The Algebraic Structures of Q-Complex Neutrosophic Soft Sets Associated with Groups and Subgroups

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Abstract

Groups and subgroups are rich algebraic structures, and both of them depend on binary operations in their work. The discussion of this paper is organized into two parts. In the first part, we define the notion of Q-complex neutrosophic soft sets (Q-CNSSs) by amalgamating two previous models of Q-complex neutrosophic set (Q-CNS) and soft set (SS) to address the issues of two-dimensionality (two variables) in a universal set under a parametric environment. Subsequently, the relation between Q-CNSSs and Q- neutrosophic soft sets (Q-NSSs) is verified. A basic set theory for this hybrid model is developed. In particular, null Q-CNSS and absolute Q-CNSS are defined. The basic operators of the complement, subset, equality, union and intersection are advanced and their properties are examined. Further, the notions of the homogeneous and completely homogeneous Q-CNSSs are proposed along with some illustrated examples. In part two, we move to study some algebraic structures of this model when we define the notions of Q-complex neutrosophic soft groups (Q-CNSG) and Q-complex neutrosophic soft subgroups (Q-CNSSG). Then, the relation between Q-CNSG and Q-neutrosophic soft group (Q-NSG) is scrutinized. Moreover, the algebraic properties of the Q-CNSG and Q-CNSSG are discussed and verified. Finally, some theories that show the relationship between the Q-CNSG and the soft group are proposed.

Kewords: Complex neutrosophic set: Soft set; Soft group; Q-neutrosophic set; Q- complex neutrosophic set

1 introduction

Zadeh[1] elaborated Cantor's binary set theory to a gradual model by presenting degrees of belonging and relationship. In an instant, this extension was applied to almost all the domains of modern mathematics.

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Consequently, some new disciplines such as fuzzy geometry, fuzzy databases, fuzzy algebraic structures, fuzzy arithmetic, fuzzy differential calculus, fuzzy topology, fuzzy relational calculus, and fuzzy decision making were appeared. Shenglin,[2] defined a more general kind of structure called an L-relation, which he studied in an abstract algebraic context. Fuzzy relations,[3] which are now used throughout fuzzy mathematics and have applications in areas such as linguistics, decision-making and clustering are special cases of L-relations when L is the unit interval [0, 1]. Later, in 1986, Atanassov[4] initiated the idea of an intuitionistic fuzzy set (IFS) by adding a false membership function to FS's membership function. FS and IFS are very solid sets to be used as prototypes for solving decision problems containing uncertainties, but in some situations, these ideas are not adequate to overcome indeterminate and inconsistent experiences in real-world problems. To fill this gap, the idea of a neutrosophic set (NS) popped up as an extension of FS and IFS by Smarandache[5,6] when he added a third membership function named indeterminate membership to both FS and IFS membership functions. On the other hand, the complex set of numbers has provided solutions to many problems that the set of real numbers cannot solve, such as evaluating the incorrect integrals that represent electrical resistance in the field of engineering. Complex numbers cover information data for some real-life problems, specifically those problems in which the time phase plays an important role in describing their data. Based on this, many researchers have developed new ideas that combine the benefits of complex numbers with the concepts of FSs and their generalizations. First of all, Ramot et al.[7] invented a complex fuzzy set (CFS), which is an amended and extended performance of ordinary FSs. Alkouri and Salleh[8] characterized a complex intuitionistic fuzzy set (CIFS) as an extension of CFS. Complex neutrosophic sets (CNSs) were developed by Ali et al.[9] as an extension of CFS and CIFS. Al-Quran et al.[10] introduced Q-complex neutrosophic sets (Q-CNSs) and defined their basic theoretical operations.
In 1999, Molodtsov[11] pointed out that there is a shortage of these concepts, as he indicated that these concepts do not have the ability to deal with uncertain data in a parameterized way. To handle this shortage, he suggested a soft set (SS) to address uncertainty in a parameterized form. The SS attracted the attention of researchers and kept them motivated to introduce more contributions; for instance, Maji et al.[12] presented the idea of fuzzy soft set (FSS) by incorporating SS and FS. Cagman and Karataş[13] constructed a generalized algorithm of decision-making based on intuitionistic fuzzy soft set (IFSS). Deli and Broumi[14] defined a relation between neutrosophic soft sets (NSSs) that allows the composition of two NSSs. Deli[15] presented the idea of NSS in interval form. Abu Qamar and Hassan[16] established a new approach called Q-neutrosophic soft sets (Q-NSSs) and tested it in dealing with decision-making problems. Consequently, many studies have been conducted on SS with complex numbers. Kumar and Bajaj[17] investigated complex intuitionistic fuzzy soft sets (CIFSSs) with some measures. Selvachandran and Singh[18] worked on addressing uncertainty in data by employing the effects of interval complex fuzzy soft sets (ICFSSs). Broumi et al.[19] presented the idea of a complex neutrosophic soft set (CNSS) by smelting both SS and NS under a complex number setting. Al-Sharqi et al.[?] worked to develop the CNSS under an interval form by proposing the interval complex neutrosophic soft set (ICNSS), further they employed this structure with some mathematical ideas like relations,[?] fuzzy parameterized,[22] similarity measures,[?] expert systems.[?] In addition to other contributions in various fields such as economics, medicine, engineering, programming, computer science, etc., see[25]-.[33]

Moreover, the idea of combining FS with algebraic structures emerged by Rosenfeld[34] when he combined the idea of a FS in group theory and defined the concept of a fuzzy subgroup. Based on this idea, a lot of research have been done by applying algebraic properties to fuzzy theory and showing how it can be used in both real and complex spaces. Smarandache and Ali[35] introduced the neutrosophic triplet group and its interesting properties. Yin et al.[36] discussed the operation properties and algebraic structures of IFSSs. Borzooei et al.[37] applied the notion of IFSS to hyper BCK algebras. As a generalization of refined neutrosophic ring, Smarandache and Abobala[38] introduced the notion of n-refined neutrosophic ring. Abed et al[?]-[?] investigated some new results of the neutrosophic multiplication module. Abu Qamar et al.[42]-[43]summarized the research progress of Q-NSS hyperstructures in algebraic structures. Al-Sharqi et al.[44] followed suit after presenting these results in a matrix framework. Consequently, the relation between group theory and CFS was demonstrated by Al-Husban and Salleh[45] when they established the notion of complex fuzzy algebra. Alsarahead and Ahmed[46] discussed the connection between CFS and subgroups. Quek et al.[47] developed the algebraic structures pertaining to groups and subgroups for the CIFSS environment. Khamis and Ahmad[48] studied complex fuzzy fields and complex fuzzy subfields when they established complex intuitionistic Q-fuzzy subfields. Gulistana et al.[49] worked on complex neutrosophic groups, which depend on the notion of complex neutrosophic space from the genuine range of three CNS membership functions [0, 1]. Gulzar et al.[50,51] discussed the notions of Q-complex fuzzy subrings, Q-complex fuzzy subrings, and complex fuzzy subfields and verified some of their properties.

This paper extends the literature in two ways. First, we extend fuzzy and soft sets theories by introducing the

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notion of Q-CNSS as a generalization of NS, CNS, CNSS, Q-CNS and Q-NSS. In reality, many phenomena and events happened periodically and NS cannot address these situations. Therefore, CNS is developed which is characterized by three complex-valued membership functions that handle information with uncertainty, incompleteness, indeterminacy and periodicity simultaneously. However, CNS lacks the adequate parameterization tool to facilitate the representation of parameters which decreases the validity of this model. Thus, the CNSS is proposed to provide a more adequate parameterization tool that can represent the problem parameters in a more comprehensive and complete manner. Both of Q-CNS and Q-NSS are generalizations of CNS and NSS by adding extra information provided by the elements of the Q-set. In depth, the rationales of introducing the Q-CNS and Q-NSS is considered as a potent motivation to the introduction of the concept of Q-CNSS. Second, we extend the fuzzy algebraic theory by introducing the concept of Q-CNSG as a generalization of Q-neutrosophic soft group (Q-NSG). Actually, Q-NSG is extended to the complex space due to the importance of the phase term in this field of study. In what follows, the phase term can be employed to accurately represent the cycles present in fuzzy algebraic structures. In the study of Q-CNSG the algebraic structures consist of amplitude and phase terms. The amplitude terms play the role of the membership functions of the Q-NSG while the phase terms can be used to represents the cycles of the algebraic structures. For example, when dealing with Q-neutrosophic alternating groups, different cycles can be represented properly and accurately using the phase term if the Q-neutrosophic alternating groups are defined in terms of Q-CNSG. This would make it easier to identify different cycles and their corresponding membership functions in a systematic manner. The desire to utilize this unique ability of the phase term present in the Q-CNSG model in the study of fuzzy algebra served as the main motivation to introduce and develop the theory of Q-CNSG in this paper. Motivated by the above discussion, the contribution of this study is as follows:

1. We create a new hybrid mathematical model called Q-complex neutrosophic soft sets (Q-CNSSs), by smelting both Q-CNS and SS in one model.
2. We discuss the characterizations of the Q-complex neutrosophic soft subgroup (Q-CNSSG) and Q-complex neutrosophic soft group (Q-CNSSG).
3. We employed these new concepts (Q-CNSS, Q-CNSSG and Q-CNSG) to develop and demonstrate many theorems and examples, which will be a new addition to knowledge in abstract algebra.

This article is spitted into the following parts. In Part 2, we provide several significant backgrounds and preliminary results regarding the concepts introduced in this work. In Part 3, we derived the notion of Q-CNSS with its basic set-theoretical. In Part 4, the ideas of Q-CNSSG and Q-CNSG are proposed, and the necessary properties of these structures are discussed and verified. The last Part presents concluding remarks and possible future works.

2 Preliminaries

Definition 2.1. [16] If U and Q are two nonempty sets. A Q-NS Υ in U is an object of the form.

$$\Upsilon = \{< (u,q); P_\Upsilon(u,q), R_\Upsilon(u,q), S_\Upsilon(u,q) >: u \in U, q \in Q\},$$

where, $P_\Upsilon, R_\Upsilon, S_\Upsilon : U \times Q \longrightarrow [0,1]$ are the truth, indeterminacy and false membership functions, respectively, and $0 \leq P_\Upsilon(u,q) + R_\Upsilon(u,q) + S_\Upsilon(u,q) \leq 3$.

Definition 2.2. [16] Let U and Q be two non empty sets and $A \subseteq E$ be a set of parameters. A pair (Λ, A) is said to be a Q-NSS in U, where $\Lambda : A \longrightarrow QNS(U)$ is a mapping, such that $QNS(U)$ is the power Q-NSs and $\Lambda(a) = \varphi$ if $a \notin A$.

