The Symbolic Plithogenic Complex Numbers

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Abstract

In this paper, we presented symbolic plithogenic complex numbers, and studied the arithmetic operations: addition, subtraction, multiplication and division. Also, we defined the conjugate, inverted and the absolute value of symbolic plithogenic complex numbers, the theories related to the conjugate of symbolic plithogenic complex numbers are proved.

Keywords: symbolic plithogenic numbers; symbolic plithogenic complex numbers; conjugate complex plithogenic numbers.

1. Introduction

As The genesis, origination, formation, development, and evolution of new entities through dynamics of contradictory and/or neutral and/or noncontradictory multiple old entities is known as plithogenic. plithogeny advocates for the integration of theories from several fields.

We use numerous "knowledges" from domains like soft sciences, hard sciences, arts and literature theories, etc. as "entities" in this study, this is what Smarandache introduced, as he presented a study on plithogeny, plithogenic Set, Logic, Probability, and Statistics [2], in addition to presenting Introduction to the symbolic plithogenic Algebraic Structures (revisited), through which he discussed several ideas, including mathematical operations on Plithogenic numbers [1]. Also, an overview of plithogenic set and symbolic plithogenic algebraic structures was discussed by him [3]. It is thought that the symbolic n-plithogenic sets are a good place to start when developing algebraic extensions for other classical structures including rings, vector spaces, modules, and equations [4-5-6-7].

Complex numbers play a significant role in daily life because they make it much easier to perform mathematical operations and give us a way to solve equations for which there are no real-number-group solutions. The electrical engineering field makes extensive use of complex numbers to calculate electric voltage and measure alternating current.

Paper is divided into four parts. provides an introduction in the first portion, which includes a review of plithogenic science. A few definitions of a plithogenic and operations with plithogenic numbers are covered in the second section. the third section defined the complex symbolic plithogenic numbers. The paper's conclusion is provided in the fourth section.

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2. Preliminaries

2.1. Definition of Plithogenic Numbers (PN) [1]

The numbers of the form $PN = a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n$ defined as above are called plithogenic numbers, where a_nP_n is called the leading (strongest) term.

2.2 Operations with Plithogenic Numbers [1]

2.2.1. Division of Symbolic Plithogenic Components

$$\frac{P_i}{P_j} = \begin{cases} x_0 + x_1P_1 + x_2P_2 + \cdots + x_jP_j + P_i & x_0 + x_1 + x_2 + \cdots + x_j = 0 \quad i > j \\ x_0 + x_1P_1 + x_2P_2 + \cdots + x_iP_i & x_0 + x_1 + x_2 + \cdots + x_i = 1 \quad i = j \\ \emptyset & i < j \end{cases}$$

where all coefficients $x_0, x_1, x_2, \dots, x_i, \dots \in$ SPS.

2.2.2. Division of Symbolic Plithogenic Numbers

Let consider two symbolic plithogenic numbers as below:

$$PN_r = a_0 + a_1P_1 + a_2P_2 + \cdots + a_rP_r$$

$$PN_s = b_0 + b_1P_1 + b_2P_2 + \cdots + b_sP_s$$

$$\frac{PN_r}{PN_s} = \begin{cases} none, one\ many & r \geq s \\ \emptyset & r < s \end{cases}$$

2.3 m-th Root of the plithogenic number [1]

$$\sqrt[m]{PN_1} = \sqrt[m]{a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n} = x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n$$

we need to find the coefficients $x_0, x_1, x_2, \dots, x_n$.

3. The symbolic plithogenic Complex numbers

Definition 1:

Let CPN is a plithogenic complex number, then we defined the standard form of it by:

$$CPN = \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + (\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n)i$$

where $\dot{a}_0, \dot{a}_1, \dot{a}_2, \dots, \dot{a}_n, \dot{b}_0, \dot{b}_1, \dot{b}_2, \dots, \dot{b}_n$ are real coefficients, such that $i^2 = -1 \Rightarrow i = \sqrt{-1}$.

