On The Symbolic n-Plithogenic Square Real Matrices For 13≤ $n \leq 14$ and Their Elementary Algebraic Properties

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Abstract

The main goal of this paper is to study the algebraic properties of the symbolic n-plithogenic matrices in two different special cases (for n=13, n=14). We present many theorems that describe the algebraic behavior of these matrices, where an algorithm for computing determinants, inverses, and eigenvalues will be provided. On the other hand, the relationships between symbolic 13-plithogenic/14-plithogenic matrices and their classical components will be derived.

Keywords: symbolic 13-plithogenic matrix; symbolic 14-plithogenic matrix; symbolic plithogenic eigenvalue; symbolic plithogenic eigenvector.

1. Introduction

The symbolic n-plithogenic algebra began with the work of Smarandache [2], where he defined for the first time the applications of symbolic n-plithogenic sets in building algebraic generalizations of well-knowm algebraic structures.

The main difference between symbolic n-plithogenic algebraic structure and n-refined neutrosophic structure is the definition of the multiplication operation, where the multiplication between the sub-indices is defined as follows:

$P_iP_j = P_{\max(i,j)}$. For more details about similar systems of neutrosophic and refined neutrosophic matrices, see [12-16].

Many authors followed his steps, where symbolic 2-plithogenic rings were defined by Merkepci.et.al [1], and then they were used to find symbolic 2-plithogenic modules [3], and symbolic 3-plithogenic structures [4-6].

Recently, the symbolic n-plithogenic matrices have been introduced for different values of n, see [7-11, 17-18]. The algebraic properties of these matrices were studied widely, especially those which are related to the diagonalization problem such as eigenvalues, eigenvectors, and inverses[19, 20,21]

In general, the symbolic n-plithogenic square real matrix is defined with the following formula:

$M = M_0 + \sum_{i=1}^{n} M_iP_i$, where M_i are m-square classical matrices with real entries.

This has motivated us to follow these efforts, where we show the concept of symbolic 13/ symbolic 14 plithogenic matrices with their elementary algebraic properties.

2. Main Discussion

Definition:

The square symbolic 13-plithogenic matrix is defined as follows:

$Z = Z_0 + \sum_{i=1}^{13} Z_iP_i$; $(Z_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 13-plithogenic matrix:

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$Z = \begin{pmatrix} -3 & -9 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 5 & 1 \end{pmatrix} P_1 + \begin{pmatrix} 2 & -1 \\ 7 & 2 \end{pmatrix} P_2 + \begin{pmatrix} 6 & -8 \\ 9 & -6 \end{pmatrix} P_3 + \begin{pmatrix} 8 & 5 \\ 6 & 1 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 9 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 4 & -1 \\ 2 & -8 \end{pmatrix} P_{12} + \begin{pmatrix} 4 & -1 \\ 2 & -9 \end{pmatrix} P_{13}.$

Definition.

Let $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$ be a symbolic 13-plithogenic matrix of size $n \times n$, hence:

$$\begin{aligned}
\det Z = \det(Z_0) &+ \left[\det\left(\sum_{i=0}^{1} Z_i\right) - \det(Z_0)\right] P_1 + \left[\det\left(\sum_{i=0}^{2} Z_i\right) - \det\left(\sum_{i=0}^{1} Z_i\right)\right] P_2 \\
&+ \left[\det\left(\sum_{i=0}^{3} Z_i\right) - \det\left(\sum_{i=0}^{2} Z_i\right)\right] P_3 + \left[\det\left(\sum_{i=0}^{4} Z_i\right) - \det\left(\sum_{i=0}^{3} Z_i\right)\right] P_4 \\
&+ \left[\det\left(\sum_{i=0}^{5} Z_i\right) - \det\left(\sum_{i=0}^{4} Z_i\right)\right] P_5 + \left[\det\left(\sum_{i=0}^{6} Z_i\right) - \det\left(\sum_{i=0}^{5} Z_i\right)\right] P_6 \\
&+ \left[\det\left(\sum_{i=0}^{7} Z_i\right) - \det\left(\sum_{i=0}^{6} Z_i\right)\right] P_7 + \left[\det\left(\sum_{i=0}^{8} Z_i\right) - \det\left(\sum_{i=0}^{7} Z_i\right)\right] P_8 \\
&+ \left[\det\left(\sum_{i=0}^{9} Z_i\right) - \det\left(\sum_{i=0}^{8} Z_i\right)\right] P_9 + \left[\det\left(\sum_{i=0}^{10} Z_i\right) - \det\left(\sum_{i=0}^{9} Z_i\right)\right] P_{10} \\
&+ \left[\det\left(\sum_{i=0}^{11} Z_i\right) - \det\left(\sum_{i=0}^{10} Z_i\right)\right] P_{11} + \left[\det\left(\sum_{i=0}^{12} Z_i\right) - \det\left(\sum_{i=0}^{11} Z_i\right)\right] P_{12} \\
&+ \left[\det\left(\sum_{i=0}^{13} Z_i\right) - \det\left(\sum_{i=0}^{12} Z_i\right)\right] P_{13}
\end{aligned}$$

Theorem1.

Let $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$ be a symbolic 13-plithogenic matrix of size $n \times n$, hence:

1. Z is invertible if and only if $\det Z$ is an invertible symbolic 13-plithogenic real number.
2. $Z^{-1} = {Z_0}^{-1} + \left[(\sum_{i=0}^{1} Z_i)^{-1} - {Z_0}^{-1}\right] P_1 + [(\sum_{i=0}^{2} U_i)^{-1} - (\sum_{i=0}^{1} Z_i)^{-1}] P_2 + [(\sum_{i=1}^{3} Z_i)^{-1} - (\sum_{i=0}^{2} Z_i)^{-1}] P_3 + [(\sum_{i=1}^{4} Z_i)^{-1} - (\sum_{i=0}^{3} Z_i)^{-1}] P_4 + \left[(\sum_{i=1}^{5} Z_i)^{-1} - (\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} Z_i)^{-1} - (\sum_{i=0}^{5} Z_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} Z_i)^{-1} - (\sum_{i=0}^{6} Z_i)^{-1}] P_7 + [(\sum_{i=1}^{8} Z_i)^{-1} - (\sum_{i=0}^{7} Z_i)^{-1}] P_8 + [(\sum_{i=1}^{9} Z_i)^{-1} - (\sum_{i=0}^{8} Z_i)^{-1}] P_9 + [(\sum_{i=1}^{10} Z_i)^{-1} - (\sum_{i=0}^{9} Z_i)^{-1}] P_{10} + [(\sum_{i=1}^{11} Z_i)^{-1} - (\sum_{i=0}^{10} Z_i)^{-1}] P_{11} + [(\sum_{i=1}^{12} Z_i)^{-1} - (\sum_{i=0}^{11} Z_i)^{-1}] P_{12} + [(\sum_{i=1}^{13} Z_i)^{-1} - (\sum_{i=0}^{12} Z_i)^{-1}] P_{13}$

Definition.

Let $q = q_0 + \sum_{i=1}^{13} q_i P_i$ be a symbolic 13-plithogenic real number and $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$ be a symbolic 13-plithogenic square real matrix, then q is called symbolic 13-plithogenic eigen value if and only if $ZX = qX$.
X is called symbolic 13-plithogenic eigenvector.

Theorem2.

Let $q = q_0 + \sum_{i=1}^{13} g_i P_i \in 13 - SP_R$, $X = X_0 + \sum_{i=1}^{13} X_i P_i$ be a symbolic 13-plithogenic real vector, then q is eigen value of $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} q_i$ is eigen value of $\sum_{i=0}^{j} Z_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 13$.

