Bipolar Triangular Neutrosophic Chromatic Numbers with the Application of traffic light system

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Abstract

For addressing issues in several domains, such as theoretical computer science, engineering, physics, combinatorics, and the medical sciences, graph theory is a crucial component of mathematics. Graph coloring is one of the new settings that is emerging in a neutrosophic chromatic number environment. In addition to introducing the idea of bipolar triangular neutrosophic chromatic graphs (BTNCG), this work also examines and demonstrates the algebraic assumption. The proposed concept has been applied in a traffic signal system to discover a new lane to avoid traffic in peak hours.

Keywords: Chromatic Number; Neutrosophic Chromatic Number; Bipolar Triangular Neutrosophic Chromatic Number; Neutrosophic Graph Coloring.

1. Introduction

Euler first put out the idea of graph theory in 1736. The theory of graphs is a very useful tool for dealing with combinatorial problems in many fields, including geometry, algebra, number theory, topology, operations research, optimization, and computer science. Fuzzy graph theory, a derivation of Eulers graph theory, was created for the first time by Rosenfeld [1] in 1975. The fuzzy set theory that Zadeh [2] initially introduced in 1965 to handle uncertainty has several practical applications. To the concept of a fuzzy set, Atanassov [3] introduced an additional component (which defines the degree of non-membership). Smarandache [4-5] introduced the notion of the neutrosophic set as a solution to issues involving unclear, shaky, and illogical data. Graph is a useful tool for simulating difficulties encountered in the actual world. In the network simulation, the items and their relationships are represented by nodes and arcs. Since they include such a wide variety of information, fuzzy graphs, intuitionistic fuzzy graphs, and neutrosophic graph theory are only a handful of the graph types we require to replicate real-world events [6–11]. The single-valued neutrosophic set (SVNS), a subclass of the neutrosophic set, was first described by Want et al. [16]. A fuzzy set can mathematically describe a group of objects having hazy boundaries [2]. Fuzzy models are beginning to be useful because they aim to reduce the differences between the traditional numerical models used in engineering and research and the symbolic models used in expert systems. Extensions of fuzzy sets whose membership degree ranges [-1,1] are known as bipolar fuzzy sets. In contrast, the membership degree [-1,0) implies that the element fulfils the implicit counter property. The membership degree [0,1] shows that the item meets a specific characteristic. Positive information represents what is deemed plausible, and negative information represents what is deemed impossibly unlikely. The truth is that a lot of problems with decision-making are brought on by bipolar judgements, which have two sides, one good and one bad. To help with decision-making, bipolar fuzzy sets are now often used in computer science, engineering, medicine, and other fields. The degrees of truth,

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falsity, and indeterminacy are related to the membership value in a neutrosophic set, but their combined total is not constrained. Bipolar neutrosophic sets were developed by Deli et al. [17] to apply the concepts of bipolar fuzzy sets to challenges involving decision-making.

Some networks from real-world issues, including transportation networks, traffic networks, communication networks, and others, have been modelled using traditional graph theory. One of the more challenging issues in a traffic network is traffic congestion. We should employ varied numbers of phases on various traffic situations, such as high, medium, or low traffic flows, to reduce traffic congestion. A graph's vertices can be used to depict the traffic lanes at an intersection. Traffic flows that come from two intersecting lanes should be handled in stages since they might result in hazardous situations. Two vertices in a graph coloring problem should be colored differently to represent two intersecting lanes. In other words, an edge should join the two vertices. The smallest phase needed for a traffic signal system may be calculated using a graph's chromatic number. Unknown factors, like high or medium traffic volume, unsafe driving techniques, etc., might create a dangerous situation where two lanes overlap. Bipolar fuzzy graphs were initially described by Akram [18]. On level graphs, Dudek and Talebi [19] described the functioning of bipolar fuzzy graphs. The concepts of neutrosophic graphs and neutrosophic soft graphs were introduced by Akram and Shahzadi [20]. Bipolar single valued neutrosophic graphs were discussed by Broumi et al. in their study [21]. Graph operations on single-valued neutrosophic graphs were addressed by Akram and Shahzadi [22]. We'll start by explaining the fundamental idea behind bipolar neutrosophic graphs. In Section 2 basic concepts have been given. In Section 3, bipolar triangular neutrosophic chromatic number has been defined with its desirable properties. In Section 4, proposed concept has been applied in traffic lighting system and in the section 5, the current work's conclusion is provided, and future directions are also considered.

2. Preliminaries

Basic definitions pertaining to the current work are provided in this section.

Definition 2.1. [15]

G: (V,E) is called a crisp graph where V is a set of objects and E is a subset of $V \times V$ such that this subset is symmetric.

Definition 2.2. [14]

A crisp graph G: (V, E) is called a neutrosophic graph $G:(\sigma,\mu)$ where $\sigma=(\sigma_1,\sigma_2,\sigma_3):V\to[0,1]$ and $\mu=(\mu_1,\mu_2,\mu_3):E\to[0,1]$ such that $\mu(xy)\le\sigma(x)\wedge\sigma(y)$ for all $xy\in E$.