3.1 Operations on symbolic plithogenic complex numbers

Let two symbolic plithogenic complex numbers:

$$CPN_1 = \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + (\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n)i$$

$$CPN_2 = \dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n + (\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n)i$$

3.1.1Addition of symbolic *plithogenic complex numbers*

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$$CPN_1 + CPN_2 = \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i + \dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n + \left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i$$

$$= \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n + \dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i$$

$$\Longrightarrow \qquad CPN_1 + CPN_2 = (\dot{a}_0 + \dot{c}_0) + \sum_{j=1}^{n}(\dot{a}_j + \dot{c}_j)\,P_j + \left(\left(\dot{b}_0 + \dot{d}_0\right) + \sum_{j=1}^{n}\left(\dot{b}_j + \dot{d}_j\right)P_j\right)i$$

Example 1

Let: $CPN_1 = 4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i$and $CPN_1 = -1 - 3P_1 + P_2 + P_4 + (7 + P_1 - P_2 + 5P_3\,)i$

then:

$$CPN_1 + CPN_2 = 4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i + (-1 - 3P_1 + P_2 + P_4 + (7 + P_1 - P_2 + 5P_3\,)i)$$

$$= 3 - 2P_1 - 4P_2 + 7P_3 + P_4 + (5 + 3P_1 + 5P_3)i$$

2.2.2. subtraction of symbolic *plithogenic complex numbers*

$$CPN_1 - CPN_2 = \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i - \left(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n + \left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right)$$

$$= \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n - (\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n) + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n - \left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)\right)i$$

$$\Longrightarrow \qquad CPN_1 - CPN_2 = (\dot{a}_0 - \dot{c}_0) + \sum_{j=1}^{n}(\dot{a}_j - \dot{c}_j)\,P_j + \left(\left(\dot{b}_0 - \dot{d}_0\right) + \sum_{j=1}^{n}\left(\dot{b}_j - \dot{d}_j\right)P_j\right)i$$

Example 2

Let: $CPN_1 = 4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i$ and $CPN_1 = -1 - 3P_1 + P_2 + P_4 + (7 + P_1 - P_2 + 5P_3\,)i$

then:

$$CPN_1 + CPN_2 = 4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i - (-1 - 3P_1 + P_2 + P_4 + (7 + P_1 - P_2 + 5P_3\,)i)$$

$$= 5 + 4P_1 - 6P_2 + 7P_3 - P_4 + (-9 + P_1 + 2P_2 - 5P_3)i$$

2.2.3. Scalar multiplication of symbolic *plithogenic complex numbers*

$$c.CPN_1 = c.\left(\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i\right)$$

$$= c.\dot{a}_0 + c.\dot{a}_1P_1 + c.\dot{a}_2P_2 + \cdots + c.\dot{a}_nP_n + \left(c.\dot{b}_0 + c.\dot{b}_1P_1 + c.\dot{b}_2P_2 + \cdots + c.\dot{b}_nP_n\right)i$$

Example 3

Let: $CPN_1 = 4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i$

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then:

$$3.CPN_1 = 3(4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i)$$

$$= 12 + 3P_1 - 15P_2 + 21P_3 + (-6 + 6P_1 + 3P_2)i$$

2.2.4. Multiplication of symbolic *plithogenic complex numbers*

$$CPN_1.CPN_2 = \left(\dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + \left(\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n\right)i\right).\left(\dot{c_0} + \dot{c_1}P_1 + \dot{c_2}P_2 + \cdots + \dot{c_n}P_n + \left(\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n\right)i\right)$$

we then multiply the terms, taking into account:$(a_iP_i).(a_jP_j) = a_i.a_j.P_{max\{i,j\}}$.