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Theorem3.

$$Z^n = {Z_0}^n + P_1\left[\left(\sum_{i=0}^{1} Z_i\right)^n - {Z_0}^n\right] + \left[\left(\sum_{i=0}^{2} Z_i\right)^n - \left(\sum_{i=0}^{1} Z_i\right)^n\right]P_2 + \left[\left(\sum_{i=1}^{3} Z_i\right)^n - \left(\sum_{i=0}^{2} Z_i\right)^n\right]P_3$$
$$+ \left[\left(\sum_{i=1}^{4} Z_i\right)^n - \left(\sum_{i=0}^{3} Z_i\right)^n\right]P_4 + \left[\left(\sum_{i=1}^{5} Z_i\right)^n - \left(\sum_{i=0}^{4} Z_i\right)^n\right]P_5 + \left[\left(\sum_{i=1}^{6} Z_i\right)^n - \left(\sum_{i=0}^{5} Z_i\right)^n\right]P_6$$
$$+ \left[\left(\sum_{i=1}^{7} Z_i\right)^n - \left(\sum_{i=0}^{6} Z_i\right)^n\right]P_7 + \left[\left(\sum_{i=1}^{8} Z_i\right)^n - \left(\sum_{i=0}^{7} Z_i\right)^n\right]P_8 + \left[\left(\sum_{i=1}^{9} Z_i\right)^n - \left(\sum_{i=0}^{8} Z_i\right)^n\right]P_9$$
$$+ \left[\left(\sum_{i=1}^{10} Z_i\right)^n - \left(\sum_{i=0}^{9} Z_i\right)^n\right]P_{10} + \left[\left(\sum_{i=1}^{11} Z_i\right)^n - \left(\sum_{i=0}^{10} Z_i\right)^n\right]P_{11}$$
$$+ \left[\left(\sum_{i=1}^{12} Z_i\right)^n - \left(\sum_{i=0}^{11} Z_i\right)^n\right]P_{12} + \left[\left(\sum_{i=1}^{13} Z_i\right)^n - \left(\sum_{i=0}^{12} Z_i\right)^n\right]P_{13}$$

Theorem4.

Let $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$ be a square 13-plithogenic invertible real matrix, then:

1). $\det(Z^{-1}) = (\det Z)^{-1}$

2). $\det Z^t = \det Z$

3). $\det(Z.C) = \det Z . \det C \,; C = C_0 + \sum_{i=1}^{13} C_i P_i$.

Proof of theorem1.

1). Let $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$, then Z is invertible if and only if there exists $T = T_0 + \sum_{i=1}^{13} T_i P_i$ such that:
$Z \times T = U_{n\times n}$, hence:

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$$\begin{cases} Z_0T_0 = U_{n\times n} \\ \sum_{i=0}^{1} Z_i \sum_{i=0}^{1} T_i - Z_0T_0 = O_{n\times n} \\ \sum_{i=0}^{2} Z_i \sum_{i=0}^{2} T_i - \sum_{i=0}^{1} Z_i \sum_{i=0}^{1} T_i = O_{n\times n} \\ \sum_{i=0}^{3} Z_i \sum_{i=0}^{3} T_i - \sum_{i=0}^{2} Z_i \sum_{i=0}^{2} T_i = O_{n\times n} \\ \sum_{i=0}^{4} Z_i \sum_{i=0}^{4} T_i - \sum_{i=0}^{3} Z_i \sum_{i=0}^{3} T_i = O_{n\times n} \\ \sum_{i=0}^{5} Z_i \sum_{i=0}^{5} T_i - \sum_{i=0}^{4} Z_i \sum_{i=0}^{4} T_i = O_{n\times n} \\ \sum_{i=0}^{6} Z_i \sum_{i=0}^{6} T_i - \sum_{i=0}^{5} Z_i \sum_{i=0}^{5} T_i = O_{n\times n} \\ \sum_{i=0}^{7} Z_i \sum_{i=0}^{7} T_i - \sum_{i=0}^{6} Z_i \sum_{i=0}^{6} T_i = O_{n\times n} \\ \sum_{i=0}^{8} Z_i \sum_{i=0}^{8} T_i - \sum_{i=0}^{7} Z_i \sum_{i=0}^{7} T_i = O_{n\times n} \\ \sum_{i=0}^{9} Z_i \sum_{i=0}^{9} T_i - \sum_{i=0}^{8} Z_i \sum_{i=0}^{8} T_i = O_{n\times n} \\ \sum_{i=0}^{10} Z_i \sum_{i=0}^{10} T_i - \sum_{i=0}^{9} Z_i \sum_{i=0}^{9} T_i = O_{n\times n} \\ \sum_{i=0}^{11} Z_i \sum_{i=0}^{11} T_i - \sum_{i=0}^{10} Z_i \sum_{i=0}^{10} T_i = O_{n\times n} \\ \sum_{i=0}^{12} Z_i \sum_{i=0}^{12} T_i - \sum_{i=0}^{11} Z_i \sum_{i=0}^{11} T_i = O_{n\times n} \\ \sum_{i=0}^{13} Z_i \sum_{i=0}^{13} T_i - \sum_{i=0}^{12} Z_i \sum_{i=0}^{12} T_i = O_{n\times n} \end{cases}$$

This implies that:

$$\begin{cases} Z_0T_0 = U_{n\times n} \\ \sum_{i=0}^{j} Z_i \sum_{i=0}^{j} T_i = U_{n\times n} \end{cases} ;\ 1 \le j \le 13$$

Hence $\det\left(\sum_{i=0}^{j} Z_i\right) \neq 0$ for all $1 \le j \le 13$, so that $\det(Z)$ is invertible in $13 - SP_R$.

2). It holds directly as follows:

$\sum_{i=0}^{j} T_i = \left(\sum_{i=0}^{j} Z_i\right)^{-1}$ for $1 \le j \le 13$, hence:

$$Z^{-1} = Z_0^{-1} + \left[(\textstyle\sum_{i=0}^{1} Z_i)^{-1} - Z_0^{-1}\right]P_1 + [(\sum_{i=0}^{2} Z_i)^{-1} - (Z)^{-1}]P_2 + [(\sum_{i=1}^{3} Z_i)^{-1} - (\sum_{i=0}^{2} Z_i)^{-1}]P_3 + [(\sum_{i=1}^{4} Z_i)^{-1} - (\sum_{i=0}^{3} Z_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5} Z_i\right)^{-1} - (\sum_{i=0}^{4} Z_i)^{-1}\right]P_5 + \left[(\sum_{i=1}^{6} Z_i)^{-1} - \left(\sum_{i=0}^{5} Z_i\right)^{-1}\right]P_6 + [(\sum_{i=1}^{7} Z_i)^{-1} - (\sum_{i=0}^{6} Z_i)^{-1}]P_7 + [(\sum_{i=1}^{8} Z_i)^{-1} - (\sum_{i=0}^{7} Z_i)^{-1}]P_8 + [(\sum_{i=1}^{9} Z_i)^{-1} - (\sum_{i=0}^{8} Z_i)^{-1}]P_9 + [(\sum_{i=1}^{10} Z_i)^{-1} - (\sum_{i=0}^{9} Z_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} Z_i)^{-1} - (\sum_{i=0}^{10} Z_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} Z_i)^{-1} - (\sum_{i=0}^{11} Z_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} Z_i)^{-1} - (\sum_{i=0}^{12} Z_i)^{-1}]P_{13}.$$

Proof of theorem2.

It is clear that q is an eigen value of Z with X as an eigen vector if and only if:

$Z.X = q.X$, which is equivalent to:

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$$\begin{cases} Z_0X_0 = g_0X_0 \\ \sum_{i=0}^{j} Z_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} q_i \sum_{i=0}^{j} X_i \end{cases}; \ 1 \le j \le 13$$

Which is equivalent to:

$\sum_{i=0}^{j} q_i$ is an eigen value of $\sum_{i=0}^{j} Z_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 13$.