Example 2.2. Neutrosophic Graph

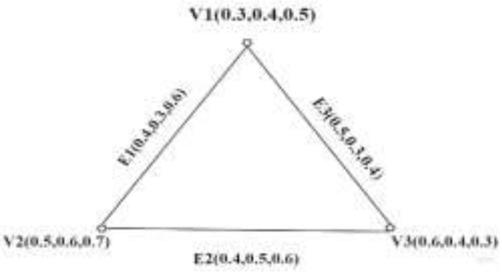

Figure 1:

Definition 2.3. [13]

A neutrosophic graph $G:(\sigma,\mu)$ is called a neutrosphic complete where it's complete and $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$.

Example 2.3. Neutrosophic Strong Graph

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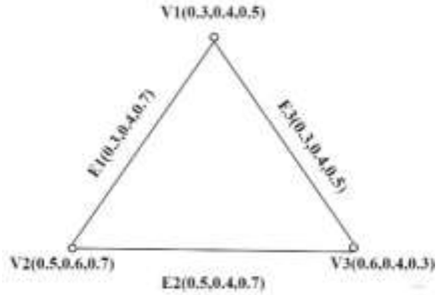

Figure 2:

Example 2.4. Neutrosophic Chromatic Graph

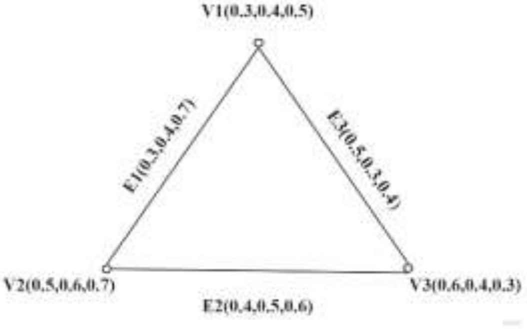

Figure 3:

The neutrosophic chromatic number is 1.4 and the chromatic number in Figure 3 is 3.

Example: 2.5.Triangular neutrosophic chromatic graph

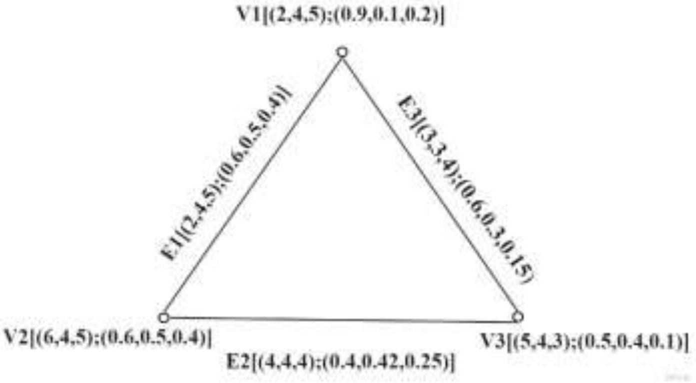

Figure 4:

In figure.4 chromatic number is 3. Triangular neutrosophic chromatic number is 2.

Definition 2.4. [24]

A bipolar neutrosophic set on a empty set 'X' is an object of the form

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$B=\left\{\left(x,Tr_B^{P+}(x),Ind_B^{P+}(x),Fal_B^{P+}(x),Tr_B^{N-}(x),Ind_B^{N-}(x),Fal_B^{N-}(x)\right):x\in X\right\}$, where $Tr_B^{P+},Ind_B^{P+},Fal_B^{P+}\to[0,1]$ and $Tr_B^{N-},Ind_B^{N-},Fal_B^{N-}\to[-1,0]$. The positive values $Tr_B^{P+},Ind_B^{P+},Fal_B^{P+}$ denote respectively the truth, indeterminacy and false membership degrees of an element $x\in X$ whereas $Tr_B^{N-},Ind_B^{N-},Fal_B^{N-}$ symbolizing the implicit counter feature of the truth, indeterminacy and false membership degrees of the element $x\in X$ corresponding to the bipolar neutrosophic set B.

Definition 2.5 [24]

A bipolar single-valued neutrosophic graph on a non-empty set 'B' is a pair $G=(E,F)$, where 'E' is a bipolar single-valued neutrosophic set on 'X' and 'F' is a bipolar single-valued neutrosophic relation in X such that

$$Tr_F^{P+}(xy)\le Tr_E^{P+}(x)\wedge Tr_E^{P+}(y),\quad Ind_F^{P+}(xy)\le Ind_E^{P+}(x)\wedge Ind_E^{P+}(y),\quad Fal_F^{P+}(xy)\le Fal_E^{P+}(x)\wedge Fal_E^{P+}(y),$$
$$Tr_F^{N-}(xy)\ge Tr_E^{N-}(x)\vee Tr_E^{N-}(y),\quad Ind_F^{N-}(xy)\ge Ind_E^{N-}(x)\vee Ind_E^{N-}(y)\quad Fal_F^{N-}(xy)\ge Fal_E^{N-}(x)\vee Fal_E^{N-}(y)$$

for all $x\in X$.