Example 4

Let: $CPN_1 = 1 + P_1 - 2P_2 + (2P_1)i$ and $CPN_1 = -2 - 3P_1 + P_2 + (1 - P_1 + 2P_3)i$

then:

$$CPN_1.CPN_2 = (1 + P_1 - 2P_2 + (2P_1)i).(-2 - 3P_1 + P_2 + (1 - P_1 + 2P_3)i)$$

$$= -2 - 3P_1 + P_2 + (1 - P_1 + 2P_3)i - 2P_1 - 3P_1^2 + P_1P_2 + \left(P_1 - P_1^2 + 2P_1P_3\right)i + 4P_2 + 6P_1P_2 - 2P_2^2 + (-2P_2 + 2P_1P_2 - 4P_2P_3)i + \left(-4P_1 - 6P_1^2 + 2P_1P_2\right)i + \left(2P_1 - 2P_1^2 + 4P_1P_3\right)i^2$$

$$= -2 - 8P_1 + 10P_2 - 4P_3 + (1 - 11P_1 + 2P_2)i$$

3.2 Conjugate of a symbolic plithogenic complex number

Let $CPN = \dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + \left(\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n\right)i$ is a symbolic plithogenic complex number, we denote the conjugate of a complex symbolic plithogenic number by $\overline{CPN}$ and define it by the following form:

$$\overline{CPN} = \dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n - \left(\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n\right)i$$

$$= \dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + \left(-\dot{b_0} - \dot{b_1}P_1 - \dot{b_2}P_2 - \cdots - \dot{b_n}P_n\right)i$$

Example 4

- $CPN = 4 + P_1 - 5P_2 + 7P_3 + (-2 + 2P_1 + P_2)i \implies \overline{CPN} = 4 + P_1 - 5P_2 + 7P_3 + (2 - 2P_1 - P_2)i$
- $CPN = P_2 - P_3 + 5P_4 + (5 + P_3 - 9P_4)i \implies \overline{CPN} = P_2 - P_3 + 5P_4 + (-5 - P_3 + 9P_4)i$
- $CPN = (-3 + 4P_1 - P_2 + 6P_3)i \implies \overline{CPN} = (3 - 4P_1 + P_2 - 6P_3)i$

Notes:

1. the conjugate of a symbolic plithogenic complex number $\overline{CPN}$ is the same symbolic plithogenic complex number CPN.

$$\overline{(\overline{CPN})} = CPN$$

2. If $CPN = \dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + \left(\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n\right)i$

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then:

- $CPN + \overline{CPN} = 2(\dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n) = 2\mathrm{Re}(CPN)$

and:

- $CPN - \overline{CPN} = 2(\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n)i = 2Im(CPN)$

where $Re(CPN)$ is the real part of the symbolic plithogenic complex number and $Im(CPN)$ is the imagine.

3. We can conclude that:

- CPN is real if and only if: $CPN = \overline{CPN}$
- CPN is imaginary if and only if: $CPN = -\overline{CPN}$

Remark 1

$$\overline{CPN_1 + CPN_2} = \overline{CPN_1} + \overline{CPN_2}$$

Proof:

Let two symbolic plithogenic complex numbers:

$$CPN_1 = \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n + (\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n)i$$

$$CPN_2 = \dot{c}_0 + \dot{c}_1 P_1 + \dot{c}_2 P_2 + \cdots + \dot{c}_n P_n + (\dot{d}_0 + \dot{d}_1 P_1 + \dot{d}_2 P_2 + \cdots + \dot{d}_n P_n)i$$

then:

$$CPN_1 + CPN_2 = \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n + (\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n)i + \dot{c}_0 + \dot{c}_1 P_1 + \dot{c}_2 P_2 + \cdots + \dot{c}_n P_n + (\dot{d}_0 + \dot{d}_1 P_1 + \dot{d}_2 P_2 + \cdots + \dot{d}_n P_n)i$$

$$= \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n + \dot{c}_0 + \dot{c}_1 P_1 + \dot{c}_2 P_2 + \cdots + \dot{c}_n P_n + (\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n + \dot{d}_0 + \dot{d}_1 P_1 + \dot{d}_2 P_2 + \cdots + \dot{d}_n P_n)i$$

