



MADM Strategy Application of Bipolar Single Valued Heptapartitioned Neutrosophic Set

Myvizhi M.¹, Ahmed M. Ali^{2*}, Ahmed Abdelhafeez³, Haitham Rizk Fadlallah⁴

¹Department of Mathematics, KPR Institute of Engineering and Technology, Coimbatore, Tamilnadu, India

² Faculty of Computers and Informatics, Zagazig University, Zagazig 44519, Sharqiyah, Egypt

^{3,4}Faculty of Information Systems and Computer Science, October 6th University, Cairo, Egypt

Emails: myvizhi.m@kpriet.ac.in; aabdelmounem@zu.edu.eg; aahafeez.scis@o6u.edu.eg; Haitham.rizk.csis@o6u.edu.eg

Abstract

The fundamental goal of this study is to propose the concept of a bipolar single-valued heptapartitioned neutrosophic set (BSVHNS). We also outline the fundamental of BSVHNS traits and illustrate a few sample theorems. We define the fundamentals of the properties of the accuracy and scoring functions for the BSVHNS. The bipolar single-valued heptapartitioned mean in neutrosophic arithmetic (BSVHMNA) operator and the bipolar single-valued heptapartitioned mean in neutrosophic geometric (BSVHMNG) operator are defined and their fundamental properties are established in this article. We develop two Multi-Attribute Decision Making (MADM) strategies in the context of the BSVHNS environment: One is BSVHNS-MADM strategy which is on the BSVHMNA operator and another one is BSVHNS-MADM strategy which is on the BSVHMNG operator. Finally, we demonstrate the effectiveness of the developed procedures using a numerical example drawn from the actual world.

Keywords: Heptapartitioned set; Heptapartitioned Neutrosophic set; Bipolar single valued Heptapartitioned set; Bipolar single valued Heptapartitioned Neutrosophic set; MADM-Strategy.

1. Introduction

Fuzzy set theory was introduced by Zadeh [31]. It has widely used in uncertain situations for solving the problems. Atanassov [2, 3] introduced the concept of an intuitionistic fuzzy (IF) set characterized by a membership function and a non-membership function. Decision making is a process that is related as final outcome of decision problems and helps decision makers (DMs) for the selection of suitable alternative or a set of alternatives. In reality, researchers often focus on decision-making problems in uncertain and inexact situations. The multiple attribute decision making (MADM) has created an efficient frame for the comparison respecting to the assessment of multiple incompatible attributes. To address the uncertainty, indeterminacy, and inconsistent nature of this actual world of mathematical objects, Smarandache [25] defined the Neutrosophic set. Fuzzy set and Intuitionistic fuzzy set are the most generalized form of neutrosophic set by including levels of indeterminacy and rejection as independent components.

Smarandache proposed the concept of NS based on the FS and its extended notions (interval valued FS, intuitionistic FS, and so on) by adding an independent indeterminacy association function to the existing IFs model presented by Atanassov. Several NS extensions and special instances have been proposed in the literature. These situations include the single valued neutrosophic sets (SVNS), interval neutrosophic sets (INs), Neutrosophic Soft Set (NSS), INSS, Refined Neutrosophic Set (RNS), bipolar neutrosophic sets (BNS), and neutrosophic cube set. NSs have recently emerged as an intriguing study area that has garnered widespread interest. The introduction of SVNSs and INs is one of the most significant advances in the research of NS.

Wang et al. [29] introduced the Single Valued Neutrosophic Set in 2010. (SVNS). In many fields, air surveillance included [8], Dispute settlement [17], decision making [9-13], error diagnosis [30], segmenting an image [15] and others, the SVNSs, as well as its variants and extensions, have been used. In the works, specifics of NS applications

and theoretical advancements are presented [4, 16, 18, 19, 26, 27].

The Bipolar Single Valued Neutrosophic Set is defined by Deli et al. [6] (SVBNS). Later, a great deal of researchers used the idea of SVBNS in the creation of models for Multi Attribute Decision Making (MADM) [1, 7, 20, 21, 22] issues. The concept of the Heptapartitioned Neutrosophic Set (HNS), which included seven separate components, was founded by Radha et al. [23] in 2021.

The Bipolar Single-Valued Heptapartitioned Neutrosophic Set (BSVHNS), created in this study by combining BSVNS and HNS, is introduced. Next, we define some of BSVHNS's fundamental characteristics. On the BSVHNS, a few illustrated instances are also given. Additionally, we suggest a few aggregation operators and demonstrate their fundamental characteristics. Also, in the BSVHNS context, we design two additional MADM techniques.

The rest of this article's description is as follows:

Several pertinent findings on HNS are displayed in Section 2. The BSVHNS is first mentioned in Section 3. The bipolar single-valued heptapartitioned mean neutrosophic arithmetic operator and the bipolar single-valued heptapartitioned mean neutrosophic geometric operator are two aggregating operators that are introduced in Section 4 of the paper. We obtain the concepts of the score function and accuracy function in the BSVHNS environment in Section 5. In the bipolar single-valued heptapartitioned neutrosophic arithmetic mean operator using BSVHNS environment, we develop a MADM method in Section 6. The bipolar single-valued heptapartitioned neutrosophic geometric mean operator is used in Section 7 to develop a MADM plan in an BSVHNS environment. We proposed MADM strategies in Section 8 by presenting a practical numerical illustration and contrasting the two MADM procedures. As a method to conclude the work, we state future study in the newly built set environment.

2. Some Preliminary Results

The main concepts of this study, it is important to review some fundamental definitions of the terms Neutrosophic Set, Bipolar Neutrosophic Set, and Heptapartitioned Neutrosophic Set.

Definition 2.1 [25] A The following is a definition of Neutrosophic Set A on X :

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$$

where $T_A, I_A, F_A : U \rightarrow [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership. Here, $T_A(x)$ and $F_A(x)$ are dependent neutrosophic elements and $I_A(x)$ is an independent neutrosophic element.

Definition 2.2 [5] Let X represent a universe. An object of the form is a QNS, A on X with independent neutrosophic components

$$A = \{ \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle, x \in X \}$$

and $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$ Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

Definition 2.3 [24] A non-empty set P shall be used. Each element of P is defined by a PNS over P by a truth-membership function $T_A(x)$, a contradiction membership function $C_A(x)$, an ignorance membership function $G_A(x)$, an unknown membership function $U_A(x)$ and a falsity membership function $F_A(x)$ such that for each $p \in P$, $0 \leq T_A(x) + C_A(x) + G_A(x) + U_A(x) + F_A(x) \leq 5$.

Definition 2.4 [23] Let R be a non-empty Universe. A Heptapartitioned neutrosophic set (HNS) A over R characterizes each element p in R by an absolute truth-membership function T_A , a relative truth membership function M_A , a contradiction membership function C_A , an ignorance membership function I_A , an unknown membership function U_A , an absolute falsity membership function F_A and a relative falsity membership function K_A such that for each $p \in R$, $T_A, M_A, C_A, I_A, U_A, F_A, K_A \in [0, 1]$ and

$$A = [p, T_A(p), M_A(p), C_A(p), I_A(p), U_A(p), F_A(p), K_A(p) : p \in R] \quad 0 \leq T_A(p) + M_A(p) + C_A(p) + I_A(p) + U_A(p) + F_A(p) + K_A(p) \leq 7.$$

Definition 2.5 [23] A Heptapartitioned neutrosophic set A is said to absolute Heptapartitioned neutrosophic set Δ if and only if its absolute truth-membership, a relative truth membership, a contradiction membership, an ignorance membership, an unknown membership, an absolute falsity membership and a relative falsity membership are defined as follows, $T_A(p) = 1, M_A(p) = 1, C_A(p) = 1, U_A(p) = 1, I_A(p) = 0, K_A(p) = 0$ and $F_A(p) = 0$.

Definition 2.6 [23] A Heptapartitioned neutrosophic set A is said to relative Heptapartitioned neutrosophic set \emptyset if and only if its absolute truth-membership, a relative truth membership a contradiction membership, an ignorance membership, an unknown membership, an absolute falsity membership and a relative falsity membership are defined as follows, $T_A(p) = 0, M_A(p) = 0, C_A(p) = 0, U_A(p) = 0, I_A(p) = 1, K_A(p) = 1$ and $F_A(p) = 1$.

Definition 2.7 [23] For any two Heptapartitioned neutrosophic sets A and B over R , A is said to be contained in

B iff $T_A(p) \leq T_B(p), M_A(p) \leq M_B(p), C_A(p) \leq C_B(p), U_A(p) \geq U_B(p), I_A(p) \geq I_B(p), K_A(p) \geq K_B(p)$ and $F_A(p) \geq F_B(p)$.

Definition 2.8 [25] The complement of Heptapartitioned neutrosophic sets A over the universe R is denoted by A^c and is defined as $A^c = [(p, F_A(p), K_A(p), I_A(p), 1 - U_A(p), C_A(p), M_A(p), T_A(p) : \forall p \in R]$.

Definition 2.9 [23] The union of any two Heptapartitioned neutrosophic sets A and B over R is denoted by $A \cup B$ and is defined as

$$A \cup B = [p, (\max(T_A(p), T_B(p)), \max(M_A(p), M_B(p)), \max(C_A(p), C_B(p)), \min(U_A(p), U_B(p)), \min(I_A(p), I_B(p)), \min(K_A(p), K_B(p)) \text{ and } \min(F_A(p), F_B(p))) : p \in R].$$

Definition 2.10 [23] The intersection of any two Heptapartitioned neutrosophic sets A and B over R is denoted by $A \cap B$ and is defined as

$$A \cap B = [p, (\min(T_A(p), T_B(p)), \min(M_A(p), M_B(p)), \min(C_A(p), C_B(p)), \max(U_A(p), U_B(p)), \max(I_A(p), I_B(p)), \max(K_A(p), K_B(p)) \text{ and } \max(F_A(p), F_B(p))) : p \in R].$$

Example 2.1 Consider two HNSs over R , given as

$$\begin{aligned} A &= [0.4, 0.3, 0.5, 0.6, 0.4, 0.2, 0.7]/p_1 + [0.6, 0.2, 0.9, 0.4, 0.7, 0.5, 0.2]/p_2 + [0.4, 0.2, 0.1, 0.9, 0.4, 0.5, 0.7]/p_3 \\ B &= [0.6, 0.5, 0.5, 0.4, 0.2, 0.4, 0.9]/p_1 + [0.2, 0.4, 0.1, 0.6, 0.4, 0.3, 0.5]/p_2 + [0.7, 0.4, 0.3, 0.4, 0.2, 0.1, 0.5]/p_3 \\ A^c &= [0.7, 0.2, 0.4, 0.4, 0.5, 0.3, 0.4]/p_1 + [0.2, 0.5, 0.7, 0.6, 0.9, 0.2, 0.6]/p_2 + [0.7, 0.5, 0.4, 0.1, 0.1, 0.2, 0.4]/p_3 \\ A \cup B &= [0.6, 0.5, 0.5, 0.4, 0.2, 0.2, 0.7]/p_1 + [0.6, 0.4, 0.9, 0.4, 0.4, 0.3, 0.2]/p_2 + [0.7, 0.4, 0.3, 0.4, 0.2, 0.1, 0.5]/p_3 \\ A \cap B &= [0.4, 0.3, 0.5, 0.6, 0.4, 0.4, 0.9]/p_1 + [0.2, 0.2, 0.1, 0.6, 0.7, 0.5, 0.5]/p_2 + [0.4, 0.2, 0.1, 0.9, 0.4, 0.5, 0.7]/p_3 \end{aligned}$$

Definition 2.11 [28] Suppose that $\lambda_1, \lambda_2, \dots, \lambda_n$ be n real numbers. The arithmetic mean (AM) of $\lambda_1, \lambda_2, \dots, \lambda_n$ is specified by $AM(\lambda_1, \lambda_2, \dots, \lambda_n) = \frac{1}{n} \sum_{i=1}^n \lambda_i$.