Proof of theorem4.

1). $detZ^{-1} = det(Z_0{}^{-1}) + P_1[det(\sum_{i=0}^{1} Z_i)^{-1} - det(Z_0{}^{-1})] + [det(\sum_{i=0}^{2} Z_i)^{-1} - det(\sum_{i=0}^{1} Z_i)^{-1}]P_2 + [det(\sum_{i=1}^{3} Z_i)^{-1} - det(\sum_{i=0}^{2} Z_i)^{-1}]P_3 + [det(\sum_{i=1}^{4} Z_i)^{-1} - det(\sum_{i=0}^{3} Z_i)^{-1}]P_4 + \left[det(\sum_{i=1}^{5} Z_i)^{-1} - det(\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[det(\sum_{i=1}^{6} Z_i)^{-1} - det(\sum_{i=0}^{5} Z_i)^{-1}\right] P_6 + [det(\sum_{i=1}^{7} Z_i)^{-1} - det(\sum_{i=0}^{6} Z_i)^{-1}]P_7 + [det(\sum_{i=1}^{8} Z_i)^{-1} - det(\sum_{i=0}^{7} Z_i)^{-1}]P_8 + [det(\sum_{i=1}^{9} Z_i)^{-1} - det(\sum_{i=0}^{8} Z_i)^{-1}]P_9 + [det(\sum_{i=1}^{10} Z_i)^{-1} - det(\sum_{i=0}^{9} Z_i)^{-1}]P_{10} + [det(\sum_{i=1}^{11} Z_i)^{-1} - det(\sum_{i=0}^{10} Z_i)^{-1}]P_{11} + [det(\sum_{i=1}^{12} Z_i)^{-1} - det(\sum_{i=0}^{11} Z_i)^{-1}]P_{12} + [det(\sum_{i=1}^{13} Z_i)^{-1} - det(\sum_{i=0}^{12} Z_i)^{-1}]P_{13} = (detZ)^{-1}$.

2). $Z^t = Z_0{}^t + Z_1{}^tP_1 + Z_2{}^tP_2 + Z_3{}^tP_3 + Z_4{}^tP_4 + Z_5{}^tP_5 + Z_6{}^tP_6 + Z_7{}^tP_7 + Z_8{}^tP_8 + Z_9{}^tP_9 + Z_{10}{}^tP_{10} + Z_{11}{}^tP_{11} + Z_{12}{}^tP_{12} + Z_{13}{}^tP_{13}$.

$detZ^t = det(Z_0{}^t) + [det(\sum_{i=0}^{1} Z_i{}^t) - det(Z_0{}^t)]P_1 + [det(\sum_{i=0}^{2} Z_i{}^t) - det(\sum_{i=0}^{1} Z_i{}^t)]P_2 + [det(\sum_{i=0}^{3} Z_i{}^t) - det(\sum_{i=0}^{2} Z_i{}^t)]P_3 + [det(\sum_{i=0}^{4} Z_i{}^t) - det(\sum_{i=0}^{3} Z_i{}^t)]P_4 + [det(\sum_{i=0}^{5} Z_i{}^t) - det\ (\sum_{i=0}^{4} Z_i{}^t)]P_5 + [det(\sum_{i=0}^{6} Z_i{}^t) - det\ (\sum_{i=0}^{5} Z_i{}^t)]P_6 + [det(\sum_{i=0}^{7} Z_i{}^t) - det\ (\sum_{i=0}^{6} Z_i{}^t)]P_7 + [det(\sum_{i=0}^{8} Z_i{}^t) - det\ (\sum_{i=0}^{7} Z_i{}^t)]P_8 + [det(\sum_{i=0}^{9} Z_i{}^t) - det\ (\sum_{i=0}^{8} Z_i{}^t)]P_9 + [det(\sum_{i=0}^{10} Z_i{}^t) - det\ (\sum_{i=0}^{9} Z_i{}^t)]P_{10} + [det(\sum_{i=0}^{11} Z_i{}^t) - det\ (\sum_{i=0}^{10} Z_i{}^t)]P_{11} + [det(\sum_{i=0}^{12} Z_i{}^t) - det\ (\sum_{i=0}^{11} Z_i{}^t)]P_{12} + [det(\sum_{i=0}^{13} Z_i{}^t) - det\ (\sum_{i=0}^{12} Z_i{}^t)]P_{13} = \det(Z_0) + [\det(\sum_{i=0}^{1} Z_i) - \det(Z_0)]P_1 + [\det(\sum_{i=0}^{2} Z_i) - \det(\sum_{i=0}^{1} Z_i)]P_2 + [\det(\sum_{i=0}^{3} Z_i) - \det(\sum_{i=0}^{2} Z_i)]P_3 + [\det(\sum_{i=0}^{4} Z_i) - \det(\sum_{i=0}^{3} Z_i)]P_4 + [\det(\sum_{i=0}^{5} Z_i) - \det(\sum_{i=0}^{4} Z_i)]P_5 + [\det(\sum_{i=0}^{6} Z_i) - \det(\sum_{i=0}^{5} Z_i)]P_6 + [\det(\sum_{i=0}^{7} Z_i) - \det(\sum_{i=0}^{6} Z_i)]P_7 + [\det(\sum_{i=0}^{8} Z_i) - \det(\sum_{i=0}^{7} Z_i)]P_8 + [\det(\sum_{i=0}^{9} Z_i) - \det(\sum_{i=0}^{8} Z_i)]P_9 + [\det(\sum_{i=0}^{10} Z_i) - \det(\sum_{i=0}^{9} Z_i)]P_{10} + [\det(\sum_{i=0}^{11} Z_i) - \det(\sum_{i=0}^{10} Z_i)]P_{11} + [\det(\sum_{i=0}^{12} Z_i) - \det(\sum_{i=0}^{11} Z_i)]P_{12} + [\det(\sum_{i=0}^{13} Z_i) - \det(\sum_{i=0}^{12} Z_i)]P_{13} = detZ$.

3). we have:

$Z.C = Z_0C_0 + [\sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i - Z_0C_0]P_1 + [\sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i - \sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i]P_2 + [\sum_{i=0}^{3} Z_i \sum_{i=0}^{3} C_i - \sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i]P_3 + [\sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i - \sum_{i=0}^{3} Z_i \sum_{i=0}^{3} B_i]P_4 + [\sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i - \sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i]P_5 + [\sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i - \sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i]P_6 + [\sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i - \sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i]P_7 + [\sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i - \sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i]P_8 + [\sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i - \sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i]P_9 + [\sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i - \sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i]P_{10} + [\sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i - \sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i]P_{11} + [\sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i - \sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i]P_{12} + [\sum_{i=0}^{13} Z_i \sum_{i=0}^{13} C_i - \sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i]P_{13}$.

$det(Z.C) = det(Z_0C_0) + [det(\sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i) - det(Z_0C_0)]P_1 + [det(\sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i) - det(\sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i)]P_2 + [det(\sum_{i=0}^{3} Z_i \sum_{i=0}^{3} C_i) - det(\sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i)]P_3 + [det(\sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i) - det(\sum_{i=0}^{3} Z_i \sum_{i=0}^{3} C_i)]P_4 + [det(\sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i) - det\ (\sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i)]P_5 + [det(\sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i) - det\ (\sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i)]P_6 + [det(\sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i) - det\ (\sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i)]P_7 + [det(\sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i) - det\ (\sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i)]P_8 + [det(\sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i) - det\ (\sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i)]P_9 + [det(\sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i) - det\ (\sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i)]P_{10} + [det(\sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i) - det\ (\sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i)]P_{11} + [det(\sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i) - det\ (\sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i)]P_{12} + [det(\sum_{i=0}^{13} Z_i \sum_{i=0}^{13} C_i) - det\ (\sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i)]P_{13} = det(Z_0)det(C_0) + [det(\sum_{i=0}^{j} Z_i).det(\sum_{i=0}^{j} C_i) - det(\sum_{i=1}^{j-1} Z_{i-1}).det(\sum_{i=1}^{j-1} C_{i-1})]P_i = det(Z)det(C); 1 \le j \le 13$.