$$\overline{CPN_1 + CPN_2} = \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n + \dot{c}_0 + \dot{c}_1 P_1 + \dot{c}_2 P_2 + \cdots + \dot{c}_n P_n - (\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n + \dot{d}_0 + \dot{d}_1 P_1 + \dot{d}_2 P_2 + \cdots + \dot{d}_n P_n)i$$

$$= \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n - (\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n)i + \dot{c}_0 + \dot{c}_1 P_1 + \dot{c}_2 P_2 + \cdots + \dot{c}_n P_n - (\dot{d}_0 + \dot{d}_1 P_1 + \dot{d}_2 P_2 + \cdots + \dot{d}_n P_n)i$$

$$= \overline{CPN_1} + \overline{CPN_2}$$

Note:

$$\overline{CPN_1 + CPN_2} == (\dot{a}_0 + \dot{c}_0) + \sum_{j=1}^{n} (\dot{a}_j + \dot{c}_j) P_j - \left((\dot{b}_0 + \dot{d}_0) + \sum_{j=1}^{n} (\dot{b}_j + \dot{d}_j) P_j \right) i$$

Theorem 1

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The conjugate of multiplication two symbolic plithogenic complex numbers is equal to the multiplication of their two conjugates.

$$\overline{CPN_1.CPN_2} = \overline{CPN_1}.\overline{CPN_2}$$

Proof:

Let two symbolic plithogenic complex numbers:

$$CPN_1 = \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i$$

$$CPN_2 = \dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n + \left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i$$

then:

$$CPN_1.CPN_2 = \left(\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i\right).\left(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n + \left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right)$$

$$\begin{aligned} &= (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n) \\ &\quad + (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)\left(\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right) \\ &\quad + \left(\left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n)i\right) \\ &\quad + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i^2 \end{aligned}$$

$$\begin{aligned} &= (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n) \\ &\quad - \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right) \\ &\quad + \left((\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right) \\ &\quad + \left(\left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n)i\right) \end{aligned}$$

$$\begin{aligned} \Longrightarrow \quad \overline{CPN_1.CPN_2} &= (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n) \\ &\quad - \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right) \\ &\quad - (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)\left(\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right) \\ &\quad - \left(\left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n)i\right) \end{aligned}$$

$$\overline{CPN_1}.\overline{CPN_2} = \left(\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n - \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i\right)\left(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n) - \left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right)$$

$$\begin{aligned} &= (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)(\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n) \\ &\quad - (\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n)\left(\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i\right) \\ &\quad - \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)\left((\dot{c}_0 + \dot{c}_1P_1 + \dot{c}_2P_2 + \cdots + \dot{c}_nP_n)i\right) \\ &\quad + \left(\left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)\left(\dot{d}_0 + \dot{d}_1P_1 + \dot{d}_2P_2 + \cdots + \dot{d}_nP_n\right)i^2\right) \end{aligned}$$

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$$= (\dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n)(\dot{c_0} + \dot{c_1}P_1 + \dot{c_2}P_2 + \cdots + \dot{c_n}P_n) - \left((\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)(\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)\right) - \left((\dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n)(\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i\right) - \left((\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)(\dot{c_0} + \dot{c_1}P_1 + \dot{c_2}P_2 + \cdots + \dot{c_n}P_n)i\right)$$

we then multiply the terms, taking into account: $(a_iP_i).(a_jP_j) = a_i.a_j.P_{\max\{i,j\}}$.

$$\Rightarrow \qquad \overline{CPN_1.CPN_2} = \overline{CPN_1}.\overline{CPN_2}$$

3.3 Division of symbolic plithogenic complex numbers

Let two symbolic plithogenic complex numbers:

$$CPN_1 = \dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)i$$

$$CPN_2 = \dot{c_0} + \dot{c_1}P_1 + \dot{c_2}P_2 + \cdots + \dot{c_n}P_n + (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i$$

then:

$$\frac{CPN_1}{CPN_2} = \frac{\dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)i}{\dot{c_0} + \dot{c_1}P_1 + \dot{c_2}P_2 + \cdots + \dot{c_n}P_n + (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i}$$

multiply the numerator and denominator by conjugate of CPN_2 we get:

$$\frac{CPN_1}{CPN_2} = \frac{\left(\dot{a_0} + \dot{a_1}P_1 + \cdots + \dot{a_n}P_n + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)i\right)\left(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n - (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i\right)}{\left(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n + (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i\right)\left(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n - (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i\right)}$$

$$= \frac{\left(\dot{a_0} + \dot{a_1}P_1 + \cdots + \dot{a_n}P_n + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)i\right)\left(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n - (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)i\right)}{(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n)^2 + (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)^2}$$

hence:

$$\frac{CPN_1}{CPN_2} = \frac{(\dot{a_0} + \dot{a_1}P_1 + \cdots + \dot{a_n}P_n)(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n) + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)(\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)}{(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n)^2 + (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)^2} + \frac{(\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n) - (\dot{a_0} + \dot{a_1}P_1 + \cdots + \dot{a_n}P_n)(\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)}{(\dot{c_0} + \dot{c_1}P_1 + \cdots + \dot{c_n}P_n)^2 + (\dot{d_0} + \dot{d_1}P_1 + \dot{d_2}P_2 + \cdots + \dot{d_n}P_n)^2} i$$

we note that numerator is a multiplication operation, taking into account that: $(a_iP_i).(a_jP_j) = a_i.a_j.P_{\max\{i,j\}}$.

Example 5

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$$\frac{1+P_1+(4+2P_1+P_3)i}{2-P_2+(P_1-3P_2\,)i}$$

Solution:

multiply the numerator and denominator by conjugate $(2-P_2-(P_1-3P_2\,)i)$ we get:

$$\frac{1+P_1+(4+2P_1+P_3)i}{2-P_2+(P_1-3P_2\,)i}=\frac{(1+P_1)+(4+2P_1+P_3)i}{2-P_2+(P_1-3P_2\,)i}\cdot\frac{(2-P_2)+(-P_1+3P_2\,)i}{2-P_2-(P_1-3P_2\,)i}$$

$$=\frac{(1+P_1)(2-P_2)+(1+P_1)(-P_1+3P_2\,)i+(2-P_2)(4+2P_1+P_3)i+(4+2P_1+P_3)(-P_1+3P_2\,)i^2}{(2-P_2)^2+(P_1-3P_2)^2}$$

$$=\frac{2-P_2+2P_1-P_2-P_1i+3P_2i-P_1i+3P_2i+8i+4P_1i+2P_3i-4P_2i-2P_2i-P_3i+4P_1-12P_2+2P_1-6P_2+P_3-3P_3}{4-4P_2+P_2+P_1-6P_2+9P_2}$$

$$=\frac{2+8P_1-20P_2-2P_3}{4+P_1}+\frac{8+2P_1+P_3}{4+P_1}i \qquad (1)$$

let's find:

$$\frac{2+8P_1-20P_2-2P_3}{4+P_1}=x_0+x_1P_1+x_2P_2+x_3P_3$$

$$2+8P_1-20P_2-2P_3=(4+P_1)(x_0+x_1P_1+x_2P_2+x_3P_3)$$

$$2+8P_1-20P_2-2P_3=4x_0+4x_1P_1+4x_2P_2+4x_3P_3+x_0P_1+x_1P_1+x_2P_2+x_3P_3$$

$$2+8P_1-20P_2-2P_3=4x_0+(x_0+5x_1)P_1+5x_2P_2+5x_3P_3$$

$$\Rightarrow\begin{cases}4x_0=2\\ x_0+5x_1=8\\ 5x_2=-20\\ 5x_3=-2\end{cases}\Rightarrow\begin{cases}x_0=\frac{1}{2}\\ x_1=\frac{3}{2}\\ x_2=-4\\ x_3=\frac{-2}{5}\end{cases}$$

then:

$$\frac{2+8P_1-20P_2-2P_3}{4+P_1}=\frac{1}{2}+\frac{3}{2}P_1-4P_2-\frac{2}{5}P_3$$

let's now find:

$$\frac{8+2P_1+P_3}{4+P_1}=x_0+x_1P_1+x_2P_2+x_3P_3$$

$$8+2P_1+P_3=(4+P_1)(x_0+x_1P_1+x_2P_2+x_3P_3)$$

$$8+2P_1+P_3=4x_0+4x_1P_1+4x_2P_2+4x_3P_3+x_0P_1+x_1P_1+x_2P_2+x_3P_3$$

$$8+2P_1+P_3=4x_0+(x_0+5x_1)P_1+5x_2P_2+5x_3P_3$$

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$$\Rightarrow \begin{cases} 4x_0 = 8 \\ x_0 + 5x_1 = 2 \\ 5x_2 = 0 \\ 5x_3 = 1 \end{cases} \Rightarrow \begin{cases} x_0 = 2 \\ x_1 = 0 \\ x_2 = 0 \\ x_3 = \frac{1}{5} \end{cases}$$

then:

$$\frac{8 + 2P_1 + P_3}{4 + P_1} = 2 + \frac{1}{5}P_3$$

by substitution in (1), we get:

$$\frac{1 + P_1 + (4 + 2P_1 + P_3)i}{2 - P_2 + (P_1 - 3P_2\ )i} = \frac{1}{2} + \frac{3}{2}P_1 - 4P_2 - \frac{2}{5}P_3 + \left(2 + \frac{1}{5}P_3\right)i$$

Let's check the answer:

$$\begin{aligned}(2 - P_2 + (P_1 - 3P_2\ )i)&\left(\frac{1}{2} + \frac{3}{2}P_1 - 4P_2 - \frac{2}{5}P_3 + \left(2 + \frac{1}{5}P_3\right)i\right) \\ &= \left(1 + P_1 + 6P_2 + \frac{2}{5}P_3 - \frac{1}{2}P_2 - \frac{3}{2}P_2 + 4P_2 + \frac{2}{5}P_3 - 8P_2 - \frac{4}{5}P_3\right) \\ &+ \left(\frac{1}{2}P_1 + \frac{3}{2}P_1 - 4P_2 - \frac{2}{5}P_3 - \frac{3}{2}P_2 - \frac{9}{2}P_2 + 12P_2 + \frac{6}{5}P_3 + 4 + \frac{2}{5}P_3 - 2P_2 - \frac{1}{5}P_3\right)i \\ &= 1 + P_1 + (4 + 2P_1 + P_3)i \quad (True)\end{aligned}$$

3.4 Inverted of symbolic plithogenic complex numbers

Let CPN_1 is a symbolic plithogenic complex number, where:

$$CPN_1 = \dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i$$

then:

$$\frac{1}{CPN_1} = \frac{1}{\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i}$$

$$= \frac{\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n - \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)i}{(\dot{c}_0 + \dot{c}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{c}_nP_n)^2 + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)^2}$$

$$= \frac{\dot{a}_0 + \dot{a}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{a}_nP_n}{(\dot{c}_0 + \dot{c}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{c}_nP_n)^2 + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)^2} - \frac{\left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)}{(\dot{c}_0 + \dot{c}_1P_1 + \dot{a}_2P_2 + \cdots + \dot{c}_nP_n)^2 + \left(\dot{b}_0 + \dot{b}_1P_1 + \dot{b}_2P_2 + \cdots + \dot{b}_nP_n\right)^2}i$$

Example 6

$$\frac{1}{1 + P_1 - P_2 + (2 + P_1 + P_2)i} = \frac{1 + P_1 - P_2 - (2 + P_1 + P_2)i}{(1 + P_1 - P_2)^2 + (2 + P_1 + P_2)^2}$$

