)}$ and $B=\frac{2I}{(1-2I)(2+3I)}$.

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$$\frac{1}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2}$$
$$=\frac{A}{((x+x_0I)-(3+I)}+\frac{B}{((x+x_0I)-(2+3I))}$$
$$+\frac{C}{((x+x_0I)-(2+3I))^2}\frac{1}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2}$$
$$=\frac{1}{(2+3I)(1-2I)}[\frac{1}{((x+x_0I)-(3+I)}]$$
$$+\frac{2I}{(1-2I)(2+3I)}[\frac{1}{((x+x_0I)-(2+3I))}]$$
$$+\frac{1}{-1+2I}[\frac{1}{((x+x_0I)-(2+3I))^2}]\int\frac{1}{((x+x_0I)-(3+I))((x+x_0I)-(2+3I))^2}$$
$$=\int\frac{A}{((x+x_0I)-(3+I)}+\int\frac{B}{((x+x_0I)-(2+3I))}+\int\frac{C}{((x+x_0I)-(2+3I))^2}$$
$$=\frac{1}{(2+3I)(1-2I)}\int[\frac{1}{((x+x_0I)-(3+I)}]$$
$$+\frac{2I}{(1-2I)(2+3I)}\int[\frac{1}{((x+x_0I)-(2+3I))}]+\frac{1}{-1+2I}\int\frac{1}{((x+x_0I)-(2+3I))^2}$$
$$=\frac{1}{(2+3I)(1-2I)}\log(x+x_0I)-(3+I))+\frac{2I}{(1-2I)(2+3I)}\log((x+x_0I)-(2+3I))+\frac{1}{-1+2I}[\frac{-1}{((x+x_0I)-(2+3I))}]$$

Type 3:Denominator consisting of quadratic term which cannot factored and repeated linear factors:

Solve: $\int\frac{1}{((x+x_0I)-(1+I))(x+x_0I)^2-(9+9I))}dx$

Solution:

These forms should be thrown into partial fraction.

Consider $\frac{1}{((x+x_0I)-(1+I))(x+x_0I)^2-(9+9I))}=\frac{A}{(x+x_0I)-(1+I)}+\frac{Bx+c}{(x+x_0I)^2-(9+9I)}$

$$\frac{1}{((x+x_0I)-(1+I))(x+x_0I)^2-(9+9I))}=\frac{A(x+x_0I)^2-(9+9I))+(Bx+c)((x+x_0I)-(1+I))}{((x+x_0I)-(1+I))(x+x_0I)^2-(9+9I))}$$

1= $A(x+x_0I)^2-(9+9I))+(Bx+c)((x+x_0I)-(1+I))$

Put $x=1$& $x_0=1$

A=(1+I)(-8+I)

Equating the coefficient of x^2on both side

B=-(1+I)(-8+I)

Equating the coefficient of x^2on both side

C=0

Therefore

$\int\frac{1}{((x+x_0I)-(1+I))(x+x_0I)^2-(9+9I))}=\int\frac{A}{(x+x_0I)-(1+I)}dx+\int\frac{Bx+c}{(x+x_0I)^2-(9+9I)}$dx

=[(1+I)(-8+I)]log((1+I)(-8+I))+$\frac{1}{2}$log($(x+x_0I)^2-(9+9I)$)

3.2 Reduction Formulae of Trigonometric Functions in Neutrosophic integrals:

Integral calculus heavily relies on the reduction formula. The amount of time and steps needed to complete the calculation are decreased. Calculating the values of the indefinite integrals using the neutrosophic integral requires a lot of time due to the uncertainty of the functions. The reduction formula can be used to assess several definite integrals. There are some integrals that direct integration techniques cannot be used to analyze. Numerous functions exist whose neutrosophic integrals are not directly attainable by using higher power integrals and whose neutrosophic integrals are not instantly reducible to one of the conventional forms. However, in some circumstances, such integrals may be linearly related to the neutrosophic integral of another expression by some algebraic formula, while the latter may be either instantly integrable or at the

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very least simple to integrate than the original function. We determine the values of the reduction formulas shown below for neutrophilic indefinite integrals in this section:

Theorem 3.2.1:
If $\int sin^n(a+bI)x\,dx$ is neutrosophic integral of higher power, then prove that the reduction formula of $\int sin^n(a+bI)x\,dx$ is $\frac{-1}{n}sin^{n-1}(a+bI)x\cos(a+bI)x + \frac{(n-1)}{n}I_{n-2}$ where $I_{n-2} = \int \sin^{n-2}x\,dx$.