Definition 2.12 [28] Suppose that $\lambda_1, \lambda_2, \dots, \lambda_n$ be n real numbers. The geometric mean (GM) of $\lambda_1, \lambda_2, \dots, \lambda_n$ is specified by $GM(\lambda_1, \lambda_2, \dots, \lambda_n) = (\prod_{i=1}^n \lambda_i)^{\frac{1}{n}}$.

3. Bipolar Single-Valued Heptapartitioned Neutrosophic Set

We obtain the idea of BSVHNS in this section. We also look into many aspects of these kinds of sets properties. A few additional instances are provided as well.

Definition 3.1 A bipolar single-valued heptapartitioned neutrosophic set H over a non-empty set ϕ is specified as: $H = \{(\lambda, T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda), T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda) : \lambda \in \phi\}$, where $T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda) \in [-1, 0]$ and $T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda) \in [0, 1]$.

The negative membership degrees $T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda)$ indicate the degree of absolute truth-membership function T_H , a relative truth membership function M_H , a contradiction membership function C_H , an ignorance membership function I_H , an unknown membership function U_H , an absolute falsity membership function F_A and a relative falsity membership function K_H respectively for $\lambda \in \phi$ corresponding to an BSVHNS. Again, the positive membership degrees, $T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda)$ indicate as same in the above membership functions of corresponding to an BSVHNS.

Example 3.1

Let $\phi = \{p, q\}$ be a fixed set. Then, $P = \{(p, -0.4, -0.2, -0.3, -0.5, -0.3, -0.2, -0.4, 0.3, 0.2, 0.4, 0.7, 0.1, 0.3, 0.5), (q, -0.3, -0.3, -0.6, -0.5, -0.2, -0.2, -0.7, 0.3, 0.2, 0.7, 0.5, 0.1, 0.6, 0.4)\}$ is an BSVHNS over λ .

Definition 3.2

Let $H = \{(\lambda, T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda), T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda) : \lambda \in \phi\}$ be an BSVHNS.

Then, $[T_H^-(\lambda), M_H^-(\lambda), C_H^-(\lambda), U_H^-(\lambda), I_H^-(\lambda), K_H^-(\lambda), F_H^-(\lambda), T_H^+(\lambda), M_H^+(\lambda), C_H^+(\lambda), U_H^+(\lambda), I_H^+(\lambda), K_H^+(\lambda), F_H^+(\lambda)]$ is called a bipolar single-valued heptapartitioned neutrosophic number (BSVHNN), for each $\lambda \in \phi$.

Definition 3.3 Suppose that

$$P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda) : \lambda \in \phi\}$$
 and

$Q = \{(\lambda, T_Q^-(\lambda), M_Q^-(\lambda), C_Q^-(\lambda), U_Q^-(\lambda), I_Q^-(\lambda), K_Q^-(\lambda), F_Q^-(\lambda), T_Q^+(\lambda), M_Q^+(\lambda), C_Q^+(\lambda), U_Q^+(\lambda), I_Q^+(\lambda), K_Q^+(\lambda), F_Q^+(\lambda)) : \lambda \in \phi\}$ be any two BSVHNS over ϕ .

Example 3.2 Given two BSVHNS's

$P = \{(p, -0.4, -0.5, -0.2, -0.5, -0.2, -0.4, -0.2, 0.2, 0.4, 0.5, 0.4, 0.5, 0.3, 0.4), (q, -0.3, -0.4, -0.3, -0.6, -0.3, -0.5, -0.1)\}$ and

$Q = \{(p, -0.4, -0.4, -0.1, -0.6, -0.4, -0.6, -0.3, 0.4, 0.7, 0.6, 0.2, 0.3, 0.1, 0.3), (q, -0.2, -0.4, -0.1, -0.8, -0.4, -0.6, -0.2, 0.1, 0.5, 0.7, 0.1, 0.2, 0.2, 0.3)\}$ over $\phi = \{p, q\}$. Then $P \subseteq Q$.

Definition 3.4 Given two BSVHNS's

$P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda)) : \lambda \in \phi\}$ and

$Q = \{(\lambda, T_Q^-(\lambda), M_Q^-(\lambda), C_Q^-(\lambda), U_Q^-(\lambda), I_Q^-(\lambda), K_Q^-(\lambda), F_Q^-(\lambda), T_Q^+(\lambda), M_Q^+(\lambda), C_Q^+(\lambda), U_Q^+(\lambda), I_Q^+(\lambda), K_Q^+(\lambda), F_Q^+(\lambda)) : \lambda \in \phi\}$ be any two BSVHNS over ϕ .

Then, the intersection of P and Q is determined by:

$$P \cap Q = \{(\lambda, \min(T_P^-(\lambda), T_Q^-(\lambda)), \min(M_P^-(\lambda), M_Q^-(\lambda)), \min(C_P^-(\lambda), C_Q^-(\lambda)), \max(U_P^-(\lambda), U_Q^-(\lambda)), \max(I_P^-(\lambda), I_Q^-(\lambda)), \max(K_P^-(\lambda), K_Q^-(\lambda)), \max(F_P^-(\lambda), F_Q^-(\lambda)), \min(T_P^+(\lambda), T_Q^+(\lambda)), \min(M_P^+(\lambda), M_Q^+(\lambda)), \min(C_P^+(\lambda), C_Q^+(\lambda)), \max(U_P^+(\lambda), U_Q^+(\lambda)), \max(I_P^+(\lambda), I_Q^+(\lambda)), \max(K_P^+(\lambda), K_Q^+(\lambda)), \max(F_P^+(\lambda), F_Q^+(\lambda)) : \lambda \in \phi\}.$$

Example 3.3

Given P and Q are two BSVHNS over $\phi = \{p, q\}$ such that

$P = \{(p, -0.7, -0.3, -0.4, -0.4, -0.3, -0.2, -0.6, 0.4, 0.5, 0.2, 0.4, 0.2, 0.3, 0.5), (q, -0.6, -0.4, -0.4, -0.3, -0.3, -0.2, -0.2, 0.3, 0.5, 0.4, 0.4, 0.7, 0.5, 0.4)\}$ and

$Q = \{(p, -0.6, -0.3, -0.5, -0.3, -0.6, -0.7, -0.5, 0.6, 0.1, 0.2, 0.2, 0.1, 0.4, 0.5), (q, -0.5, -0.4, -0.4, -0.5, -0.7, -0.5, -0.4, 0.5, 0.5, 0.7, 0.4, 0.4, 0.1, 0.2)\}$.

Then, their intersection is

$P \cap Q = \{(p, -0.7, -0.3, -0.5, -0.3, -0.3, -0.2, -0.5, 0.4, 0.1, 0.2, 0.4, 0.2, 0.4, 0.5), (q, -0.6, -0.4, -0.4, -0.3, -0.3, -0.2, -0.2, 0.3, 0.5, 0.4, 0.4, 0.7, 0.5, 0.4)\}$.

Definition 3.5 Given P and Q are two BSVHNS,

$P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda)) : \lambda \in \phi\}$ and

and

$Q = \{(\lambda, T_Q^-(\lambda), M_Q^-(\lambda), C_Q^-(\lambda), U_Q^-(\lambda), I_Q^-(\lambda), K_Q^-(\lambda), F_Q^-(\lambda), T_Q^+(\lambda), M_Q^+(\lambda), C_Q^+(\lambda), U_Q^+(\lambda), I_Q^+(\lambda), K_Q^+(\lambda), F_Q^+(\lambda)) : \lambda \in \phi\}$ be any two BSVHNS over ϕ . Then, the union of P and Q is defined by:

$$P \cup Q = \{(\lambda, \max(T_P^-(\lambda), T_Q^-(\lambda)), \max(M_P^-(\lambda), M_Q^-(\lambda)), \max(C_P^-(\lambda), C_Q^-(\lambda)), \min(U_P^-(\lambda), U_Q^-(\lambda)), \min(I_P^-(\lambda), I_Q^-(\lambda)), \min(K_P^-(\lambda), K_Q^-(\lambda)), \min(F_P^-(\lambda), F_Q^-(\lambda)), \max(T_P^+(\lambda), T_Q^+(\lambda)), \max(M_P^+(\lambda), M_Q^+(\lambda)), \max(C_P^+(\lambda), C_Q^+(\lambda)), \min(U_P^+(\lambda), U_Q^+(\lambda)), \min(I_P^+(\lambda), I_Q^+(\lambda)), \min(K_P^+(\lambda), K_Q^+(\lambda)), \min(F_P^+(\lambda), F_Q^+(\lambda)) : \lambda \in \phi\}.$$

Example 3.4

Given P and Q are two BSVHNS s over $\phi = \{p, q\}$ such that

$P = \{(p, -0.3, -0.5, -0.2, -0.6, -0.2, -0.4, -0.3, 0.2, 0.3, 0.5, 0.3, 0.2, 0.4, 0.6), (q, -0.2, -0.4, -0.4, -0.5, -0.2, -0.3, -0.4, 0.2, 0.4, 0.5, 0.2, 0.2, 0.3, 0.8)\}$ and

$Q = \{(p, -0.3, -0.2, -0.4, -0.6, -0.2, -0.1, -0.5, 0.3, 0.1, 0.3, 0.6, 0.3, 0.1, 0.6), (q, -0.4, -0.5, -0.5, -0.4, -0.3, -0.4, -0.1, 0.4, 0.2, 0.3, 0.2, 0.1, 0.3, 0.5)\}$.

Then, their union is

$P \cup Q = \{(p, -0.3, -0.2, -0.2, -0.6, -0.2, -0.4, -0.5, 0.3, 0.3, 0.5, 0.3, 0.2, 0.1, 0.6), (q, -0.2, -0.4, -0.4, -0.5, -0.3, -0.4, -0.4, 0.4, 0.4, 0.5, 0.2, 0.1, 0.3, 0.5)\}$.

Definition 3.6

Let $P = \{(\lambda, T_P^-(\lambda), M_P^-(\lambda), C_P^-(\lambda), U_P^-(\lambda), I_P^-(\lambda), K_P^-(\lambda), F_P^-(\lambda), T_P^+(\lambda), M_P^+(\lambda), C_P^+(\lambda), U_P^+(\lambda), I_P^+(\lambda), K_P^+(\lambda), F_P^+(\lambda)) : \lambda \in \phi\}$ be an BSVHNSs over ϕ . Then, P^c is defined as:

$P^c = \{(\lambda, F_P^-(\lambda), K_P^-(\lambda), I_P^-(\lambda), 1 - U_P^-(\lambda), C_P^-(\lambda), M_P^-(\lambda), T_P^-(\lambda), F_P^+(\lambda), K_P^+(\lambda), I_P^+(\lambda), 1 - U_P^+(\lambda), C_P^+(\lambda), M_P^+(\lambda), T_P^+(\lambda)) : \lambda \in \phi\}$.