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$$\frac{1}{1+P_1-P_2+(2+P_1+P_2)i}=\frac{1+P_1-P_2}{5+8P_1+4P_2}+\frac{-2-P_1-P_2}{5+8P_1+4P_2}i \qquad (2)$$

let's find:

$$\frac{1+P_1-P_2}{5+8P_1+4P_2}=x_0+x_1P_1+x_2P_2$$

$$1+P_1-P_2=(5+8P_1+4P_2)(x_0+x_1P_1+x_2P_2)$$

$$1+P_1-P_2=5x_0+5x_1P_1+5x_2P_2+8x_0P_1+8x_1{P_1}^2+8x_2P_1P_2+4x_0P_2+4x_1P_1P_2+4x_2{P_2}^2$$

$$1+P_1-P_2=5x_0+(8x_0+13x_1)P_1+(4x_0+4x_1+17x_2)P_2$$

$$\Rightarrow\begin{cases}5x_0=1\\8x_0+13x_1=1\\4x_0+4x_1+17x_2=-1\end{cases}\Rightarrow\begin{cases}x_0=\frac{1}{5}\\x_1=\frac{-3}{65}\\x_2=\frac{-21}{221}\end{cases}$$

then:

$$\frac{1+P_1-P_2}{5+8P_1+4P_2}=\frac{1}{5}-\frac{3}{65}P_1-\frac{21}{221}P_2$$

let's now find:

$$\frac{-2-P_1-P_2}{5+8P_1+4P_2}=x_0+x_1P_1+x_2P_2$$

$$-2-P_1-P_2=(5+8P_1+4P_2)(x_0+x_1P_1+x_2P_2)$$

$$-2-P_1-P_2=5x_0+5x_1P_1+5x_2P_2+8x_0P_1+8x_1{P_1}^2+8x_2P_1P_2+4x_0P_2+4x_1P_1P_2+4x_2{P_2}^2$$

$$-2-P_1-P_2=5x_0+(8x_0+13x_1)P_1+(4x_0+4x_1+17x_2)P_2$$

$$\Rightarrow\begin{cases}5x_0=-2\\8x_0+13x_1=-1\\4x_0+4x_1+17x_2=-1\end{cases}\Rightarrow\begin{cases}x_0=\frac{-2}{5}\\x_1=\frac{11}{65}\\x_2=\frac{-1}{221}\end{cases}$$

then:

$$\frac{-2-P_1-P_2}{5+8P_1+4P_2}=-\frac{2}{5}+\frac{11}{65}P_1-\frac{1}{221}P_2$$

by substitution in (2), we get:

$$\frac{1}{1+P_1-P_2+(2+P_1+P_2)i}=\frac{1}{5}-\frac{3}{65}P_1-\frac{21}{221}P_2+\left(-\frac{2}{5}+\frac{11}{65}P_1-\frac{1}{221}P_2\right)i$$

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4. The absolute value of a symbolic plithogenic complex number

Let CPN_1 is a symbolic plithogenic complex number, where:

$$CPN_1 = \dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)i$$

the absolute value of a symbolic plithogenic complex number defined by the following form:

$$|CPN_1| = \sqrt{(\dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n)^2 + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)^2}$$

Example 7

Let $CPN_1 = 2 - P_1 + P_2 i$, then:

$$|CPN_1| = \sqrt{(\dot{a_0} + \dot{a_1}P_1 + \dot{a_2}P_2 + \cdots + \dot{a_n}P_n)^2 + (\dot{b_0} + \dot{b_1}P_1 + \dot{b_2}P_2 + \cdots + \dot{b_n}P_n)^2}$$

$$= \sqrt{(2 - P_1)^2 + (P_2)^2}$$

$$= \sqrt{4 - 4P_1 + P_1 + P_2}$$

$$= \sqrt{4 - 3P_1 + P_2}$$

$$\sqrt{4 - 3P_1 + P_2} \equiv x_0 + x_1P_1 + x_2P_2$$

$$4 - 3P_1 + P_2 \equiv (x_0 + x_1P_1 + x_2P_2)^2$$

$$4 - 3P_1 + P_2 \equiv {x_0}^2 + 2x_0x_1P_1 + {x_1}^2{P_1}^2 + 2x_0x_2P_2 + 2x_1x_2P_1P_2 + {x_2}^2{P_2}^2$$