Definition.

Let $Z = Z_0 + \sum_{i=1}^{13} Z_iP_i$ be a symbolic 13-plithogenic real square matrix, then:

Z is called orthogonal if and only if $Z^t = Z^{-1}$.

Theorem5.

Z is orthogonal if and only if $\sum_{i=0}^{j} Z_i$; $0 \le j \le 13$ are orthogonal.

Proof of theorem5.

Z is orthogonal if and only if $Z^t = Z^{-1}$, hence:

$Z_0{}^t + \sum_{i=1}^{13} Z_i{}^tP_i = Z_0{}^{-1} + [(\sum_{i=0}^{1} Z_i)^{-1} - U_0{}^{-1}]P_1 + [(\sum_{i=0}^{2} Z_i)^{-1} - (\sum_{i=0}^{1} Z_i)^{-1}]P_2 + [(\sum_{i=1}^{3} `Z_i)^{-1} - (\sum_{i=0}^{2} Z_i)^{-1}]P_3 + [(\sum_{i=1}^{4} Z_i)^{-1} - (\sum_{i=0}^{3} Z_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} Z_i)^{-1} - (\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} Z_i)^{-1} - (\sum_{i=0}^{5} Z_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} Z_i)^{-1} - (\sum_{i=0}^{6} Z_i)^{-1}]P_7 + [(\sum_{i=1}^{8} Z_i)^{-1} - (\sum_{i=0}^{7} Z_i)^{-1}]P_8 + [(\sum_{i=1}^{9} Z_i)^{-1} - (\sum_{i=0}^{8} Z_i)^{-1}]P_9 + [(\sum_{i=1}^{10} Z_i)^{-1} - (\sum_{i=0}^{9} Z_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} Z_i)^{-1} - (\sum_{i=0}^{10} Z_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} Z_i)^{-1} - (\sum_{i=0}^{11} Z_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} Z_i)^{-1} - (\sum_{i=0}^{12} Z_i)^{-1}]P_{13}$, thus:

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$$\begin{cases} {Z_0}^t = {Z_0}^{-1} \\ {Z_1}^t = \left(\sum_{i=0}^{1} Z_i\right)^{-1} - {Z_0}^{-1} \\ {Z_2}^t = \left(\sum_{i=0}^{2} Z_i\right)^{-1} - \left(\sum_{i=0}^{1} Z_i\right)^{-1} \\ {U_3}^t = \left(\sum_{i=0}^{3} Z_i\right)^{-1} - \left(\sum_{i=0}^{2} Z_i\right)^{-1} \\ {U_4}^t = \left(\sum_{i=0}^{4} Z_i\right)^{-1} - \left(\sum_{i=0}^{3} Z_i\right)^{-1} \\ {U_5}^t = \left(\sum_{i=0}^{5} Z_i\right)^{-1} - \left(\sum_{i=0}^{4} Z_i\right)^{-1} \\ {U_6}^t = \left(\sum_{i=0}^{6} Z_i\right)^{-1} - \left(\sum_{i=0}^{5} Z_i\right)^{-1} \\ {U_7}^t = \left(\sum_{i=0}^{7} Z_i\right)^{-1} - \left(\sum_{i=0}^{6} Z_i\right)^{-1} \\ {U_8}^t = \left(\sum_{i=0}^{8} Z_i\right)^{-1} - \left(\sum_{i=0}^{7} Z_i\right)^{-1} \\ {U_9}^t = \left(\sum_{i=0}^{9} Z_i\right)^{-1} - \left(\sum_{i=0}^{8} Z_i\right)^{-1} \\ {U_{10}}^t = \left(\sum_{i=0}^{10} Z_i\right)^{-1} - \left(\sum_{i=0}^{9} Z_i\right)^{-1} \\ {U_{11}}^t = \left(\sum_{i=0}^{11} Z_i\right)^{-1} - \left(\sum_{i=0}^{10} Z_i\right)^{-1} \\ {U_{12}}^t = \left(\sum_{i=0}^{12} Z_i\right)^{-1} - \left(\sum_{i=0}^{11} Z_i\right)^{-1} \\ {U_{13}}^t = \left(\sum_{i=0}^{13} Z_i\right)^{-1} - \left(\sum_{i=0}^{12} Z_i\right)^{-1} \end{cases}$$

This implies that:

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$$\begin{cases} Z_0{}^t = Z_0{}^{-1} \\ \sum_{i=0}^{1} Z_i{}^t = (\sum_{i=0}^{1} Z_i)^{-1} \\ \sum_{i=0}^{2} Z_i{}^t = (\sum_{i=0}^{2} Z_i)^{-1} \\ \sum_{i=0}^{3} Z_i{}^t = (\sum_{i=0}^{3} Z_i)^{-1} \\ \sum_{i=0}^{4} Z_i{}^t = (\sum_{i=0}^{4} Z_i)^{-1} \\ \sum_{i=0}^{5} Z_i{}^t = (\sum_{i=0}^{5} Z_i)^{-1} \\ \sum_{i=0}^{6} Z_i{}^t = (\sum_{i=0}^{6} Z_i)^{-1} \\ \sum_{i=0}^{7} Z_i{}^t = (\sum_{i=0}^{7} Z_i)^{-1}, \\ \sum_{i=0}^{8} Z_i{}^t = (\sum_{i=0}^{8} Z_i)^{-1} \\ \sum_{i=0}^{9} Z_i{}^t = (\sum_{i=0}^{9} Z_i)^{-1} \\ \sum_{i=0}^{10} Z_i{}^t = (\sum_{i=0}^{10} Z_i)^{-1} \\ \sum_{i=0}^{11} Z_i{}^t = (\sum_{i=0}^{11} Z_i)^{-1} \\ \sum_{i=0}^{12} Z_i{}^t = (\sum_{i=0}^{12} Z_i)^{-1} \\ \sum_{i=0}^{13} Z_i{}^t = (\sum_{i=0}^{13} Z_i)^{-1} \end{cases}$$

Definition:

The square symbolic 14-plithogenic matrix is defined as follows:

$Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$; $(Z_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 13-plithogenic matrix:

$$Z = \begin{pmatrix} -3 & -9 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 5 & 1 \end{pmatrix} P_1 + \begin{pmatrix} 2 & -1 \\ 7 & 2 \end{pmatrix} P_2 + \begin{pmatrix} 6 & -8 \\ 9 & -6 \end{pmatrix} P_3 + \begin{pmatrix} 8 & 5 \\ 6 & 1 \end{pmatrix} P_4 + \begin{pmatrix} -5 & -5 \\ -5 & -2 \end{pmatrix} P_5 + \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix} P_6 + \begin{pmatrix} -1 & 7 \\ 9 & 8 \end{pmatrix} P_7 + \begin{pmatrix} 12 & 11 \\ 65 & -1 \end{pmatrix} P_8 + \begin{pmatrix} -1 & 9 \\ -1 & 0 \end{pmatrix} P_{10} + \begin{pmatrix} 8 & -1 \\ 7 & 5 \end{pmatrix} P_{11} + \begin{pmatrix} 4 & -1 \\ 2 & -8 \end{pmatrix} P_{12} + \begin{pmatrix} 4 & -1 \\ 2 & -9 \end{pmatrix} P_{13} + \begin{pmatrix} 4 & -1 \\ 2 & -8 \end{pmatrix} P_{14}.$$

Definition.

