



# The General Exponential form of a Symbolic Plithogenic Complex Numbers

Yaser Ahmad Alhasan<sup>1,\*</sup>, Raja Abdullah Abdulfatah<sup>2</sup>, Suliman Sheen<sup>3</sup>

<sup>1</sup>Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia

<sup>2</sup>Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia

<sup>3</sup>Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia

Emails: [y.alhasan@psau.edu.sa](mailto:y.alhasan@psau.edu.sa); [r.abdulfatah@psau.edu.sa](mailto:r.abdulfatah@psau.edu.sa); [S.almleh@psau.edu.sa](mailto:S.almleh@psau.edu.sa)

\*Corresponding author: [y.alhasan@psau.edu.sa](mailto:y.alhasan@psau.edu.sa)

## Abstract

In this study, we defined a symbolic plithogenic complex number's general exponential form. A symbolic plithogenic complex number's general trigonometric form was defined. Theories have been supported by evidence showing how to find the general exponential form's conjugate for symbolic plithogenic complex numbers, division for symbolic plithogenic complex numbers, multiplication for two symbolic plithogenic complex numbers, and inversion for symbolic plithogenic complex numbers.

**Keywords:** symbolic plithogenic complex numbers; trigonometric form; the exponential form.

## 1. Introduction

As The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. plithogeny advocates for the integration of theories from several fields.

We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on plithogeny, plithogenic Set, Logic, Probability, and Statistics [2], in addition to presenting Introduction to the symbolic plithogenic Algebraic Structures (revisited), through which he discussed several ideas, including mathematical operations on Plithogenic numbers [1]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7]. Alhasan and Abdulfatah presented a study on the symbolic plithogenic complex numbers[8].

Complex numbers play a significant role in daily life because they make it much easier to perform mathematical operations and give us a way to solve equations for which there are no real-number-group solutions. The electrical engineering field makes extensive use of complex numbers to calculate electric voltage and measure alternating current.

Paper is divided into four parts. Gives a review of plithogenic science as well as an introduction in the first section. In the second section, a few plithogenic definitions and plithogenic number operations are given. In the third segment, the general exponential form of a symbolic plithogenic complex numbers were defined. the fourth section offers the paper's conclusion.

## 2. Preliminaries

### 2.1. The symbolic plithogenic Complex numbers[8]

#### Definition 1:

Let  $CPN$  is a plithogenic complex number, then we defined the standard form of it by:

$$CPN = a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n + (b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n)i$$

where  $a_0, a_1, a_2, \dots, a_n, b_0, b_1, b_2, \dots, b_n$  are real coefficients, such that  $i^2 = -1 \Rightarrow i = \sqrt{-1}$ .

### 2.2. The absolute value of a symbolic plithogenic complex number[8]

Let  $CPN_1$  is a symbolic plithogenic complex number, where:

$$CPN_1 = a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n + (b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n)i$$

the absolute value of a symbolic plithogenic complex number defined by the following form:

$$|CPN_1| = \sqrt{(a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n)^2 + (b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n)^2}$$

## 3. The general exponential form of a symbolic plithogenic complex numbers

#### Theorem 1

The general exponential form of the symbolic plithogenic complex numbers is given by the formula:

$$CPN = |CPN| e^{i(\theta_0 + \theta_1P_1 + \theta_2P_2 + \dots + \theta_nP_n)} = r e^{i(\theta_0 + \theta_1P_1 + \theta_2P_2 + \dots + \theta_nP_n)}$$

Whereas  $r = |CPN|$  is the absolute value of a symbolic plithogenic complex numbers.

Proof:

We know that:

$$CPN = a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n + (b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n)i$$

$$CPN = r \left( \frac{a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n}{r} + \frac{b_0 + b_1P_1 + b_2P_2 + \dots + b_nP_n}{r} i \right)$$

$$CPN = r (\cos(\theta_0 + \theta_1P_1 + \theta_2P_2 + \dots + \theta_nP_n) + \sin(\theta_0 + \theta_1P_1 + \theta_2P_2 + \dots + \theta_nP_n) i)$$

$$\cos(\theta_0 + \theta_1P_1 + \theta_2P_2 + \dots + \theta_nP_n) = \frac{x}{r} = \frac{a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n}{r}$$

$$\sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) = \frac{y}{r} = \frac{\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \dots + \dot{b}_n P_n}{r}$$

hence we get:

$$CPN = r e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$$

### 3.1 The general Trigonometric form of a symbolic plithogenic complex numbers

#### Definition 1:

The following formula:

$$CPN = r [\cos(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) + \sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) i]$$

is called the general trigonometric form of a symbolic plithogenic complex numbers.

