Unveiling an Innovative Approach to Q-Complex Neutrosophic Soft Rings

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Abstract

In this paper, our aim is to investigate the algebraic structures within the Q-complex neutrosophic soft model. We introduce two fundamental concepts: the Q-complex neutrosophic soft ring (Q-CNSR) and the Q-complex neutrosophic soft ideal (Q-CNSI). Q-CNSRs combine the properties of Q-complex neutrosophic soft sets (Q-CNSSs) with ring theory, effectively capturing uncertainty and indeterminacy present in ring operations through the incorporation of Q-complex neutrosophic membership values. Additionally, we define Q-CNSIs as subsets of Q-CNSRs that possess distinctive properties and hold significant roles in ring theory. Furthermore, we discuss and verify the specific algebraic properties of Q-CNSR and Q-CNSI. By examining these properties, we gain a deeper understanding of the algebraic behavior of Q-CNSR and Q-CNSI. In particular, we shed light on the relationship between Q-CNSRs and soft rings. This provides insights into how Q-CNSR relates to the broader framework of soft ring, highlighting the unique features and contributions of Q-complex neutrosophic soft structures in the realm of algebraic analysis. We have also verified the relations between Q-CNSR and Q-neutrosophic soft ring (Q-NSR), as well as between Q-CNSI and Q-neutrosophic soft ideal (Q-NSI). Through this comprehensive exploration, our objective is to advance the understanding of Q-CNSR and Q-CNSI, thereby contributing to the field of algebraic analysis and its application in handling uncertainty and vagueness.

Keywords: Complex neutrosophic soft set; Q-complex neutrosophic soft set; Q-neutrosophic soft ideal; Q-neutrosophic soft ring; Q-neutrosophic soft set.

1 introduction

In real-life situations, human thinking faces many situations that are fully hidden, uncertain, and impartial. To translate these positions and to handle the outlined uncertainties, Smarandache[1] provided the definition of

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a neutrosophic set (NS), since the preserve is not able to handle the outlined issues. On the other hand, one of the Russian researchers set out to introduce a new mathematical tool called the soft set (SS).[2] This set is distinguished by giving a more accurate description of the data on daily life issues. The NSs and SSs attracted the attention of researchers around the world to present many research works that have wide applications.
As a combination of SS and NS in several environments, scholars proposed different models with a more powerful ability to process real-life problems. For instance,[3,4] discusses the topic in the context of soft computing, while[6,7] covers its import to analysis and[8,9] to graph theory, complex analysis,[10,11] and algebraic structures.[12] Additionally, Palanikumar et al.[13] proposed many methods to solve design-making problems. A multi-criteria decision-making approach was introduced by Broumi et al.[14] when they extended NSs to plithogenic set. Al-Sharqi et al.[15] came up with NSS in matrix form and showed its application to real-life problems. Working on NSSs Al-Quran et al.[16] also presented some studies with some applications. Some researchers[17–19] also give similarity measures between NS-sets and some of their properties and applications.

On the opposite side, Ali and Smarandache[20] coined the concept of complex neutrosophic set (CNS) as an extension of NSs, but in the context of complex space. A CNS is defined by three complex-valued functions: the truth member (TM) function, representing uncertainty with periodicity; the indeterminacy member (IM) function, representing indeterminacy with periodicity; and the falsity member (FM) function, representing falsity with periodicity. Building upon this, Broumi et al.[21,22] combined the SS framework with CNS, giving rise to the concept of complex neutrosophic soft set (CNSS). CNSS provides an approximate description of objects under consideration, considering various parameters. On the other hand, Abu Qamar and Hassan[23] proposed a novel approach called Q-neutrosophic soft sets (Q-NSSs), which has been successfully applied to decision-making problems. Moreover, their remarkable contributions extended beyond decision-making. In the realm of group theory, Abu Qamar and Hassan[23] introduced the concept of Q-neutrosophic soft groups, conducting an extensive investigation into their properties and fundamental characteristics. Their pioneering research also encompassed the study of rings and fields within the paradigm of Q-Neutrosophic Soft Settings, as evidenced by their notable works. The exploration of algebraic structures within uncertainty sets in complex space has captivated the attention of numerous researchers. Alsarahead and Ahmad[24] extended the concept of fuzzy subgroup to the complex realm, resulting in the introduction of the distinguished complex fuzzy subgroup. Their contributions did not stop there, as they further introduced the notions of complex fuzzy subring,[25] complex fuzzy soft rings (CFSR),[26] complex intuitionistic fuzzy subrings[27] and complex intuitionistic fuzzy ideal.[28] The extension of Q-NSSs to the complex space was achieved by Al-Quran et al.,[29,30] who introduced the concept of Q-complex neutrosophic soft sets (Q-CNSSs). In their work, the authors not only pioneered Q-CNSSs but also conducted a comprehensive study of their algebraic structures. They defined the significant notions of Q-complex neutrosophic soft groups (Q-CNSG) and Q-complex neutrosophic soft subgroups (Q-CNSSG). In this article, we aim to delve deeper into the exploration of algebraic structures within Q-CNSSs. To enrich this investigation, we introduce the novel concepts of Q-CNSRs and Q-CNSIs. This pioneering research expands the horizons of knowledge, unraveling the intricate algebraic structures inherent in Q-CNSSs within the complex space.

2 Preliminaries

Within this section, we provide an overview of the fundamental principles underlying Q-NSS, Q-CNSS with their operations, Q-NSR, Q-NSI, CFSR and CFSI. These essential concepts and operations serve as the building blocks for our forthcoming analysis in this article.

Definition 2.1. [23] Assuming that Y and Q are non-empty sets, and $\mathbb{A}$ represents a set of parameters. The Q-NSS $(\mathbb{F}, \mathbb{A})$ in Y is distinctly characterized as follows:

$(\mathbb{F}, \mathbb{A}) = \{< a; \Gamma_{F(\mathfrak{a})}(y,q), \Lambda_{F(\mathfrak{a})}(y,q), \Omega_{F(\mathfrak{a})}(y,q) >: a \in \mathbb{A}, y \in Y, q \in Q\}$. The resolute functions $\Gamma_{F(\mathfrak{a})}(y,q), \Lambda_{F(\mathfrak{a})}(y,q)$, and $\Omega_{F(\mathfrak{a})}(y,q)$, unyieldingly, represent the TM, IM and FM functions, respectively.

Definition 2.2. [29] Let us assume that W and Q are two non-empty sets, and $\mathbb{A}$ is a set of parameters. A Q-CNSS $(\mathbb{H}, \mathbb{A})$ in W is defined as follows.

$$(\mathbb{H}, \mathbb{A}) = \{< \mathfrak{a}; \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q) >: \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q\},$$

where $\forall \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q$, $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q) = \Omega_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(r,q)}$, are, respectively, the complex-valued TM, IM and FM functions.