$$4 - 3P_1 + P_2 \equiv {x_0}^2 + 2x_0x_1P_1 + {x_1}^2P_1 + 2x_0x_2P_2 + 2x_1x_2P_2 + {x_2}^2P_2$$

$$4 - 3P_1 + P_2 \equiv {x_0}^2 + (2x_0x_1 + {x_1}^2)P_1 + (2x_0x_2 + 2x_1x_2 + {x_2}^2)P_2$$

$$\begin{cases} {x_0}^2 = 4 \quad \Longrightarrow \begin{cases} x_0 = 2 \\ x_0 = -2 \end{cases} \\ 2x_0x_1 + {x_1}^2 = -3 \quad (3) \\ 2x_0x_2 + 2x_1x_2 + {x_2}^2 = 1 \quad (4) \end{cases}$$

since the absolute value is positive, we take: $x_0 = 2$

by substitution in (3):

$$4x_1 + {x_1}^2 = -3 \quad \Longrightarrow \quad {x_1}^2 + 4x_1 + 3 = 0$$

$$x_1 = -1 \,, x_1 = -3$$

let's find x_2:

- case1: $x_0 = 2 \ and \ x_1 = -1$

By substitution in (4):

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$$4x_2 - 2x_2 + {x_2}^2 = 1$$

$$ {x_2}^2 + 2x_2 - 1 = 0$$

$$x_2 = -1 + \sqrt{2}\,, x_2 = -1 - \sqrt{2}$$

- case2: $x_0 = 2\ and\ x_1 = -3$

by substitution in (4):

$$4x_2 - 4x_2 - 6{x_2}^2 = 1$$

$$ {x_2}^2 - 2x_2 - 1 = 0$$

$$x_2 = 1 + \sqrt{2}\,, x_2 = 1 - \sqrt{2}$$

hence:

$$(x_0, x_1, x_2) = \left(2, -1, -1 + \sqrt{2}\right)$$

$$or \qquad = \left(2, -1, -1 - \sqrt{2}\right)$$

$$or \qquad = \left(2, -3, 1 + \sqrt{2}\right)$$

$$or \qquad = \left(2, -3, 1 - \sqrt{2}\right)$$

since the absolute value is positive, therefore:

$$|CPN_1| = |2 - P_1 + P_2 i|$$

$$= \sqrt{4 - 3P_1 + P_2} = 2 - P_1 + (-1 + \sqrt{2})P_2$$

Theorem 2

Let CPN_1 is a symbolic plithogenic complex number, then:

$$CPN_1.\overline{CPN_1} = |CPN_1|^2$$

Proof:

$$CPN_1 = \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n + \left(\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n\right)i$$

$$\Longrightarrow \ \overline{CPN_1} = \dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n - \left(\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n\right)$$

$$CPN_1.\overline{CPN_1} = \left(\dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n + \left(\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n\right)i\right)\Big(\dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n - \left(\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n\right)\Big)$$

$$= (\dot{a}_0 + \dot{a}_1 P_1 + \dot{a}_2 P_2 + \cdots + \dot{a}_n P_n)^2 + (\dot{b}_0 + \dot{b}_1 P_1 + \dot{b}_2 P_2 + \cdots + \dot{b}_n P_n)^2$$

$$= |CPN_1|^2$$

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$$\Rightarrow \quad CPN_1.\overline{CPN_1} = |CPN_1|^2$$

Example 8

Let $1 + P_1 - P_2 + (2 + P_1 + P_2)i$, find $CPN_1.\overline{CPN_1}$.

Solution:

$$CPN_1.\overline{CPN_1} = |CPN_1|^2$$

$$= (1 + P_1 - P_2)^2 + (2 + P_1 + P_2)^2$$

$$= 1 + 3P_1 - 3P_2 + 4 + 5P_1 + 7P_2$$

$$= 5 + 8P_1 + 4P_2$$

4. Conclusions

This paper is important in that it presented the concept of symbolic plithogenic complex number in a way that is easy to understand by any researcher. In addition to harnessing the rules established by Florentine Smarandache, accurate results were obtained in this field, through the examples we provided to illustrate this.

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