Example 2.5.

We consider an example of a bipolar neutrosophic graph such that $X=(x_1,x_2,x_3)$. Let 'E' be a bipolar neutrosophic set on 'X' and 'F' be a bipolar neutrosophic relation in X' given in Table 1. Routine calculation shown that $G=(E,F)$ is a bipolar neutrosophic graph. The bipolar neutosophic graph 'G' is shown in Figure 1.

Table 1:

	x_1	x_2	x_3	x_1x_2	x_2x_3	x_3x_1
	Bipolar Neutrosophic Set on X			Bipolar Neutrosophic Relation on X		
Tr_E^{P+}	0.5	0.3	0.6	0.3	0.6	0.5
Ind_E^{P+}	0.3	0.5	0.4	0.4	0.5	0.4
Fal_E^{P+}	0.2	0.3	0.2	0.4	0.4	0.3
Tr_E^{N-}	-0.3	-0.4	-0.5	-0.4	-0.4	-0.5
Ind_E^{N-}	-0.5	-0.2	-0.3	-0.3	-0.5	-0.4
Fal_E^{N-}	-0.1	-0.6	-0.1	-0.5	-0.5	-0.2

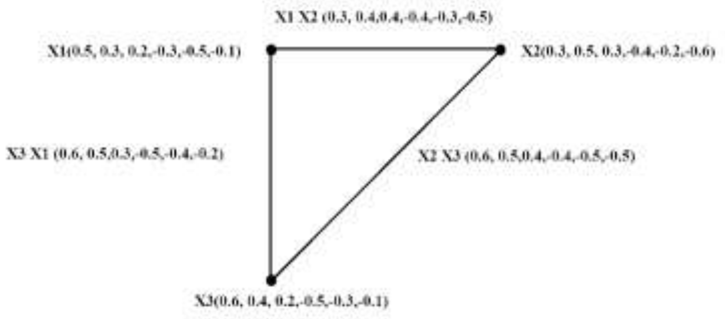

Figure 5:

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Definition 2.14.[23]

Let $A=\left\langle (a_1,a_2,a_3);Tr,Ind,Fal \right\rangle$ be a Single valued triangular neutrosophic number. Then, the score function S(A) of the same is defined as follows: $S(A)=\frac{1}{9}\left[a_1+a_2+a_3\right]\times\left[2+Tr-Ind-Fal\right]$.

3. Proposed definition of Bipolar Triangular Neutrosophic chromatic graph and chromatic number [BTNCG-CN]

The definition of BTNCG-CN has been proposed in this section.

Definition 3.1.

Let $G=(C,D)$ be a bipolar neutrosophic graph. Chromatic number is minimum number of distinct colors which are used to color the vertices which have neutrosophic strong edge. Neutrosophic cardinality of the bipolar triangular set $BT_{NCN}=\left\langle (a_1,a_2,a_3);\left(Tr_B^{P+},Ind_B^{P+},Fal_B^{P+}\rightarrow[0,1]\ \ \&\ \ Tr_B^{N-},Ind_B^{N-},Fal_B^{N-}\rightarrow[-1,0]\right)\right\rangle$ of this distinct color when it is minimum amid all these bipolar triangular sets, is called bipolar triangular neutrosophic chromatic number with respect with first order.

Example: 3.2. Bipolar Triangular neutrosophic chromatic graph

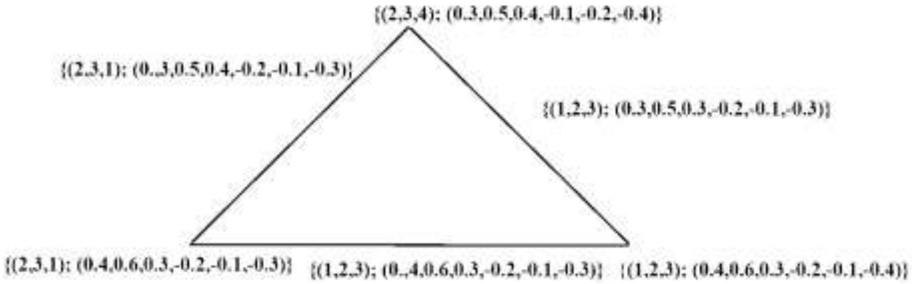

Figure 6:

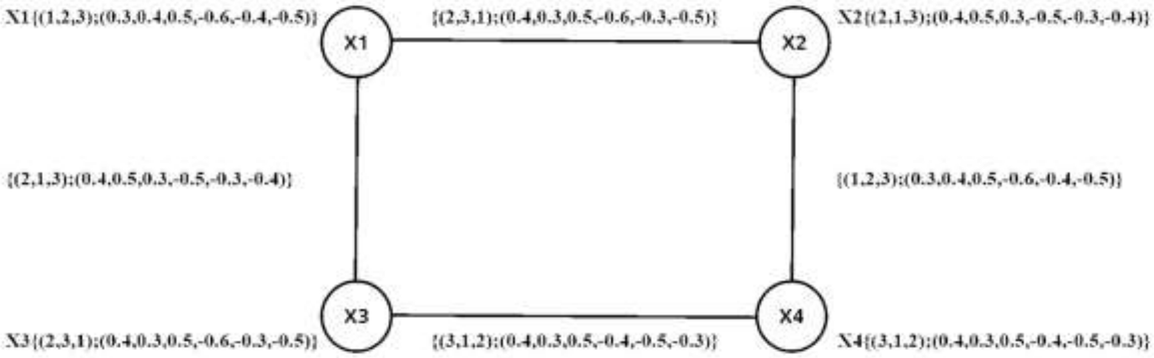

Figure 7:

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Proposition 3.3

Let $G=(C,D)$ be a bipolar neutrosophic complete. Then chromatic number is ‘n’ and bipolar triangular neutrosophic chromatic number is bipolar neutrosophic order.

Proof.

The edges are all neutrosophically robust. Each vertex contains an edge with n vertices. 'n' is therefore a neutrosophic chromatic number. Neutrosophic cardinality to 'V' is a bipolar triangular neutrosophic chromatic number because each vertex differs in color from each other. BTNCN is hence of neutrosophic order.

Proposition 3.4.

Let $G=(C,D)$ be a bipolar neutrosophic strong path. Then chromatic number is two and bipolar triangular neutrosophic chromatic number is

$\min\limits_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle \right\}$ as well as for indeterminacy and falsity membership function.

Proof.

Neutrosophic strong paths feature separate colors with alternate colors for each vertex that share an edge. Therefore, if 'x' and 'y' are two vertices that share an edge, 'x' and 'y' have different colors. The chromatic number is thus two. A vertex with the lowest value among all the other vertices that share the same color as it serves as the color's representative. Thus,

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle \right\}$$

similarly for indeterminacy and falsity membership function as follows

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle \right\}$$

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Fal^{P+}}(x), C_{Fal^{N-}}(x) \right\rangle + \left\langle C_{Fal^{P+}}(y), C_{Fal^{N-}}(y) \right\rangle \right\}$$

Proposition 3.5.

Let $G=(C,D)$ be a bipolar neutrosophic strong cycle. Then chromatic number is two and bipolar triangular neutrosophic chromatic number is

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle \right\}.$$

Proof.

The edges are all neutrosophically robust. Due to the cycle's even vertices and alternate vertice coloring, the vertices that share an edge have distinct colors. So, two is the neutrosophic chromatic number. Each color is represented by the vertex that has the lowest value among its neighboring vertices that share the same color. Thus,

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle \right\}$$

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similarly for indeterminacy and falsity membership function as follows

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle \right\}$$

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Fal^{P+}}(x), C_{Fal^{N-}}(x) \right\rangle + \left\langle C_{Fal^{P+}}(y), C_{Fal^{N-}}(y) \right\rangle \right\}$$

Proposition 3.6.

Let $G = (C, D)$ be a neutrosophic strong cycle. Then chromatic number is three and BTNCN is

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle + \left\langle C_{Tr^{P+}}(z), C_{Tr^{N-}}(z) \right\rangle \right\}$$
$$\min_{x, y \text{ and } z \text{ have different colors}} \left\{ \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle + \left\langle C_{Ind^{P+}}(z), C_{Ind^{N-}}(z) \right\rangle \right\}.$$
$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Fal^{P+}}(x), C_{Fal^{N-}}(x) \right\rangle + \left\langle C_{Fal^{P+}}(y), C_{Fal^{N-}}(y) \right\rangle + \left\langle C_{Fal^{P+}}(z), C_{Fal^{N-}}(z) \right\rangle \right\}$$

Proof.

At the end, two vertices with alternative vertex coloring share the same color and edge. Chromatic is thus three. Since there are three colors, the vertices with the lowest values in each color serve as representatives.

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle + \left\langle C_{Tr^{P+}}(z), C_{Tr^{N-}}(z) \right\rangle \right\}$$
$$\min_{x, y \text{ and } z \text{ have different colors}} \left\{ \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle + \left\langle C_{Ind^{P+}}(z), C_{Ind^{N-}}(z) \right\rangle \right\}$$
$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Fal^{P+}}(x), C_{Fal^{N-}}(x) \right\rangle + \left\langle C_{Fal^{P+}}(y), C_{Fal^{N-}}(y) \right\rangle + \left\langle C_{Fal^{P+}}(z), C_{Fal^{N-}}(z) \right\rangle \right\}$$

Proposition 3.7

Let $G = (C, D)$ be a bipolar neutrosophic strong star with 'c' as a center. Then neutrosophic chromatic number is two and bipolar triangular neutrosophic chromatic number is

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle \right\}.$$

Proof.