Definition 2.3. [29] The union of two Q-CNSSs $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ in W is a Q-CNSS $(\mathbb{M},\mathbb{E})$, where $\mathbb{E}=\mathbb{A}\cup\mathbb{B}$ and $\forall\, e\in\mathbb{E},\forall\,(r,q)\in W\times Q$,

$$\mathcal{T}_{\mathbb{M}(e)}(r,q)=\begin{cases}\Gamma_{\mathbb{H}(e)}(r,q).e^{i2\pi\mu_{\mathbb{H}(e)}(r,q)} & ,\text{if } e\in\mathbb{A}-\mathbb{B}\\ \Gamma_{\mathbb{G}(e)}(r,q).e^{i2\pi\mu_{\mathbb{G}(e)}(r,q)} & ,\text{if } e\in\mathbb{B}-\mathbb{A}\\ (\Gamma_{\mathbb{H}(e)}(r,q)\vee\Gamma_{\mathbb{G}(e)}(r,q)).e^{i2\pi(\mu_{\mathbb{H}(e)}(r,q)\vee\mu_{\mathbb{G}(e)}(r,q))} & ,\text{if } e\in\mathbb{A}\cap\mathbb{B},\end{cases}$$

$$\mathcal{I}_{\mathbb{M}(e)}(r,q)=\begin{cases}\Lambda_{\mathbb{H}(e)}(r,q).e^{i2\pi\nu_{\mathbb{H}(e)}(r,q)} & ,\text{if } e\in\mathbb{A}-\mathbb{B}\\ \Lambda_{\mathbb{G}(e)}(r,q).e^{i2\pi\nu_{\mathbb{G}(e)}(r,q)} & ,\text{if } e\in\mathbb{B}-\mathbb{A}\\ (\Lambda_{\mathbb{H}(e)}(r,q)\wedge\Lambda_{\mathbb{G}(e)}(r,q)).e^{i2\pi(\nu_{\mathbb{H}(e)}(r,q)\wedge\nu_{\mathbb{G}(e)}(r,q))} & ,\text{if } e\in\mathbb{A}\cap\mathbb{B},\end{cases}$$

$$\mathcal{F}_{\mathbb{M}(e)}(r,q)=\begin{cases}\Omega_{\mathbb{H}(e)}(r,q).e^{i2\pi\omega_{\mathbb{H}(e)}(r,q)} & ,\text{if } e\in\mathbb{A}-\mathbb{B}\\ \Omega_{\mathbb{G}(e)}(r,q).e^{i2\pi\omega_{\mathbb{G}(e)}(r,q)} & ,\text{if } e\in\mathbb{B}-\mathbb{A}\\ (\Omega_{\mathbb{H}(e)}(r,q)\wedge\Omega_{\mathbb{G}(e)}(r,q)).e^{i2\pi(\omega_{\mathbb{H}(e)}(r,q)\wedge\omega_{\mathbb{G}(e)}(r,q))} & ,\text{if } e\in\mathbb{A}\cap\mathbb{B}.\end{cases}$$

Here, $\vee$ represents the maximum operator and $\wedge$ represents the minimum operator. The union of $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ is denoted as $(\mathbb{M},\mathbb{E})$, i.e., $(\mathbb{H},\mathbb{A})\cup(\mathbb{G},\mathbb{B})=(\mathbb{M},\mathbb{E})$.

Definition 2.4. [30] The intersection of two Q-CNSSs $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ in W is a Q-CNSS $(\mathbb{M},\mathbb{E})$, where $\mathbb{E}=\mathbb{A}\cap\mathbb{B}$ and $\forall\, e\in\mathbb{E},\forall\,(r,q)\in W\times Q$, the membership degrees of $(\mathbb{M},\mathbb{E})$ are:

$$\mathcal{T}_{\mathbb{M}(e)}(r,q)=(\Gamma_{\mathbb{H}(e)}(r,q)\wedge\Gamma_{\mathbb{G}(e)}(r,q)).e^{i2\pi(\mu_{\mathbb{H}(e)}(r,q)\wedge\mu_{\mathbb{G}(e)}(r,q))},$$

$$\mathcal{I}_{\mathbb{M}(e)}(r,q)=(\Lambda_{\mathbb{H}(e)}(r,q)\vee\Lambda_{\mathbb{G}(e)}(r,q)).e^{i2\pi(\nu_{\mathbb{H}(e)}(r,q)\vee\nu_{\mathbb{G}(e)}(r,q))},$$

$$\mathcal{F}_{\mathbb{M}(e)}(r,q)=(\Omega_{\mathbb{H}(e)}(r,q)\vee\Omega_{\mathbb{G}(e)}(r,q)).e^{i2\pi(\omega_{\mathbb{H}(e)}(r,q)\vee\omega_{\mathbb{G}(e)}(r,q))}.$$

Here, $\vee$ represents the maximum operator and $\wedge$ represents the minimum operator. The intersection of $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ is denoted as $(\mathbb{M},\mathbb{E})$, i.e., $(\mathbb{H},\mathbb{A})\cap(\mathbb{G},\mathbb{B})=(\mathbb{M},\mathbb{E})$.

This part provides a clear explanation of the relationship between Q-CNSS and Q-NSS.

Remark 2.5. [30] Consider two non-empty sets, W and Q, and let $(\mathbb{H},\mathbb{A})$ be a Q-CNSS in W. The Q-CNSS $(\mathbb{H},\mathbb{A})$ can be expressed as a set of elements in the form $\{<\mathfrak{a};\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q)>:\mathfrak{a}\in\mathbb{A},r\in W,q\in Q\}$, where $\mathfrak{a}\in\mathbb{A},r\in W$, and $q\in Q$. Within this context, the complex-valued TM, IM, and FM functions can be defined as follows: $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q)=\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q)=\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q)=\Omega_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(r,q)}$. Based on this, $(\mathbb{H},\mathbb{A})$ generates two real Q-NSSs in W using the following formulations:

(1) The Q-NSS $(\mathfrak{h},\mathbb{A})$ is defined in the form of $\{<a;\Gamma_{\mathfrak{h}(\mathfrak{a})}(r,q),\Lambda_{\mathfrak{h}(\mathfrak{a})}(r,q),\Omega_{\mathfrak{h}(\mathfrak{a})}(r,q)>:a\in\mathbb{A},r\in W,q\in Q\}$, where $a\in\mathbb{A},r\in W$, and $q\in Q$. Within this context, $\Gamma_{\mathfrak{h}(\mathfrak{a})}(r,q),\Lambda_{\mathfrak{h}(\mathfrak{a})}(r,q)$, and $\Omega_{\mathfrak{h}(\mathfrak{a})}(r,q)$ represent the amplitude terms associated with the complex valued membership functions $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q)$, respectively.