Example 3.5

Given $P = \{(p, -0.4, -0.2, -0.3, -0.5, -0.3, -0.2, -0.4, 0.3, 0.2, 0.4, 0.7, 0.1, 0.3, 0.5), (q, -0.3, -0.3, -0.6, -0.5, -0.2, -0.2, -0.7, 0.3, 0.2, 0.7, 0.5, 0.1, 0.6, 0.4)\}$ be an BSVHNS over $\lambda = \{p, q\}$.

Then, P^c is

$P^c = \{(p, -0.4, -0.2, -0.3, -0.5, -0.3, -0.2, -0.4, 0.5, 0.3, 0.1, 0.3, 0.4, 0.2, 0.3), (q, -0.7, -0.2, -0.2, -0.5, -0.6, -0.3, -0.3, 0.4, 0.6, 0.1, 0.5, 0.7, 0.2, 0.3)\}$.

Definition 3.7

The null BSVHNS (0^{BHN}) and the absolute BSVHNS (1^{BHN}) over ϕ are specified as given below:

- (i) $0^{BHN} = \{(\lambda, 1, 1, 1, 0, 0, 0, 0, -1, -1, -1, 0, 0, 0, 0) : \lambda \in \phi\}$;
- (ii) $1^{BHN} = \{(\lambda, 0, 0, 0, -1, -1, -1, -1, 0, 0, 0, 1, 1, 1, 1) : \lambda \in \phi\}$;

It is clearly know that,

- (i) $0^{BHN} \subseteq X \subseteq 1^{BHN}$, where X is an BSVHNS over ϕ ;
- (ii) $0^{BHNc} = 1^{BHN}$ & $1^{BHNc} = 0^{BHN}$;
- (iii) $0^{BHN} \cup 1^{BHN} = 1^{BHN}$;
- (iv) $0^{BHN} \cap 1^{BHN} = 0^{BHN}$.

Definition 3.8 Given

$\lambda = [T_{\phi}^{-}(\lambda), M_{\phi}^{-}(\lambda), C_{\phi}^{-}(\lambda), U_{\phi}^{-}(\lambda), I_{\phi}^{-}(\lambda), K_{\phi}^{-}(\lambda), F_{\phi}^{-}(\lambda), T_{\phi}^{+}(\lambda), M_{\phi}^{+}(\lambda), C_{\phi}^{+}(\lambda), U_{\phi}^{+}(\lambda), I_{\phi}^{+}(\lambda), K_{\phi}^{+}(\lambda), F_{\phi}^{+}(\lambda)]$ and $\gamma = [T_{\phi}^{-}(\gamma), M_{\phi}^{-}(\gamma), C_{\phi}^{-}(\gamma), U_{\phi}^{-}(\gamma), I_{\phi}^{-}(\gamma), K_{\phi}^{-}(\gamma), F_{\phi}^{-}(\gamma), T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\gamma)]$ be two BSVHNSs. Then,

$$(i)k. \lambda = [-(-T_{\phi}^{-}(\lambda))^k, -(-M_{\phi}^{-}(\lambda))^k, -(-C_{\phi}^{-}(\lambda))^k, -(1 - (1 - (-U_{\phi}^{-}(\lambda))^k)), -(-I_{\phi}^{-}(\lambda))^k, -(-K_{\phi}^{-}(\lambda))^k, -(-F_{\phi}^{-}(\lambda))^k, 1 - (1 - (T_{\phi}^{+}(\lambda))^k), 1 - (1 - (M_{\phi}^{+}(\lambda))^k), 1 - (1 - (C_{\phi}^{+}(\lambda))^k), (U_{\phi}^{+}(\lambda))^k, 1 - (1 - (I_{\phi}^{+}(\lambda))^k), 1 - (1 - (K_{\phi}^{+}(\lambda))^k), 1 - (1 - (F_{\phi}^{+}(\lambda))^k)]$$

where $k > 0$.

$$(ii)\lambda^k = [-(-T_{\phi}^{-}(\lambda))^k, -(-M_{\phi}^{-}(\lambda))^k, -(-C_{\phi}^{-}(\lambda))^k, -(1 - (1 - (-U_{\phi}^{-}(\lambda))^k)), -(-I_{\phi}^{-}(\lambda))^k, -(-K_{\phi}^{-}(\lambda))^k, -(-F_{\phi}^{-}(\lambda))^k, 1 - (1 - (T_{\phi}^{+}(\lambda))^k), 1 - (1 - (M_{\phi}^{+}(\lambda))^k), 1 - (1 - (C_{\phi}^{+}(\lambda))^k), (U_{\phi}^{+}(\lambda))^k, 1 - (1 - (I_{\phi}^{+}(\lambda))^k), 1 - (1 - (K_{\phi}^{+}(\lambda))^k), 1 - (1 - (F_{\phi}^{+}(\lambda))^k)]$$

where $k > 0$.

$$(iii) \lambda + \gamma = [-(-T_{\phi}^{-}(\lambda) - T_{\phi}^{-}(\gamma) - T_{\phi}^{-}(\lambda).T_{\phi}^{-}(\gamma)), -(-M_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\gamma) - M_{\phi}^{-}(\lambda).M_{\phi}^{-}(\gamma)), -(-C_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\gamma) - C_{\phi}^{-}(\lambda).C_{\phi}^{-}(\gamma)), -U_{\phi}^{-}(\lambda).U_{\phi}^{-}(\gamma), -(-I_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\gamma) - I_{\phi}^{-}(\lambda).I_{\phi}^{-}(\gamma)), -(-K_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\gamma) - K_{\phi}^{-}(\lambda).K_{\phi}^{-}(\gamma)), -(-F_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\gamma) - F_{\phi}^{-}(\lambda).F_{\phi}^{-}(\gamma)), T_{\phi}^{+}(\lambda).T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\lambda).M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\lambda).C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\lambda) + U_{\phi}^{+}(\gamma) - U_{\phi}^{+}(\lambda).U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\lambda).I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\lambda).K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\lambda).F_{\phi}^{+}(\gamma)]$$

$$(iv)\lambda. \gamma = [-T_{\phi}^{-}(\lambda).T_{\phi}^{-}(\gamma), -M_{\phi}^{-}(\lambda).M_{\phi}^{-}(\gamma), -C_{\phi}^{-}(\lambda).C_{\phi}^{-}(\gamma), -(-U_{\phi}^{-}(\lambda) - U_{\phi}^{-}(\gamma) - U_{\phi}^{-}(\lambda).U_{\phi}^{-}(\gamma)), -I_{\phi}^{-}(\lambda).I_{\phi}^{-}(\gamma), -K_{\phi}^{-}(\lambda).K_{\phi}^{-}(\gamma), -F_{\phi}^{-}(\lambda).F_{\phi}^{-}(\gamma), T_{\phi}^{+}(\lambda) + T_{\phi}^{+}(\gamma) - T_{\phi}^{+}(\lambda).T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\lambda) + M_{\phi}^{+}(\gamma) - M_{\phi}^{+}(\lambda).M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\lambda) + C_{\phi}^{+}(\gamma) - C_{\phi}^{+}(\lambda).C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\lambda).U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\lambda) + I_{\phi}^{+}(\gamma) - I_{\phi}^{+}(\lambda).I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\lambda) + K_{\phi}^{+}(\gamma) - K_{\phi}^{+}(\lambda).K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\lambda) + F_{\phi}^{+}(\gamma) - F_{\phi}^{+}(\lambda).F_{\phi}^{+}(\gamma)]$$

4. Bipolar Single-Valued Heptapartitioned Neutrosophic Operators of Aggregation

Definition 4.1 Assume that

$\lambda_i = [T_{\phi}^{-}(\lambda_i), M_{\phi}^{-}(\lambda_i), C_{\phi}^{-}(\lambda_i), U_{\phi}^{-}(\lambda_i), I_{\phi}^{-}(\lambda_i), K_{\phi}^{-}(\lambda_i), F_{\phi}^{-}(\lambda_i), T_{\phi}^{+}(\lambda_i), M_{\phi}^{+}(\lambda_i), C_{\phi}^{+}(\lambda_i), U_{\phi}^{+}(\lambda_i), I_{\phi}^{+}(\lambda_i), K_{\phi}^{+}(\lambda_i), F_{\phi}^{+}(\lambda_i)]$, $i = 1, 2, 3, \dots, n$ be a group of BSVHNSs, over ϕ . The bipolar single-valued heptapartitioned mean in neutrosophic arithmetic (BSVHMNA) operator is determined as follows:

$$BSVHMNA(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n) = \frac{1}{n} \sum_{i=1}^n \lambda_i \dots \dots \dots (1)$$

Theorem 4.1 Assume that

$\lambda_i = [T_{\phi}^{-}(\lambda_i), M_{\phi}^{-}(\lambda_i), C_{\phi}^{-}(\lambda_i), U_{\phi}^{-}(\lambda_i), I_{\phi}^{-}(\lambda_i), K_{\phi}^{-}(\lambda_i), F_{\phi}^{-}(\lambda_i), T_{\phi}^{+}(\lambda_i), M_{\phi}^{+}(\lambda_i), C_{\phi}^{+}(\lambda_i), U_{\phi}^{+}(\lambda_i), I_{\phi}^{+}(\lambda_i), K_{\phi}^{+}(\lambda_i), F_{\phi}^{+}(\lambda_i)]$, $i = 1, 2, 3, \dots, n$ be a group of BSVHNSs, over ϕ . The combined value BSVHMNA $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n)$ is also an BSVHNS.