Definition 2.3. [43] If (Λ, A) is a Q-NSS in U. Then (Λ, A) is a Q-NSG in G if $\forall s, t \in G, q \in Q$ and $a \in A$ it satisfies:

(1) $P_{\Lambda(a)}(st,q) \geq \min\{P_{\Lambda(a)}(s,q), P_{\Lambda(a)}(t,q)\}$, $R_{\Lambda(a)}(st,q) \leq \max\{R_{\Lambda(a)}(s,q), R_{\Lambda(a)}(t,q)\}$, $S_{\Lambda(a)}(st,q) \leq \max\{S_{\Lambda(a)}(s,q), S_{\Lambda(a)}(t,q)\}$.

(2) $P_{\Lambda(a)}(s^{-1},q) \geq P_{\Lambda(a)}(s,q),\ R_{\Lambda(a)}(s^{-1},q) \leq R_{\Lambda(a)}(s,q),\ S_{\Lambda(a)}(s^{-1},q) \leq S_{\Lambda(a)}(s,q)$.

Alsarahead and Ahmad[46] defined the concept of complex fuzzy soft group as follows.

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Definition 2.4. Let G be a group and (H, E) be a homogeneous complex fuzzy soft set over G. Then (H, E) is said to be a complex fuzzy soft group over G if and only if the following hold:

(1) $\mu_{H(e)}(uv) \geq \min\{\mu_{H(e)}(u), \mu_{H(e)}(v)\}, \forall e \in E$ and $u, v \in G$,

(2) $\mu_{H(e)}(u^{-1}) \geq \mu_{H(e)}(u), \forall e \in E$ and $u \in G$.

Quek et al.[47] defined the notion of a complex intuitionistic fuzzy subgroup of group G and used it to define the notion of a complex intuitionistic fuzzy soft group of a group G as follows.

Definition 2.5. Let $N = \{(u, \mu_N(u), \nu_N(u)) : u \in G\}$ be a complex intuitionistic fuzzy set on G. Then N is said to be a complex intuitionistic fuzzy subgroup of G if the following conditions hold for all $u, v \in G$.

(1) $\mu_N(uv) \geq \min\{\mu_N(u), \mu_N(v)\}$,

(2) $\nu_N(uv) \leq \max\{\nu_N(u), \nu_N(v)\}$,

(3) $\mu_N(u^{-1}) \geq \mu_N(u)$,

(4) $\nu_N(u^{-1}) \leq \nu_N(u)$.

Moreover, let M and N be two complex intuitionistic fuzzy subgroups of G with $M \subseteq N$. In this case, M is said to be a complex intuitionistic fuzzy subgroup of N.

Definition 2.6. [47] Let (G, A) be a complex intuitionistic fuzzy soft set on G. Then (G, A) is said to be complex intuitionistic fuzzy soft group on G if $G(a)$ is a complex intuitionistic fuzzy subgroup of G for all $a \in A$.

Ali and Smarandache[9] conceptualized CNS as follows.

Definition 2.7. [18] Let a universe of discourse X, a CNS S in X is characterized by a T-MF $T_S(x)$, an I-MF $I_S(x)$, and a F-MF $F_S(x)$ that assign an element $x \in X$ a complex-valued grade of $T_S(x)$, $I_S(x)$, and $F_S(x)$ in S . By definition, the values $T_S(x)$, $I_S(x)$, $F_S(x)$ and their sum may all within the unit circle in the complex plane and are of the form, $T_S(x) = p_S(x).e^{j\mu_S(x)}$, $I_S(x) = q_S(x).e^{j\nu_S(x)}$ and $F_S(x) = r_S(x).e^{j\omega_S(x)}$, each of $p_S(x), q_S(x), r_S(x)$ and $\mu_S(x), \nu_S(x), \omega_S(x)$ are, respectively, real valued and $p_S(x), q_S(x), r_S(x) \in [0, 1]$ such that $0^- \leq P_S(x) + q_S(x) + r_S(x) \leq 3^+$.

Definition 2.8. [10] Let U and Q be two non empty sets. A Q-complex neutrosophic set (Q-CNS) M in U is defined as follows.

$$M = \{< (u, q); T_M(u, q), I_M(u, q), F_M(u, q) >: u \in U, q \in Q\},$$

where $T_M(u, q), I_M(u, q)$ and $F_M(u, q)$ are complex-valued truth, indeterminate and false membership functions of the form $T_M(u, q) = P_M(u, q)e^{j2\pi\mu_\Lambda(u,q)}$, $I_M(u, q) = R_M(u, q)e^{j2\pi\nu_M(u,q)}$, $F_M(u, q) = S_M(u, q)e^{j2\pi\omega_M(u,q)}$, each of $P_M(u, q), R_M(u, q), S_M(u, q)$ and $\mu_M(u, q), \nu_M(u, q), \omega_M(u, q)$ are, respectively, real valued and $P_M, R_M, S_M, \mu_M, \nu_M, \omega_M : U \times Q \longrightarrow [0, 1]$, such that $0 \leq P_M(u, q) + R_M(u, q) + S_M(u, q) \leq 3$ and $0 \leq \mu_M(u, q) + \nu_M(u, q) + \omega_M(u, q) \leq 3$. The set of all Q-complex neutrosophic sets in U is denoted by Q-CNS(U).

Definition 2.9. [10]

Let $M = \{< (u, q); T_M(u, q), I_M(u, q), F_M(u, q) >: u \in U, q \in Q\}$ be a Q-CNS in a non empty set U. Then, the complement of a Q-CNS M is denoted as $c(M)$ and is defined by

$c(M) = \{< (u, q); T^c_M(u, q), I^c_M(u, q), F^c_M(u, q) >: u \in U, q \in Q\}$, where,

$$T^c_M(u, q) = c(P_M(u, q)e^{j2\pi{\mu'}_M(u,q)}) = P^c_M(u, q)e^{j2\pi{\mu'}^c_M(u,q)} = S_M(u, q)e^{j2\pi(1-{\mu'}_M(u,q))},$$

$$I^c_M(u, q) = c(R_M(u, q)e^{j2\pi{\nu'}_M(u,q)}) = R^c_M(u, q)e^{j2\pi{\nu'}^c_M(u,q)} = (1 - R_M(u, q))e^{j2\pi(1-{\nu'}_M(u,q))},$$

$$F^c_M(u, q) = c(S_M(u, q)e^{j2\pi{\omega'}_M(u,q)}) = S^c_M(u, q)e^{j2\pi{\omega'}^c_M(u,q)} = P_M(u, q)e^{j2\pi(1-{\omega'}_M(u,q))}.$$

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Definition 2.10. [10] A Q-CNS A is contained in another Q-CNS B i.e. $A \subseteq B$, if and only if, $\forall u \in U$ and $q \in Q$, the following conditions are satisfied:

1. $T_A(u,q) \leq T_B(u,q)$ such that $P_A(u,q) \leq P_B(u,q)$ and $\mu_A(u,q) \leq \mu_B(u,q)$,
2. $I_A(u,q) \geq I_B(u,q)$ such that $R_A(u,q) \geq R_B(u,q)$ and $\nu_A(u,q) \geq \nu_B(u,q)$,
3. $F_A(u,q) \geq F_B(u,q)$ such that $S_A(u,q) \geq S_B(u,q)$ and $\omega_A(u,q) \geq \omega_B(u,q)$.

3 Q-Complex Neutrosophic Soft Set

This section presents the development of Q-CNSS. This generalization is a combination of Q-CNS and SS. This section is developed to achieve the first objective of this research. We start with the introduction of a formal definition of Q-CNSS.

Definition 3.1. Assume that U and Q be two non empty sets and Υ be a set of parameters. A Q-complex neutrosophic soft set (Q-CNSS) (Λ, Υ) in U is defined as follows.

$$(\Lambda_Q, \Upsilon) = \{< \varepsilon; T_{\Lambda_Q(\varepsilon)}(u,q), I_{\Lambda_Q(\varepsilon)}(u,q), F_{\Lambda_Q(\varepsilon)}(u,q) >: \varepsilon \in \Upsilon, u \in U, q \in Q\},$$

where $\forall \varepsilon \in \Upsilon, u \in U, q \in Q$, $T_{\Lambda_Q(\varepsilon)}(u,q) = P_{\Lambda_Q(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda_Q(\varepsilon)}(u,q)}$, $I_{\Lambda_Q(\varepsilon)}(u,q) = R_{\Lambda_Q(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda_Q(\varepsilon)}(u,q)}$ and $F_{\Lambda_Q(\varepsilon)}(u,q) = S_{\Lambda_Q(\varepsilon)}(u,q)e^{j2\pi\omega_{\Lambda_Q(\varepsilon)}(u,q)}$ are, respectively, the complex-valued truth, indeterminate and false membership functions.

This part illustrates the relation between the Q-CNSS and Q-NSS.

Remark 3.2. Let U and Q be two non empty sets and (Λ, Υ) be a Q-CNSS in U such that $(\Lambda, \Upsilon) = \{< \varepsilon; T_{\Lambda(\varepsilon)}(u,q), I_{\Lambda(\varepsilon)}(u,q), F_{\Lambda(\varepsilon)}(u,q) >: \varepsilon \in \Upsilon, u \in U, q \in Q\}$, where $T_{\Lambda(\varepsilon)}(u,q) = P_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}$, $I_{\Lambda(\varepsilon)}(u,q) = R_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$ and $F_{\Lambda(\varepsilon)}(u,q) = S_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\omega_{\Lambda(\varepsilon)}(u,q)}$, representing the complex valued truth membership function, complex valued indeterminacy membership function, and complex valued falsity membership function, respectively, $\forall \varepsilon \in \Upsilon, u \in U, q \in Q$. Then (Λ, Υ) yields two real Q-NSSs in U as follows:

(1) The Q-NSS $(H, \Upsilon) = \{< \varepsilon; P_{\Lambda(\varepsilon)}(u,q), R_{\Lambda(\varepsilon)}(u,q), S_{\Lambda(\varepsilon)}(u,q) >: \varepsilon \in \Upsilon, u \in U, q \in Q\}$, where $\forall \varepsilon \in \Upsilon, u \in U, q \in Q, P_{\Lambda(\varepsilon)}(u,q), R_{\Lambda(\varepsilon)}(u,q)$, and $S_{\Lambda(\varepsilon)}(u,q)$, respectively, represent the amplitude terms of the complex valued membership functions $T_{\Lambda(\varepsilon)}(u,q), I_{\Lambda(\varepsilon)}(u,q)$ and $F_{\Lambda(\varepsilon)}(u,q)$.