(2) The Q-NSS $(\mathfrak{K},\mathbb{A})$ is defined in the form of $\{<a;\mu_{\mathfrak{K}(\mathfrak{a})}(r,q),\nu_{\mathfrak{K}(\mathfrak{a})}(r,q),\omega_{\mathfrak{K}(\mathfrak{a})}(r,q)>:a\in\mathbb{A},r\in W,q\in Q\}$, where $a\in\mathbb{A},r\in W$, and $q\in Q$. Within this context, $\mu_{\mathfrak{K}(\mathfrak{a})}(r,q),\nu_{\mathfrak{K}(\mathfrak{a})}(r,q)$, and $\omega_{\mathfrak{K}(\mathfrak{a})}(r,q)$ represent the phase terms associated with the complex valued membership functions $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q)$, respectively.

In this section, we discuss various concepts related to homogeneity in Q-CNSSs.

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Definition 2.6. [29] Consider two non-empty sets, W and Q. Let $(\mathbb{H}, \mathbb{A})$ be a Q-CNSS in W, characterized by the complex-valued membership functions as follows:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r, q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r, q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}$ and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q) = \Omega_{\mathbb{H}(\mathfrak{a})}(r, q).e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(r,q)}$. Then, $\forall\ r, s \in W$ and $\mathfrak{a} \in \mathbb{A}$, the following conditions hold for a set $(\mathbb{H}, \mathbb{A})$ to be considered a homogeneous Q-CNSS:

(1) $\Gamma_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \Gamma_{\mathbb{H}(\mathfrak{a})}(s, q)$ if and only if $\mu_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \mu_{\mathbb{H}(\mathfrak{a})}(s, q)$,

(2) $\Lambda_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \Lambda_{\mathbb{H}(\mathfrak{a})}(s, q)$ if and only if $\nu_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \nu_{\mathbb{H}(\mathfrak{a})}(s, q)$,

(3) $\Omega_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \Omega_{\mathbb{H}(\mathfrak{a})}(s, q)$ if and only if $\omega_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \omega_{\mathbb{H}(\mathfrak{a})}(s, q)$.

In summary, a homogeneous Q-CNSS $(\mathbb{H}, \mathbb{A})$ in W is characterized by the given complex-valued membership functions, satisfying the specified conditions related to the amplitude terms and their associated parameters.

Example 2.7. Consider two non-empty sets, $W = \{r, s\}$ and $Q = \{q_1, q_2\}$, and $\mathbb{A} = \{\mathfrak{a}_1, \mathfrak{a}_2\}$ as a set of parameters. Let's define the Q-CNSS $(\mathbb{H}, \mathbb{A})$ as follows:

$$(\mathbb{H}, \mathbb{A}) = \Big\{\Big(\mathfrak{a}_1, \{\frac{\langle 0.3e^{j2\Pi(0.4)}, 0.5e^{j2\Pi(0.6)}, 0.4e^{j2\Pi(0.7)}\rangle}{(r,q_1)}, \frac{\langle 0.4e^{j2\Pi(0.5)}, 0.6e^{j2\Pi(0.6)}, 0.4e^{j2\Pi(0.8)}\rangle}{(r,q_2)}, \frac{\langle 0.2e^{j2\Pi(0.3)}, 0.6e^{j2\Pi(0.7)}, 0.5e^{j2\Pi(0.8)}\rangle}{(s,q_1)}, \frac{\langle 0.4e^{j2\Pi(0.5)}, 0.7e^{j2\Pi(0.8)}, 0.9e^{j2\Pi(0.9)}\rangle}{(s,q_2)}\}\Big), \Big(\mathfrak{a}_2, \{\frac{\langle 0.9e^{j2\Pi(0.6)}, 0.7e^{j2\Pi(0.7)}, 0.8e^{j2\Pi(0.8)}\rangle}{(r,q_1)}, \frac{\langle 0.3e^{j2\Pi(0.4)}, 0.5e^{j2\Pi(0.3)}, 0.2e^{j2\Pi(0.6)}\rangle}{(r,q_2)}, \frac{\langle 0.9e^{j2\Pi(0.6)}, 0.6e^{j2\Pi(0.7)}, 0.4e^{j2\Pi(0.8)}\rangle}{(s,q_1)}, \frac{\langle 0.9e^{j2\Pi(0.6)}, 0.5e^{j2\Pi(0.6)}, 0.4e^{j2\Pi(0.8)}\rangle}{(s,q_2)}\}\Big)\Big\}.$$

From the given definition, it is evident that the Q-CNSS $(\mathbb{H}, \mathbb{A})$ exhibits homogeneity.

Definition 2.8. [29] Consider $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ as two Q-CNSSs in W, which are characterized by the following complex-valued membership functions:

For $(\mathbb{H}, \mathbb{A})$: $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r, q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r, q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}$ and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q) = \Omega_{\mathbb{H}(\mathfrak{a})}(r, q).e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(r,q)}$.
For $(\mathbb{G}, \mathbb{B})$: $\mathcal{T}_{\mathbb{G}(\mathfrak{a})}(r, q) = \Gamma_{\mathbb{G}(\mathfrak{a})}(r, q)e^{i2\pi\mu_{\mathbb{G}(\mathfrak{a})}(r,q)}$, $\mathcal{I}_{\mathbb{G}(\mathfrak{a})}(r, q) = \Lambda_{\mathbb{G}(\mathfrak{a})}(r, q)e^{i2\pi\nu_{\mathbb{G}(\mathfrak{a})}(r,q)}$ and $\mathcal{F}_{\mathbb{G}(\mathfrak{a})}(r, q) = \Omega_{\mathbb{G}(\mathfrak{a})}(r, q).e^{i2\pi\omega_{\mathbb{G}(\mathfrak{a})}(r,q)}$.

Q-CNSS $(\mathbb{H}, \mathbb{A})$ is said to be homogeneous with $(\mathbb{G}, \mathbb{B})$ if and only if for all $\mathfrak{a} \in \mathbb{A} \cap \mathbb{B}$, $r \in W$ and $q \in Q$, we have

(1) $\Gamma_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \Gamma_{\mathbb{G}(\mathfrak{a})}(r, q)$ if and only if $\mu_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \mu_{\mathbb{G}(\mathfrak{a})}(r, q)$,

(2) $\Lambda_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \Lambda_{\mathbb{G}(\mathfrak{a})}(r, q)$ if and only if $\nu_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \nu_{\mathbb{G}(\mathfrak{a})}(r, q)$,

(3) $\Omega_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \Omega_{\mathbb{G}(\mathfrak{a})}(r, q)$ if and only if $\omega_{\mathbb{H}(\mathfrak{a})}(r, q) \leq \omega_{\mathbb{G}(\mathfrak{a})}(r, q)$.