Proof: Assume that

$\lambda_i = [T_{\phi}^{-}(\lambda_i), M_{\phi}^{-}(\lambda_i), C_{\phi}^{-}(\lambda_i), U_{\phi}^{-}(\lambda_i), I_{\phi}^{-}(\lambda_i), K_{\phi}^{-}(\lambda_i), F_{\phi}^{-}(\lambda_i), T_{\phi}^{+}(\lambda_i), M_{\phi}^{+}(\lambda_i), C_{\phi}^{+}(\lambda_i), U_{\phi}^{+}(\lambda_i), I_{\phi}^{+}(\lambda_i), K_{\phi}^{+}(\lambda_i), F_{\phi}^{+}(\lambda_i)]$, $i = 1, 2, 3, \dots, n$ be a finite group of BSVHNSs, over ϕ . Therefore, λ_i is an BSVHNS. Now,

$$\begin{aligned} \sum_{i=1}^2 \lambda_i &= (\lambda_1 + \lambda_2) \\ &= [(-T_{\phi}^{-}(\lambda_1) - T_{\phi}^{-}(\lambda_2) - T_{\phi}^{-}(\lambda_1) \cdot T_{\phi}^{-}(\lambda_2)), -(-M_{\phi}^{-}(\lambda_1) - M_{\phi}^{-}(\lambda_2) - M_{\phi}^{-}(\lambda_1) \cdot M_{\phi}^{-}(\lambda_2)), \\ &\quad -(-C_{\phi}^{-}(\lambda_1) - C_{\phi}^{-}(\lambda_2) - C_{\phi}^{-}(\lambda_1) \cdot C_{\phi}^{-}(\lambda_2)), -U_{\phi}^{-}(\lambda_1) \cdot U_{\phi}^{-}(\lambda_2), -(-I_{\phi}^{-}(\lambda_1) - I_{\phi}^{-}(\lambda_2) - I_{\phi}^{-}(\lambda_1) \cdot I_{\phi}^{-}(\lambda_2)), \\ &\quad -(-K_{\phi}^{-}(\lambda_1) - K_{\phi}^{-}(\lambda_2) - K_{\phi}^{-}(\lambda_1) \cdot K_{\phi}^{-}(\lambda_2)), -(-F_{\phi}^{-}(\lambda_1) - F_{\phi}^{-}(\lambda_2) - F_{\phi}^{-}(\lambda_1) \cdot F_{\phi}^{-}(\lambda_2)), \\ &\quad (T_{\phi}^{+}(\lambda_1) \cdot T_{\phi}^{+}(\lambda_2), M_{\phi}^{+}(\lambda_1) \cdot M_{\phi}^{+}(\lambda_2), C_{\phi}^{+}(\lambda_1) \cdot C_{\phi}^{+}(\lambda_2), U_{\phi}^{+}(\lambda_1) + U_{\phi}^{+}(\lambda_2) - U_{\phi}^{+}(\lambda_1) \cdot U_{\phi}^{+}(\lambda_2), \\ &\quad I_{\phi}^{+}(\lambda_1) \cdot I_{\phi}^{+}(\lambda_2), K_{\phi}^{+}(\lambda_1) \cdot K_{\phi}^{+}(\lambda_2), F_{\phi}^{+}(\lambda_1) \cdot F_{\phi}^{+}(\lambda_2))] \\ &= [T_{\phi}^{-}(\lambda_1, \lambda_2), M_{\phi}^{-}(\lambda_1, \lambda_2), C_{\phi}^{-}(\lambda_1, \lambda_2), U_{\phi}^{-}(\lambda_1, \lambda_2), I_{\phi}^{-}(\lambda_1, \lambda_2), K_{\phi}^{-}(\lambda_1, \lambda_2), F_{\phi}^{-}(\lambda_1, \lambda_2), T_{\phi}^{+}(\lambda_1, \lambda_2), M_{\phi}^{+}(\lambda_1, \lambda_2), \\ &\quad C_{\phi}^{+}(\lambda_1, \lambda_2), U_{\phi}^{+}(\lambda_1, \lambda_2), I_{\phi}^{+}(\lambda_1, \lambda_2), K_{\phi}^{+}(\lambda_1, \lambda_2), F_{\phi}^{+}(\lambda_1, \lambda_2)] \text{ (say), which is an BSVHNN.} \end{aligned}$$

Assume that $\sum_{i=1}^n \lambda_i$ is an BSVHNN over λ for $n = m$,

$$\begin{aligned} \text{i.e., } \sum_{i=1}^m \lambda_i &= [T_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), M_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), C_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), U_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), \\ &\quad I_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), K_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), F_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), T_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), M_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), \\ &\quad C_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), U_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), I_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), K_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), F_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m)] \end{aligned}$$

is an BSVHNN.

$$\begin{aligned} \sum_{i=1}^{m+1} \lambda_i &= \sum_{i=1}^m \lambda_i + \lambda_{m+1} \\ &= [T_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), M_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), C_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), U_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), I_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), \\ &\quad I_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), K_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), F_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m), T_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), M_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), \\ &\quad C_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), U_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), I_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), K_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m), F_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m)] \\ &\quad + [T_{\phi}^{-}(\lambda_{m+1}), M_{\phi}^{-}(\lambda_{m+1}), C_{\phi}^{-}(\lambda_{m+1}), U_{\phi}^{-}(\lambda_{m+1}), I_{\phi}^{-}(\lambda_{m+1}), I_{\phi}^{-}(\lambda_{m+1}), K_{\phi}^{-}(\lambda_{m+1}), F_{\phi}^{-}(\lambda_{m+1}), \\ &\quad T_{\phi}^{+}(\lambda_{m+1}), M_{\phi}^{+}(\lambda_{m+1}), C_{\phi}^{+}(\lambda_{m+1}), U_{\phi}^{+}(\lambda_{m+1}), I_{\phi}^{+}(\lambda_{m+1}), K_{\phi}^{+}(\lambda_{m+1}), F_{\phi}^{+}(\lambda_{m+1})] \\ &= \left[-\left(-T_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) - T_{\phi}^{-}(\lambda_{m+1}) - T_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot T_{\phi}^{-}(\lambda_{m+1}) \right), -\left(-M_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \right. \right. \\ &\quad \left. \left. - M_{\phi}^{-}(\lambda_{m+1}) - M_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot M_{\phi}^{-}(\lambda_{m+1}), \right. \right. \\ &\quad \left. \left. -\left(-C_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) - C_{\phi}^{-}(\lambda_{m+1}) - C_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot C_{\phi}^{-}(\lambda_{m+1}) \right), \right. \right. \\ &\quad \left. \left. -U_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot U_{\phi}^{-}(\lambda_{m+1}), -\left(-I_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) - I_{\phi}^{-}(\lambda_{m+1}) \right. \right. \right. \\ &\quad \left. \left. - I_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot I_{\phi}^{-}(\lambda_{m+1}) \right), -\left(-K_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) - K_{\phi}^{-}(\lambda_{m+1}) \right. \right. \\ &\quad \left. \left. - K_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot K_{\phi}^{-}(\lambda_{m+1}) \right), -\left(-F_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) - F_{\phi}^{-}(\lambda_{m+1}) \right. \right. \\ &\quad \left. \left. - F_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot F_{\phi}^{-}(\lambda_{m+1}) \right), T_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot T_{\phi}^{+}(\lambda_{m+1}), M_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot M_{\phi}^{+}(\lambda_{m+1}), \right. \\ &\quad \left. C_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot C_{\phi}^{+}(\lambda_{m+1}), \left(U_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) + U_{\phi}^{+}(\lambda_{m+1}) \right. \right. \\ &\quad \left. \left. - U_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot U_{\phi}^{+}(\lambda_{m+1}) \right), \right. \\ &\quad \left. I_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot I_{\phi}^{+}(\lambda_{m+1}), K_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot K_{\phi}^{+}(\lambda_{m+1}), F_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_m) \cdot F_{\phi}^{+}(\lambda_{m+1}) \right)] \\ &= [T_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), M_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), C_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), U_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), \\ &\quad I_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), K_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), F_{\phi}^{-}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), T_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), \\ &\quad M_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), C_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), U_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), I_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), \\ &\quad K_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1}), F_{\phi}^{+}(\lambda_1, \lambda_2, \dots, \lambda_{m+1})] \text{ (say) which is an BSVHNN.} \end{aligned}$$

Therefore, $\sum_{i=1}^{m+1} \lambda_i$ is an BSVHNN. This becomes, $\sum_{i=1}^n \lambda_i$ is an BSVHNN for $n = m + 1$.

Hence, $\sum_{i=1}^n \lambda_i$ is an BSVHNN for $n = 1$ and $n = 2$.

Again, $\sum_{i=1}^n \lambda_i$ is an BSVHNN for $n = m + 1$, whenever it is an BSVHNN for $n = m$. Therefore, based on the idea of mathematical induction, we can say that $\sum_{i=1}^n \lambda_i$ is an BSVHNN for each n . Now, from definition 3.8, we can tell that $\frac{1}{n} \sum_{i=1}^n \lambda_i$ is an BSVHNN. Hence, $BSVHMNA(\lambda_1, \lambda_2, \dots, \lambda_m) = \frac{1}{n} \sum_{i=1}^n \lambda_i$ is an BSVHNN.

Example 4.1 Assume that

$$p = (-0.8, -0.4, -0.6, -0.8, -0.4, -0.5, -0.3, 0.4, 0.2, 0.7, 0.8, 0.2, 0.3, 0.3)$$

$$\text{and } q = (-0.7, -0.4, -0.6, -0.3, -0.6, -0.4, -0.6, 0.4, 0.3, 0.6, 0.4, 0.4, 0.5, 0.8)$$

be two BSVHNNs.

Then, BSVHMNA is given by,

$$(p, q) = 0.5(p + q) =$$

$$\begin{aligned} &0.5(-0.94, -0.64, -0.84, -0.24, -0.76, -0.7, -0.72, 0.16, 0.06, 0.42, 0.88, 0.08, 0.15, 0.24) = \\ &(-0.9847, -0.8944, -0.9573, -0.0663, -0.9337, -0.9147, -0.9212, 0.0427, 0.0153, 0.1273, \\ &0.9685, 0.0206, 0.0398, 0.0663). \text{ It is also an BSVHNN.} \end{aligned}$$

$(\lambda_1, \lambda_2, \dots, \lambda_n) = (\prod_{i=1}^n \lambda_i)^{\frac{1}{n}}$ is an BSVHNN.

Example 4.2 Assume that

$p = (-0.8, -0.4, -0.6, -0.8, -0.4, -0.5, -0.3, 0.4, 0.2, 0.7, 0.8, 0.2, 0.3, 0.3)$

and $q = (-0.7, -0.4, -0.6, -0.3, -0.6, -0.4, -0.6, 0.4, 0.3, 0.6, 0.4, 0.4, 0.5, 0.8)$

be two BSVHNNs. Then, BSVHGNM

$(p, q) = (p, q)^{0.5} =$

$(-0.56, -0.16, -0.36, -0.86, -0.24, -0.20, -0.18, 0.64, 0.44, 0.88, 0.32, 0.52, 0.65, 0.86)^{0.5} =$

$(-0.7483, -0.4, -0.6, -0.9274, -0.4899, -0.4472, -0.4243, 0.8, 0.6633, 0.9381, 0.5657, 0.7211, 0.8062, 0.9274)$. It is also an BSVHNN.

5. Score and Accuracy Functions under the BSVHNS Environment

Definition 5.1 Suppose that

$\lambda = [T_{\phi}^{-}(\lambda), M_{\phi}^{-}(\lambda), C_{\phi}^{-}(\lambda), U_{\phi}^{-}(\lambda), I_{\phi}^{-}(\lambda), K_{\phi}^{-}(\lambda), F_{\phi}^{-}(\lambda), T_{\phi}^{+}(\lambda), M_{\phi}^{+}(\lambda), C_{\phi}^{+}(\lambda), U_{\phi}^{+}(\lambda), I_{\phi}^{+}(\lambda), K_{\phi}^{+}(\lambda), F_{\phi}^{+}(\lambda)]$

be an BSVHNN over ϕ . Then, the score function and accuracy function are determined by:

$$SF(\lambda) = \frac{-T_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) + 1 - M_{\phi}^{+}(\lambda) + 1 - C_{\phi}^{+}(\lambda) - U_{\phi}^{+}(\lambda) + 1 - I_{\phi}^{+}(\lambda) + 1 - K_{\phi}^{+}(\lambda) + 1 - F_{\phi}^{+}(\lambda)}{14}$$

$$AF(\lambda) = \frac{-T_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + U_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) - T_{\phi}^{+}(\lambda) - C_{\phi}^{+}(\lambda) + U_{\phi}^{+}(\lambda) - K_{\phi}^{+}(\lambda) - F_{\phi}^{+}(\lambda)}{5}$$

Example 5.1 Given

$\lambda = (-0.8, -0.4, -0.6, -0.8, -0.4, -0.5, -0.3, 0.4, 0.2, 0.7, 0.8, 0.2, 0.3, 0.3)$ be an BSVHNN as specified in Example 4.1. Then, $SF(\lambda) = 0.55$ and $AF(\lambda) = 0.1$.