(2) The Q-NSS $(K, \Upsilon) = \{< \varepsilon; \mu_{\Lambda(\varepsilon)}(u,q), \nu_{\Lambda(\varepsilon)}(u,q), \omega_{\Lambda(\varepsilon)}(u,q) >: \varepsilon \in \Upsilon, u \in U, q \in Q\}$, where $\forall \varepsilon \in \Upsilon, u \in U, q \in Q, \mu_{\Lambda(\varepsilon)}(u,q), \nu_{\Lambda(\varepsilon)}(u,q)$, and $\omega_{\Lambda(\varepsilon)}(u,q)$, respectively, represent the phase terms of the complex valued membership functions $T_{\Lambda(\varepsilon)}(u,q), I_{\Lambda(\varepsilon)}(u,q)$ and $F_{\Lambda(\varepsilon)}(u,q)$.

Definition 3.3. For a Q-CNSS (Λ, Υ) in U. (Λ, Υ) is said to be a null Q-CNSS denoted by $(\Lambda, \Upsilon)_\Phi$, if $T_{\Lambda(\varepsilon)}(u,q) = 0, I_{\Lambda(\varepsilon)}(u,q) = 1$ and $F_{\Lambda(\varepsilon)}(u,q) = 1, \forall \varepsilon \in \Upsilon, u \in U$, and $q \in Q$.

Definition 3.4. For a Q-CNSS (Λ, Υ) in U. (Λ, Υ) is said to be an absolute Q-CNSS denoted by $(\Lambda, \Upsilon)_\Psi$, if $T_{\Lambda(\varepsilon)}(u,q) = 1, I_{\Lambda(\varepsilon)}(u,q) = 0$ and $F_{\Lambda(\varepsilon)}(u,q) = 0, \forall \varepsilon \in \Upsilon, u \in U$, and $q \in Q$.

Definition 3.5. Let U and Q be two non empty sets, (Λ, Υ) and (Γ, Ψ) be two Q-CNSSs in U. (Λ, Υ) is said to be Q-complex neutrosophic soft subset of (Γ, Ψ) if

1. $\Upsilon \subset \Psi$,
2. $\forall \varepsilon \in \Upsilon, \Lambda(\varepsilon)$ is a Q-complex neutrosophic subset of $\Gamma(\varepsilon)$.

Definition 3.6. Let (Λ_Q, Υ) be a Q-CNSS in U. Then the complement of (Λ, Υ) is denoted by $c(\Lambda, \Upsilon)$ and is defined as:
$c\left(\Lambda_Q, \Upsilon\right) = \left\{\left\langle \varepsilon; cT_{\Lambda_Q(\varepsilon)}\left(u, q\right)\ , cI_{\Lambda_Q(\varepsilon)}\left(u, q\right),\ cF_{\Lambda_Q(\varepsilon)}\left(u, q\right)\right\rangle : \varepsilon \in \Upsilon, u \in U, q \in Q\right\}$
where

$$cT_{\Lambda_Q(\varepsilon)}\left(u, q\right) = cP_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi c\mu_{\Lambda_Q(\varepsilon)}(u,q)} = S_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\left(1-\mu_{\Lambda_Q(\varepsilon)}(u,q)\right)},$$

$$cI_{\Lambda_Q(\varepsilon)}\left(u, q\right) = cR_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi c\nu_{\Lambda_Q(\varepsilon)}(u,q)} = 1 - R_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\left(1-\nu_{\Lambda_Q(\varepsilon)}(u,q)\right)},$$

$$cF_{\Lambda_Q(\varepsilon)}\left(u, q\right) = cS_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi c\omega_{\Lambda_Q(\varepsilon)}(u,q)} = P_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\left(1-\omega_{\Lambda_Q(\varepsilon)}(u,q)\right)}.$$

Proposition 3.7. *If $(\Lambda_Q, \Upsilon) \in$ Q-CNSS(U), then $c(c(\Lambda_Q, \Upsilon)) = (\Lambda_Q, \Upsilon)$.*

Proof. From the above definition 3.6, we will simply deal with the complex-valued truth membership function.Then,
$T_{\Lambda_Q(\varepsilon)}\left(u, q\right)$
Take the complement c

$$cT_{\Lambda_Q(\varepsilon)}\left(u, q\right) = cP_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi c\mu_{\Lambda_Q(\varepsilon)}(u,q)} = S_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\left(1-\mu_{\Lambda_Q(\varepsilon)}(u,q)\right)}$$

Take the complement c gain

$$c(cT_{\Lambda_Q(\varepsilon)}\left(u, q\right)) = c(cP_{\Lambda_Q(\varepsilon)}\left(u, q\right)).e^{2\pi c(c\mu_{\Lambda_Q(\varepsilon)}(u,q))} = cS_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\left(1-c\mu_{\Lambda_Q(\varepsilon)}(u,q)\right)}$$

$= cS_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\left(1-(1-c\mu_{\Lambda_Q(\varepsilon)}(u,q))\right)} = P_{\Lambda_Q(\varepsilon)}\left(u, q\right).e^{2\pi\mu_{\Lambda_Q(\varepsilon)}(u,q)} = T_{\Lambda_Q(\varepsilon)}\left(u, q\right)$
The proofs for the complex-valued identity and falsity membership functions can be similarly constructed. ☐

Definition 3.8. The union of two Q-CNSSs (Λ, Υ) and (Γ, Ψ) in U is a Q-CNSS (K, C), where $C = \Upsilon \cup \Psi$ and $\forall\, \epsilon \in C, \forall\,(u, q) \in U \times Q$,

$$T_{K(\epsilon)}(u, q) = \begin{cases} P_{\Lambda(\epsilon)}(u, q).e^{j2\pi\mu_{\Lambda(\epsilon)}(u,q)} & ,\text{if } \epsilon \in \Upsilon - \Psi \\ P_{\Gamma(\epsilon)}(u, q).e^{j2\pi\mu_{\Gamma(\epsilon)}(u,q)} & ,\text{if } \epsilon \in \Psi - \Upsilon \\ (P_{\Lambda(\epsilon)}(u, q) \vee P_{\Gamma(\epsilon)}(u, q)).e^{j2\pi(\mu_{\Lambda(\epsilon)}(u,q) \vee \mu_{\Gamma(\epsilon)}(u,q))} & ,\text{if } \epsilon \in \Upsilon \cap \Psi, \end{cases}$$

$$I_{K(\epsilon)}(u, q) = \begin{cases} R_{\Lambda(\epsilon)}(u, q).e^{j2\pi\nu_{\Lambda(\epsilon)}(u,q)} & ,\text{if } \epsilon \in \Upsilon - \Psi \\ R_{\Gamma(\epsilon)}(u, q).e^{j2\pi\nu_{\Gamma(\epsilon)}(u,q)} & ,\text{if } \epsilon \in \Psi - \Upsilon \\ (R_{\Lambda(\epsilon)}(u, q) \wedge R_{\Gamma(\epsilon)}(u, q)).e^{j2\pi(\nu_{\Lambda(\epsilon)}(u,q) \wedge \nu_{\Gamma(\epsilon)}(u,q))} & ,\text{if } \epsilon \in \Upsilon \cap \Psi, \end{cases}$$

$$F_{K(\epsilon)}(u, q) = \begin{cases} S_{\Lambda(\epsilon)}(u, q).e^{j2\pi\omega_{\Lambda(\epsilon)}(u,q)} & ,\text{if } \epsilon \in \Upsilon - \Psi \\ S_{\Gamma(\epsilon)}(u, q).e^{j2\pi\omega_{\Gamma(\epsilon)}(u,q)} & ,\text{if } \epsilon \in \Psi - \Upsilon \\ (S_{\Lambda(\epsilon)}(u, q) \wedge S_{\Gamma(\epsilon)}(u, q)).e^{j2\pi(\omega_{\Lambda(\epsilon)}(u,q) \wedge \omega_{\Gamma(\epsilon)}(u,q))} & ,\text{if } \epsilon \in \Upsilon \cap \Psi, \end{cases}$$

where $\vee = \max$, and $\wedge = \min$. The union $(\Lambda, \Upsilon)\, \tilde{\cup}\, (\Gamma, \Psi)$ = (K, C).

Definition 3.9. The Intersection of two Q-CNSSs (Λ, Υ) and (Γ, Ψ) in U is a Q-CNSS (K, C), where $C = \Upsilon \cap \Psi$ and $\forall\, \epsilon \in C, \forall\,(u, q) \in U \times Q$, the membership degrees of (K, C) are:

$T_{K(\epsilon)}(u, q) = (P_{\Lambda(\epsilon)}(u, q) \wedge P_{\Gamma(\epsilon)}(u, q)).e^{j2\pi(\mu_{\Lambda(\epsilon)}(u,q) \wedge \mu_{\Gamma(\epsilon)}(u,q))}$,

$I_{K(\epsilon)}(u, q) = (R_{\Lambda(\epsilon)}(u, q) \vee R_{\Gamma(\epsilon)}(u, q)).e^{j2\pi(\nu_{\Lambda(\epsilon)}(u,q) \vee \nu_{\Gamma(\epsilon)}(u,q))}$,

$F_{K(\epsilon)}(u, q) = (S_{\Lambda(\epsilon)}(u, q) \vee S_{\Gamma(\epsilon)}(u, q)).e^{j2\pi(\omega_{\Lambda(\epsilon)}(u,q) \vee \omega_{\Gamma(\epsilon)}(u,q))}$.

where $\vee = \max$, and $\wedge = \min$. The intersection $(\Lambda, \Upsilon)\, \tilde{\cap}\, (\Gamma, \Psi)$ = (K, C).