Let $Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$ be a symbolic 14-plithogenic matrix of size $n \times n$, hence:

$$\begin{aligned} \det Z = \det(Z_0) &+ \left[\det\left(\sum_{i=0}^{1} Z_i\right) - \det(Z_0)\right] P_1 + \left[\det\left(\sum_{i=0}^{2} Z_i\right) - \det\left(\sum_{i=0}^{1} Z_i\right)\right] P_2 \\ &+ \left[\det\left(\sum_{i=0}^{3} Z_i\right) - \det\left(\sum_{i=0}^{2} Z_i\right)\right] P_3 + \left[\det\left(\sum_{i=0}^{4} Z_i\right) - \det\left(\sum_{i=0}^{3} Z_i\right)\right] P_4 \\ &+ \left[\det\left(\sum_{i=0}^{5} Z_i\right) - \det\left(\sum_{i=0}^{4} Z_i\right)\right] P_5 + \left[\det\left(\sum_{i=0}^{6} Z_i\right) - \det\left(\sum_{i=0}^{5} Z_i\right)\right] P_6 \\ &+ \left[\det\left(\sum_{i=0}^{7} Z_i\right) - \det\left(\sum_{i=0}^{6} Z_i\right)\right] P_7 + \left[\det\left(\sum_{i=0}^{8} Z_i\right) - \det\left(\sum_{i=0}^{7} Z_i\right)\right] P_8 \\ &+ \left[\det\left(\sum_{i=0}^{9} Z_i\right) - \det\left(\sum_{i=0}^{8} Z_i\right)\right] P_9 + \left[\det\left(\sum_{i=0}^{10} Z_i\right) - \det\left(\sum_{i=0}^{9} Z_i\right)\right] P_{10} \\ &+ \left[\det\left(\sum_{i=0}^{11} Z_i\right) - \det\left(\sum_{i=0}^{10} Z_i\right)\right] P_{11} + \left[\det\left(\sum_{i=0}^{12} Z_i\right) - \det\left(\sum_{i=0}^{11} Z_i\right)\right] P_{12} \\ &+ \left[\det\left(\sum_{i=0}^{13} Z_i\right) - \det\left(\sum_{i=0}^{12} Z_i\right)\right] P_{13} + \left[\det\left(\sum_{i=0}^{14} Z_i\right) - \det\left(\sum_{i=0}^{13} Z_i\right)\right] P_{14} \end{aligned}$$

Theorem6.

Let $Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$ be a symbolic 14-plithogenic matrix of size $n \times n$, hence:

1. Z is invertible if and only if $\det Z$ is an invertible symbolic 13-plithogenic real number.
2. $Z^{-1} = Z_0{}^{-1} + [(\sum_{i=0}^{1} Z_i)^{-1} - Z_0{}^{-1}]P_1 + [(\sum_{i=0}^{2} U_i)^{-1} - (\sum_{i=0}^{1} Z_i)^{-1}]P_2 + [(\sum_{i=1}^{3} Z_i)^{-1} - (\sum_{i=0}^{2} Z_i)^{-1}]P_3 + [(\sum_{i=1}^{4} Z_i)^{-1} - (\sum_{i=0}^{3} Z_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} Z_i)^{-1} - (\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} Z_i)^{-1} - (\sum_{i=0}^{5} Z_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} Z_i)^{-1} - (\sum_{i=0}^{6} Z_i)^{-1}]P_7 + [(\sum_{i=1}^{8} Z_i)^{-1} - (\sum_{i=0}^{7} Z_i)^{-1}]P_8 + [(\sum_{i=1}^{9} Z_i)^{-1} - (\sum_{i=0}^{8} Z_i)^{-1}]P_9 + [(\sum_{i=1}^{10} Z_i)^{-1} - (\sum_{i=0}^{9} Z_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} Z_i)^{-1} - (\sum_{i=0}^{10} Z_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} Z_i)^{-1} - (\sum_{i=0}^{11} Z_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} Z_i)^{-1} - (\sum_{i=0}^{12} Z_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} Z_i)^{-1} - (\sum_{i=0}^{13} Z_i)^{-1}]P_{14}$

Definition.

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Let $q = q_0 + \sum_{i=1}^{14} q_i P_i$ be a symbolic 14-plithogenic real number and $Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$ be a symbolic 14-plithogenic square real matrix, then q is called symbolic 14-plithogenic eigen value if and only if $ZX = qX$.
X is called symbolic 14-plithogenic eigenvector.

Theorem7.

Let $q = q_0 + \sum_{i=1}^{13} q_i P_i \in 14 - SP_R$, $X = X_0 + \sum_{i=1}^{14} X_i P_i$ be a symbolic 14-plithogenic real vector, then q is eigen value of $Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} q_i$ is eigen value of $\sum_{i=0}^{j} Z_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \le j \le 14$.

Theorem8.

$$Z^n = {Z_0}^n + P_1\left[\left(\sum_{i=0}^{1} Z_i\right)^n - {Z_0}^n\right] + \left[\left(\sum_{i=0}^{2} Z_i\right)^n - \left(\sum_{i=0}^{1} Z_i\right)^n\right]P_2 + \left[\left(\sum_{i=1}^{3} Z_i\right)^n - \left(\sum_{i=0}^{2} Z_i\right)^n\right]P_3$$
$$+ \left[\left(\sum_{i=1}^{4} Z_i\right)^n - \left(\sum_{i=0}^{3} Z_i\right)^n\right]P_4 + \left[\left(\sum_{i=1}^{5} Z_i\right)^n - \left(\sum_{i=0}^{4} Z_i\right)^n\right]P_5 + \left[\left(\sum_{i=1}^{6} Z_i\right)^n - \left(\sum_{i=0}^{5} Z_i\right)^n\right]P_6$$
$$+ \left[\left(\sum_{i=1}^{7} Z_i\right)^n - \left(\sum_{i=0}^{6} Z_i\right)^n\right]P_7 + \left[\left(\sum_{i=1}^{8} Z_i\right)^n - \left(\sum_{i=0}^{7} Z_i\right)^n\right]P_8 + \left[\left(\sum_{i=1}^{9} Z_i\right)^n - \left(\sum_{i=0}^{8} Z_i\right)^n\right]P_9$$
$$+ \left[\left(\sum_{i=1}^{10} Z_i\right)^n - \left(\sum_{i=0}^{9} Z_i\right)^n\right]P_{10} + \left[\left(\sum_{i=1}^{11} Z_i\right)^n - \left(\sum_{i=0}^{10} Z_i\right)^n\right]P_{11}$$
$$+ \left[\left(\sum_{i=1}^{12} Z_i\right)^n - \left(\sum_{i=0}^{11} Z_i\right)^n\right]P_{12} + \left[\left(\sum_{i=1}^{13} Z_i\right)^n - \left(\sum_{i=0}^{12} Z_i\right)^n\right]P_{13}$$
$$+ \left[\left(\sum_{i=1}^{14} Z_i\right)^n - \left(\sum_{i=0}^{13} Z_i\right)^n\right]P_{14}$$

Theorem9.

Let $Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$ be a square 14-plithogenic invertible real matrix, then:
1). $\det(Z^{-1}) = (\det Z)^{-1}$
2). $\det Z^t = \det Z$
3). $\det(Z.C) = \det Z . \det C$; $C = C_0 + \sum_{i=1}^{14} C_i P_i$.

Definition.

Let $Z = Z_0 + \sum_{i=1}^{13} Z_i P_i$ be a symbolic 13-plithogenic real square matrix, then:
Z is called orthogonal if and only if $Z^t = Z^{-1}$.