Definition 5.2. Given

$\lambda = [T_{\phi}^{-}(\lambda), M_{\phi}^{-}(\lambda), C_{\phi}^{-}(\lambda), U_{\phi}^{-}(\lambda), I_{\phi}^{-}(\lambda), K_{\phi}^{-}(\lambda), F_{\phi}^{-}(\lambda), T_{\phi}^{+}(\lambda), M_{\phi}^{+}(\lambda), C_{\phi}^{+}(\lambda), U_{\phi}^{+}(\lambda), I_{\phi}^{+}(\lambda), K_{\phi}^{+}(\lambda), F_{\phi}^{+}(\lambda)]$

and

$\gamma = [T_{\phi}^{-}(\gamma), M_{\phi}^{-}(\gamma), C_{\phi}^{-}(\gamma), U_{\phi}^{-}(\gamma), I_{\phi}^{-}(\gamma), K_{\phi}^{-}(\gamma), F_{\phi}^{-}(\gamma), T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\gamma)]$

be any two BSVHNNs over ϕ . Then,

(i) $SF(\lambda) > SF(\gamma) \Rightarrow \lambda > \gamma$;

(ii) $SF(\lambda) = SF(\gamma), AF(\lambda) > AF(\gamma) \Rightarrow \lambda > \gamma$;

(iii) $SF(\lambda) = SF(\gamma), AF(\lambda) = AF(\gamma), T_{\phi}^{+}(\lambda) > T_{\phi}^{+}(\gamma), T_{\phi}^{-}(\lambda) < T_{\phi}^{-}(\gamma) \Rightarrow \lambda > \gamma$.

Theorem 5.1

An BSVHNN has bounds on both its score function and accuracy function.

Proof: Suppose that

$\gamma = [T_{\phi}^{-}(\gamma), M_{\phi}^{-}(\gamma), C_{\phi}^{-}(\gamma), U_{\phi}^{-}(\gamma), I_{\phi}^{-}(\gamma), K_{\phi}^{-}(\gamma), F_{\phi}^{-}(\gamma), T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\gamma)]$

be an BSVHNN.

Therefore, $-1 \leq T_{\phi}^{-}(\gamma) \leq 0, -1 \leq M_{\phi}^{-}(\gamma) \leq 0, -1 \leq C_{\phi}^{-}(\gamma) \leq 0, -1 \leq U_{\phi}^{-}(\gamma) \leq 0, -1 \leq I_{\phi}^{-}(\gamma) \leq 0, -1 \leq K_{\phi}^{-}(\gamma) \leq 0, -1 \leq F_{\phi}^{-}(\gamma) \leq 0, 0 \leq T_{\phi}^{+}(\gamma) \leq 1, 0 \leq M_{\phi}^{+}(\gamma) \leq 1, 0 \leq C_{\phi}^{+}(\gamma) \leq 1, 0 \leq U_{\phi}^{+}(\gamma) \leq 1, 0 \leq I_{\phi}^{+}(\gamma) \leq 1, 0 \leq K_{\phi}^{+}(\gamma) \leq 1, 0 \leq F_{\phi}^{+}(\gamma) \leq 1$.

This implies, $0 \leq 1 + T_{\phi}^{-}(\gamma) + T_{\phi}^{+}(\gamma) \leq 2, 0 \leq 1 + M_{\phi}^{-}(\gamma) + M_{\phi}^{+}(\gamma) \leq 2, 0 \leq 1 + C_{\phi}^{-}(\gamma) + C_{\phi}^{+}(\gamma) \leq 2, 0 \leq 1 + U_{\phi}^{-}(\gamma) + U_{\phi}^{+}(\gamma) \leq 2, 0 \leq 1 + I_{\phi}^{-}(\gamma) + I_{\phi}^{+}(\gamma) \leq 2, 0 \leq 1 + K_{\phi}^{-}(\gamma) + K_{\phi}^{+}(\gamma) \leq 2, 0 \leq 1 + F_{\phi}^{-}(\gamma) + F_{\phi}^{+}(\gamma) \leq 2$.

Therefore,

$$0 \leq -T_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) - M_{\phi}^{-}(\lambda) + 1 - M_{\phi}^{+}(\lambda) - C_{\phi}^{-}(\lambda) + 1 - C_{\phi}^{+}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - U_{\phi}^{+}(\lambda) - I_{\phi}^{-}(\lambda) + 1 - I_{\phi}^{+}(\lambda) - K_{\phi}^{-}(\lambda) + 1 - K_{\phi}^{+}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - F_{\phi}^{+}(\lambda) \leq 14$$

$$\Rightarrow 0 \leq -T_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) + 1 - M_{\phi}^{+}(\lambda) + 1 - C_{\phi}^{+}(\lambda) - U_{\phi}^{+}(\lambda) + 1 - I_{\phi}^{+}(\lambda) + 1 - K_{\phi}^{+}(\lambda) + 1 - F_{\phi}^{+}(\lambda) \leq 14$$

$\Rightarrow 0 \leq$

$$\frac{-T_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) + 1 - M_{\phi}^{+}(\lambda) + 1 - C_{\phi}^{+}(\lambda) - U_{\phi}^{+}(\lambda) + 1 - I_{\phi}^{+}(\lambda) + 1 - K_{\phi}^{+}(\lambda) + 1 - F_{\phi}^{+}(\lambda)}{14} \leq 1$$

1

$$SF(\lambda) = \frac{-T_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) + 1 - M_{\phi}^{+}(\lambda) + 1 - C_{\phi}^{+}(\lambda) - U_{\phi}^{+}(\lambda) + 1 - I_{\phi}^{+}(\lambda) + 1 - K_{\phi}^{+}(\lambda) + 1 - F_{\phi}^{+}(\lambda)}{14}$$

$\Rightarrow 0 \leq SF(\lambda) \leq 1$.

The score function is hence bounded.

Again,

$$\begin{aligned}
 & -1 \leq T_{\phi}^{-}(\gamma) + T_{\phi}^{+}(\gamma) \leq 1, -1 \leq M_{\phi}^{-}(\gamma) + M_{\phi}^{+}(\gamma) \leq 1, -1 \leq C_{\phi}^{-}(\gamma) + C_{\phi}^{+}(\gamma) \leq 1, \\
 & -1 \leq U_{\phi}^{-}(\gamma) + U_{\phi}^{+}(\gamma) \leq 1, -1 \leq I_{\phi}^{-}(\gamma) + I_{\phi}^{+}(\gamma) \leq 1, -1 \leq K_{\phi}^{-}(\gamma) + K_{\phi}^{+}(\gamma) \leq 1, \\
 & -1 \leq F_{\phi}^{-}(\gamma) + F_{\phi}^{+}(\gamma) \leq 1.
 \end{aligned}$$

This implies

$$\begin{aligned}
 & -5 \leq -T_{\phi}^{-}(\gamma) - T_{\phi}^{+}(\gamma) - C_{\phi}^{-}(\gamma) - C_{\phi}^{+}(\gamma) + U_{\phi}^{-}(\gamma) + U_{\phi}^{+}(\gamma) - K_{\phi}^{-}(\gamma) - K_{\phi}^{+}(\gamma) - F_{\phi}^{-}(\gamma) - F_{\phi}^{+}(\gamma) \leq 5 \\
 \Rightarrow & -1 \leq \frac{-T_{\phi}^{-}(\gamma) - C_{\phi}^{-}(\gamma) + U_{\phi}^{-}(\gamma) - K_{\phi}^{-}(\gamma) - F_{\phi}^{-}(\gamma) - T_{\phi}^{+}(\gamma) - C_{\phi}^{+}(\gamma) + U_{\phi}^{+}(\gamma) - K_{\phi}^{+}(\gamma) - F_{\phi}^{+}(\gamma)}{5} \leq 1 \\
 \Rightarrow & -1 \leq AF(\gamma) \leq 1
 \end{aligned}$$

Hence, the accuracy function is bounded.

Theorem 5.2

The score function and accuracy function of an BSVHNN are monotonic increasing.

Proof: Given that

$$\lambda = [T_{\phi}^{-}(\lambda), M_{\phi}^{-}(\lambda), C_{\phi}^{-}(\lambda), U_{\phi}^{-}(\lambda), I_{\phi}^{-}(\lambda), K_{\phi}^{-}(\lambda), F_{\phi}^{-}(\lambda), T_{\phi}^{+}(\lambda), M_{\phi}^{+}(\lambda), C_{\phi}^{+}(\lambda), U_{\phi}^{+}(\lambda), I_{\phi}^{+}(\lambda), K_{\phi}^{+}(\lambda), F_{\phi}^{+}(\lambda)]$$

and

$$\gamma = [T_{\phi}^{-}(\gamma), M_{\phi}^{-}(\gamma), C_{\phi}^{-}(\gamma), U_{\phi}^{-}(\gamma), I_{\phi}^{-}(\gamma), K_{\phi}^{-}(\gamma), F_{\phi}^{-}(\gamma), T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\gamma)]$$

be any two BSVHNNs over ϕ such that $\lambda \subseteq \gamma$.

Therefore,

$$\begin{aligned}
 T_{\phi}^{-}(\lambda) \geq T_{\phi}^{-}(\gamma), M_{\phi}^{-}(\lambda) \geq M_{\phi}^{-}(\gamma), C_{\phi}^{-}(\lambda) \geq C_{\phi}^{-}(\gamma), U_{\phi}^{-}(\lambda) \leq U_{\phi}^{-}(\gamma), I_{\phi}^{-}(\lambda) \geq I_{\phi}^{-}(\gamma), K_{\phi}^{-}(\lambda) \geq K_{\phi}^{-}(\gamma), F_{\phi}^{-}(\lambda) \\
 \geq F_{\phi}^{-}(\gamma), T_{\phi}^{+}(\lambda) \geq T_{\phi}^{+}(\gamma), M_{\phi}^{+}(\lambda) \geq M_{\phi}^{+}(\gamma), C_{\phi}^{+}(\lambda) \geq C_{\phi}^{+}(\gamma), U_{\phi}^{+}(\lambda) \leq U_{\phi}^{+}(\gamma), I_{\phi}^{+}(\lambda) \\
 \geq I_{\phi}^{+}(\gamma), K_{\phi}^{+}(\lambda) \geq K_{\phi}^{+}(\gamma), F_{\phi}^{+}(\lambda) \geq F_{\phi}^{+}(\gamma).
 \end{aligned}$$