Proposition 3.10. *Let* $(\Lambda_Q, \Upsilon), (\Theta_Q, \Gamma)$ *and* $(\Xi_Q, \Psi) \in$*Q-CNSS(U).Then the following properties are fulfilled*

Definition 3.11. Let U and Q be two non empty sets, (Λ, Υ) and (Γ, Ψ) be two Q-CNSSs in U which are characterized by the complex valued membership functions respectively :

$T_{\Lambda(\varepsilon)}(u,q) = P_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)},\ I_{\Lambda(\varepsilon)}(u,q) = R_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$ and
$F_{\Lambda(\varepsilon)}(u,q) = S_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\omega_{\Lambda(\varepsilon)}(u,q)}$ and $T_{\Gamma(\varepsilon)}(u,q) = P_{\Gamma(\varepsilon)}(u,q)e^{j2\pi\mu_{\Gamma(\varepsilon)}(u,q)},\ I_{\Gamma(\varepsilon)}(u,q) = R_{\Gamma(\varepsilon)}(u,q)e^{j2\pi\nu_{\Gamma(\varepsilon)}(u,q)}$ and
$F_{\Gamma(\varepsilon)}(u,q) = S_{\Gamma(\varepsilon)}(u,q)e^{j2\pi\omega_{\Gamma(\varepsilon)}(u,q)}$. Then

(i) A set (Λ, Υ) is said to be homogeneous Q-CNSS if for all $\varepsilon \in \Upsilon$ and $u, v \in U$ and $q \in Q$, we have

(1) $P_{\Lambda(\varepsilon)}(u,q) \leq P_{\Lambda(\varepsilon)}(v,q)$ if and only if $\mu_{\Lambda(\varepsilon)}(u,q) \leq \mu_{\Lambda(\varepsilon)}(v,q)$,

(2) $R_{\Lambda(\varepsilon)}(u,q) \leq R_{\Lambda(\varepsilon)}(v,q)$ if and only if $\nu_{\Lambda(\varepsilon)}(u,q) \leq \nu_{\Lambda(\varepsilon)}(v,q)$,

(3) $S_{\Lambda(\varepsilon)}(u,q) \leq S_{\Lambda(\varepsilon)}(v,q)$ if and only if $\omega_{\Lambda(\varepsilon)}(u,q) \leq \omega_{\Lambda(\varepsilon)}(v,q)$,

(ii) A Q-CNSS (Λ, Υ) is said to be completely homogeneous Q-CNSS if it is homogeneous and if and only if $\forall \varepsilon, \delta \in \Upsilon$, $u \in U$ and $q \in Q$, we have

(1) $P_{\Lambda(\varepsilon)}(u,q) \leq P_{\Lambda(\delta)}(u,q)$ if and only if $\mu_{\Lambda(\varepsilon)}(u,q) \leq \mu_{\Lambda(\delta)}(u,q)$,

(2) $R_{\Lambda(\varepsilon)}(u,q) \leq R_{\Lambda(\delta)}(u,q)$ if and only if $\nu_{\Lambda(\varepsilon)}(u,q) \leq \nu_{\Lambda(\delta)}(u,q)$,

(3) $S_{\Lambda(\varepsilon)}(u,q) \leq S_{\Lambda(\delta)}(u,q)$ if and only if $\omega_{\Lambda(\varepsilon)}(u,q) \leq \omega_{\Lambda(\delta)}(u,q)$,

(iii) A Q-CNSS (Λ, Υ) is said to be homogeneous with (Γ, Ψ) if and only if for all $\varepsilon \in \Upsilon \cap \Psi$ and for all $u \in U$ and $q \in Q$, we have

(1) $P_{\Lambda(\varepsilon)}(u,q) \leq P_{\Gamma(\varepsilon)}(u,q)$ if and only if $\mu_{\Lambda(\varepsilon)}(u,q) \leq \mu_{\Gamma(\varepsilon)}(u,q)$,

(2) $R_{\Lambda(\varepsilon)}(u,q) \leq R_{\Gamma(\varepsilon)}(u,q)$ if and only if $\nu_{\Lambda(\varepsilon)}(u,q) \leq \nu_{\Gamma(\varepsilon)}(u,q)$,

(3) $S_{\Lambda(\varepsilon)}(u,q) \leq S_{\Gamma(\varepsilon)}(u,q)$ if and only if $\omega_{\Lambda(\varepsilon)}(u,q) \leq \omega_{\Gamma(\varepsilon)}(u,q)$,

Example 3.12. Let $U = \{u_1, u_2\}$ and $Q = \{q_1, q_2\}$ be two non empty sets and $\Upsilon = \{\varepsilon_1, \varepsilon_2\}$, $\Psi = \{\varepsilon_1, \varepsilon_3, \varepsilon_4\}$ be two sets of parameters that are subsets from the universal set of parameters E. Suppose that the two Q-CNSSs (Λ, Υ) and (Γ, Ψ) are defined as follows:

$$(\Lambda, \Upsilon) = \Bigg\{\Bigg(\varepsilon_1, \Big\{\frac{\langle 0.2e^{j2\Pi(0.3)}, 0.4e^{j2\Pi(0.5)}, 0.3e^{j2\Pi(0.6)}\rangle}{(u_1,q_1)}, \frac{\langle 0.3e^{j2\Pi(0.4)}, 0.5e^{j2\Pi(0.5)}, 0.3e^{j2\Pi(0.7)}\rangle}{(u_1,q_2)}, \frac{\langle 0.1e^{j2\Pi(0.2)}, 0.5e^{j2\Pi(0.7)}, 0.4e^{j2\Pi(0.8)}\rangle}{(u_2,q_1)}, \frac{\langle 0.3e^{j2\Pi(0.4)}, 0.6e^{j2\Pi(0.7)}, 0.9e^{j2\Pi(0.8)}\rangle}{(u_2,q_2)}\Big\}\Bigg), \Bigg(\varepsilon_2, \Big\{\frac{\langle 0.8e^{j2\Pi(0.5)}, 0.6e^{j2\Pi(0.6)}, 0.7e^{j2\Pi(0.7)}\rangle}{(u_1,q_1)}, \frac{\langle 0.2e^{j2\Pi(0.3)}, 0.4e^{j2\Pi(0.2)}, 0.1e^{j2\Pi(0.7)}\rangle}{(u_1,q_2)}, \frac{\langle 0.8e^{j2\Pi(0.5)}, 0.5e^{j2\Pi(0.6)}, 0.3e^{j2\Pi(0.7)}\rangle}{(u_2,q_1)}, \frac{\langle 0.9e^{j2\Pi(0.5)}, 0.4e^{j2\Pi(0.6)}, 0.3e^{j2\Pi(0.7)}\rangle}{(u_2,q_2)}\Big\}\Bigg)\Bigg\}.$$

$$(\Gamma, \Psi) = \Bigg\{\Bigg(\varepsilon_1, \Big\{\frac{\langle 0.4e^{j2\Pi(0.5)}, 0.4e^{j2\Pi(0.6)}, 0.1e^{j2\Pi(0.5)}\rangle}{(u_1,q_1)}, \frac{\langle 0.3e^{j2\Pi(0.7)}, 0.6e^{j2\Pi(0.9)}, 0.5e^{j2\Pi(0.8)}\rangle}{(u_1,q_2)}, \frac{\langle 0.9e^{j2\Pi(0.9)}, 0.5e^{j2\Pi(0.7)}, 0.9e^{j2\Pi(0.9)}\rangle}{(u_2,q_1)}, \frac{\langle 0.7e^{j2\Pi(0.5)}, 0.1e^{j2\Pi(0.6)}, 0.3e^{j2\Pi(0.5)}\rangle}{(u_2,q_2)}\Big\}\Bigg), \Bigg(\varepsilon_3, \Big\{\frac{\langle 0.2e^{j2\Pi(0.1)}, 0.4e^{j2\Pi(0.2)}, 0.7e^{j2\Pi(0.6)}\rangle}{(u_1,q_1)}, \frac{\langle 0.1e^{j2\Pi(0.8)}, 0.5e^{j2\Pi(0.7)}, 0.7e^{j2\Pi(0.2)}\rangle}{(u_1,q_2)}, \frac{\langle 0.2e^{j2\Pi(0.3)}, 0.4e^{j\Pi(0.5)}, 0.3e^{j2\Pi(0.6)}\rangle}{(u_2,q_1)}, \frac{\langle 0.1e^{j2\Pi(0.2)}, 0.5e^{j2\Pi(0.7)}, 0.4e^{j2\Pi(0.8)}\rangle}{(u_2,q_2)}\Big\}\Bigg), \Bigg(\varepsilon_4, \Big\{\frac{\langle 0.3e^{j2\Pi(0.4)}, 0.4e^{j2\Pi(0.6)}, 0.9e^{j2\Pi(0.8)}\rangle}{(u_1,q_1)}, \frac{\langle 0.8e^{j2\Pi(0.5)}, 0.6e^{j\Pi(0.6)}, 0.7e^{j2\Pi(0.7)}\rangle}{(u_1,q_2)}, \frac{\langle 0.2e^{j2\Pi(0.3)}, 0.4e^{j2\Pi(0.2)}, 0.1e^{j2\Pi(0.7)}\rangle}{(u_2,q_1)}, \frac{\langle 0.8e^{j2\Pi(0.5)}, 0.5e^{j2\Pi(0.6)}, 0.3e^{j2\Pi(0.7)}\rangle}{(u_2,q_2)}\Big\}\Bigg)\Bigg\}.$$

It can clearly be shown from the definition that (Λ, Υ) is homogeneous and completely homogeneous. (Γ, Ψ) is not homogeneous therefore, it is not completely homogeneous. It is also clear that (Λ, Υ) is homogeneous with (Γ, Ψ).

4 Q-Complex Neutrosophic Soft Group

Definition 4.1. Let G be a group and (Λ, Υ) be a homogeneous Q-CNSS in G. Then (Λ, Υ) is said to be a Q-complex neutrosophic soft group (Q-CNSG) in G if and only if, for all $\varepsilon \in \Upsilon$, $u, v \in G$ and $q \in Q$, the following hold:

1. $T_{\Lambda(\varepsilon)}(uv, q) \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v, q)\}$,
2. $I_{\Lambda(\varepsilon)}(uv, q) \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v, q)\}$,
3. $F_{\Lambda(\varepsilon)}(uv, q) \leq \max\{F_{\Lambda(\varepsilon)}(u, q), F_{\Lambda(\varepsilon)}(v, q)\}$,
4. $T_{\Lambda(\varepsilon)}(u^{-1}, q) \geq T_{\Lambda(\varepsilon)}(u, q)$,
5. $I_{\Lambda(\varepsilon)}(u^{-1}, q) \leq I_{\Lambda(\varepsilon)}(u, q)$,
6. $F_{\Lambda(\varepsilon)}(u^{-1}, q) \leq F_{\Lambda(\varepsilon)}(u, q)$.

The following examples illustrate the definition of the Q-CNSG.