Definition 2.9. [23] A Q-NS ring over $(\mathbb{R}, +, .)$ is defined as a Q-NSS $(\mathbb{F}, \mathbb{A})$, satisfying the following conditions for all $y, z \in \mathbb{R}$, $q \in Q$ and $\mathfrak{a} \in \mathbb{A}$:

1. $\Gamma_{F(\mathfrak{a})}(y - z, q) \geq \min\{\Gamma_{F(\mathfrak{a})}(y, q), \Gamma_{F(\mathfrak{a})}(z, q)\}$,
2. $\Lambda_{F(\mathfrak{a})}(y - z, q) \leq \max\{\Lambda_{F(\mathfrak{a})}(y, q), \Lambda_{F(\mathfrak{a})}(z, q)\}$,
3. $\Omega_{F(\mathfrak{a})}(y - z, q) \leq \max\{\Omega_{F(\mathfrak{a})}(y, q), \Omega_{F(\mathfrak{a})}(z, q)\}$.
4. $\Gamma_{F(\mathfrak{a})}(y.z, q) \geq \min\{\Gamma_{F(\mathfrak{a})}(y, q), \Gamma_{F(\mathfrak{a})}(z, q)\}$,
5. $\Lambda_{F(\mathfrak{a})}(y.z, q) \leq \max\{\Lambda_{F(\mathfrak{a})}(y, q), \Lambda_{F(\mathfrak{a})}(z, q)\}$,
6. $\Omega_{F(\mathfrak{a})}(y.z, q) \leq \max\{\Omega_{F(\mathfrak{a})}(y, q), \Omega_{F(\mathfrak{a})}(z, q)\}$.

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Definition 2.10. [30] A Q-NSS over $(\mathbb{R}, +, .)$ is defined as a Q-NS ideal over $(\mathbb{R}, +, .)$, if it is satisfying the following conditions for all $y, z \in \mathbb{R}$, $q \in Q$ and $\mathfrak{a} \in \mathbb{A}$:

1. $\Gamma_{F(\mathfrak{a})}(y - z, q) \geq \min\{\Gamma_{F(\mathfrak{a})}(y, q), \Gamma_{F(\mathfrak{a})}(z, q)\}$,
2. $\Lambda_{F(\mathfrak{a})}(y - z, q) \leq \max\{\Lambda_{F(\mathfrak{a})}(y, q), \Lambda_{F(\mathfrak{a})}(z, q)\}$,
3. $\Omega_{F(\mathfrak{a})}(y - z, q) \leq \max\{\Omega_{F(\mathfrak{a})}(y, q), \Omega_{F(\mathfrak{a})}(z, q)\}$.
4. $\Gamma_{F(\mathfrak{a})}(y.z, q) \geq \max\{\Gamma_{F(\mathfrak{a})}(y, q), \Gamma_{F(\mathfrak{a})}(z, q)\}$,
5. $\Lambda_{F(\mathfrak{a})}(y.z, q) \leq \min\{\Lambda_{F(\mathfrak{a})}(y, q), \Lambda_{F(\mathfrak{a})}(z, q)\}$,
6. $\Omega_{F(\mathfrak{a})}(y.z, q) \leq \min\{\Omega_{F(\mathfrak{a})}(y, q), \Omega_{F(\mathfrak{a})}(z, q)\}$.

Definition 2.11. [24] Let $(\mathfrak{C}, \mathfrak{A}) = \{< u; \mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r) >: u \in \mathfrak{A}, r \in \mathbb{R}\}$ be a homogeneous complex fuzzy soft set over a ring $(\mathbb{R}, +, .)$. Then $(\mathfrak{C}, \mathfrak{A})$ is said to be a complex fuzzy soft ring shortly (CFSR) over $\mathbb{R}$ if and only if the following hold:

1. $\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r - s) \geq \min\{\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r), \Gamma_{\mathfrak{C}(\mathfrak{u})}(s)\}$,
2. $\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r.s) \geq \min\{\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r), \Gamma_{\mathfrak{C}(\mathfrak{u})}(s)\}$.

Definition 2.12. [25] Let $(\mathfrak{C}, \mathfrak{A}) = \{< u; \mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r) >: u \in \mathfrak{A}, r \in \mathbb{R}\}$ be a homogeneous complex fuzzy soft set over a ring $(\mathbb{R}, +, .)$. Then $(\mathfrak{C}, \mathfrak{A})$ is said to be a complex fuzzy soft ideal shortly (CFSI) over $\mathbb{R}$ if and only if the following hold:

1. $\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r - s) \geq \min\{\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r), \Gamma_{\mathfrak{C}(\mathfrak{u})}(s)\}$,
2. $\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r.s) \geq \max\{\mathcal{T}_{\mathfrak{C}(\mathfrak{u})}(r), \Gamma_{\mathfrak{C}(\mathfrak{u})}(s)\}$.

3 Q-Complex Neutrosophic Soft Ring

In this section, we define the notion of Q-CNSR and its algebraic properties.

Definition 3.1. Assume $(\mathbb{H}, \mathbb{A})$ is a Q-CNSS in $(\mathbb{R}, +, .)$. It is stated that $(\mathbb{H}, \mathbb{A})$ is categorized as a Q-CNSR in $(\mathbb{R}, +, .)$ if and only if, for every $\mathfrak{a} \in \mathbb{A}$, $q \in Q$ and all $r, s \in \mathbb{R}$, the following conditions are fulfilled:

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-r, q) \geq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q)$,
2. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-r, q) \leq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q)$,
3. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-r, q) \leq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q)$,
4. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
5. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
6. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r.s, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
7. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r + s, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
8. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r + s, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
9. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r + s, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$.

Definition 3.2. Consider two Q-CNSSs denoted as $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{K}, \mathbb{B})$. It is asserted that $(\mathbb{H}, \mathbb{A})$ is designated as a Q-CNS subring of $(\mathbb{K}, \mathbb{B})$ only when the following conditions are met with absolute certainty:

1 $(\mathbb{H}, \mathbb{A}) \subseteq (\mathbb{K}, \mathbb{B})$, where $\subseteq$ is a Q-CNS subset.

2 Both $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{K}, \mathbb{B})$ are Q-CNSRs.

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Theorem 3.3. *Consider a Q-CNSS denoted as* $(\mathbb{H}, \mathbb{A})$ *in* $(\mathbb{R}, +, .)$*. It is asserted that* $(\mathbb{H}, \mathbb{A})$ *can be classified as a Q-CNSR in* $(\mathbb{R}, +, .)$ *if and only if, for every* $\mathfrak{a} \in \mathbb{A}$, $q \in Q$, *and* r , s *from* $\mathbb{R}$, *the following conditions are satisfied:*

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
2. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
3. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-s, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
4. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
5. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,
6. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r.s, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$.