### 3.2 Multiplying two symbolic plithogenic complex numbers by the general exponential form

#### Theorem 2

Let  $CPN_1 = r_1 e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$  and  $CPN_2 = r_2 e^{i(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)}$

Then:

$$CPN_1 \cdot CPN_2 = r_1 r_2 e^{i(\theta_0 + \vartheta_0 + (\theta_1 + \vartheta_1) P_1 + (\theta_2 + \vartheta_2) P_2 + \dots + (\theta_n + \vartheta_n) P_n)}$$

Proof:

$$CPN_1 \cdot CPN_2 = r_1 e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)} \cdot r_2 e^{i(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)}$$

$$\begin{aligned} CPN_1 \cdot CPN_2 &= r_1 r_2 \left( (\cos(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) \right. \\ &\quad \left. + i \sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)) (\cos(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n) \right. \\ &\quad \left. + i \sin(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)) \right) \end{aligned}$$

$$\begin{aligned} CPN_1 \cdot CPN_2 &= r_1 r_2 \left( [\cos(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) \cos(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n) \right. \\ &\quad \left. - \sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) \sin(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)] \right. \\ &\quad \left. + i [\sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) \cos(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n) \right. \\ &\quad \left. + \cos(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) \sin(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)] \right) \end{aligned}$$

$$\begin{aligned} CPN_1 \cdot CPN_2 &= r_1 r_2 \left( \cos \left( (\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) + (\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n) \right) \right. \\ &\quad \left. + i \sin \left( (\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) + (\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n) \right) \right) \end{aligned}$$

$$\begin{aligned} CPN_1 \cdot CPN_2 &= r_1 r_2 \left( \cos \left( (\theta_0 + \vartheta_0 + (\theta_1 + \vartheta_1) P_1 + (\theta_2 + \vartheta_2) P_2 + \dots + (\theta_n + \vartheta_n) P_n) \right) \right. \\ &\quad \left. + i \sin \left( (\theta_0 + \vartheta_0 + (\theta_1 + \vartheta_1) P_1 + (\theta_2 + \vartheta_2) P_2 + \dots + (\theta_n + \vartheta_n) P_n) \right) \right) \end{aligned}$$

hence:

$$CPN_1 \cdot CPN_2 = r_1 r_2 e^{i(\theta_0 + \theta_0 + (\theta_1 + \theta_1)P_1 + (\theta_2 + \theta_2)P_2 + \dots + (\theta_n + \theta_n)P_n)}$$

**Example 1**

$$\text{Let } CPN_1 = 3e^{i(\frac{\pi}{6} + \frac{\pi}{3}P_1 + \frac{2\pi}{3}P_2)} \text{ and } CPN_2 = 2e^{i(\frac{\pi}{2} - \frac{\pi}{3}P_1 + \frac{\pi}{4}P_2 - \frac{\pi}{12}P_3)}$$

then:

$$CPN_1 \cdot CPN_2 = 6e^{i(\frac{2\pi}{3} + \frac{5\pi}{12}P_2 - \frac{\pi}{12}P_3)}$$

**3.3 Conjugate of a symbolic plithogenic complex numbers by the general exponential form****Theorem 3**

Let  $CPN = r e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$ , then the conjugate of a symbolic plithogenic complex number by the general exponential form is given by form:

$$\overline{CPN} = r e^{-i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$$

Proof:

We know that:

$$\overline{CPN} = a_0 + a_1 P_1 + a_2 P_2 + \dots + a_n P_n - (b_0 + b_1 P_1 + b_2 P_2 + \dots + b_n P_n) i$$

$$\overline{CPN} = r \left( \frac{a_0 + a_1 P_1 + a_2 P_2 + \dots + a_n P_n}{r} - \frac{b_0 + b_1 P_1 + b_2 P_2 + \dots + b_n P_n}{r} i \right)$$

$$\overline{CPN} = r (\cos(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) - \sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) i)$$

$$\overline{CPN} = r (\cos(-(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)) + \sin(-(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) i))$$

hence:

$$\overline{CPN} = r e^{-i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$$

**3.4 Inverted symbolic plithogenic complex number by the general exponential form**

Let  $CPN = r e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$ , then

$$\frac{1}{CPN} = \frac{1}{r} e^{-i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)}$$

Proof:

$$\frac{1}{CPN} = \frac{1}{r (\cos(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) + \sin(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n) i)}$$

multiplying the numerator and denominator by the conjugate:

$(\cos(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) - \sin(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) i)$ , we get:

$$\frac{1}{z} = \frac{1}{r} \frac{(\cos(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) - \sin(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) i)}{(\cos^2(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) + \sin^2(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n))} \quad (*)$$

but:

$$\begin{aligned} \cos(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) - \sin(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) i &= e^{-i(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n)} \\ \cos^2(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) + \sin^2(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n) &= 1 \end{aligned}$$

by substitution in (\*):

$$\frac{1}{CPN} = \frac{1}{r} e^{-i(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n)}$$

**Special case:**

When  $r = 1$ , then:

$$\frac{1}{CPN} = e^{-i(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n)}$$