Theorem10.

Z is orthogonal if and only if $\sum_{i=0}^{j} Z_i$; $0 \le j \le 13$ are orthogonal.

Proof of theorem6.

1). Let $Z = Z_0 + \sum_{i=1}^{14} Z_i P_i$, then Z is invertible if and only if there exists $T = T_0 + \sum_{i=1}^{14} T_i P_i$ such that:
$Z \times T = U_{n \times n}$, hence:

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$$\begin{cases} Z_0 T_0 = U_{n\times n} \\ \sum_{i=0}^{1} Z_i \sum_{i=0}^{1} T_i - Z_0 T_0 = O_{n\times n} \\ \sum_{i=0}^{2} Z_i \sum_{i=0}^{2} T_i - \sum_{i=0}^{1} Z_i \sum_{i=0}^{1} T_i = O_{n\times n} \\ \sum_{i=0}^{3} Z_i \sum_{i=0}^{3} T_i - \sum_{i=0}^{2} Z_i \sum_{i=0}^{2} T_i = O_{n\times n} \\ \sum_{i=0}^{4} Z_i \sum_{i=0}^{4} T_i - \sum_{i=0}^{3} Z_i \sum_{i=0}^{3} T_i = O_{n\times n} \\ \sum_{i=0}^{5} Z_i \sum_{i=0}^{5} T_i - \sum_{i=0}^{4} Z_i \sum_{i=0}^{4} T_i = O_{n\times n} \\ \sum_{i=0}^{6} Z_i \sum_{i=0}^{6} T_i - \sum_{i=0}^{5} Z_i \sum_{i=0}^{5} T_i = O_{n\times n} \\ \sum_{i=0}^{7} Z_i \sum_{i=0}^{7} T_i - \sum_{i=0}^{6} Z_i \sum_{i=0}^{6} T_i = O_{n\times n} \\ \sum_{i=0}^{8} Z_i \sum_{i=0}^{8} T_i - \sum_{i=0}^{7} Z_i \sum_{i=0}^{7} T_i = O_{n\times n} \\ \sum_{i=0}^{9} Z_i \sum_{i=0}^{9} T_i - \sum_{i=0}^{8} Z_i \sum_{i=0}^{8} T_i = O_{n\times n} \\ \sum_{i=0}^{10} Z_i \sum_{i=0}^{10} T_i - \sum_{i=0}^{9} Z_i \sum_{i=0}^{9} T_i = O_{n\times n} \\ \sum_{i=0}^{11} Z_i \sum_{i=0}^{11} T_i - \sum_{i=0}^{10} Z_i \sum_{i=0}^{10} T_i = O_{n\times n} \\ \sum_{i=0}^{12} Z_i \sum_{i=0}^{12} T_i - \sum_{i=0}^{11} Z_i \sum_{i=0}^{11} T_i = O_{n\times n} \\ \sum_{i=0}^{13} Z_i \sum_{i=0}^{13} T_i - \sum_{i=0}^{12} Z_i \sum_{i=0}^{12} T_i = O_{n\times n} \\ \sum_{i=0}^{14} Z_i \sum_{i=0}^{14} T_i - \sum_{i=0}^{13} Z_i \sum_{i=0}^{13} T_i = O_{n\times n} \end{cases}$$

This implies that:

$$\begin{cases} Z_0 T_0 = U_{n\times n} \\ \sum_{i=0}^{j} Z_i \sum_{i=0}^{j} T_i = U_{n\times n} \end{cases} ;\ 1 \le j \le 14$$

Hence $\det\left(\sum_{i=0}^{j} Z_i\right) \neq 0$ for all $1 \le j \le 14$, so that $\det(Z)$ is invertible in $14 - SP_R$.

2). It holds directly as follows:

$\sum_{i=0}^{j} T_i = \left(\sum_{i=0}^{j} Z_i\right)^{-1}$ for $1 \le j \le 14$, hence:

$$Z^{-1} = {Z_0}^{-1} + \left[\left(\sum_{i=0}^{1} Z_i\right)^{-1} - {Z_0}^{-1}\right] P_1 + \left[\left(\sum_{i=0}^{2} Z_i\right)^{-1} - (Z)^{-1}\right] P_2 + \left[\left(\sum_{i=1}^{3} Z_i\right)^{-1} - \left(\sum_{i=0}^{2} Z_i\right)^{-1}\right] P_3 + \left[\left(\sum_{i=1}^{4} Z_i\right)^{-1} - \left(\sum_{i=0}^{3} Z_i\right)^{-1}\right] P_4 + \left[\left(\sum_{i=1}^{5} Z_i\right)^{-1} - \left(\sum_{i=0}^{4} Z_i\right)^{-1}\right] P_5 + \left[\left(\sum_{i=1}^{6} Z_i\right)^{-1} - \left(\sum_{i=0}^{5} Z_i\right)^{-1}\right] P_6 + \left[\left(\sum_{i=1}^{7} Z_i\right)^{-1} - \left(\sum_{i=0}^{6} Z_i\right)^{-1}\right] P_7 + \left[\left(\sum_{i=1}^{8} Z_i\right)^{-1} - \left(\sum_{i=0}^{7} Z_i\right)^{-1}\right] P_8 + \left[\left(\sum_{i=1}^{9} Z_i\right)^{-1} - \left(\sum_{i=0}^{8} Z_i\right)^{-1}\right] P_9 + \left[\left(\sum_{i=1}^{10} Z_i\right)^{-1} - \left(\sum_{i=0}^{9} Z_i\right)^{-1}\right] P_{10} + \left[\left(\sum_{i=1}^{11} Z_i\right)^{-1} - \left(\sum_{i=0}^{10} Z_i\right)^{-1}\right] P_{11} + \left[\left(\sum_{i=1}^{12} Z_i\right)^{-1} - \left(\sum_{i=0}^{11} Z_i\right)^{-1}\right] P_{12} + \left[\left(\sum_{i=1}^{13} Z_i\right)^{-1} - \left(\sum_{i=0}^{12} Z_i\right)^{-1}\right] P_{13}. \left[\left(\sum_{i=1}^{14} Z_i\right)^{-1} - \left(\sum_{i=0}^{13} Z_i\right)^{-1}\right] P_{14}.$$

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Proof of theorem7.

It is clear that q is an eigen value of Z with X as an eigen vector if and only if:

$Z.X = q.X$, which is equivalent to:

$$\begin{cases} Z_0X_0 = g_0X_0 \\ \sum_{i=0}^{j} Z_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} q_i \sum_{i=0}^{j} X_i \end{cases}; \ 1 \le j \le 14$$

Which is equivalent to:

$\sum_{i=0}^{j} q_i$ is an eigen value of $\sum_{i=0}^{j} Z_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 14$.

Proof of theorem9.