Proof. $\Rightarrow$ Consider that $(\mathbb{H}, \mathbb{A})$ is a Q-CNSR in $(\mathbb{R}, +, .)$. Then,
$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r+(-s), q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-s, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r+(-s), q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-s, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-s, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r+(-s), q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-s, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$.

Given that $(\mathbb{H}, \mathbb{A})$ is a Q-CNSR, it automatically satisfies conditions 4, 5, and 6.

$\Leftarrow$ To prove the converse, let us assume that conditions 1-6 are fulfilled.

To establish the validity of the first three conditions of Definition (3.1), we demonstrate the following properties for the additive identity "0" in the set $(\mathbb{R}, +, .)$:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-r, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q)$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-r, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q)$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-r, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q)$.

Consequently, $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-r, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0-r, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q)$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-r, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0-r, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q)$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-r, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0-r, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q)$.

Conditions from 4-6 in Definition (3.1) have already satisfied.

To establish the validity of conditions 7-9 in Definition (3.1), we will proceed by demonstrating that

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r+s, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-(-s), q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-s, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r+s, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-(-s), q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-s, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r+s, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-(-s), q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-s, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$.
With this, we have concluded the proof. □

Theorem 3.4. *Let* $(\mathbb{R}, +, .)$ *be a ring, and let* $(\mathbb{H}, \mathbb{A}) = \{< \mathfrak{a}; \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q) >: \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q\}$ *be homogeneous Q-CNSS in* $(\mathbb{R}, +, .)$*. Suppose* $(\mathbb{H}, \mathbb{A})$ *generates the two Q-NSSs* $(\mathfrak{h}, \mathbb{A}) = \{< a; \Gamma_{\mathfrak{h}(\mathfrak{a})}(r, q), \Lambda_{\mathfrak{h}(\mathfrak{a})}(r, q), \Omega_{\mathfrak{h}(\mathfrak{a})}(r, q) >: a \in \mathbb{A}, r \in W, q \in Q\}$ *and* $(\mathfrak{K}, \mathbb{A}) = \{< a; \mu_{\mathfrak{K}(\mathfrak{a})}(r, q), \nu_{\mathfrak{K}(\mathfrak{a})}(r, q), \omega_{\mathfrak{K}(\mathfrak{a})}(r, q) >: a \in \mathbb{A}, r \in W, q \in Q\}$*. Then,* $(\mathbb{H}, \mathbb{A})$ *is a Q-CNS subring of* $\mathbb{R}$ *if and only if both* $(\mathfrak{h}, \mathbb{A})$ *and* $(\mathfrak{K}, \mathbb{A})$ *are Q-NS subrings.*

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Proof. To establish the validity of the first direction of this theorem, it is necessary to demonstrate compliance with the six conditions outlined in Definition (2.9).

$\Rightarrow$ Consider that $(\mathbb{H}, \mathbb{A})$ is a Q-CNS subring of $\mathbb{R}$, then for all $a \in \mathbb{A}, r, s \in \mathbb{R}, q \in Q$, we have,

$\Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q), \mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

Likewise, we can derive,

$\Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q), \nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

By employing a similar approach, we can obtain $\Omega_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\Omega_{\mathbb{H}(\mathfrak{a})}(r,q), \Omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\omega_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\omega_{\mathbb{H}(\mathfrak{a})}(r,q), \omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. Therefore, conditions 1, 2, and 3 are satisfied.

Next, $\Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q), \mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

Similarly, we can obtain

$\Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q), \nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

In the same way we get,

$\Omega_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\Omega_{\mathbb{H}(\mathfrak{a})}(r,q), \Omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\omega_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\omega_{\mathbb{H}(\mathfrak{a})}(r,q), \omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Therefore, conditions 4, 5, and 6 are satisfied. Which implies that $(\mathfrak{h}, \mathbb{A})$ and $(\mathfrak{K}, \mathbb{A})$ are Q-NS subrings.

In order to prove the second direction of this theorem, it should satisfy previously defined six conditions listed in Theorem (3.3).

$\Leftarrow$ Suppose that $(\mathfrak{h}, \mathbb{A})$ and $(\mathfrak{K}, \mathbb{A})$ are two Q-NS subrings. To prove that $(\mathbb{H}, \mathbb{A})$ is a Q-CNS subring, we have to show that:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

In a similar manner : $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.$
$e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.
($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

In the same manner we show that $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. Therefore, conditions 1, 2, and 3 are satisfied.

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To verify the validity of the conditions 4, 5, and 6, we have to show that:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H,\mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Similarly, we can obtain

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H,\mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Using the same steps, we can show that $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Thus, the six conditions listed in Theorem (3.3) have been verified. Which proves that $(\mathbb{H},\mathbb{A})$ is Q-CNS subring. □

Theorem 3.5. *Consider a ring $(\mathbb{R},+,.)$ and let $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ be two Q-CNSSs in $\mathbb{R}$, where $(\mathbb{H},\mathbb{A})$ is homogeneous with $(\mathbb{G},\mathbb{B})$. If both $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ are Q-CNSRs in $\mathbb{R}$, then their intersection $(\mathbb{H},\mathbb{A}) \cap (\mathbb{G},\mathbb{B})$ is also a Q-CNSR in $\mathbb{R}$.*

Proof. Suppose $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ are two Q-CNSRs. Then, the generated Q-NSSs are also Q-NSRs. Let's begin by establishing the validity of the first three conditions of Theorem (3.3).

First, We examine the complex-valued truth membership function of the intersection.

For all $e \in \mathbb{A} \cap \mathbb{B}$, $q \in Q$, and $r,s \in \mathbb{R}$,

$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) = \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q)}$
$= \min\{\Gamma_{\mathbb{H}(e)}(r-s,q), \Gamma_{\mathbb{G}(e)}(r-s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r-s,q),\mu_{\mathbb{G}(e)}(r-s,q)\}}$
$\geq \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{H}(e)}(s,q)\}, \min\{\Gamma_{\mathbb{G}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{H}(e)}(s,q)\},\min\{\mu_{\mathbb{G}(e)}(r,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\},\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\}}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}}\}$ ($(\mathbb{H},\mathbb{A})$ is homogeneous with $(\mathbb{G},\mathbb{B})$)
$= \min\{\Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Second: We will verify whether the condition for the indeterminacy membership function of the intersection is met.

$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) = \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q)}$
$= \max\{\Lambda_{\mathbb{H}(e)}(r-s,q), \Lambda_{\mathbb{G}(e)}(r-s,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(r-s,q),\nu_{\mathbb{G}(e)}(r-s,q)\}}$
$\leq \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{H}(e)}(s,q)\}, \max\{\Lambda_{\mathbb{G}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{H}(e)}(s,q)\},\max\{\nu_{\mathbb{G}(e)}(r,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\},\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\}}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}}\}$ ($(\mathbb{H},\mathbb{A})$ is homogeneous with $(\mathbb{G},\mathbb{B})$)
$= \max\{\Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$

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$= \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Third: Using the same steps as in the case of indeterminacy membership function, we obtain:

$\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) \leq \max\{\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Conditions 4-6 of Theorem (3.3) can be examined as follows.