Example 4.2. Let us take into consideration the classical group $G = \{1, -1, i, -i\}$ with the natural multiplication. Define the homogeneous Q-CNSS (Λ, Υ) on G as follows.

$$\begin{aligned}&(\Lambda, \Upsilon) = \\ &\Big\{\Big(\varepsilon_1, \{\tfrac{\langle 0.3e^{j2\Pi(0.4)},\ 0.4e^{j2\Pi(0.5)},\ 0.7e^{j2\Pi(0.7)}\rangle}{(1,q)}, \tfrac{\langle 0.1e^{j2\Pi(0.2)},\ 0.4e^{j2\Pi(0.6)},\ 0.3e^{j2\Pi(0.7)}\rangle}{(-1,q)}, \\ &\tfrac{\langle 0.2e^{j2\Pi(0.3)},\ 0.5e^{j2\Pi(0.7)},\ 0.9e^{j2\Pi(0.8)}\rangle}{(i,q)}, \tfrac{\langle 0.2e^{j2\Pi(0.3)},\ 0.5e^{j2\Pi(0.7)},\ 0.9e^{j2\Pi(0.8)}\rangle}{(-i,q)}\}\Big), \\ &\Big(\varepsilon_2, \{\tfrac{\langle 0.9e^{j2\Pi(0.7)},\ 0.3e^{j2\Pi(0.1)},\ 0.1e^{j2\Pi(0.4)}\rangle}{(1,q)}, \tfrac{\langle 0.3e^{j2\Pi(0.4)},\ 0.5e^{j2\Pi(0.7)},\ 0.2e^{j2\Pi(0.4)}\rangle}{(-1,q)}, \\ &\tfrac{\langle 0.8e^{j2\Pi(0.6)},\ 0.4e^{j2\Pi(0.5)},\ 0.3e^{j2\Pi(0.6)}\rangle}{(i,q)}, \tfrac{\langle 0.8e^{j2\Pi(0.6)},\ 0.4e^{j2\Pi(0.5)},\ 0.3e^{j2\Pi(0.6)}\rangle}{(-i,q)}\}\Big)\Big\}.\end{aligned}$$

It is clear that the Q-CNSS (Λ, Υ) satisfies all the conditions listed in Definition 4.1. Thus it is a Q-CNSG.

Example 4.3. Consider the 3^{th} symmetric group $S_3 = \{1, (12), (23), (13), (123), (132)\}$ with the permutations of $M = \{1, 2, 3\}$ under the composition of functions as a binary operation. Consider the parameters set $\Psi = \{\varepsilon_1, \varepsilon_2\}$. Suppose $T_1 = 0.3e^{j2\Pi(0.4)}, T_2 = 0.3e^{j2\Pi(0.5)}, T_3 = 0.4e^{j2\Pi(0.5)},\ I_1 = 0.5e^{j2\Pi(0.7)}, I_2 = 0.2e^{j2\Pi(0.7)}, I_3 = 0.2e^{j2\Pi(0.6)}$ and $F_1 = 0.6e^{j2\Pi(0.5)}, F_2 = 0.6e^{j2\Pi(0.4)}, F_3 = 0.3e^{j2\Pi(0.4)}$. It is to be noted that $T_1 \leq T_2 \leq T_3$, $I_1 \geq I_2 \geq I_3$ and $F_1 \geq F_2 \geq F_3$. Define the homogeneous Q-CNSS (Γ, Ψ) on G as follows.

$$\begin{aligned}&(\Gamma, \Psi) = \\ &\Big\{\Big(\varepsilon_1, \{\tfrac{\langle T_3,I_3,F_3\rangle}{(1,q)}, \tfrac{\langle T_1,I_1,F_1\rangle}{((12),q)}, \tfrac{\langle T_1,I_1,F_1\rangle}{((13),q)}, \tfrac{\langle T_1,I_1,F_1\rangle}{((23),q)}, \tfrac{\langle T_2,I_2,F_2\rangle}{((123),q)}, \tfrac{\langle T_2,I_2,F_2\rangle}{((132),q)}\}\Big), \Big(\varepsilon_2, \{\tfrac{\langle T_3,I_3,F_3\rangle}{(1,q)}, \tfrac{\langle T_2,I_2,F_2\rangle}{((12),q)}, \\ &\tfrac{\langle T_1,I_1,F_1\rangle}{((13),q)}, \tfrac{\langle T_1,I_1,F_1\rangle}{((23),q)}, \tfrac{\langle T_1,I_1,F_1\rangle}{((123),q)}, \tfrac{\langle T_1,I_1,F_1\rangle}{((132),q)}\}\Big)\Big\}.\end{aligned}$$

It is clear form Definition 4.1 that the Q-CNSS (Γ, Ψ) is Q-CNSG.

Definition 4.4. Let (Λ, Υ) and (Γ, Ψ) be two Q-CNSSs, then (Λ, Υ) is said to be a Q-complex neutrosophic soft subgroup of (Γ, Ψ) if the following conditions are satisfied:

(1)$(\Lambda, \Upsilon) \subseteq (\Gamma, \Psi)$ where $\subseteq$ is a Q-complex neutrosophic soft subset.

(2) (Λ, Υ) and (Γ, Ψ) are both Q-CNSGs

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The following theorem demonstrates the relation between Q-CNSG and Q-NSGs.

Theorem 4.5. *Let G be a group and* $(\Lambda,\Upsilon)=\{<\varepsilon;T_{\Lambda(\varepsilon)}(u,q),I_{\Lambda(\varepsilon)}(u,q),F_{\Lambda(\varepsilon)}(u,q)>:\varepsilon\in\Upsilon,u\in U,q\in Q\}$ *be a homogeneous Q-complex neutrosophic soft set in G which yields the two Q-NSSs* $(H,\Upsilon)=\{<\varepsilon;P_{\Lambda(\varepsilon)}(u,q),R_{\Lambda(\varepsilon)}(u,q),S_{\Lambda(\varepsilon)}(u,q)>:\varepsilon\in\Upsilon,u\in U,q\in Q\}$ *and* $(K,\Upsilon)=\{<\varepsilon;\mu_{\Lambda(\varepsilon)}(u,q),\nu_{\Lambda(\varepsilon)}(u,q),\omega_{\Lambda(\varepsilon)}(u,q)>:\varepsilon\in\Upsilon,u\in U,q\in Q\}$. *Then (Λ,Υ) is a Q-CNSG of G if and only if both of (H,Υ) and (K,Υ) are Q-NSGs.*

Proof. In order to prove the first direction of this theorem, it should satisfy previously defined six conditions listed in Definition (2.3).

$\Rightarrow$ Firstly, suppose (Λ,Υ) is a Q-CNSG and $u,v\in G$. Then for all $\varepsilon\in\Upsilon$, we have:

$P_{\Lambda(\varepsilon)}(uv,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(uv,q)}=T_{\Lambda(\varepsilon)}(uv,q)\geq\min\{T_{\Lambda(\varepsilon)}(u,q),T_{\Lambda(\varepsilon)}(v,q)\}=\min\{P_{\Lambda(\varepsilon)}(u,q)$ $e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)},P_{\Lambda(\varepsilon)}(v,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(v,q)}\}=\min\{P_{\Lambda(\varepsilon)}(u,q),P_{\Lambda(\varepsilon)}(v,q)\}e^{j2\pi\min\{\mu_{\Lambda(\varepsilon)}(u,q),\mu_{\Lambda(\varepsilon)}(v,q)\}}$. Since (Λ,Υ) is homogeneous, so $P_{\Lambda(\varepsilon)}(uv,q)\geq\min\{P_{\Lambda(\varepsilon)}(u,q),P_{\Lambda(\varepsilon)}(v,q)\}$, and $\mu_{\Lambda(\varepsilon)}(uv,q)\geq\min\{\mu_{\Lambda(\varepsilon)}(u,q),\mu_{\Lambda(\varepsilon)}(v,q)\}$.

Similarly, we can obtain

$R_{\Lambda(\varepsilon)}(uv,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(uv,q)}=I_{\Lambda(\varepsilon)}(uv,q)\leq\max\{I_{\Lambda(\varepsilon)}(u,q),I_{\Lambda(\varepsilon)}(v,q)\}=\max\{R_{\Lambda(\varepsilon)}(u,q)$ $e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)},R_{\Lambda(\varepsilon)}(v,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(v,q)}\}=\max\{R_{\Lambda(\varepsilon)}(u,q),R_{\Lambda(\varepsilon)}(v,q)\}e^{j2\pi\max\{\nu_{\Lambda(\varepsilon)}(u,q),\nu_{\Lambda(\varepsilon)}(v,q)\}}$. Since (Λ,Υ) is homogeneous, so $R_{\Lambda(\varepsilon)}(uv,q)\leq\max\{R_{\Lambda(\varepsilon)}(u,q),R_{\Lambda(\varepsilon)}(v,q)\}$, and $\nu_{\Lambda(\varepsilon)}(uv,q)\leq\max\{\nu_{\Lambda(\varepsilon)}(u,q),\nu_{\Lambda(\varepsilon)}(v,q)\}$.

In the same way we get $S_{\Lambda(\varepsilon)}(uv,q)\leq\max\{S_{\Lambda(\varepsilon)}(u,q),S_{\Lambda(\varepsilon)}(v,q)\}$, and $\omega_{\Lambda(\varepsilon)}(uv,q)\leq\max\{\omega_{\Lambda(\varepsilon)}(u,q),\omega_{\Lambda(\varepsilon)}(v,q)\}$. Thus, conditions 1,2 and 3 hold.

Next, $P_{\Lambda(\varepsilon)}(u^{-1},q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u^{-1},q)}=T_{\Lambda(\varepsilon)}(u^{-1},q)\geq T_{\Lambda(\varepsilon)}(u,q)=P_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}$ which implies $P_{\Lambda(\varepsilon)}(u^{-1},q)\geq P_{\Lambda(\varepsilon)}(u,q)$ and $\mu_{\Lambda(\varepsilon)}(u^{-1},q)\geq\mu_{\Lambda(\varepsilon)}(u,q)$. ($(\Lambda,\Upsilon)$ is homogeneous).

Similarly,

$R_{\Lambda(\varepsilon)}(u^{-1},q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u^{-1},q)}=I_{\Lambda(\varepsilon)}(u^{-1},q)\leq I_{\Lambda(\varepsilon)}(u,q)=R_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$. Since (Λ,Υ) is homogeneous, we obtain $R_{\Lambda(\varepsilon)}(u^{-1},q)\leq R_{\Lambda(\varepsilon)}(u,q)$ and $\nu_{\Lambda(\varepsilon)}(u^{-1},q)\leq\nu_{\Lambda(\varepsilon)}(u,q)$.

Similarly, we can get,

$S_{\Lambda(\varepsilon)}(u^{-1},q)\leq S_{\Lambda(\varepsilon)}(u,q)$ and $\omega_{\Lambda(\varepsilon)}(u^{-1},q)\leq\omega_{\Lambda(\varepsilon)}(u,q)$.