It is known that,

$$\begin{aligned}
 SF(\lambda) &= \frac{-T_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) + 1 - M_{\phi}^{+}(\lambda) + 1 - C_{\phi}^{+}(\lambda) - U_{\phi}^{+}(\lambda) + 1 - I_{\phi}^{+}(\lambda) + 1 - K_{\phi}^{+}(\lambda) + 1 - F_{\phi}^{+}(\lambda)}{14} \\
 SF(\gamma) &= \frac{-T_{\phi}^{-}(\gamma) - M_{\phi}^{-}(\gamma) - C_{\phi}^{-}(\gamma) + 1 + U_{\phi}^{-}(\gamma) - I_{\phi}^{-}(\gamma) - K_{\phi}^{-}(\gamma) - F_{\phi}^{-}(\gamma) + 1 - T_{\phi}^{+}(\gamma) + 1 - M_{\phi}^{+}(\gamma) + 1 - C_{\phi}^{+}(\gamma) - U_{\phi}^{+}(\gamma) + 1 - I_{\phi}^{+}(\gamma) + 1 - K_{\phi}^{+}(\gamma) + 1 - F_{\phi}^{+}(\gamma)}{14} \\
 AF(\lambda) &= \frac{-T_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + U_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) - T_{\phi}^{+}(\lambda) - C_{\phi}^{+}(\lambda) + U_{\phi}^{+}(\lambda) - K_{\phi}^{+}(\lambda) - F_{\phi}^{+}(\lambda)}{5} \\
 AF(\gamma) &= \frac{-T_{\phi}^{-}(\gamma) - C_{\phi}^{-}(\gamma) + U_{\phi}^{-}(\gamma) - K_{\phi}^{-}(\gamma) - F_{\phi}^{-}(\gamma) - T_{\phi}^{+}(\gamma) - C_{\phi}^{+}(\gamma) + U_{\phi}^{+}(\gamma) - K_{\phi}^{+}(\gamma) - F_{\phi}^{+}(\gamma)}{5}
 \end{aligned}$$

Now,

$$\begin{aligned}
 & SF(\lambda) - SF(\gamma) \\
 & \frac{-T_{\phi}^{-}(\lambda) - M_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + 1 + U_{\phi}^{-}(\lambda) - I_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) + 1 - T_{\phi}^{+}(\lambda) + 1 - M_{\phi}^{+}(\lambda) + 1 - C_{\phi}^{+}(\lambda) - U_{\phi}^{+}(\lambda) + 1 - I_{\phi}^{+}(\lambda) + 1 - K_{\phi}^{+}(\lambda) + 1 - F_{\phi}^{+}(\lambda)}{14} \\
 & - \frac{-T_{\phi}^{-}(\gamma) - M_{\phi}^{-}(\gamma) - C_{\phi}^{-}(\gamma) + 1 + U_{\phi}^{-}(\gamma) - I_{\phi}^{-}(\gamma) - K_{\phi}^{-}(\gamma) - F_{\phi}^{-}(\gamma) + 1 - T_{\phi}^{+}(\gamma) + 1 - M_{\phi}^{+}(\gamma) + 1 - C_{\phi}^{+}(\gamma) - U_{\phi}^{+}(\gamma) + 1 - I_{\phi}^{+}(\gamma) + 1 - K_{\phi}^{+}(\gamma) + 1 - F_{\phi}^{+}(\gamma)}{14} \\
 & \geq 0 \text{ [since } \lambda \subseteq \gamma \text{]}
 \end{aligned}$$

This implies, $SF(\gamma) \geq SF(\lambda)$, i.e., The scoring function increases monotonically.

Now,

$$\begin{aligned}
 AF(\gamma) \geq AF(\lambda) &= \frac{-T_{\phi}^{-}(\gamma) - C_{\phi}^{-}(\gamma) + U_{\phi}^{-}(\gamma) - K_{\phi}^{-}(\gamma) - F_{\phi}^{-}(\gamma) - T_{\phi}^{+}(\gamma) - C_{\phi}^{+}(\gamma) + U_{\phi}^{+}(\gamma) - K_{\phi}^{+}(\gamma) - F_{\phi}^{+}(\gamma)}{5} \\
 & - \frac{-T_{\phi}^{-}(\lambda) - C_{\phi}^{-}(\lambda) + U_{\phi}^{-}(\lambda) - K_{\phi}^{-}(\lambda) - F_{\phi}^{-}(\lambda) - T_{\phi}^{+}(\lambda) - C_{\phi}^{+}(\lambda) + U_{\phi}^{+}(\lambda) - K_{\phi}^{+}(\lambda) - F_{\phi}^{+}(\lambda)}{5} \\
 & \geq 0 \text{ [since } \gamma \subseteq \lambda \text{]}
 \end{aligned}$$

This implies, $AF(\gamma) \geq AF(\lambda)$, i.e., the accuracy function increases monotonically.

In light of this, the accuracy and score functions are monotonically growing functions.

6. BSVHNS – MADM Strategy based on BSVHMNA Operator

Suppose that $E = \{E_1, E_2 \dots \dots \dots E_n\}$ be a fixed set of alternatives, and $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2 \dots \dots \dots \mathcal{D}_m\}$ be a collection of attributes. Each alternative is evaluated by the decision maker who is involved in the decision-making process E_i ($i = 1, 2 \dots \dots \dots n$) over the attribute P_j ($j = 1, 2, \dots \dots m$) in terms of BSVHNNs. A decision matrix can express the entire evaluation information of all alternatives. The proposed BSVHNS-MADM plan (see Figure 1) is using the following steps:

Step 1 : Create the decision matrix with BSVHNSs.

Each alternative's whole evaluation information E_i ($i = 1, 2, \dots, n$) based on the attributes \mathcal{D}_j ($j = 1, 2, \dots, m$) is expressed in terms of BSVHNS

$$EI_{(E_i, \mathcal{D}_j)} = \{(\mathcal{D}_j, T_{ij}^-(E_i, \mathcal{D}_j), M_{ij}^-(E_i, \mathcal{D}_j), C_{ij}^-(E_i, \mathcal{D}_j), U_{ij}^-(E_i, \mathcal{D}_j), I_{ij}^-(E_i, \mathcal{D}_j), K_{ij}^-(E_i, \mathcal{D}_j), F_{ij}^-(E_i, \mathcal{D}_j), T_{ij}^+(E_i, \mathcal{D}_j), M_{ij}^+(E_i, \mathcal{D}_j), C_{ij}^+(E_i, \mathcal{D}_j), U_{ij}^+(E_i, \mathcal{D}_j), I_{ij}^+(E_i, \mathcal{D}_j), K_{ij}^+(E_i, \mathcal{D}_j), F_{ij}^+(E_i, \mathcal{D}_j))\}$$

indicate the evaluation data of E ($i = 1, 2, \dots, n$) base on \mathcal{D}_j ($j = 1, 2, \dots, m$).

Then the Decision Matrix ($DM[E|\mathcal{D}]$) can be stated as

$$DM[E|\mathcal{D}] =$$

Step 2 : In this step, the decision maker determines the aggregation values $(E_i | \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_m) =$

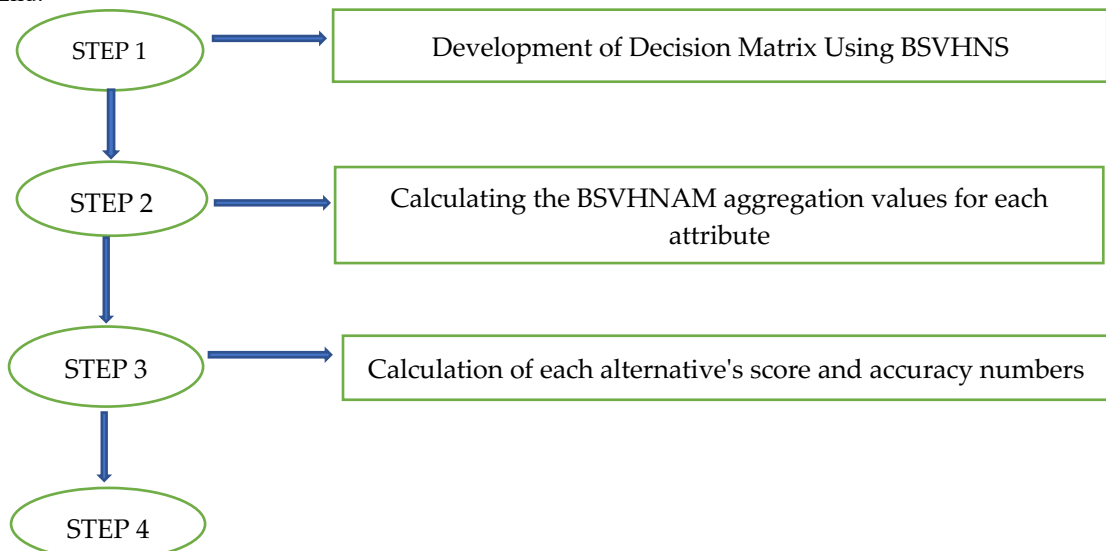
	\mathcal{D}_1	\mathcal{D}_2	...	\mathcal{D}_m
E_1	$[T_{11}^-(E_1, \mathcal{D}_1), M_{11}^-(E_1, \mathcal{D}_1), C_{11}^-(E_1, \mathcal{D}_1), U_{11}^-(E_1, \mathcal{D}_1), I_{11}^-(E_1, \mathcal{D}_1), K_{11}^-(E_1, \mathcal{D}_1), F_{11}^-(E_1, \mathcal{D}_1), T_{11}^+(E_1, \mathcal{D}_1), M_{11}^+(E_1, \mathcal{D}_1), C_{11}^+(E_1, \mathcal{D}_1), U_{11}^+(E_1, \mathcal{D}_1), I_{11}^+(E_1, \mathcal{D}_1), K_{11}^+(E_1, \mathcal{D}_1), F_{11}^+(E_1, \mathcal{D}_1)]$	$[T_{12}^-(E_1, \mathcal{D}_2), M_{12}^-(E_1, \mathcal{D}_2), C_{12}^-(E_1, \mathcal{D}_2), U_{12}^-(E_1, \mathcal{D}_2), I_{12}^-(E_1, \mathcal{D}_2), K_{12}^-(E_1, \mathcal{D}_2), F_{12}^-(E_1, \mathcal{D}_2), T_{12}^+(E_1, \mathcal{D}_2), M_{12}^+(E_1, \mathcal{D}_2), C_{12}^+(E_1, \mathcal{D}_2), U_{12}^+(E_1, \mathcal{D}_2), I_{12}^+(E_1, \mathcal{D}_2), K_{12}^+(E_1, \mathcal{D}_2), F_{12}^+(E_1, \mathcal{D}_2)]$...	$[T_{1m}^-(E_1, \mathcal{D}_m), M_{1m}^-(E_1, \mathcal{D}_m), C_{1m}^-(E_1, \mathcal{D}_m), U_{1m}^-(E_1, \mathcal{D}_m), I_{1m}^-(E_1, \mathcal{D}_m), K_{1m}^-(E_1, \mathcal{D}_m), F_{1m}^-(E_1, \mathcal{D}_m), T_{1m}^+(E_1, \mathcal{D}_m), M_{1m}^+(E_1, \mathcal{D}_m), C_{1m}^+(E_1, \mathcal{D}_m), U_{1m}^+(E_1, \mathcal{D}_m), I_{1m}^+(E_1, \mathcal{D}_m), K_{1m}^+(E_1, \mathcal{D}_m), F_{1m}^+(E_1, \mathcal{D}_m)]$
E_2	$[T_{21}^-(E_2, \mathcal{D}_1), M_{21}^-(E_2, \mathcal{D}_1), C_{21}^-(E_2, \mathcal{D}_1), U_{21}^-(E_2, \mathcal{D}_1), I_{21}^-(E_2, \mathcal{D}_1), K_{21}^-(E_2, \mathcal{D}_1), F_{21}^-(E_2, \mathcal{D}_1), T_{21}^+(E_2, \mathcal{D}_1), M_{21}^+(E_2, \mathcal{D}_1), C_{21}^+(E_2, \mathcal{D}_1), U_{21}^+(E_2, \mathcal{D}_1), I_{21}^+(E_2, \mathcal{D}_1), K_{21}^+(E_2, \mathcal{D}_1), F_{21}^+(E_2, \mathcal{D}_1)]$	$[T_{22}^-(E_2, \mathcal{D}_2), M_{22}^-(E_2, \mathcal{D}_2), C_{22}^-(E_2, \mathcal{D}_2), U_{22}^-(E_2, \mathcal{D}_2), I_{22}^-(E_2, \mathcal{D}_2), K_{22}^-(E_2, \mathcal{D}_2), F_{22}^-(E_2, \mathcal{D}_2), T_{22}^+(E_2, \mathcal{D}_2), M_{22}^+(E_2, \mathcal{D}_2), C_{22}^+(E_2, \mathcal{D}_2), U_{22}^+(E_2, \mathcal{D}_2), I_{22}^+(E_2, \mathcal{D}_2), K_{22}^+(E_2, \mathcal{D}_2), F_{22}^+(E_2, \mathcal{D}_2)]$...	$[T_{2m}^-(E_2, \mathcal{D}_m), M_{2m}^-(E_2, \mathcal{D}_m), C_{2m}^-(E_2, \mathcal{D}_m), U_{2m}^-(E_2, \mathcal{D}_m), I_{2m}^-(E_2, \mathcal{D}_m), K_{2m}^-(E_2, \mathcal{D}_m), F_{2m}^-(E_2, \mathcal{D}_m), T_{2m}^+(E_2, \mathcal{D}_m), M_{2m}^+(E_2, \mathcal{D}_m), C_{2m}^+(E_2, \mathcal{D}_m), U_{2m}^+(E_2, \mathcal{D}_m), I_{2m}^+(E_2, \mathcal{D}_m), K_{2m}^+(E_2, \mathcal{D}_m), F_{2m}^+(E_2, \mathcal{D}_m)]$
...
E_n	$[T_{n1}^-(E_n, \mathcal{D}_1), M_{n1}^-(E_n, \mathcal{D}_1), C_{n1}^-(E_n, \mathcal{D}_1), U_{n1}^-(E_n, \mathcal{D}_1), I_{n1}^-(E_n, \mathcal{D}_1), K_{n1}^-(E_n, \mathcal{D}_1), F_{n1}^-(E_n, \mathcal{D}_1), T_{n1}^+(E_n, \mathcal{D}_1), M_{n1}^+(E_n, \mathcal{D}_1), C_{n1}^+(E_n, \mathcal{D}_1), U_{n1}^+(E_n, \mathcal{D}_1), I_{n1}^+(E_n, \mathcal{D}_1), K_{n1}^+(E_n, \mathcal{D}_1), F_{n1}^+(E_n, \mathcal{D}_1)]$	$[T_{n2}^-(E_n, \mathcal{D}_2), M_{n2}^-(E_n, \mathcal{D}_2), C_{n2}^-(E_n, \mathcal{D}_2), U_{n2}^-(E_n, \mathcal{D}_2), I_{n2}^-(E_n, \mathcal{D}_2), K_{n2}^-(E_n, \mathcal{D}_2), F_{n2}^-(E_n, \mathcal{D}_2), T_{n2}^+(E_n, \mathcal{D}_2), M_{n2}^+(E_n, \mathcal{D}_2), C_{n2}^+(E_n, \mathcal{D}_2), U_{n2}^+(E_n, \mathcal{D}_2), I_{n2}^+(E_n, \mathcal{D}_2), K_{n2}^+(E_n, \mathcal{D}_2), F_{n2}^+(E_n, \mathcal{D}_2)]$...	$[T_{nm}^-(E_n, \mathcal{D}_m), M_{nm}^-(E_n, \mathcal{D}_m), C_{nm}^-(E_n, \mathcal{D}_m), U_{nm}^-(E_n, \mathcal{D}_m), I_{nm}^-(E_n, \mathcal{D}_m), K_{nm}^-(E_n, \mathcal{D}_m), F_{nm}^-(E_n, \mathcal{D}_m), T_{nm}^+(E_n, \mathcal{D}_m), M_{nm}^+(E_n, \mathcal{D}_m), C_{nm}^+(E_n, \mathcal{D}_m), U_{nm}^+(E_n, \mathcal{D}_m), I_{nm}^+(E_n, \mathcal{D}_m), K_{nm}^+(E_n, \mathcal{D}_m), F_{nm}^+(E_n, \mathcal{D}_m)]$