For the complex-valued truth membership function of the intersection, we obtain

$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) = \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q)}$
$= \min\{\Gamma_{\mathbb{H}(e)}(r.s,q), \Gamma_{\mathbb{G}(e)}(r.s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r.s,q),\mu_{\mathbb{G}(e)}(r.s,q)\}}$
$\geq \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{H}(e)}(s,q)\}, \min\{\Gamma_{\mathbb{G}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{H}(e)}(s,q)\},\min\{\mu_{\mathbb{G}(e)}(r,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\},\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\}}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}}\}$ (($\mathbb{H}, \mathbb{A}$) is homogeneous with ($\mathbb{G}, \mathbb{B}$))
$= \min\{\Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) \geq \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

For the complex-valued indeterminacy membership function of the intersection, we obtain:

$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) = \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q)}$
$= \max\{\Lambda_{\mathbb{H}(e)}(r.s,q), \Lambda_{\mathbb{G}(e)}(r.s,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(r.s,q),\nu_{\mathbb{G}(e)}(r.s,q)\}}$
$\leq \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{H}(e)}(s,q)\}, \max\{\Lambda_{\mathbb{G}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{H}(e)}(s,q)\},\max\{\nu_{\mathbb{G}(e)}(r,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\},\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\}}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}}\}$ (($\mathbb{H}, \mathbb{A}$) is homogeneous with ($\mathbb{G}, \mathbb{B}$))
$= \max\{\Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) \leq \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

In the similar way, we can show that $\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) \leq \max\{\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Thus, the six conditions listed in Theorem (3.3) have been verified. Which proves that the intersection ($\mathbb{H}, \mathbb{A}$) $\cap$ ($\mathbb{G}, \mathbb{B}$) is a Q-CNSR.

□

Definition 3.6. Consider W and Q are two non empty sets and ($\mathbb{H}, \mathbb{A}$) is a Q-CNSS in W such that $(\mathbb{H}, \mathbb{A}) = \{<\mathfrak{a}; \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q)>: \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q\}$, where $\forall \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q$, $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q) = \Omega_{\mathbb{H}(\mathfrak{a})}(r,q)$. $e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(r,q)}$,Then $\forall \alpha, \beta \in [0,1]$, the (α,β)-level set of ($\mathbb{H}, \mathbb{A}$), denoted by $(\mathbb{H}, \mathbb{A})_{(\alpha,\beta)}$ is a soft set in W defined as:

$(\mathbb{H}, \mathbb{A})_{(\alpha,\beta)} = \{(\mathfrak{a}, \mathbb{H}_{(\alpha,\beta)}(\mathfrak{a})) : \mathfrak{a} \in \mathbb{A}, \mathbb{H}_{(\alpha,\beta)}(\mathfrak{a}) \in \rho(W \times Q)\}$, where,

$\mathbb{H}_{(\alpha,\beta)}(\mathfrak{a}) = \{r \in W, q \in Q : \Gamma_{\mathbb{H}(\mathfrak{a})}(r,q) \geq \alpha, \Lambda_{\mathbb{H}(\mathfrak{a})}(r,q) \leq \alpha, \Omega_{\mathbb{H}(\mathfrak{a})}(r,q) \leq \alpha$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r,q) \geq \beta, \nu_{\mathbb{H}(\mathfrak{a})}(r,q) \leq \beta, \omega_{\mathbb{H}(\mathfrak{a})}(r,q) \leq \beta\}, \forall \mathfrak{a} \in \mathbb{A}$.

If $\alpha = \beta$, then $(\mathbb{H}, \mathbb{A})_{(\alpha,\beta)}$ is called $\alpha-$level set of ($\mathbb{H}, \mathbb{A}$), denoted by $(\mathbb{H}, \mathbb{A})_\alpha$ and defined as:

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$(\mathbb{H},\mathbb{A})_\alpha=\{(\mathfrak{a},\mathbb{H}_\alpha(\mathfrak{a})):\mathfrak{a}\in\mathbb{A},\mathbb{H}_\alpha(\mathfrak{a})\in\rho(W\times Q)\}$, where,

$\mathbb{H}_\alpha(\mathfrak{a})=\{r\in W,q\in Q:\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q)\geq\alpha,\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\alpha,\Omega_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\alpha$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r,q)\geq\alpha,\nu_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\alpha,\omega_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\alpha\},\forall\mathfrak{a}\in\mathbb{A}$.

The next theorem serves as a vital link between Q-CNSR and soft ring.

Theorem 3.7. *Suppose $(\mathbb{H},\mathbb{A})$ is a Q-CNSR over $(\mathbb{R},+,.)$. Then, for arbitrary $\alpha,\beta\in[0,1]$, the (α,β)-level set $(\mathbb{H},\mathbb{A})_{(\alpha,\beta)}$ constitutes a soft ring of $(\mathbb{R},+,.)$.*

Proof. Suppose $(\mathbb{H},\mathbb{A})$ is a Q-CNSR over $(\mathbb{R},+,.)$. For arbitrary $\alpha,\beta\in[0,1]$, let $\mathfrak{a}\in\mathbb{A}$, $r,s\in\mathbb{H}_{(\alpha,\beta)}(\mathfrak{a})$ and $q\in Q$. Then, we have

$\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q)\geq\alpha,\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\alpha,\Omega_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\alpha$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r,q)\geq\beta,\nu_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\beta,\omega_{\mathbb{H}(\mathfrak{a})}(r,q)\leq\beta$ and $\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\geq\alpha,\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\leq\alpha,\Omega_{\mathbb{H}(\mathfrak{a})}(s,q)\leq\alpha$ and $\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\geq\beta,\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\leq\beta,\omega_{\mathbb{H}(\mathfrak{a})}(s,q)\leq\beta$.

To show that $r-s\in\mathbb{H}_{(\alpha,\beta)}(\mathfrak{a})$ and $r.s\in\mathbb{H}_{(\alpha,\beta)}(\mathfrak{a})$, $\forall\mathfrak{a}\in\mathbb{A}$, we begin with complex valued truth membership function as follows.

Since $(\mathbb{H},\mathbb{A})$ is a Q-CNSR over $\mathbb{R}$, then we have

$\Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q)}=\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q)\geq\min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}=\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)},$
$\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\}=\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q),\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q)\geq\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q),\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}\geq\min\{\alpha,\alpha\}=\alpha$

and $\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q)\geq\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}\geq\min\{\beta,\beta\}=\beta$.