To summarize:

(1) $P_{\Lambda(\varepsilon)}(uv,q)\geq\min\{P_{\Lambda(\varepsilon)}(u,q),P_{\Lambda(\varepsilon)}(v,q)\}$,(2) $R_{\Lambda(\varepsilon)}(uv,q)\leq\max\{R_{\Lambda(\varepsilon)}(u,q),R_{\Lambda(\varepsilon)}$ $(v,q)\}$, (3) $S_{\Lambda(\varepsilon)}(uv,q)\leq\max\{S_{\Lambda(\varepsilon)}(u,q),S_{\Lambda(\varepsilon)}(v,q)\}$, (4) $P_{\Lambda(\varepsilon)}(u^{-1},q)\geq P_{\Lambda(\varepsilon)}(u,q)$, (5) $R_{\Lambda(\varepsilon)}(u^{-1},q)\leq R_{\Lambda(\varepsilon)}(u,q)$, (6) $S_{\Lambda(\varepsilon)}(u^{-1},q)\leq S_{\Lambda(\varepsilon)}(u,q)$, which implies that (H,Υ) is a Q-NSG.

We also obtain:

(1)$\mu_{\Lambda(\varepsilon)}(uv,q)\geq\min\{\mu_{\Lambda(\varepsilon)}(u,q),\mu_{\Lambda(\varepsilon)}(v,q)\}$, (2) $\nu_{\Lambda(\varepsilon)}(uv,q)\leq\max\{\nu_{\Lambda(\varepsilon)}(u,q),\nu_{\Lambda(\varepsilon)}(v,q)$ $\}$, (3) $\omega_{\Lambda(\varepsilon)}(uv,q)\leq\max\{\omega_{\Lambda(\varepsilon)}(u,q),\omega_{\Lambda(\varepsilon)}(v,q)\}$, (4) $\mu_{\Lambda(\varepsilon)}(u^{-1},q)\geq\mu_{\Lambda(\varepsilon)}(u,q)$. ($(\Lambda,\Upsilon)$, (5) $\nu_{\Lambda(\varepsilon)}(u^{-1},q)\leq\nu_{\Lambda(\varepsilon)}(u,q)$, (6) $\omega_{\Lambda(\varepsilon)}(u^{-1},q)\leq\omega_{\Lambda(\varepsilon)}(u,q)$, Which implies that (K,Υ) is a Q-NSG.

$\Leftarrow$ Conversely, suppose that (H, Υ) and (K, Υ) are two Q-NSGs. Then for all $\varepsilon \in \Upsilon$, we have:

(1) $P_{\Lambda(\varepsilon)}(uv, q) \geq \min\{P_{\Lambda(\varepsilon)}(u, q), P_{\Lambda(\varepsilon)}(v, q)\}$,(2) $R_{\Lambda(\varepsilon)}(uv, q) \leq \max\{R_{\Lambda(\varepsilon)}(u, q), R_{\Lambda(\varepsilon)}(v, q)\}$, (3) $S_{\Lambda(\varepsilon)}(uv, q) \leq \max\{S_{\Lambda(\varepsilon)}(u, q), S_{\Lambda(\varepsilon)}(v, q)\}$, (4) $P_{\Lambda(\varepsilon)}(u^{-1}, q) \geq P_{\Lambda(\varepsilon)}(u, q)$, (5) $R_{\Lambda(\varepsilon)}(u^{-1}, q) \leq R_{\Lambda(\varepsilon)}(u, q)$, (6) $S_{\Lambda(\varepsilon)}(u^{-1}, q) \leq S_{\Lambda(\varepsilon)}(u, q)$,

and,

(1)$\mu_{\Lambda(\varepsilon)}(uv, q) \geq \min\{\mu_{\Lambda(\varepsilon)}(u, q), \mu_{\Lambda(\varepsilon)}(v, q)\}$, (2) $\nu_{\Lambda(\varepsilon)}(uv, q) \leq \max\{\nu_{\Lambda(\varepsilon)}(u, q), \nu_{\Lambda(\varepsilon)}(v, q)\}$, (3) $\omega_{\Lambda(\varepsilon)}(uv, q) \leq \max\{\omega_{\Lambda(\varepsilon)}(u, q), \omega_{\Lambda(\varepsilon)}(v, q)\}$, (4) $\mu_{\Lambda(\varepsilon)}(u^{-1}, q) \geq \mu_{\Lambda(\varepsilon)}(u, q)$. $((\Lambda, \Upsilon)$, (5) $\nu_{\Lambda(\varepsilon)}(u^{-1}, q) \leq \nu_{\Lambda(\varepsilon)}(u, q)$, (6) $\omega_{\Lambda(\varepsilon)}(u^{-1}, q) \leq \omega_{\Lambda(\varepsilon)}(u, q)$.

To prove that (Λ, Υ) is a Q-CNSG, we have to show that:

$T_{\Lambda(\varepsilon)}(uv, q) =$
$P_{\Lambda(\varepsilon)}(uv, q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(uv,q)} \geq \min\{P_{\Lambda(\varepsilon)}(u, q), P_{\Lambda(\varepsilon)}(v, q)\}.e^{j2\pi \min\{\mu_{\Lambda(\varepsilon)}(u,q),\mu_{\Lambda(\varepsilon)}(v,q)\}} =$
$min\{P_{\Lambda(\varepsilon)}(u, q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}, P_{\Lambda(\varepsilon)}(v, q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(v,q)}\}$. ((Λ, Υ) is homogeneous)

Thus, we obtain $T_{\Lambda(\varepsilon)}(uv, q) \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v, q)\}$.

In a similar manner :

$I_{\Lambda(\varepsilon)}(uv, q) =$
$R_{\Lambda(\varepsilon)}(uv, q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(uv,q)} \leq \max\{R_{\Lambda(\varepsilon)}(u, q), R_{\Lambda(\varepsilon)}(v, q)\}.e^{j2\pi\ max\{\nu_{\Lambda(\varepsilon)}(u,q),\nu_{\Lambda(\varepsilon)}(v,q)\}} =$
$\max\{R_{\Lambda(\varepsilon)}(u, q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}, R_{\Lambda(\varepsilon)}(v, q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(v,q)}\}$. ((Λ, Υ) is homogeneous)

Thus, we obtain $I_{\Lambda(\varepsilon)}(uv, q) \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v, q)\}$.

In the same manner we show that $F_{\Lambda(\varepsilon)}(uv, q) \leq \max\{F_{\Lambda(\varepsilon)}(u, q), F_{\Lambda(\varepsilon)}(v, q)\}$.

On the other hand.

$T_{\Lambda(\varepsilon)}(u^{-1}, q) = P_{\Lambda(\varepsilon)}(u^{-1}, q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u^{-1},q)} \geq P_{\Lambda(\varepsilon)}(u, q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}$ (homogeneity)

$= T_{\Lambda(\varepsilon)}(u, q)$.

Also,

$I_{\Lambda(\varepsilon)}(u^{-1}, q) = R_{\Lambda(\varepsilon)}(u^{-1}, q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u^{-1},q)} \leq R_{\Lambda(\varepsilon)}(u, q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$ (homogeneity)

$= I_{\Lambda(\varepsilon)}(u, q)$.

Similarly it can easily be proven that; $F_{\Lambda(\varepsilon)}(u^{-1}, q) \leq F_{\Lambda(\varepsilon)}(u, q)$.

Thus (Λ, Υ) is a Q-CNSG. This completes the proof.

□

Theorem 4.6. *Let G be a group and (Λ, Υ) be a homogeneous Q-CNSS in G. Then (Λ, Υ) is a Q-CNSG if and only if for all $\varepsilon \in \Upsilon, u, v \in G$. and $q \in Q$.*

(1) $T_{\Lambda(\varepsilon)}(uv^{-1}, q) \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v, q)\}$,

(2) $I_{\Lambda(\varepsilon)}(uv^{-1}, q) \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v, q)\}$,

(3) $F_{\Lambda(\varepsilon)}(uv^{-1}, q) \leq \max\{F_{\Lambda(\varepsilon)}(u, q), F_{\Lambda(\varepsilon)}(v, q)\}$.

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Proof. $\Rightarrow$ (1) Let (Λ, Υ) be a Q-CNSG in G, $u, v \in G$ and $q \in Q$. Then for all $\varepsilon \in \Upsilon$, we will prove the two parts related to the complex truth as well as indeterminacy membership functions.

We begin with the truth membership proof as follows:

Since (Λ, Υ) is a Q-CNSG, we have

$T_{\Lambda(\varepsilon)}(uv, q) \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v, q)\}$, therefore, $T_{\Lambda(\varepsilon)}(uv^{-1}, q) \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v^{-1}, q)\} \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v, q)\}$, since $T_{\Lambda(\varepsilon)}(u^{-1}, q) \geq T_{\Lambda(\varepsilon)}(u, q)$.

Next , the indeterminacy membership can be proved as follows:

$I_{\Lambda(\varepsilon)}(uv, q) \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v, q)\}$, therefore, $I_{\Lambda(\varepsilon)}(uv^{-1}, q) \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v^{-1}, q)\} \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v, q)\}$, since $I_{\Lambda(\varepsilon)}(u^{-1}, q) \leq I_{\Lambda(\varepsilon)}(u, q)$.

The falsity membership can be proven in the same manner as has been done with indeterminacy membership function.

$\Leftarrow$ Conversely, let e be the unit of G. Since G is a group, then:

$T_{\Lambda(\varepsilon)}(u^{-1}, q) = T_{\Lambda(\varepsilon)}(e.u^{-1}, q) \geq \min\{T_{\Lambda(\varepsilon)}(e, q), T_{\Lambda(\varepsilon)}(u, q)\}$,

$= \min\{T_{\Lambda(\varepsilon)}(u.u^{-1}, q), T_{\Lambda(\varepsilon)}(u, q)\} \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(u, q)\} = T_{\Lambda(\varepsilon)}(u, q)$. Thus, $T_{\Lambda(\varepsilon)}(u^{-1}, q) \geq T_{\Lambda(\varepsilon)}(u, q)$.

Now, we can similarly prove that;

$I_{\Lambda(\varepsilon)}(u^{-1}, q) \leq I_{\Lambda(\varepsilon)}(u, q)$ as follows:

$I_{\Lambda(\varepsilon)}(u^{-1}, q) = I_{\Lambda(\varepsilon)}(e.u^{-1}, q) \leq \max\{I_{\Lambda(\varepsilon)}(e, q), I_{\Lambda(\varepsilon)}(u, q)\}$,

$= \max\{I_{\Lambda(\varepsilon)}(u.u^{-1}, q), I_{\Lambda(\varepsilon)}(u, q)\} \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(u, q)\} = I_{\Lambda(\varepsilon)}(u, q)$.

The falsity membership can be proven in the same manner as it has been done with indeterminacy membership function.