BSVHNAM ($\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_m$) of all the attributes for each alternative by using eqn (1). After the determination of aggregation values BSVHMNA ($\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_m$), the decision maker makes an aggregate decision matrix aggregate-DM.

Step 3 : In this step, the decision maker determine the score and accuracy values of each alternative by using the equation (3) and (4).

Step 4 : In this step, the decision maker ranks the alternatives by using Definition 5.1 and Definition 5.2.

Step 5 : End.



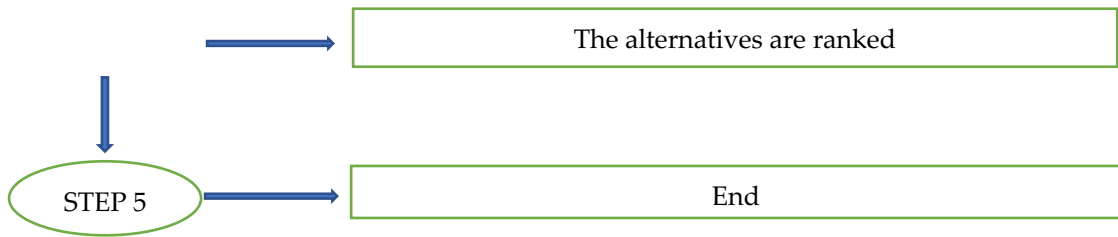


Figure 1 Flow chart of the BSVHNS-MADM Strategy based on BSVHMNA Operator

7. BSVHNS-MADM Strategy based on BSVHMNG Operator

Consider the same MADM problem that was discussed in Section 6. The proposed BSVHNS-MADM scheme (see Figure 2) can then be stated as follows:

Step 1 : Create the decision-making matrix with BSVHNSs.

It is comparable to step 1 in Section 6.

Step 2 : The decision makers determine the aggregation values in this step $(E_i | \mathcal{D}_1, \mathcal{D}_2, \dots \dots \mathcal{D}_m) = BSVHMNG(\mathcal{D}_1, \mathcal{D}_2, \dots \dots \mathcal{D}_m)$ of all the attributes for each alternative by using the equation (1). After the determination of aggregation values $BSVHMNG(\mathcal{D}_1, \mathcal{D}_2, \dots \dots \mathcal{D}_m)$, The decision maker makes an aggregate DM for the decision makers.

Step 3 : In this stage, the decision maker uses the equations (3) and (4) to calculate the score and accuracy values for each alternative.

Step 4 : The decision maker ranks the choices using Definitions 5.1 and 5.2 in this step.

Step 5 : End.

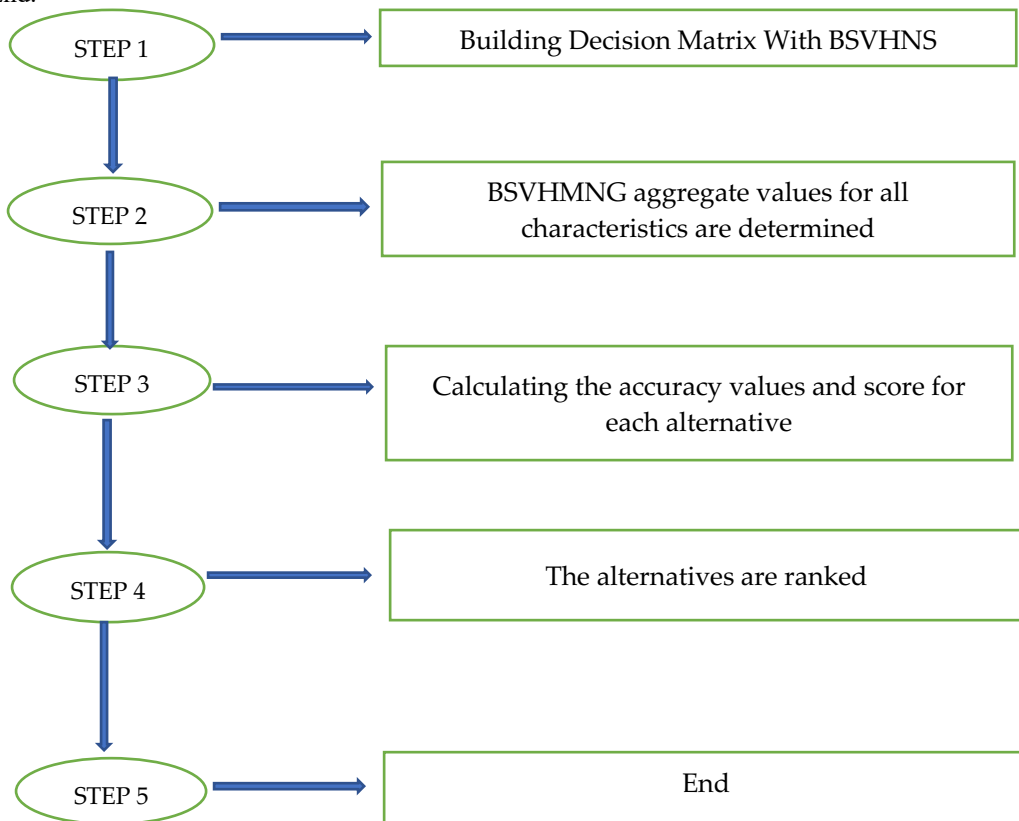


Figure 2: Flow chart of the BSVHNS-MADM Strategy based on BSVHMNG Operator

8. BSVHNS - MADM Strategy Validation

At this section, we offer a genuine scenario of "selection for good equipment in best hospital" to validate the suggested BSVHNS-MADM strategies based on both BSVHMNA and BSVHMNG operators. A good hospital should focus on

making the patient's experience as seamless as possible, from appointment booking to discharge. Every government / private hospital requires Hospital Stretchers, Room rent per day, scanning process, X-Ray, ECG, and so on for the benefit of hospital users. To purchase a specific or all items, hospitals must select an appropriate private concern for providing some features. As a result, selecting the best private hospitals for acquiring the necessary items can be considered a MADM problem.

For the selection of suitable private hospital, the decision maker selects four major attributes namely \mathcal{D}_1 : The price of the products; \mathcal{D}_2 : Product high quality; \mathcal{D}_3 : Company support; \mathcal{D}_4 : Safety

Table 1

In Table 2, We compute the aggregate values $(E_i | \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$ of all attributes for each alternative E_i , by using the BSVHMNA operator.

By using equation (2), we get $SF(E_1) = 0.63574$; $SF(E_2) = 0.635164$; $SF(E_3) = 0.7245$.

Therefore, $SF(E_2) < SF(E_1) < SF(E_3)$.