For indeterminacy membership term, we have:

$\Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q)}=\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q)\leq\max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}=\max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)},$
$\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\}=\max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q),\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q)\leq\max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q),\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}\leq\max\{\alpha,\alpha\}=\alpha$.

and $\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q)\leq\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}\leq\max\{\beta,\beta\}=\beta$.

In the same manner we show that $\Omega_{\mathbb{H}(\mathfrak{a})}(r-s,q)\leq\max\{\Omega_{\mathbb{H}(\mathfrak{a})}(r,q),\Omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}\leq\max\{\alpha,\alpha\}=\alpha$.

and $\omega_{\mathbb{H}(\mathfrak{a})}(r-s,q)\leq\max\{\omega_{\mathbb{H}(\mathfrak{a})}(r,q),\omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}\leq\max\{\beta,\beta\}=\beta$. This implies that $r-s\in\mathbb{H}_{(\alpha,\beta)}(\mathfrak{a})$. This satisfies the first condition of the soft ring definition. In order to fulfill the second condition, we begin with complex valued truth membership function as follows.

let $r,s\in\mathbb{H}_{(\alpha,\beta)}(\mathfrak{a})$. Then, $\forall\mathfrak{a}\in\mathbb{A}$, we have:

$\Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q)}=\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q)\geq\min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}=\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)},$
$\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\}=\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q),\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,

$\Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q)\geq\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q),\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}\geq\min\{\alpha,\alpha\}=\alpha$ and

$\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q)\geq\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}\geq\min\{\beta,\beta\}=\beta$.

Similarly, the process can be applied in the same manner for the complex-valued indeterminacy membership function, as outlined below:

$\Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q)}=\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q)\leq\max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}=\max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)},$
$\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\}=\max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q),\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,

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$\Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\} \leq \max\{\alpha,\alpha\} = \alpha$ and

$\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q), \nu_{\mathbb{H}(\mathfrak{a})}(s,q)\} \leq \max\{\beta,\beta\} = \beta.$

By following a similar approach, we can demonstrate that $\Omega_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \alpha$ and $\omega_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \beta$.

This indicates that $r.s \in \mathbb{H}_{(\alpha,\beta)}(\mathfrak{a}) \forall \mathfrak{a} \in \mathbb{A}$, fulfilling the second condition of the soft ring definition. Consequently, $(\mathbb{H},\mathbb{A})_{(\alpha,\beta)}$ qualifies as a soft ring in $\mathbb{R}$. With this, the proof is concluded.

□

4 Q-Complex Neutrosophic Soft Ideal

In this section, we define the notion of Q-CNSI and its algebraic properties.

Definition 4.1. Consider a Q-CNSS denoted as $(\mathbb{H},\mathbb{A})$ over a ring $(\mathbb{R},+,.)$. Then $(\mathbb{H},\mathbb{A})$ can be classified as a Q-CNS ideal over $(\mathbb{R},+,.)$ if for every $\mathfrak{a} \in \mathbb{A}$, $q \in Q$, and r , s from $\mathbb{R}$, the following conditions are satisfied:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\},$

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\},$

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)\},$

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \max\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\},$

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\},$

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)\},$

Theorem 4.2. *Let $(\mathbb{R},+,.)$ be a ring, and let $(\mathbb{H},\mathbb{A}) = \{< \mathfrak{a}; \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q) >: \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q\}$ be homogeneous Q-CNSS in $(\mathbb{R},+,.)$. Suppose $(\mathbb{H},\mathbb{A})$ generates the two Q-NSSs $(\mathfrak{h},\mathbb{A}) = \{< a; \Gamma_{\mathfrak{h}(\mathfrak{a})}(r,q), \Lambda_{\mathfrak{h}(\mathfrak{a})}(r,q), \Omega_{\mathfrak{h}(\mathfrak{a})}(r,q) >: a \in \mathbb{A}, r \in W, q \in Q\}$ and $(\mathfrak{K},\mathbb{A}) = \{< a; \mu_{\mathfrak{K}(\mathfrak{a})}(r,q), \nu_{\mathfrak{K}(\mathfrak{a})}(r,q),$ $\omega_{\mathfrak{K}(\mathfrak{a})}(r,q) >: a \in \mathbb{A}, r \in W, q \in Q\}$. Then, $(\mathbb{H},\mathbb{A})$ is a Q-CNS ideal of $\mathbb{R}$ if and only if both $(\mathfrak{h},\mathbb{A})$ and $(\mathfrak{K},\mathbb{A})$ are Q-NS ideals.*

Proof. In order to establish the validity of the first direction of this theorem, it is imperative to provide evidence of meeting the six conditions delineated in Definition (2.10).

⇒ Consider that $(\mathbb{H},\mathbb{A})$ is a Q-CNSI over $\mathbb{R}$, then for all $a \in \mathbb{A}, r,s \in \mathbb{R}, q \in Q$, we have,

$\Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)},$ $\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q), \mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H},\mathbb{A})$ is homogeneous).

Likewise, we can derive,

$\Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)},$ $\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus,
$\Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q), \nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H},\mathbb{A})$ is homogeneous).

By employing a similar approach, we can obtain $\Omega_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\Omega_{\mathbb{H}(\mathfrak{a})}(r,q), \Omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\omega_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\omega_{\mathbb{H}(\mathfrak{a})}(r,q), \omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. Therefore, conditions 1, 2, and 3 are satisfied.

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Next, $\Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \max\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \max\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)},$ $\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus, $\Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \max\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \max\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q), \mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

Similarly, we can obtain

$\Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\} = \min\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)},$ $\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}}$. Thus, $\Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q), \nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

In the same way we get,

$\Omega_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\Omega_{\mathbb{H}(\mathfrak{a})}(r,q), \Omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$ and $\omega_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\omega_{\mathbb{H}(\mathfrak{a})}(r,q), \omega_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Therefore, conditions 4, 5, and 6 are satisfied. Which implies that $(\mathfrak{h}, \mathbb{A})$ and $(\mathfrak{K}, \mathbb{A})$ are Q-CNSIs.

In order to prove the second direction of this theorem, it should satisfy previously defined six conditions listed in Theorem (3.8).

$\Leftarrow$ Suppose that $(\mathfrak{h}, \mathbb{A})$ and $(\mathfrak{K}, \mathbb{A})$ are two Q-CNSIs. To prove that $(\mathbb{H}, \mathbb{A})$ is a Q-CNSI, we have to show that:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

In a similar manner : $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r-s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r-s,q)} \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.$ $e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

In the same manner we show that $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r-s,q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. Therefore, conditions 1, 2, and 3 are satisfied.