The edges are all neutrosophically robust. Every vertex has a shared edge, including the center vertex. As a result, it is a distinct color from the other vertices. As a result, each color has a single vertex that bears that color. There is no common edge connecting any non-center vertices together. So they are the same color. A non-center vertex with the lowest value among all non-center vertices serves as the color's exemplar. Hence,

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle \right\}$$

Similiarly for indeterminacy and falsity as well.

Proposition 3.8.

Let $G = (C, D)$ be a neutrosophic strong wheel with 'c' as a center. Then neutrosophic chromatic number is three where bipolar triangle neutrosophic cycle has even number as its length and BTNCN is

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$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle \right\}.$$

Proof.

The central vertex is characterized by a particular coloring. As a result, it solely serves as an illustration of this color. A bipolar triangular neutrosophic cycle is produced when non-center vertices with adjacent edges have two colors. As a result, the neutrosophic strong wheel has unique hues for vertices that share an edge—one color for the central vertex and two colors for the other vertices. Therefore, when the non-center vertices produce an odd cycle, the chromatic number is three. Therefore,

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle \right\}$$

$$\min_{x, y \text{ and } z \text{ have different colors}} \left\{ \left\langle C_{Ind^{P+}}(c), C_{Ind^{N-}}(c) \right\rangle + \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle \right\}$$

$$\min_{x \text{ and } y \text{ have different colors}} \left\{ \left\langle C_{Fal^{P+}}(c), C_{Fal^{N-}}(c) \right\rangle + \left\langle C_{Fal^{P+}}(x), C_{Fal^{N-}}(x) \right\rangle + \left\langle C_{Fal^{P+}}(y), C_{Fal^{N-}}(y) \right\rangle \right\}$$

Proposition 3.9.

Let $G=(C,D)$ be a bipolar triangular neutrosophic strong wheel with ‘c’ as a center. Then neutrosophic chromatic number is four where bipolar triangular neutrosophic cycle has odd number as its length and bipolar triangular neutrosophic chromatic number is

$$\min_{x, y \text{ are non-center vertices and have different colors}} \left\{ \begin{array}{l} \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \\ \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle + \left\langle C_{Tr^{P+}}(z), C_{Tr^{N-}}(z) \right\rangle \end{array} \right\}$$

$$\min_{x, y \text{ are non-center vertices and have different colors}} \left\{ \begin{array}{l} \left\langle C_{Ind^{P+}}(c), C_{Ind^{N-}}(c) \right\rangle + \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle + \\ \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle + \left\langle C_{Ind^{P+}}(z), C_{Ind^{N-}}(z) \right\rangle \end{array} \right\}$$

$$\min_{x, y \text{ are non-center vertices and have different colors}} \left\{ \begin{array}{l} \left\langle C_{Fal^{P+}}(c), C_{Fal^{N-}}(c) \right\rangle + \left\langle C_{Fal^{P+}}(x), C_{Fal^{N-}}(x) \right\rangle + \\ \left\langle C_{Fal^{P+}}(y), C_{Fal^{N-}}(y) \right\rangle + \left\langle C_{Fal^{P+}}(z), C_{Fal^{N-}}(z) \right\rangle \end{array} \right\}$$

Proof.

Non-center vertices create odd bipolar triangular neutrosophic strong cycles, and all edges are bipolar neutrosophic strong. The chromatic number of the odd bipolar triangular neutrosophic strong cycle is three. The edges of every non-center vertex are identical. Because of this, non-center vertices have various colors. The neutrosophic chromatic number is four as a result. One representation serves as the center vertex and the other three serve as non-center vertices in the bipolar triangular neutrosophic chromatic number formed by four color representatives. So,

$$\min_{x, y \text{ are non-center vertices and have different colors}} \left\{ \begin{array}{l} \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle + \\ \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle + \left\langle C_{Tr^{P+}}(z), C_{Tr^{N-}}(z) \right\rangle \end{array} \right\}$$

Similarly

$$\min_{x, y \text{ are non-center vertices and have different colors}} \left\{ \begin{array}{l} \left\langle C_{Ind^{P+}}(c), C_{Ind^{N-}}(c) \right\rangle + \left\langle C_{Ind^{P+}}(x), C_{Ind^{N-}}(x) \right\rangle \\ + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle + \left\langle C_{Ind^{P+}}(y), C_{Ind^{N-}}(y) \right\rangle \end{array} \right\}$$

$$\min_{x\,,y\text{ are non-center vertices and have different colors}} \left\{ \begin{array}{l} \left\langle C_{Tr^{P+}}(c), C_{Tr^{N-}}(c) \right\rangle + \left\langle C_{Tr^{P+}}(x), C_{Tr^{N-}}(x) \right\rangle \\ + \left\langle C_{Tr^{P+}}(y), C_{Tr^{N-}}(y) \right\rangle + \left\langle C_{Tr^{P+}}(z), C_{Tr^{N-}}(z) \right\rangle \end{array} \right\}$$

Proposition 3.10.