Proof:

Consider $\int sin^n(a+bI)x\,dx = \int sin^{n-1}(a+bI)x\sin(a+bI)x\,dx$
Let u = $sin^{n-1}(a+bI)x$ and $\frac{dv}{dx} = \sin(a+bI)x$

So, $\frac{du}{dx} = (n-1)\sin^{n-2}(a+bI)x\cos(a+bI)x$; v = $-\cos(a+bI)x$
$\int sin^n(a+bI)x\,dx = -sin^{n-1}(a+bI)x\cos(a+bI)x + \int \cos(a+bI)x\,(n-1)sin^{n-2}(a+bI)x\cos(a+bI)x\,dx$ $-sin^{n-1}(a+bI)x\cos(a+bI)x + \int \cos^2(a+bI)x\,(n-1)sin^{n-2}(a+bI)x\,dx$

$= -sin^{n-1}(a+bI)x\cos(a+bI)x + \int[1 - sin^2(a+bI)x](n-1)sin^{n-2}(a+bI)x\,dx$

$= -sin^{n-1}(a+bI)x\cos(a+bI)x + \int(n-1)sin^{n-2}(a+bI)x\,dx - \int(n-1)sin^2x\,sin^{n-2}(a+bI)x\,dx$
$= -sin^{n-1}(a+bI)x\cos(a+bI)x + \int(n-1)sin^{n-2}(a+bI)x\,dx - \int(n-1)sin^n(a+bI)x\,dx$

$\int n\,sin^n(a+bI)x\,dx = -sin^{n-1}(a+bI)x\cos(a+bI)x + \int(n-1)sin^{n-2}(a+bI)x\,dx$
$\int sin^n(a+bI)x\,dx = -1/n\,sin^{n-1}(a+bI)x\cos(a+bI)x + \int(n-1)/n\,sin^{n-2}(a+bI)x\,dx$

$I_n = \frac{-1}{n}sin^{n-1}(a+bI)x\cos(a+bI)x + \frac{(n-1)}{n}I_{n-2}$

Theorem 3.2.2: If $\int cos^n(a+bI)x\,dx$ is neutrosophic integral of higher power, then prove that the reduction formula of $\int cos^n(a+bI)x\,dx$ is $\frac{1}{n}cos^{n-1}(a+bI)x\sin(a+bI)x + \frac{(n-1)}{n}I_{n-2}$ where $I_{n-2} = \int cos^{n-2}(a+bI)x\,dx$

Proof:

Consider $I_n = \int cos^n(a+bI)x\,dx = \int cos^{n-1}(a+bI)x\,cos(a+bI)x\,dx$

Let u = $cos^{n-1}(a+bI)x$ and $\frac{dv}{dx} = cos(a+bI)x$
So, $\frac{du}{dx} = (n-1)cos^{n-2}(a+bI)x\,sin(a+bI)x$; $v = sin(a+bI)x$
$\int cos^n(a+bI)x\,dx = cos^{n-1}(a+bI)x\sin(a+bI)x - \int \sin(a+bI)x\,(n-1)cos^{n-2}(a+bI)x\cos(a+bI)x\,dx$

$= cos^{n-1}(a+bI)x\sin(a+bI)x + \int \sin^2(a+bI)x\,(n-1)cos^{n-2}(a+bI)x\,dx$

$= cos^{n-1}(a+bI)x\sin(a+bI)x + \int[1 - cos^2(a+bI)x](n-1)cos^{n-2}(a+bI)x\,dx$

$= cos^{n-1}(a+bI)x\sin(a+bI)x + \int(n-1)cos^{n-2}(a+bI)x\,dx - \int(n-1)cos^2x\,cos^{n-2}(a+bI)x\,dx$

$= cos^{n-1}(a+bI)x\sin(a+bI)x + \int(n-1)cos^{n-2}(a+bI)x\,dx - \int(n-1)cos^n(a+bI)x\,dx$

$\int n\,cos^n(a+bI)x\,dx = = cos^{n-1}(a+bI)x\sin(a+bI)x + \int(n-1)cos^{n-2}(a+bI)x\,dx$
$\int cos^n(a+bI)x\,dx = -1/n\,cos^{n-1}(a+bI)x\sin(a+bI)x + \int(n-1)/n\,cos^{n-2}(a+bI)x\,dx$

$I_n = \frac{1}{n}cos^{n-1}(a+bI)x\sin(a+bI)x + \frac{(n-1)}{n}I_{n-2}$.
The ultimate integral is I_0 or I_1
n is even: $I_0 = \int dx = x + C$ when n = 0 and
n is odd: $I_1 = \int cosxdx = \sin x + C$ when n = 1.