To verify the validity of the conditions 4, 5, and 6, we have to show that:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} \geq \max\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\max\{\mu_{\mathbb{H}(\mathfrak{a})}(r,q),\mu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \max\{\Gamma_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \max\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \geq \max\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Similarly, we can obtain

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(r.s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r.s,q)} \leq \min\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)\}.e^{i\min\{\nu_{\mathbb{H}(\mathfrak{a})}(r,q),\nu_{\mathbb{H}(\mathfrak{a})}(s,q)\}} = \min\{\Lambda_{\mathbb{H}(\mathfrak{a})}(r,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(r,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}\} = \min\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Using the same steps, we can show that $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r.s,q) \leq \min\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)\}$.

Thus, the six conditions listed in Theorem (3.8) have been verified. Which proves that $(\mathbb{H}, \mathbb{A})$ is Q-CNSI. □

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Theorem 4.3. *Let $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ be two Q-CNSIs over a ring $\mathbb{R}$, where $(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$. Then their intersection $(\mathbb{H}, \mathbb{A}) \cap (\mathbb{G}, \mathbb{B})$ is also a Q-CNSI in $\mathbb{R}$.*

Proof. Suppose $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are two Q-CNSIs over a ring $\mathbb{R}$. Let's begin by establishing the validity of the first three conditions of Definition (3.8).

First, We examine the complex-valued truth membership function of the intersection.

For all $e \in \mathbb{A} \cap \mathbb{B}$, $q \in Q$, and $r, s \in \mathbb{R}$,

$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) = \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q)}$
$= \min\{\Gamma_{\mathbb{H}(e)}(r-s,q), \Gamma_{\mathbb{G}(e)}(r-s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r-s,q),\mu_{\mathbb{G}(e)}(r-s,q)\}}$
$\geq \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{H}(e)}(s,q)\}, \min\{\Gamma_{\mathbb{G}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{H}(e)}(s,q)\},\min\{\mu_{\mathbb{G}(e)}(r,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\},\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\}}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}}\}$ ($(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$)
$= \min\{\Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) \geq \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Second: We will verify whether the condition for the indeterminacy membership function of the intersection is met.

$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) = \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q)}$
$= \max\{\Lambda_{\mathbb{H}(e)}(r-s,q), \Lambda_{\mathbb{G}(e)}(r-s,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(r-s,q),\nu_{\mathbb{G}(e)}(r-s,q)\}}$
$\leq \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{H}(e)}(s,q)\}, \max\{\Lambda_{\mathbb{G}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{H}(e)}(s,q)\},\max\{\nu_{\mathbb{G}(e)}(r,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\},\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\}}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}}\}$ ($(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$)
$= \max\{\Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) \leq \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Third: Using the same steps as in the case of indeterminacy membership function, we obtain:

$\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r-s,q) \leq \max\{\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Conditions 4-6 of Definition (3.8) can be examined as follows.

For the complex-valued truth membership function of the intersection, we obtain

$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) = \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q)}$
$= \min\{\Gamma_{\mathbb{H}(e)}(r.s,q), \Gamma_{\mathbb{G}(e)}(r.s,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(r.s,q),\mu_{\mathbb{G}(e)}(r.s,q)\}}$
$\geq \min\{\max\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{H}(e)}(s,q)\}, \max\{\Gamma_{\mathbb{G}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\min\{\max\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{H}(e)}(s,q)\},\max\{\mu_{\mathbb{G}(e)}(r,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$\geq \max\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{H}(e)}(s,q)\}, \min\{\Gamma_{\mathbb{G}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{H}(e)}(s,q)\},\min\{\mu_{\mathbb{G}(e)}(r,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi\max\{\min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\},\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \max\{\min\{\Gamma_{\mathbb{H}(e)}(r,q), \Gamma_{\mathbb{G}(e)}(r,q)\}.$

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$e^{i2\pi \min\{\mu_{\mathbb{H}(e)}(r,q),\mu_{\mathbb{G}(e)}(r,q)\}}, \min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi \min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}}\}$ ($(\mathbb{H},\mathbb{A})$ is homogeneous with $(\mathbb{G},\mathbb{B})$)
$= \max\{\Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \max\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) \geq \max\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

For the complex-valued indeterminacy membership function of the intersection, we obtain:

$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) = \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q)}$
$= \max\{\Lambda_{\mathbb{H}(e)}(r.s,q), \Lambda_{\mathbb{G}(e)}(r.s,q)\}.e^{i2\pi \max\{\nu_{\mathbb{H}(e)}(r.s,q),\nu_{\mathbb{G}(e)}(r.s,q)\}}$
$\leq \max\{\min\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{H}(e)}(s,q)\}, \min\{\Lambda_{\mathbb{G}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi \max\{\min\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{H}(e)}(s,q)\},\min\{\nu_{\mathbb{G}(e)}(r,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$\leq \min\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{H}(e)}(s,q)\}, \max\{\Lambda_{\mathbb{G}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi \min\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{H}(e)}(s,q)\},\max\{\nu_{\mathbb{G}(e)}(r,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}\}.$
$e^{i2\pi \min\{\max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\},\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}\}}$
$= \min\{\max\{\Lambda_{\mathbb{H}(e)}(r,q), \Lambda_{\mathbb{G}(e)}(r,q)\}.$
$e^{i2\pi \max\{\nu_{\mathbb{H}(e)}(r,q),\nu_{\mathbb{G}(e)}(r,q)\}}, \max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}.e^{i2\pi \max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}}\}$ ($(\mathbb{H},\mathbb{A})$ is homogeneous with $(\mathbb{G},\mathbb{B})$)
$= \min\{\Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(r,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(r,q)}, \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}\}$
$= \min\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$. Thus,
$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) \leq \min\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

In the similar way, we can show that $\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r.s,q) \leq \min\{\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)\}$.

Thus, the six conditions listed in Definition (3.8) have been verified. Which proves that the intersection $(\mathbb{H},\mathbb{A})$ $\cap$ $(\mathbb{G},\mathbb{B})$ is a Q-CNSI.

□

5 Conclusion

In this paper, we investigated various algebraic structures within the model of Q-CNSS by introducing the notions of Q-CNSR and Q-CNSI. Our discussion has focused on verifying and examining the algebraic properties specific to Q-CNSRs and Q-CNSIs, enabling a deeper understanding of their algebraic behavior. We have shed light on the relationship between Q-CNSRs and soft rings. Additionally, we have verified the relationships between Q-CNSRs and Q-NSRs, as well as between Q-CNSIs and Q-NSIs. This comprehensive exploration aims to enhance the understanding of Q-CNSRs and Q-CNSIs, ultimately contributing to the field of algebraic analysis. Moving forward, our future research will focus on investigating additional algebraic structures related to Q-CNSSs, specifically exploring the realm of Q-complex neutrosophic soft fields. By extending our analysis to this new domain, we aim to deepen our understanding of the algebraic behaviors and properties exhibited by Q-CNSSs within this framework.

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