Now, we proceed to the next conditions of the Q-CNSG.

We first start by proving the truth membership function as follows:

$T_{\Lambda(\varepsilon)}(uv, q) = T_{\Lambda(\varepsilon)}(u.(v^{-1})^{-1}, q) \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v^{-1}, q)\} \geq \min\{T_{\Lambda(\varepsilon)}(u, q), T_{\Lambda(\varepsilon)}(v, q)\}$,

Now, for the indeterminacy membership, we have :

$I_{\Lambda(\varepsilon)}(uv, q) = I_{\Lambda(\varepsilon)}(u.(v^{-1})^{-1}, q) \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v^{-1}, q)\} \leq \max\{I_{\Lambda(\varepsilon)}(u, q), I_{\Lambda(\varepsilon)}(v, q)\}$,

The falsity membership part can be proven in the same manner as it has been done with indeterminacy membership part. This completes the proof.

□

Theorem 4.7. *Let G be a group and (Λ, Υ) be a Q-CNSG in G, where e is the unit element of G. Then, for all $\varepsilon \in \Upsilon, u \in G$ and $q \in Q$, the following properties are satisfied:*

(1) $T_{\Lambda(\varepsilon)}(e, q) \geq T_{\Lambda(\varepsilon)}(u, q)$, (2) $F_{\Lambda(\varepsilon)}(e, q) \leq F_{\Lambda(\varepsilon)}(u, q)$, (3) $I_{\Lambda(\varepsilon)}(e, q) \leq I_{\Lambda(\varepsilon)}(u, q)$, (4) $T_{\Lambda(\varepsilon)}(u^{-1}, q) = T_{\Lambda(\varepsilon)}(u, q)$, (5) $F_{\Lambda(\varepsilon)}(u^{-1}, q) = F_{\Lambda(\varepsilon)}(u, q)$, (6) $I_{\Lambda(\varepsilon)}(u^{-1}, q) = I_{\Lambda(\varepsilon)}(u, q)$.

Proof. Part I: Properties (1 - 3).

Proof of property 1: Let e be a unit element in G and $u \in G$ be arbitrary, then by Definition 4.1.

$T_{\Lambda(\varepsilon)}(e,q) = T_{\Lambda(\varepsilon)}(u.u^{-1},q) \geq \min\{T_{\Lambda(\varepsilon)}(u,q), T_{\Lambda(\varepsilon)}(u,q)\} = T_{\Lambda(\varepsilon)}(u,q)$. Thus $T_{\Lambda(\varepsilon)}(e,q) \geq T_{\Lambda(\varepsilon)}(u,q)$.

For falsity membership.

$F_{\Lambda(\varepsilon)}(e,q) = F_{\Lambda(\varepsilon)}(u.u^{-1},q) \leq \max\{F_{\Lambda(\varepsilon)}(u,q), F_{\Lambda(\varepsilon)}(u,q)\} = F_{\Lambda(\varepsilon)}(u,q)$. Thus $F_{\Lambda(\varepsilon)}(e,q) \leq F_{\Lambda(\varepsilon)}(u,q)$.

Similarly, the property for indeterminacy membership function can be proven in the same manner.

Part (II): Properties (4 - 6).

Let $u \in G$, and $q \in Q$ be given, since (Λ, Υ) is a Q-CNSG. Then :

$T_{\Lambda(\varepsilon)}(u^{-1},q) \geq T_{\Lambda(\varepsilon)}(u,q)$. To prove that: $T_{\Lambda(\varepsilon)}(u^{-1},q) = T_{\Lambda(\varepsilon)}(u,q)$, we need to show that:

$T_{\Lambda(\varepsilon)}(u^{-1},q) \leq T_{\Lambda(\varepsilon)}(u,q)$,

Now, $T_{\Lambda(\varepsilon)}(u,q) = T_{\Lambda(\varepsilon)}((u^{-1})^{-1},q) \geq T_{\Lambda(\varepsilon)}(u^{-1},q)$, so $T_{\Lambda(\varepsilon)}(u^{-1},q) \leq T_{\Lambda(\varepsilon)}(u,q)$. Proving that $T_{\Lambda(\varepsilon)}(u^{-1},q) = T_{\Lambda(\varepsilon)}(u,q)$.

For the falsity membership:

Given that

$F_{\Lambda(\varepsilon)}(u^{-1},q) \leq F_{\Lambda(\varepsilon)}(u,q)$, we need to show that:$F_{\Lambda(\varepsilon)}(u^{-1},q) \geq F_{\Lambda(\varepsilon)}(u,q)$.

To prove the equality:

$F_{\Lambda(\varepsilon)}(u,q) = F_{\Lambda(\varepsilon)}((u^{-1})^{-1},q) \leq F_{\Lambda(\varepsilon)}(u^{-1},q)$, proving that $F_{\Lambda(\varepsilon)}(u^{-1},q) = F_{\Lambda(\varepsilon)}(u,q)$.

For Indeterminacy, it can be proven in the same way as in falsity. This completes the proof. □

Theorem 4.8. *Let G be a group and* (Λ, Υ), (Γ, Ψ) *are two homogeneous Q-CNSSs in G. If* (Λ, Υ) *and* (Γ, Ψ) *are two Q-CNSGs in G. Then the intersection* $(\Lambda, \Upsilon) \cap (\Gamma, \Psi)$ *so is Q-CNSG.*

Proof. Let $u, v \in G$, $q \in Q$ and $\varepsilon \in \Upsilon \cup \Psi$. By Theorem 4.5, it is enough to show that:

$T_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q) \geq \min\{T_{\Lambda\cap\Gamma(\varepsilon)}(u,q), T_{\Lambda\cap\Gamma(\varepsilon)}(v,q)\}$,

$I_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q) \leq \max\{I_{\Lambda\cap\Gamma(\varepsilon)}(u,q), I_{\Lambda\cap\Gamma(\varepsilon)}(v,q)\}$.

$F_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q) \leq \max\{F_{\Lambda\cap\Gamma(\varepsilon)}(u,q), F_{\Lambda\cap\Gamma(\varepsilon)}(v,q)\}$,

First consider the complex valued truth membership function of the intersection

$T_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q) = P_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q).e^{j2\pi\mu_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q))} = \min\{P_{\Lambda(\varepsilon)}(uv^{-1},q), P_{\Gamma(\varepsilon)}(uv^{-1},q)$
$\}.e^{j2\pi\min\{\mu_{\Lambda(\varepsilon)}(uv^{-1},q),\mu_{\Gamma(\varepsilon)}(uv^{-1},q)\}} = \min\{P_{\Lambda(\varepsilon)}(uv^{-1},q).e^{j2\pi\mu_{\Lambda(\varepsilon)}(uv^{-1},q)}, P_{\Gamma(\varepsilon)}(uv^{-1},q).$
$e^{j2\pi\mu_{\Gamma(\varepsilon)}(uv^{-1},q)}\} \geq \min\{\min\{P_{\Lambda(\varepsilon)}(u,q).e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}, P_{\Lambda(\varepsilon)}(v,q).e^{j2\pi\mu_{\Lambda(\varepsilon)}(v,q)}\}, \min\{P_{\Gamma(\varepsilon)}$
$(u,q).e^{j2\pi\mu_{\Gamma(\varepsilon)}(u,q)}, P_{\Gamma(\varepsilon)}(v,q).e^{j2\pi\mu_{\Gamma(\varepsilon)}(v,q)}\}\} = \min\{\min\{P_{\Lambda(\varepsilon)}(u,q).e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}, P_{\Gamma(\varepsilon)}(u,q).$
$e^{j2\pi\mu_{\Gamma(\varepsilon)}(u,q)}\}, \min\{P_{\Lambda(\varepsilon)}(v,q).e^{j2\pi\mu_{\Lambda(\varepsilon)}(v,q)}, P_{\Gamma(\varepsilon)}(v,q).e^{j2\pi\mu_{\Gamma(\varepsilon)}(v,q)}\}\},$
$= \min\{P_{\Lambda\cap\Gamma(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda\cap\Gamma(\varepsilon)}(u,q)}, P_{\Lambda\cap\Gamma(\varepsilon)}(v,q)e^{j2\pi\mu_{\Lambda\cap\Gamma(\varepsilon)}(v,q)}\},$
$= \min\{T_{\Lambda\cap\Gamma(\varepsilon)}(u,q), T_{\Lambda\cap\Gamma(\varepsilon)}(v,q)\}.$

For complex valued indeterminacy membership function of the intersection,

$I_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q) = R_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q).e^{j2\pi\nu_{\Lambda\cap\Gamma(\varepsilon)}(uv^{-1},q))} = \max\{R_{\Lambda(\varepsilon)}(uv^{-1},q), R_{\Gamma(\varepsilon)}(uv^{-1},q)$
$\}.e^{j2\pi\max\{\nu_{\Lambda(\varepsilon)}(uv^{-1},q),\nu_{\Gamma(\varepsilon)}(uv^{-1},q)\}} = \max\{R_{\Lambda(\varepsilon)}(uv^{-1},q).e^{j2\pi\nu_{\Lambda(\varepsilon)}(uv^{-1},q)}, R_{\Gamma(\varepsilon)}(uv^{-1},q).$
$e^{j2\pi\nu_{\Gamma(\varepsilon)}(uv^{-1},q)}\} \leq \max\{\max\{R_{\Lambda(\varepsilon)}(u,q).e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}, R_{\Lambda(\varepsilon)}(v,q).e^{j2\pi\nu_{\Lambda(\varepsilon)}(v,q)}\}, \max\{R_{\Gamma(\varepsilon)}$
$(u,q).e^{j2\pi\nu_{\Gamma(\varepsilon)}(u,q)}, R_{\Gamma(\varepsilon)}(v,q).e^{j2\pi\nu_{\Gamma(\varepsilon)}(v,q)}\}\} = \max\{\max\{R_{\Lambda(\varepsilon)}(u,q).e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}, R_{\Gamma(\varepsilon)}(u,q).$
$e^{j2\pi\nu_{\Gamma(\varepsilon)}(u,q)}\}, \max\{R_{\Lambda(\varepsilon)}(v,q).e^{j2\pi\nu_{\Lambda(\varepsilon)}(v,q)}, R_{\Gamma(\varepsilon)}(v,q).e^{j2\pi\nu_{\Gamma(\varepsilon)}(v,q)}\}\},$
$= \max\{R_{\Lambda\cap\Gamma(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda\cap\Gamma(\varepsilon)}(u,q)}, R_{\Lambda\cap\Gamma(\varepsilon)}(v,q)e^{j2\pi\nu_{\Lambda\cap\Gamma(\varepsilon)}(v,q)}\},$
$= \max\{I_{\Lambda\cap\Gamma(\varepsilon)}(u,q), I_{\Lambda\cap\Gamma(\varepsilon)}(v,q)\}.$