Let $G=(C,D)$ be a bipolar neutrosophic strong. Then neutrosophic chromatic number is 1 if and only if $G=(C,D)$ is neutrosophic empty.

Proof.

First, we consider neutrosophic Chromatic number is 1. To prove neutrosophic is empty.

Let chromatic number be 1. It implies there is no vertex which has same edge with a vertex. So, there is no neutrosophic strong edge. Since $G=(C,D)$ is neutrosophic strong, $G=(C,D)$ is a neutrosophic empty.

Reversely, Consider Neutrosophic is empty and strong. Then prove chromatic number is 1.

Let $G=(C,D)$ be neutrosophic empty and neutrosophic strong. There is therefore no edge. It suggests that there isn't a single common neutrosophic strong edge for each vertex. It implies that each vertex has a single color. So, one of the n vertices is chosen to represent this color. Neutrosophic Chromatic number is thus 1.

Proposition 3.11.

Let $G=(C,D)$ be a bipolar neutrosophic strong. Then neutrosophic chromatic number is 'n' if and only if $G=(C,D)$ is bipolar triangular neutrosophic complete.

Proof.

First, we consider Neutrosophic Chromatic number is n then to prove chromatic number is 'n'.

Let chromatic number be 'n'. So any given vertex has n vertices which have common edge with them and every of them have common edge with each other. It implies every vertex has 'n' vertices which have common edge with them. Since $G=(C,D)$ is neutrosophic complete.

Reversely, consider $G=(C,D)$ is neutrosophic complete. To prove chromatic number is 'n'.

Suppose $G=(C,D)$ is bipolar neutrosophic complete. Every vertex has 'n' vertices which have common edge with them. Since all edges are neutrosophic strong, the minimum number of colors are 'n'. Thus, chromatic number is 'n'.

Proposition 3.12.

Let $G=(C,D)$ be bipolar triangular neutrosophic graph. Then neutrosophic chromatic number is at most the number of vertices and bipolar triangular neutrosophic chromatic number is at most neutrosophic order.

Proof.

When each vertex serves as a representation of a different color, the chromatic number is equal to the number of vertices, and this occurs when the neutrosophic full chromatic number is 'n'. When each vertex has a unique color, the triangle's neutrosophic number is in neutrosophic order and is sharp for neutrosophic completion.

Proposition 3.13

Let $G=(C,D)$ be a bipolar neutrosophic r-regular. Then neutrosophic chromatic number is at most r+1.

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Proof.

$G=(C,D)$ is a bipolar neutrosophic r-regular. So any of vertex has ‘r’ vertices which have common edge with it. If these vertices have no common edge with each other, for instance neutrosophic star, chromatic number is two. But since the vertices have common edge with each other, chromatic number is r+1, for instance, neutrosophic complete.

4. Applications in Traffic signal system

In this section proposed concept has been applied in traffic signal system [25].

It is recommended that traffic flows at crossings be organized using bipolar triangular neutrosophic graphs with a bipolar triangular chromatic number. There is a restriction that only certain traffic lanes are allowed at times, and we may employ two different traffic signal configurations since certain lanes see higher traffic volumes during peak hours. The number of stages needed will depend on whether there is low, medium, or high traffic intensity at the intersection.

Let's think about how traffic moves through an intersection as depicted in Figure. Two traffic movements, from D to C and from B to D, are initially allowed at the crossroads. There are many residential areas on the road B and numerous public facilities, such as schools or workplaces, on the road A. Therefore, during peak hours (noon or afternoon), there would be more people travelling back to their homes on the route B. It will result in heavy traffic in the lanes AB. The same cause also accounts for peak-hour traffic on Lane CB. As a result, a new regulation allows cars from AB and CB to operate during peak hours.

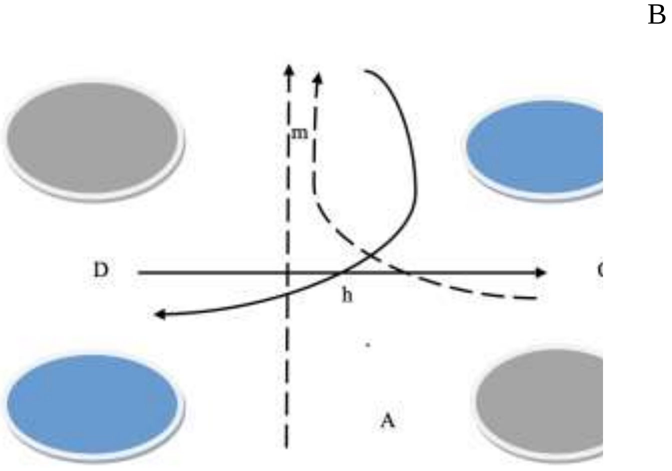

Figure 8: The flow of traffic at a crossroads

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We provide a method to describe a traffic signal system using a bipolar triangular neutrosophic graph and neutrosophic chromatic number as follows:

1. **Describing vertices and edges**

A vertex is displayed for each lane. A given level may collide with two lanes. The two linked vertices should be connected by an edge with a truth, indeterminacy, and falsity membership degree in both the positive and negative side, and two conflict lanes should be organized on distinct colors. The truth, indeterminacy, and falsity membership degrees of an edge serve as indicators for the degree of conflict between two vertices or the possibility that a car travelling in two distinct lanes will collide.