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Theorem3.2.3: If$\int cosec^n(a+bI)xdx$ is neutrosophic integral of higher power, then prove that the reduction formula of $\int cosec^n(a+bI)xdx$ is $\frac{-1}{n-1}cosec^{n-2}(a+bI)x\ \cot(a+bI)x+\frac{n-2}{n-1}I_{n-2}$ where $I_{n-2}=\int cosec^{n-2}(a+bI)xdx.$

Proof:

$I_n=\int cosec^n(a+bI)x$ dx = $\int cos^{n-2}(a+bI)xcosec^2(a+bI)x\ dx$

Let u = $cosec^{n-2}(a+bI)x$ and $\frac{dv}{dx}=cosec^2(a+bI)$x

So, $\frac{du}{dx}$ = (n-2)$cosec^{n-3}(a+bI)x\ (-cosec(a+bI)xcot(a+bI)x\ ;\ v\ =\ -cot(a+bI)x$

$\int cosec^n(a+bI)x$ dx = $cosec^{n-2}(a+bI))$x $(-\ cot(a+bI)x)$+ $\int cot(a+bI)x$ (n-2)$cosec^{n-3}(a+bI)x(-cosec(a+bI)xcot(a+bI)x$ dx

= $-cosec^{n-2}(a+bI)x\ \cot(a+bI)$x –(n-2) $\int($ $cosec^2(a+bI)$x-1)$cosec^{n-2}(a+bI)x$ dx

= $-cosec^{n-2}(a+bI)x\ \cot(a+bI)$x –(n-2) $\int($ $cosec^n(a+bI)$x+(n-2)$\int cosec^{n-2}(a+bI)x$ dx

=$-cosec^{n-2}(a+bI)x\ \cot(a+bI)$x –(n-2) $I_{\ n}$+(n-2)$I_{\ n-2}$

= $\frac{-1}{n-1}cosec^{n-2}(a+bI)x\ \cot(a+bI)$x+$\frac{n-2}{n-1}I_{\ n-2}$.

The ultimate integral is $I_0\ or\ I_1$

n is even: $I_0=\int dx=x+C$ when n = 0 and

n is odd: $I_1=\int cosecxdx=\log(cosecx-cotx)+C$ when n = 1.

Theorem 3.2.4: If $\int tan^n(a+bI)xdx$ is neutrosophic integral of higher power, then prove that the reduction formula of$\int tan^n(a+bI)xdx$ is $\frac{(\tan^{n-1}(a+bI)x)}{n-1}-I_{n-2}$ where $I_{n-2}=\int tan^{n-2}(a+bI)xdx.$

Proof:

Let $I_{\ n}\ =\int tan^n(a+bI)xdx=\int tan^{n-2}(a+bI)xtan^2(a+bI)xdx$

$$=\int tan^{n-2}(a+bI)x\ (sec^2(a+bI)x-1)dx$$

$$=\int tan^{n-2}(a+bI)xsec^2(a+bI)x-\int tan^{n-2}(a+bI)xdx$$

$$=\int tan^{n-2}(a+bI)xd(tan(a+bI)x)-I_{\ n-2}$$

$$=\frac{(\tan^{n-1}(a+bI)x)}{n-1}-I_{n-2}$$

The ultimate integral is $I_0\ or\ I_1$

n is even: $I_0=\int dx=x+C$ when n = 0 and

n is odd: $I_1=\int tandx=-\log(cosx)+C$ when n = 1.

Theorem 3.2.5: If $\int sec^n(a+bI)xdx$is neutrosophic integral of higher power, then prove that the reduction formula of$\int sec^n(a+bI)$xdx is $\frac{1}{n-1}sec^{n-2}(a+bI)xtan(a+bI)x$+$\frac{n-2}{n-1}I_{n-2}$ where $I_{n-2}=\int sec^{n-2}(a+bI)xdx.$

Proof:

$I_{\ n}=\int sec^n(a+bI)x$ dx = $\int sec^{n-2}(a+bI)xsec^2(a+bI)$x dx

Let u = $sec^{n-2}(a+bI)x$ and $\frac{dv}{dx}=sec^2(a+bI)$x

So, du = (n-2)$sec^{n-3}(a+bI)x\ (sec(a+bI)x\ tan(a+bI)x\ dx\ ;\ v\ =tan(a+bI)x$

$\int sec^n(a+bI)x$ dx = $sec^{n-2}(a+bI))xtan(a+bI)x)$ $-\int tan(a+bI)x$ (n-2)$sec^{n-3}(a+bI)xsec(a+bI)x\ tan(a+bI)x\ dx$

= $sec^{n-2}(a+bI)xtan(a+bI)x$ –(n-2) $\int(\ tan^2(a+bI)x-1)sec^{n-2}(a+bI)xdx$

= $sec^{n-2}(a+bI)xtan(a+bI)x$ –(n-2) $\int(\ sec^n(a+bI)x$+(n-2)$\int sec^{n-2}(a+bI)x$dx

=$sec^{n-2}(a+bI)xtan(a+bI)x$ –(n-2) $I_{\ n}$+(n-2)$I_{\ n-2}$

= $\frac{1}{n-1}sec^{n-2}(a+bI)xtan(a+bI)x$+$\frac{n-2}{n-1}I_{\ n-2}$.