For falsity, it can be proven in the same way as in indeterminacy. This completes the proof. □

Definition 4.9. Let U and Q be two non empty sets and (Λ,Υ) be a Q-CNSS in U such that $(\Lambda,\Upsilon) = \{<\varepsilon; T_{\Lambda(\varepsilon)}(u,q), I_{\Lambda(\varepsilon)}(u,q), F_{\Lambda(\varepsilon)}(u,q) >: \varepsilon\in\Upsilon, u\in U, q\in Q\}$, where $\forall\varepsilon\in\Upsilon, u\in U, q\in Q, T_{\Lambda(\varepsilon)}(u,q) = P_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}$, $I_{\Lambda(\varepsilon)}(u,q) = R_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$ and $F_{\Lambda(\varepsilon)}(u,q) = S_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\omega_{\Lambda(\varepsilon)}(u,q)}$. Then $\forall\alpha,\beta\in[0,1]$, the (α,β)-level set of (Λ,Υ), denoted by $(\Lambda,\Upsilon)_{(\alpha,\beta)}$ is a soft set in U defined as:

$$(\Lambda,\Upsilon)_{(\alpha,\beta)} = \{(\varepsilon,\Lambda_{(\alpha,\beta)}(\varepsilon)) : \varepsilon\in\Upsilon, \Lambda_{(\alpha,\beta)}(\varepsilon)\in\rho(U\times Q)\}, \text{ where,}$$

$\Lambda_{(\alpha,\beta)}(\varepsilon) = \{u\in U, q\in Q : P_{\Lambda(\varepsilon)}(u,q)\geq\alpha, R_{\Lambda(\varepsilon)}(u,q)\leq\alpha, S_{\Lambda(\varepsilon)}(u,q)\leq\alpha$ and $\mu_{\Lambda(\varepsilon)}(u,q)\geq\beta, \nu_{\Lambda(\varepsilon)}(u,q)\leq\beta, \omega_{\Lambda(\varepsilon)}(u,q)\leq\beta\}, \forall\varepsilon\in\Upsilon$.

If $\alpha=\beta$, then $(\Lambda,\Upsilon)_{(\alpha,\alpha)}$ is called $\alpha-$level set of (Λ,Υ), denoted by $(\Lambda,\Upsilon)_\alpha$ and defined as:

$(\Lambda,\Upsilon)_\alpha = \{(\varepsilon,\Lambda_\alpha(\varepsilon)) : \varepsilon\in\Upsilon, \Lambda_\alpha(\varepsilon)\in\rho(U\times Q)\}$, where,

$\Lambda_\alpha(\varepsilon) = \{u\in U, q\in Q : P_{\Lambda(\varepsilon)}(u,q)\geq\alpha, R_{\Lambda(\varepsilon)}(u,q)\leq\alpha, S_{\Lambda(\varepsilon)}(u,q)\leq\alpha$ and $\mu_{\Lambda(\varepsilon)}(u,q)\geq\alpha, \nu_{\Lambda(\varepsilon)}(u,q)\leq\alpha, \omega_{\Lambda(\varepsilon)}(u,q)\leq\alpha\}, \forall\varepsilon\in\Upsilon$.

The next theorem shows the relationship between Q-CNSG and soft group.

Theorem 4.10. *Let* (Λ,Υ) *be a Q-CNSG in G, then* $\forall\varepsilon\in\Upsilon$ *and for arbitrary*

$\alpha,\beta\in[0,1]$, *the* (α,β)*-level set* $(\Lambda,\Upsilon)_{(\alpha,\beta)}$ *is a soft group in G.*

Proof. Let (Λ,Υ) be a Q-CNSG. For arbitrary $\alpha,\beta\in[0,1]$, let $\varepsilon\in\Upsilon, u, v\in\Lambda_{(\alpha,\beta)}(\varepsilon)$ and $q\in Q$.

Then, we have

$P_{\Lambda(\varepsilon)}(u,q)\geq\alpha, R_{\Lambda(\varepsilon)}(u,q)\leq\alpha, S_{\Lambda(\varepsilon)}(u,q)\leq\alpha$ and $\mu_{\Lambda(\varepsilon)}(u,q)\geq\beta, \nu_{\Lambda(\varepsilon)}(u,q)\leq\beta, \omega_{\Lambda(\varepsilon)}(u,q)\leq\beta$ and $P_{\Lambda(\varepsilon)}(v,q)\geq\alpha, R_{\Lambda(\varepsilon)}(v,q)\leq\alpha, S_{\Lambda(\varepsilon)}(v,q)\leq\alpha$ and $\mu_{\Lambda(\varepsilon)}(v,q)\geq\beta, \nu_{\Lambda(\varepsilon)}(v,q)\leq\beta, \omega_{\Lambda(\varepsilon)}(v,q)\leq\beta$.

To show that $uv\in\Lambda_{(\alpha,\beta)}(\varepsilon)$ and $u^{-1}\in\Lambda_{(\alpha,\beta)}(\varepsilon)$, $\forall\varepsilon\in\Upsilon$, we begin with complex valued truth membership function as follows.

$P_{\Lambda(\varepsilon)}(uv,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(uv,q)} = T_{\Lambda(\varepsilon)}(uv,q)\geq\min\{T_{\Lambda(\varepsilon)}(u,q), T_{\Lambda(\varepsilon)}(v,q)\} = \min\{P_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)},$
$P_{\Lambda(\varepsilon)}(v,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(v,q)}\} = \min\{P_{\Lambda(\varepsilon)}(u,q), P_{\Lambda(\varepsilon)}(v,q)\}.$

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$e^{j2\pi \min\{\mu_{\Lambda(\varepsilon)}(u,q),\mu_{\Lambda(\varepsilon)}(v,q)\}}$. This implies that: $P_{\Lambda(\varepsilon)}(uv,q) \geq \min\{P_{\Lambda(\varepsilon)}(u,q), P_{\Lambda(\varepsilon)}(v,q)\} \geq \min\{\alpha,\alpha\} = \alpha$ and $\mu_{\Lambda(\varepsilon)}(uv,q) \geq \min\{\mu_{\Lambda(\varepsilon)}(u,q), \mu_{\Lambda(\varepsilon)}(v,q)\} \geq \min\{\beta,\beta\} = \beta$.

For indeterminacy membership term, we have:

$R_{\Lambda(\varepsilon)}(uv,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(uv,q)} = I_{\Lambda(\varepsilon)}(uv,q) \leq \max\{I_{\Lambda(\varepsilon)}(u,q), I_{\Lambda(\varepsilon)}(v,q)\} = \max\{R_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$, $R_{\Lambda(\varepsilon)}(v,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(v,q)}\} = \max\{R_{\Lambda(\varepsilon)}(u,q), R_{\Lambda(\varepsilon)}(v,q)\}$.
$e^{j2\pi \max\{\nu_{\Lambda(\varepsilon)}(u,q),\nu_{\Lambda(\varepsilon)}(v,q)\}}$. This implies that: $R_{\Lambda(\varepsilon)}(uv,q) \leq \max\{R_{\Lambda(\varepsilon)}(u,q), R_{\Lambda(\varepsilon)}(v,q)\} \leq \max\{\alpha,\alpha\} = \alpha$ and $\nu_{\Lambda(\varepsilon)}(uv,q) \leq \max\{\nu_{\Lambda(\varepsilon)}(u,q), \nu_{\Lambda(\varepsilon)}(v,q)\} \leq \max\{\beta,\beta\} = \beta$.

In the same manner we show that

$S_{\Lambda(\varepsilon)}(uv,q) \leq \alpha$ and $\omega_{\Lambda(\varepsilon)}(uv,q) \leq \beta$. This implies that $uv \in \Lambda_{(\alpha,\beta)}(\varepsilon)$ which satisfies the first condition of the soft group definition. To satisfy the second condition, we begin with complex valued truth membership function as follows.

Let $u \in \Lambda_{(\alpha,\beta)}(\varepsilon)$, then we have $P_{\Lambda(\varepsilon)}(u^{-1},q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u^{-1},q)} = T_{\Lambda(\varepsilon)}(u^{-1},q) \geq T_{\Lambda(\varepsilon)}(u,q) =$ $P_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\mu_{\Lambda(\varepsilon)}(u,q)}$. This implies that $P_{\Lambda(\varepsilon)}(u^{-1},q) \geq P_{\Lambda(\varepsilon)}(u,q) \geq \alpha$ and $\mu_{\Lambda(\varepsilon)}(u^{-1},q) \geq \mu_{\Lambda(\varepsilon)}(u,q) \geq \beta$.

It can also be done in the same way for complex valued indeterminacy membership function as follows:

$R_{\Lambda(\varepsilon)}(u^{-1},q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u^{-1},q)} = I_{\Lambda(\varepsilon)}(u^{-1},q) \leq I_{\Lambda(\varepsilon)}(u,q) = R_{\Lambda(\varepsilon)}(u,q)e^{j2\pi\nu_{\Lambda(\varepsilon)}(u,q)}$. This implies that $R_{\Lambda(\varepsilon)}(u^{-1},q) \leq R_{\Lambda(\varepsilon)}(u,q) \leq \alpha$ and $\nu_{\Lambda(\varepsilon)}(u^{-1},q) \leq \nu_{\Lambda(\varepsilon)}(u,q) \leq \beta$.

In the same manner we show that $S_{\Lambda(\varepsilon)}(u^{-1},q) \leq \alpha$ and $\omega_{\Lambda(\varepsilon)}(u^{-1},q) \leq \beta$. This implies that $u^{-1} \in \Lambda_{(\alpha,\beta)}(\varepsilon), \forall \varepsilon \in \Upsilon$ which satisfies the second condition of the soft group definition. Thus, $(\Lambda,\Upsilon)_{(\alpha,\beta)}$ is a soft group in G. This completes the proof. $\square$

5 Conclusions

In this paper, we proposed a new hybrid concept of Q-CNSS by smelting both Q-CNS and SS in one model in order to collect the pros of both Q-CNS and SS in one model. The essential operations of this model have been suggested. Furthermore, some algebraic structures on this model were investigated when we proposed the notions of Q-CNSG and Q-CNS subgroup. Based on these structures, we derived the essential theorems that show the relationship of these structures with soft groups.Finally, for further work on these topics, We recommend developing these tools by integrating them with some other mathematical structures, such as the hypersoft set, algebraic structures, topological structures, and other ideas[52] -.[61]

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