2. **Error-proofing vehicle volume data**

The vehicles are classified into compact vehicle (CV) hefty automobile (HA), and motor bike (MB). According to [23], we count volume of vehicles on each lane follow: $(1.0+CV)+(1.3*HA)+(0.2*MB)$. For example, if we have data of volume of vehicles as presented in Table 2. And the volume of vehicles lies in a range [0,d], then we divide the range [0,d] into 3 sub intervals that represents compact vehicle with degree ρ_{CV}, hefty automobile with the degree of ρ_{MC} or high volume with the degree ρ_{HA}. The information can be collected by keeping a close watch on the number of vehicles during quiet, medium, or peak periods. Figure 4 illustrates the method of fuzzily classifying the data of volumes on Table IV into low, medium, and large volumes.

3. **Representing bipolar neutrosophic model of a traffic light system**

In order to obtain a bipolar triangular neutrosophic graph $G=(C,D)$, we need to determine membership, indeterminacy, non-membership degree of each edge by using a rule as follows: if the lanes XY and UV intersect each other, then the degree of conflict $\mu(XY,UV)$ is equal to the maximum between the degree of volume on lane XY (μ_{XY}) and the degree of volume on lane UV (μ_{UV}). Another rule to determine membership degree of each edge is as follows: if the lanes XY and UV do not intersect each other, then the degree of conflict $\mu(XY,UV)$ is equal to the minimum between the degree $\{XY,UV\}$ is equal to the minimum between the degree $(\rho_{XY,}\rho_{UV})$.

Let us consider Table 4, volume of vehicle on lane DC classified as high volume with degree $\rho_{DC}=1$ and the volume on lane BD is also high with degree $\rho_{BD}=1$. Since the lanes DC and BD cross each other, there is an edge (DC,BD) with the degree on conflict of the $\mu(DC,BD)=\max(\rho_{DC},\rho_{BD})=1$. Another illustration, volume of vehicle on lane AB is classified as medium volume with degree $\rho_{AB}=0.6$ and volume on lane CB is a high volume with degree $\rho_{CB}=1$. Since the lanes DC and BD do not intersect each other, there is an edge (AB, CD) with the degree of conflict is $\mu(AB,CB)=\min(\rho_{AB},\rho_{CB})=0.6$.

The rule mentioned in 3) are satisfied in crisp graph case i.e., if two lanes XY and UV do not cross each other, then there is no edge connected the two lanes. In other words, $\mu(XY,UV)=0=\min(\rho_{XY},\rho_{UV})$. Otherwise, two lanes XY and UV cross each other, then there is an edge connected the two lanes and $\mu(XY,UV)=1=\max(\rho_{XY},\rho_{UV})$.

Figure 6 (on the left side) depicts the edge (DC, BD) with the degree of high, and Figure 7 (also on the left side) depicts the edge (CB, AB) with the degree of medium. The traffic flows in Fig. may be represented as a bipolar triangular neutrosophic graph in Fig. (on the right side), which is a different color neutrosophic graphs since there is a new rule that allows vehicles from AB and CB at peak time.

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Table 2:

No.	**The number of automobiles**				
	Lanes	Volume	Level	$\rho(x)$ (In the form of Bipolar triangular neutrosophic number)	$\rho(x)$ Using [2.14]
1	AB	948	M	{(1,2,3);(0.4,0.3,0.2,-0.3,-0.2,-0.1)}	0.6
2	BD	2223	H	{(2,3,4);(0.5,0.5,0.3,-0.1,-0.2,-0.3)}	1
3	CB	2214	H	{(2,3,4);(0.6,0.5,0.4,-0.2,-0.3,-0.4)}	1
4	DC	2226	H	{(1,2,4);(0.5,0.5,0.4,-0.1,-0.2,-0.2)}	1