**Example 2**

$$\text{Let } CPN = 5e^{i(\frac{\pi}{4} + \frac{5\pi}{3}P_1 - \frac{3\pi}{4}P_2 + \frac{7\pi}{4}P_3 - \frac{11\pi}{12}P_4)}$$

then:

$$\frac{1}{CPN} = \frac{1}{5} e^{-i(\frac{\pi}{4} + \frac{5\pi}{3}P_1 - \frac{3\pi}{4}P_2 + \frac{7\pi}{4}P_3 - \frac{11\pi}{12}P_4)}$$

**Example 3**

$$\text{Let } CPN = e^{i(\frac{-3\pi}{4} + \frac{2\pi}{3}P_2 - \frac{\pi}{6}P_3 + \frac{\pi}{2}P_4)}$$

then:

$$\frac{1}{CPN} = e^{-i(\frac{-3\pi}{4} + \frac{2\pi}{3}P_2 - \frac{\pi}{6}P_3 + \frac{\pi}{2}P_4)}$$

### 3.5 Division two symbolic plithogenic complex numbers by the general exponential form

**Theorem 4**

$$\text{Let } CPN_1 = r_1 e^{i(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n)} \text{ and } CPN_2 = r_2 e^{i(\dot{\theta}_0 + \dot{\theta}_1 P_1 + \dot{\theta}_2 P_2 + \dots + \dot{\theta}_n P_n)}$$

then:

$$\frac{CPN_1}{CPN_2} = \frac{r_1}{r_2} e^{i(\dot{\theta}_0 - \dot{\theta}_0 + (\dot{\theta}_1 - \dot{\theta}_1)P_1 + (\dot{\theta}_2 - \dot{\theta}_2)P_2 + \dots + (\dot{\theta}_n - \dot{\theta}_n)P_n)}$$

Proof:

$$\frac{CPN_1}{CPN_2} = CPN_1 \cdot \frac{1}{CPN_2}$$

$$\frac{CPN_1}{CPN_2} = r_1 e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)} \cdot \frac{1}{r_2 e^{-i(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)}}$$

$$\frac{CPN_1}{CPN_2} = \frac{r_1}{r_2} e^{i(\theta_0 + \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_n P_n)} \cdot e^{-i(\vartheta_0 + \vartheta_1 P_1 + \vartheta_2 P_2 + \dots + \vartheta_n P_n)}$$

hence:

$$\frac{CPN_1}{CPN_2} = \frac{r_1}{r_2} e^{i(\theta_0 - \vartheta_0 + (\theta_1 - \vartheta_1)P_1 + (\theta_2 - \vartheta_2)P_2 + \dots + (\theta_n - \vartheta_n)P_n)}$$

#### Example 4

Let  $CPN_1 = (P_1 + P_2)e^{i(\frac{\pi}{6} + \frac{\pi}{3}P_1 + \frac{2\pi}{3}P_2)}$  and  $CPN_2 = (2 + P_2)e^{i(\frac{\pi}{2} - \frac{\pi}{3}P_1 + \frac{\pi}{4}P_2 - \frac{\pi}{12}P_3)}$

then:

$$\begin{aligned} \frac{CPN_1}{CPN_2} &= \frac{P_1 + P_2}{2 + P_2} e^{i(\frac{\pi}{3} + \frac{2\pi}{3}P_1 + \frac{5\pi}{12}P_2 + \frac{\pi}{12}P_3)} \\ &= \left(\frac{1}{2}P_1 + \frac{1}{6}P_2\right) e^{i(\frac{\pi}{3} + \frac{2\pi}{3}P_1 + \frac{5\pi}{12}P_2 + \frac{\pi}{12}P_3)} \end{aligned}$$

where:

$$\frac{P_1 + P_2}{2 + P_2} = x_0 + x_1 P_1 + x_2 P_2$$

$$P_1 + P_2 = (2 + P_2)(x_0 + x_1 P_1 + x_2 P_2)$$

$$P_1 + P_2 = 2x_0 + 2x_1 P_1 + 2x_2 P_2 + x_0 P_2 + x_1 P_2 + x_2 P_2$$

$$P_1 + P_2 = 2x_0 + 2x_1 P_1 + (x_0 + x_1 + 3x_2)P_2$$

$$x_0 = 0, x_1 = \frac{1}{2}, x_2 = \frac{1}{6}$$

hence:

$$\frac{P_1 + P_2}{2 + P_2} = \frac{1}{2}P_1 + \frac{1}{6}P_2$$

#### 5. Conclusions

In this artical, we defined the general exponential form of a symbolic plithogenic complex number as, studied the inverted symbolic plithogenic complex number by the general exponential form, and performed arithmetic operations

on it, including multiplication, division, conjugate, and of a symbolic plithogenic complex numbers by the general exponential form. This article is regarded as one of the key studies on symbolic plithogenic complex numbers.

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