1). $detZ^{-1} = det(Z_0^{-1}) + P_1[det(\sum_{i=0}^{1} Z_i)^{-1} - det(Z_0^{-1})] + [det(\sum_{i=0}^{2} Z_i)^{-1} - det(\sum_{i=0}^{1} Z_i)^{-1}]P_2 + [det(\sum_{i=1}^{3} Z_i)^{-1} - det(\sum_{i=0}^{2} Z_i)^{-1}]P_3 + [det(\sum_{i=1}^{4} Z_i)^{-1} - det(\sum_{i=0}^{3} Z_i)^{-1}]P_4 + \left[det(\sum_{i=1}^{5} Z_i)^{-1} - det(\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[det(\sum_{i=1}^{6} Z_i)^{-1} - det(\sum_{i=0}^{5} Z_i)^{-1}\right] P_6 + [det(\sum_{i=1}^{7} Z_i)^{-1} - det(\sum_{i=0}^{6} Z_i)^{-1}]P_7 + [det(\sum_{i=1}^{8} Z_i)^{-1} - det(\sum_{i=0}^{7} Z_i)^{-1}]P_8 + [det(\sum_{i=1}^{9} Z_i)^{-1} - det(\sum_{i=0}^{8} Z_i)^{-1}]P_9 + [det(\sum_{i=1}^{10} Z_i)^{-1} - det(\sum_{i=0}^{9} Z_i)^{-1}]P_{10} + [det(\sum_{i=1}^{11} Z_i)^{-1} - det(\sum_{i=0}^{10} Z_i)^{-1}]P_{11} + [det(\sum_{i=1}^{12} Z_i)^{-1} - det(\sum_{i=0}^{11} Z_i)^{-1}]P_{12} + [det(\sum_{i=1}^{13} Z_i)^{-1} - det(\sum_{i=0}^{12} Z_i)^{-1}]P_{13} + [det(\sum_{i=1}^{14} Z_i)^{-1} - det(\sum_{i=0}^{13} Z_i)^{-1}]P_{14} = (detZ)^{-1}$.

2). $Z^t = Z_0{}^t + Z_1{}^tP_1 + Z_2{}^tP_2 + Z_3{}^tP_3 + Z_4{}^tP_4 + Z_5{}^tP_5 + Z_6{}^tP_6 + Z_7{}^tP_7 + Z_8{}^tP_8 + Z_9{}^tP_9 + Z_{10}{}^tP_{10} + Z_{11}{}^tP_{11} + Z_{12}{}^tP_{12} + Z_{13}{}^tP_{13} + Z_{14}{}^tP_{14}$.

$detZ^t = det(Z_0{}^t) + [det(\sum_{i=0}^{1} Z_i{}^t) - det(Z_0{}^t)]P_1 + [det(\sum_{i=0}^{2} Z_i{}^t) - det(\sum_{i=0}^{1} Z_i{}^t)]P_2 + [det(\sum_{i=0}^{3} Z_i{}^t) - det(\sum_{i=0}^{2} Z_i{}^t)]P_3 + [det(\sum_{i=0}^{4} Z_i{}^t) - det(\sum_{i=0}^{3} Z_i{}^t)]P_4 + [det(\sum_{i=0}^{5} Z_i{}^t) - det\ (\sum_{i=0}^{4} Z_i{}^t)]P_5 + [det(\sum_{i=0}^{6} Z_i{}^t) - det\ (\sum_{i=0}^{5} Z_i{}^t)]P_6 + [det(\sum_{i=0}^{7} Z_i{}^t) - det\ (\sum_{i=0}^{6} Z_i{}^t)]P_7 + [det(\sum_{i=0}^{8} Z_i{}^t) - det\ (\sum_{i=0}^{7} Z_i{}^t)]P_8 + [det(\sum_{i=0}^{9} Z_i{}^t) - det\ (\sum_{i=0}^{8} Z_i{}^t)]P_9 + [det(\sum_{i=0}^{10} Z_i{}^t) - det\ (\sum_{i=0}^{9} Z_i{}^t)]P_{10} + [det(\sum_{i=0}^{11} Z_i{}^t) - det\ (\sum_{i=0}^{10} Z_i{}^t)]P_{11} + [det(\sum_{i=0}^{12} Z_i{}^t) - det\ (\sum_{i=0}^{11} Z_i{}^t)]P_{12} + [det(\sum_{i=0}^{13} Z_i{}^t) - det\ (\sum_{i=0}^{12} Z_i{}^t)]P_{13} + [det(\sum_{i=0}^{14} Z_i{}^t) - det\ (\sum_{i=0}^{13} Z_i{}^t)]P_{14} = \det(Z_0) + [\det(\sum_{i=0}^{1} Z_i) - \det(Z_0)]P_1 + [\det(\sum_{i=0}^{2} Z_i) - \det(\sum_{i=0}^{1} Z_i)]P_2 + [\det(\sum_{i=0}^{3} Z_i) - \det(\sum_{i=0}^{2} Z_i)]P_3 + [\det(\sum_{i=0}^{4} Z_i) - \det(\sum_{i=0}^{3} Z_i)]P_4 + [\det(\sum_{i=0}^{5} Z_i) - \det(\sum_{i=0}^{4} Z_i)]P_5 + [\det(\sum_{i=0}^{6} Z_i) - \det(\sum_{i=0}^{5} Z_i)]P_6 + [\det(\sum_{i=0}^{7} Z_i) - \det(\sum_{i=0}^{6} Z_i)]P_7 + [\det(\sum_{i=0}^{8} Z_i) - \det(\sum_{i=0}^{7} Z_i)]P_8 + [\det(\sum_{i=0}^{9} Z_i) - \det(\sum_{i=0}^{8} Z_i)]P_9 + [\det(\sum_{i=0}^{10} Z_i) - \det(\sum_{i=0}^{9} Z_i)]P_{10} + [\det(\sum_{i=0}^{11} Z_i) - \det(\sum_{i=0}^{10} Z_i)]P_{11} + [\det(\sum_{i=0}^{12} Z_i) - \det(\sum_{i=0}^{11} Z_i)]P_{12} + [\det(\sum_{i=0}^{13} Z_i) - \det(\sum_{i=0}^{12} Z_i)]P_{13} + [\det(\sum_{i=0}^{14} Z_i) - \det(\sum_{i=0}^{13} Z_i)]P_{14} = detZ$.

3). we have:

$Z.C = Z_0C_0 + [\sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i - Z_0C_0]P_1 + [\sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i - \sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i]P_2 + [\sum_{i=0}^{3} Z_i \sum_{i=0}^{3} C_i - \sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i]P_3 + [\sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i - \sum_{i=0}^{3} Z_i \sum_{i=0}^{3} B_i]P_4 + [\sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i - \sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i]P_5 + [\sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i - \sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i]P_6 + [\sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i - \sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i]P_7 + [\sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i - \sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i]P_8 + [\sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i - \sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i]P_9 + [\sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i - \sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i]P_{10} + [\sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i - \sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i]P_{11} + [\sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i - \sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i]P_{12} + [\sum_{i=0}^{13} Z_i \sum_{i=0}^{13} C_i - \sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i]P_{13} + [\sum_{i=0}^{14} Z_i \sum_{i=0}^{14} C_i - \sum_{i=0}^{13} Z_i \sum_{i=0}^{13} C_i]P_{14}$.

$det(Z.C) = det(Z_0C_0) + [det(\sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i) - det(Z_0C_0)]P_1 + [det(\sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i) - det(\sum_{i=0}^{1} Z_i \sum_{i=0}^{1} C_i)]P_2 + [det(\sum_{i=0}^{3} Z_i \sum_{i=0}^{3} C_i) - det(\sum_{i=0}^{2} Z_i \sum_{i=0}^{2} C_i)]P_3 + [det(\sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i) - det(\sum_{i=0}^{3} Z_i \sum_{i=0}^{3} C_i)]P_4 + [det(\sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i) - det\ (\sum_{i=0}^{4} Z_i \sum_{i=0}^{4} C_i)]P_5 + [det(\sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i) - det\ (\sum_{i=0}^{5} Z_i \sum_{i=0}^{5} C_i)]P_6 + [det(\sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i) - det\ (\sum_{i=0}^{6} Z_i \sum_{i=0}^{6} C_i)]P_7 + [det(\sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i) - det\ (\sum_{i=0}^{7} Z_i \sum_{i=0}^{7} C_i)]P_8 + [det(\sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i) - det\ (\sum_{i=0}^{8} Z_i \sum_{i=0}^{8} C_i)]P_9 + [det(\sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i) - det\ (\sum_{i=0}^{9} Z_i \sum_{i=0}^{9} C_i)]P_{10} + [det(\sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i) - det\ (\sum_{i=0}^{10} Z_i \sum_{i=0}^{10} C_i)]P_{11} + [det(\sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i) - det\ (\sum_{i=0}^{11} Z_i \sum_{i=0}^{11} C_i)]P_{12} + [det(\sum_{i=0}^{13} Z_i \sum_{i=0}^{13} C_i) - det\ (\sum_{i=0}^{12} Z_i \sum_{i=0}^{12} C_i)]P_{13} + [det(\sum_{i=0}^{14} Z_i \sum_{i=0}^{14} C_i) - det\ (\sum_{i=0}^{13} Z_i \sum_{i=0}^{13} C_i)]P_{14} = det(Z_0)det(C_0) + [det(\sum_{i=0}^{j} Z_i).det(\sum_{i=0}^{j} C_i) - det(\sum_{i=1}^{j-1} Z_{i-1}).det(\sum_{i=1}^{j-1} C_{i-1})]P_i = det(Z)det(C); 1 \le j \le 14$.