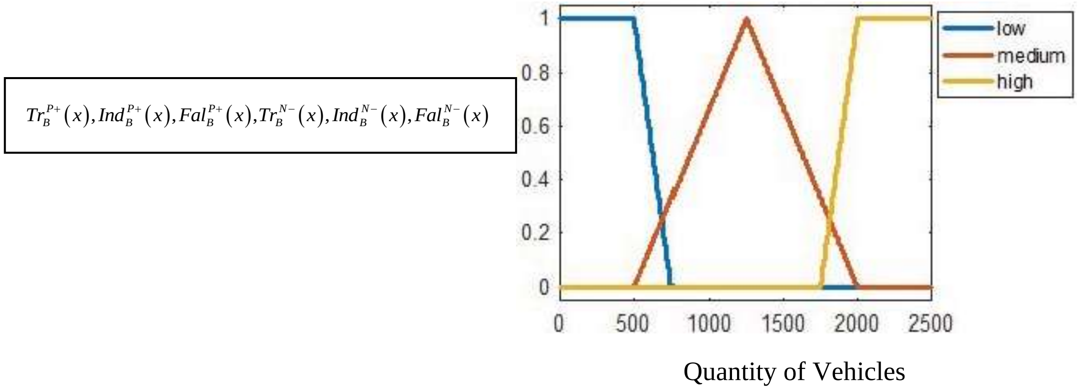

Figure 9: the number of automobiles

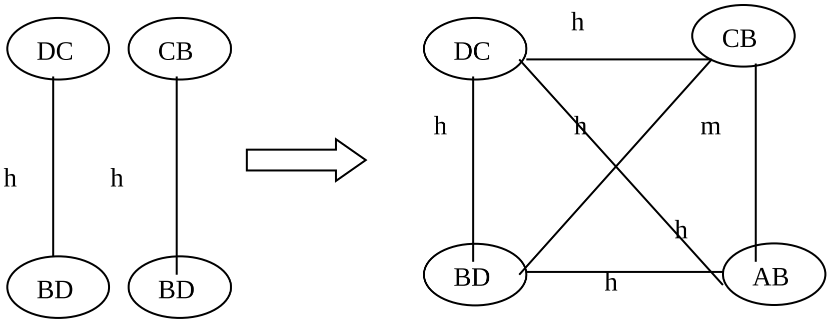

Figure10: illustrates a bipolar triangular neutrosophic chromatic graph model of traffic flows.

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4. **4.** Determining neutrosophic chromatic number of triangular bipolar neutrosophic graphs. We obtain neutrosophic chromatic number of in figure 6 as follows.

$$\chi = \left\{ \left(\left(a_1, a_2, a_3 \right), Tr_\chi^{P+}, Ind_\chi^{P+}, Fal_\chi^{P+} \rightarrow [0,1] \quad \& \quad Tr_\chi^{N-}, Ind_\chi^{N-}, Fal_\chi^{N-} \rightarrow [-1,0] \right) \right\}$$

$$\chi = \begin{cases} (1,2,3);(0.5,0.4,0.3,-0.2,-0.3,0.1),(2,3,4);(0.6,0.5,0.4,-0.2,-0.3,-0.1), \\ (2,3,4),(0.4,0.3,0.2,-0.3,-0.2,-0.1) \end{cases}$$

Using the definition 2.14 converting the above equation into the single valued function as follows:

$$\chi = \{(1,0),(2,0),(3,0.4),(4,1)\}$$

5. Determining optimum number of phases.

The number of phases needed is the number ‘k’ in the neutrosophic chromatic number $\left\{ \left(\left(a_1, a_2, a_3 \right), Tr_\chi^{P+}, Ind_\chi^{P+}, Fal_\chi^{P+} \rightarrow [0,1] \quad \& \quad Tr_\chi^{N-}, Ind_\chi^{N-}, Fal_\chi^{N-} \rightarrow [-1,0] \right) \right\}$ and $\left\langle Tr_\chi^{P+}, Ind_\chi^{P+}, Fal_\chi^{P+} \rightarrow [0,1] \quad \& \quad Tr_\chi^{N-}, Ind_\chi^{N-}, Fal_\chi^{N-} \rightarrow [-1,0] \right\rangle$ represents a possibility that condition is safe (there are no accidents). The value $\left\langle Tr_\chi^{P+}, Ind_\chi^{P+}, Fal_\chi^{P+} \rightarrow [0,1] \quad \& \quad Tr_\chi^{N-}, Ind_\chi^{N-}, Fal_\chi^{N-} \rightarrow [-1,0] \right\rangle$ is a degree of safety on k-phase.

1) The phase k=1 means that the condition is not safe, and it is indicated by the possibility is

$$\left\langle Tr_\chi^{P+}, Ind_\chi^{P+}, Fal_\chi^{P+} \rightarrow [0,1] \quad \& \quad Tr_\chi^{N-}, Ind_\chi^{N-}, Fal_\chi^{N-} \rightarrow [-1,0] \right\rangle = 0$$

2) The traffic flows may be set up as follows if we utilize k=3 phases:

Traffic from DC receives the green light on the first phase (while traffic from the other lanes receives the red light), followed by traffic from BD on the second phase and CB and AB on the third phase. This phase can be employed when traffic intensity is medium, and the likelihood of a safe situation is 0.4.

3) Like other phases, k=4 phases can be interpreted. If we employ k=4 stages, there is a 1 percent chance of a secure situation. If there is a lot of traffic, this phase may be employed.

5. Conclusion

One of the new settings that is emerging in a neutrosophic chromatic number environment in graph coloring. Such a important graph coloring about using bipolar triangular neutrosophic chromatic number is presented in this paper with its desirable proposition. The proposed concept has been applied in the traffic signal system to prove the effectiveness of the proposed concept. With various neutrosophic number types, we can expand the idea of chromatic number in the future.

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