The ultimate integral is $I_0\ or\ I_1$

n is even: $I_0=\int dx=x+C$ when n = 0and

n is odd: $I_1=\int secdx=\log(secx+tanx)+C$ when n = 1.

Theorem 3.2.6: If$\int cot^n(a+bI)x\ dx$ is neutrosophic integral of higher power, then prove that the reduction formula of $\int cot^n(a+bI)x\ dx$ is $-\left(\frac{cot^{n-1}(a+bI)x}{n-1}\right)-I_{n-2}$ where

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$$I_{n-2} = \int cot^{n-2}(a + bI)x\, dx.$$

Proof:

Let $I_{\,n} = \int cot^{n}(a + bI)xdx$

$= \int cot^{n-2}(a + bI)xcot^{2}(a + bI)xdx$

$= \int cot^{n-2}(a + bI)x(\text{cosec}^{2}(a + bI)x - 1)dx$

$= -\int \cot^{n-2}(a + bI)x\ \text{cosec}^{2}(a + bI)x - \int \cot^{n-2}(a + bI)xdx$

$= -\int \cot^{n-2}(a + bI)xd(cot(a + bI)x) - I_{\,n-2}$

$= -\dfrac{cot^{n-1}(a + bI)x}{n - 1} - I_{\,n-2}$

$= -\left(\dfrac{cot^{n-1}(a + bI)x}{n - 1}\right) - I_{n-2}$

The ultimate integral is $I_0\ or\ I_1$

n is even: $I_0 = \int dx = x + C$ when n = 0 and

n is odd: $I_1 = \int cotdx = \log(sinx) + C$ when n = 1.

Results on Reduction formula of Neutrosophic integrals:

Neutrosophic Integral	Reduction Formula	Base Integrals
$\int sin^{n}(a + bI)x$ dx.	$\frac{-1}{n} sin^{n-1}(a + bI)$x cos $(a + bI)$x + $\frac{(n-1)}{n} I_{n-2}$	n is even: $I_0 = \int dx = x + C$ when n = 0 and n is odd: $I_1 = \int sinxdx = -cosx + C$ when n = 1.
$\int cos^{n}(a + bI)x$dx	$\frac{1}{n} cos^{n-1}(a + bI)$x sin $(a + bI)$x + $\frac{(n-1)}{n} I_{n-2}$	n is even: $I_0 = \int dx = x + C$ when n = 0 and n is odd: $I_1 = \int cosxdx = \sin x + C$ when n = 1.
$\int cosec^{n}(a + bI)x$ dx	$\frac{-1}{n-1} cosec^{n-2}(a + bI)x$ $\cot(a + bI)$x+$\frac{n-2}{n-1} I_{\,n-2}$	n is even: $I_0 = \int dx = x + C$ when n = 0 and n is odd: $I_1 = \int cosecxdx = \log(cosecx - cotx) + C$ when n = 1.
$\int tan^{n}(a + bI)x$ dx	$\dfrac{(\tan^{n-1}(a + bI)x)}{n - 1} - I_{n-2}$	n is even: $I_0 = \int dx = x + C$ when n = 0 n is odd: $I_1 = \int tandx = -\log(cosx) + C$ when n = 1
$\int sec^{n}(a + bI)x$dx	$\frac{1}{n-1} sec^{n-2}(a + bI)xtan(a + bI)x$+$\frac{n-2}{n-1} I_{\,n-2}$	n is even: $I_0 = \int dx = x + C$ when n = 0 n is odd: $I_1 = \int secdx = \log(secx + tanx) + C$ when n = 1.
$\int cot^{n}(a + bI)x\ dx$	$-\left(\dfrac{cot^{n-1}(a + bI)x}{n - 1}\right) - I_{n-2}$	n is even: $I_0 = \int dx = x + C$ when n = 0 n is odd: $I_1 = \int cotdx = \log(sinx) + C$ when n = 1

4. Results and Conclusion:

Finding the integral values using the trigonometric, logarithmic, and exponential functions relies heavily on the reduction formula. With appropriate examples in a neutrosophic setting, the reduction formula and partial fraction methods of integration are presented here. An effective mathematical method for evaluating and simplifying integrals including trigonometric Neutrosophic functions is the Reduction Formula for Trigonometric Functions in Neutrosophic Integrals. The area of any irregular closed surfaces, the summation of a series, and the solution of differential equations containing indeterminacy and uncertainty can all be solved using this reduction formula and partial fraction methods. This partial fraction method and neutrosophic integrals reduction formula can also be used to solve incorrect neutrosophic integrals, alter the order of integration of neutrosophic integrals, and compute neutrosophic multiple integrals.

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