Proof of theorem10.

Z is orthogonal if and only if $Z^t = Z^{-1}$, hence:

$Z_0{}^t + \sum_{i=1}^{14} Z_i{}^tP_i = Z_0{}^{-1} + [(\sum_{i=0}^{1} Z_i)^{-1} - U_0{}^{-1}]P_1 + [(\sum_{i=0}^{2} Z_i)^{-1} - (\sum_{i=0}^{1} Z_i)^{-1}]P_2 + [(\sum_{i=1}^{3} `Z_i)^{-1} - (\sum_{i=0}^{2} Z_i)^{-1}]P_3 + [(\sum_{i=1}^{4} Z_i)^{-1} - (\sum_{i=0}^{3} Z_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} Z_i)^{-1} - (\sum_{i=0}^{4} Z_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} Z_i)^{-1} - (\sum_{i=0}^{5} Z_i)^{-1}\right] P_6 + [(\sum_{i=1}^{7} Z_i)^{-1} - (\sum_{i=0}^{6} Z_i)^{-1}]P_7 + [(\sum_{i=1}^{8} Z_i)^{-1} - (\sum_{i=0}^{7} Z_i)^{-1}]P_8 + [(\sum_{i=1}^{9} Z_i)^{-1} - (\sum_{i=0}^{8} Z_i)^{-1}]P_9 + [(\sum_{i=1}^{10} Z_i)^{-1} - (\sum_{i=0}^{9} Z_i)^{-1}]P_{10} + [(\sum_{i=1}^{11} Z_i)^{-1} - (\sum_{i=0}^{10} Z_i)^{-1}]P_{11} + [(\sum_{i=1}^{12} Z_i)^{-1} - (\sum_{i=0}^{11} Z_i)^{-1}]P_{12} + [(\sum_{i=1}^{13} Z_i)^{-1} - (\sum_{i=0}^{12} Z_i)^{-1}]P_{13} + [(\sum_{i=1}^{14} Z_i)^{-1} - (\sum_{i=0}^{13} Z_i)^{-1}]P_{14}$, thus:

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$$\begin{cases} {Z_0}^t = {Z_0}^{-1} \\ {Z_1}^t = \left(\sum_{i=0}^{1} Z_i\right)^{-1} - {Z_0}^{-1} \\ {Z_2}^t = \left(\sum_{i=0}^{2} Z_i\right)^{-1} - \left(\sum_{i=0}^{1} Z_i\right)^{-1} \\ {U_3}^t = \left(\sum_{i=0}^{3} Z_i\right)^{-1} - \left(\sum_{i=0}^{2} Z_i\right)^{-1} \\ {U_4}^t = \left(\sum_{i=0}^{4} Z_i\right)^{-1} - \left(\sum_{i=0}^{3} Z_i\right)^{-1} \\ {U_5}^t = \left(\sum_{i=0}^{5} Z_i\right)^{-1} - \left(\sum_{i=0}^{4} Z_i\right)^{-1} \\ {U_6}^t = \left(\sum_{i=0}^{6} Z_i\right)^{-1} - \left(\sum_{i=0}^{5} Z_i\right)^{-1} \\ {U_7}^t = \left(\sum_{i=0}^{7} Z_i\right)^{-1} - \left(\sum_{i=0}^{6} Z_i\right)^{-1} \\ {U_8}^t = \left(\sum_{i=0}^{8} Z_i\right)^{-1} - \left(\sum_{i=0}^{7} Z_i\right)^{-1} \\ {U_9}^t = \left(\sum_{i=0}^{9} Z_i\right)^{-1} - \left(\sum_{i=0}^{8} Z_i\right)^{-1} \\ {U_{10}}^t = \left(\sum_{i=0}^{10} Z_i\right)^{-1} - \left(\sum_{i=0}^{9} Z_i\right)^{-1} \\ {U_{11}}^t = \left(\sum_{i=0}^{11} Z_i\right)^{-1} - \left(\sum_{i=0}^{10} Z_i\right)^{-1} \\ {U_{12}}^t = \left(\sum_{i=0}^{12} Z_i\right)^{-1} - \left(\sum_{i=0}^{11} Z_i\right)^{-1} \\ {U_{13}}^t = \left(\sum_{i=0}^{13} Z_i\right)^{-1} - \left(\sum_{i=0}^{12} Z_i\right)^{-1} \\ {U_{14}}^t = \left(\sum_{i=0}^{14} Z_i\right)^{-1} - \left(\sum_{i=0}^{13} Z_i\right)^{-1} \end{cases}$$

This implies that:

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$$\begin{cases} Z_0{}^t = Z_0{}^{-1} \\ \sum_{i=0}^{1} Z_i{}^t = (\sum_{i=0}^{1} Z_i)^{-1} \\ \sum_{i=0}^{2} Z_i{}^t = (\sum_{i=0}^{2} Z_i)^{-1} \\ \sum_{i=0}^{3} Z_i{}^t = (\sum_{i=0}^{3} Z_i)^{-1} \\ \sum_{i=0}^{4} Z_i{}^t = (\sum_{i=0}^{4} Z_i)^{-1} \\ \sum_{i=0}^{5} Z_i{}^t = (\sum_{i=0}^{5} Z_i)^{-1} \\ \sum_{i=0}^{6} Z_i{}^t = (\sum_{i=0}^{6} Z_i)^{-1} \\ \sum_{i=0}^{7} Z_i{}^t = (\sum_{i=0}^{7} Z_i)^{-1}, \\ \sum_{i=0}^{8} Z_i{}^t = (\sum_{i=0}^{8} Z_i)^{-1} \\ \sum_{i=0}^{9} Z_i{}^t = (\sum_{i=0}^{9} Z_i)^{-1} \\ \sum_{i=0}^{10} Z_i{}^t = (\sum_{i=0}^{10} Z_i)^{-1} \\ \sum_{i=0}^{11} Z_i{}^t = (\sum_{i=0}^{11} Z_i)^{-1} \\ \sum_{i=0}^{12} Z_i{}^t = (\sum_{i=0}^{12} Z_i)^{-1} \\ \sum_{i=0}^{13} Z_i{}^t = (\sum_{i=0}^{13} Z_i)^{-1} \\ \sum_{i=0}^{14} Z_i{}^t = (\sum_{i=0}^{14} Z_i)^{-1} \end{cases}$$

3. Conclusion

Tin this work, we have found the algebraic properties of the symbolic n-plithogenic matrices in two different special cases (for n=13, n=14), where we have established many theorems that describe the algebraic behavior of these matrices, such as determinants, inverses, and eigenvalues. Also, the relationships between symbolic 13-plithogenic/14-plithogenic matrices and their classical components are presented.

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