The ranking order is determined as follows: $E_2 < E_1 < E_3$.

As a result, E_3 is the best hospital in terms of quality goods and services among the alternatives (hospitals).

In table 3, we calculate the aggregation values $(E_i | \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$ of all attributes for each alternative E_i , by using the BSVHMNG operator.

Table 2: Aggregate – DM

DM	\mathcal{D}_1	\mathcal{D}_2	\mathcal{D}_3	\mathcal{D}_4
E_1	(-0.2, -0.4, -0.5, -0.7, -0.1, -0.3, -0.2, 0.3, 0.6, 0.5, 0.2, 0.4, 0.1, 0.3)	(-0.7, -0.2, -0.3, -0.4, -0.5, -0.3, -0.6, 0.2, 0.4, 0.5, 0.4, 0.3, 0.6, 0.8)	(-0.4, -0.6, -0.2, -0.1, -0.5, -0.5, -0.3, 0.8, 0.7, 0.4, 0.2, 0.5, 0.3, 0.6)	(-0.2, -0.1, -0.8, -0.1, -0.6, -0.3, -0.1, 0.3, 0.2, 0.4, 0.3, 0.7, 0.5, 0.4)
E_2	(-0.2, -0.6, -0.7, -0.8, -0.4, -0.2, -0.2, 0.3, 0.4, 0.3, 0.1, 0.3, 0.1, 0.3)	(-0.6, -0.4, -0.4, -0.6, -0.4, -0.3, -0.5, 0.4, 0.2, 0.4, 0.4, 0.5, 0.7, 0.6)	(-0.7, -0.3, -0.1, -0.1, -0.6, -0.3, -0.1, 0.9, 0.6, 0.3, 0.1, 0.7, 0.4, 0.7)	(-0.5, -0.1, -0.8, -0.2, -0.7, -0.5, -0.4, 0.5, 0.4, 0.5, 0.3, 0.6, 0.6, 0.4)
E_3	(-0.4, -0.3, -0.5, -0.9, -0.2, -0.1, -0.5, 0.5, 0.2, 0.2, 0.1, 0.4, 0.2, 0.6)	(-0.4, -0.2, -0.2, -0.6, -0.7, -0.2, -0.5, 0.7, 0.5, 0.5, 0.4, 0.6, 0.4, 0.2)	(-0.7, -0.3, -0.1, -0.4, -0.6, -0.3, -0.7, 1.0, 0.6, 0.3, 0.1, 0.7, 0.4, 0.4)	(-0.4, -0.6, -0.9, -0.1, -0.5, -0.4, -0.8, 0.5, 0.4, 0.5, 0.2, 0.4, 0.3, 0.5)

	$(E_i \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$
E_1	(-0.9699, -0.9537, -0.9857, -0.2300, -0.9767, -0.9541, -0.3299, 0.0036, 0.4281, 0.4472, 0.9102, 0.4527, 0.3080, 0.4899)
E_2	(-0.9878, -0.9598, -0.9918, -0.3130, -0.9890, -0.9469, -0.3183, 0.0138, 0.3722, 0.3663, 0.9013, 0.5009, 0.3600, 0.4738)
E_3	(-0.9834, -0.9583, -0.9909, -0.2378, -0.9878, -0.9139, -0.6501, 0.047, 0.3936, 0.3499, 0.8842, 0.5091, 0.3130, 0.3936)

By using equation (2), we get $SF(E_1) = 0.38315$; $SF(E_2) = 0.3941$; $SF(E_3) = 0.4123$.

Therefore, $SF(E_1) < SF(E_2) < SF(E_3)$.

The ranking order is determined as follows: $E_1 < E_2 < E_3$.

As a result, E_3 is the best hospital for getting good services.

Table 3: Aggregate – DM

Table 4 : Ranking order of alternatives

Strategies	Ranking order	Best alternative
BSVHNS – MADM strategy based on BHNAM operator	$E_2 < E_1 < E_3$	E_3
BSVHNS – MADM strategy based on BHNGM operator	$E_1 < E_2 < E_3$	E_3

Both BSVHNS - MADM techniques provide the same ranking order of the alternatives (See table 4), with E_3 being the best hospital for receiving decent treatment.

9. Sensitivity Analysis

Sensitivity analysis is a financial model that investigates how changes in other variables known as input variables

	$(E_i \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4)$
E_1	(-0.3253, -0.2632, -0.3936, -0.3821, -0.3499, -0.3409, -0.2449, 0.4708, 0.5101, 0.4523, 0.2632, 0.4991, 0.4042, 0.5719)
E_2	(-0.4527, -0.2913, -0.3869, -0.5101, -0.5091, -0.3080, -0.2515, 0.6193, 0.4174, 0.3808, 0.1861, 0.5473, 0.4955, 0.5262)
E_3	(-0.4601, -0.3224, -0.3080, -0.2865, -0.4527, -0.2213, -0.6117, 1.000, 0.4434, 0.3883, 0.1682, 0.5441, 0.3299, 0.4434)

affect target variables. It is a technique for predicting the outcome of a choice based on a set of variables. By building a given collection of variables, an analyst can determine how changes in one variable affect the outcome. In this model, we have reduced uncertainty to select the best hospital in the MADM scheme.

10. Comparative Analysis

Surapati Pramnik utilized five values and the MADM scheme in his Pentapartitioned neutrosophic set, whereas I used seven values and so many attributes in my Heptapartitioned neutrosophic set. I discovered score and accuracy functions for identifying the best service at the nearest hospital utilizing the MADM scheme by employing Bipolar Heptapartitioned Neutrosophic Set.

11. Conclusion

We define the concept of BSVHNS and demonstrate its fundamental attributes in this essay. We determine the BSVHNS' score and accuracy functions and demonstrate their fundamental characteristics. In this section, we construct two aggregation operators the bipolar single-valued heptapartitioned neutrosophic arithmetic mean operator and the bipolar single-valued heptapartitioned neutrosophic geometric mean operator and demonstrate their fundamental characteristics. We create two new MADM scheme based on these two operators and provide a numerical illustration in an BSVHNS environment to demonstrate the usefulness of BSVHNS in MADM.

References

[1] Abdel-Basset, M., Gamal, A., Son, L.H., & Smarandache, F. (2020). A bipolar neutrosophic multi criteria decision making framework for professional selection. Applied Sciences, 10(4), 1202. doi:10.3390/app10041202.

[2] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87–96.

[3] Atanassov, K. T. (1999). Intuitionistic fuzzy sets. Heidelberg: Springer-Verlag.

[4] S. Manivasan, P. Kalidass, Applications in KU-algebras based on BMBJ-neutrosophic Structures, International Journal of Neutrosophic Science, Vol. 20 , No. 4 , (2023) : 223-231 (Doi : https://doi.org/10.54216/IJNS.200420)

[5] Chahhterjee, R., Majumdar, P., Samanta, S.K. (2016). On some similarity measures and entropy on quadripartithioned single valued neutrosophic sets. J. Int. Fuzzy Syst., 30, 2475–2485.

[6] Deli, I., Ali, M., Smarandache, F. (2015). Bipolar neutrosophic sets and their application based on multi-criteria decision-making problems. proceedings of the 2015 International Conference on Advanced Mechatronic Systems, Beijing, China, August, 22-24.

[7] Dey, P.P., Pramanik, S., & Giri, B.C. (2016). TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds.), New trends in neutrosophic theory and applications (pp. 65-77). Brussels: Pons Editions.

[8] Fan, E., Hu, K., & Li, X. (2019). Review of neutrosophic-set theory-based multiple-target tracking methods in uncertain situations. In 2019 IEEE International Conference on Artificial Intelligence and Computer

- Applications (ICAICA) (pp. 19-27). IEEE.
- [9] P. Arulpandy, Bipolar neutrosophic soft continuity mappings, Journal of Neutrosophic and Fuzzy Systems, Vol. 6, No. 1, (2023) : 29-38 (Doi : <https://doi.org/10.54216/JNFS.060104>)
- [10] Jana, C., Pal, M., Karaaslan, F., & Wang, J.Q. (2020). Trapezoidal neutrosophic aggregation operators and their application to the multi-attribute decision-making process. *Scientica Iranica*, 27(3), 1655-1673.
- [11] Karaaslan, F., & Hunu, F. (2020). Type-2 single-valued neutrosophic sets and their applications in multi-criteria group decision making based on TOPSIS method. *Journal of Ambient Intelligence and Humanized Computing*, 11(10), 4113-4132.
- [12] Karaaslan, F., & Hayat, K. (2018). Some new operations on single-valued neutrosophic matrices and their applications in multi-criteria group decision making. *Applied Intelligence*, 48(2), 4594-4614.
- [13] Karaaslan, F. (2018). Multi-criteria decision-making method based on similarity measures under single-valued neutrosophic refined and interval neutrosophic refined environments. *International Journal of Intelligent Systems*, 33(5), 928-952.
- [14] Karaaslan, F. (2018). Gaussian Single-valued neutrosophic number and its application in multi-attribute decision making. *Neutrosophic Sets and Systems*, 22, 2018, 101-117.
- [15] Koundal, D., Gupta, S., & Singh, S. (2016). Applications of neutrosophic sets in medical image denoising and segmentation. In F. Smarandache, & S. Pramanik (Eds.), *New trends in neutrosophic theory and application* (pp.257-275). Brussels, Belgium: Pons Editions.
- [16] Peng, X., & Dai, J. (2020). A bibliometric analysis of neutrosophic set: Two decades review from 1998 to 2017. *Artificial Intelligence Review*, 53(1), 199-255.
- [17] Pramanik, S., & Roy, T.K. (2014). Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. *Neutrosophic sets and systems*, 2, 82-101.
- [18] Pramanik, S., Mallick, R., & Dasgupta, A. (2018). Contributions of selected Indian researchers to multi-attribute decision making in neutrosophic environment. *Neutrosophic Sets and Systems*, 20, 108-131.
- [19] Pramanik, S. (2020). Rough neutrosophic set: an overview. In F. Smarandache, & S. Broumi, (Eds.), *Neutrosophic theories in communication, management and information technology* (pp.275-311). New York. Nova Science Publishers.
- [20] Pramanik, S., Dalapati, S., Alam, S., & Roy, T.K. (2018). TODIM method for group decision making under bipolar neutrosophic set environment. In F. Smarandache, & S. Pramanik (Eds., vol.2), *New trends in neutrosophic theory and applications* (pp. 140-155). Brussels: Pons Editions.
- [21] Shilpi Pal, Avishek Chakraborty, Triangular Neutrosophic-based EOQ model for non-Instantaneous Deteriorating Item under Shortages, *American Journal of Business and Operations Research*, Vol. 1, No. 1, (2020) : 28-35 (Doi : <https://doi.org/10.54216/AJBOR.010103>)
- [22] Pramanik, S., Dey, P.P., Giri, B.C., & Smarandache, F. (2017). Bipolar neutrosophic projection-based models for solving multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 15, 70-79.
- [23] Radha, R., Stanis Arul Mary, A. (2021). Heptapartitioned Neutrosophic Set. *International Journal of Creative Research Thoughts*, 9(2), 222-230.
- [24] Rama Mallick, Surapati Pramanik. (2020). Pentapartitioned neutrosophic set and its properties, *Neutrosophic sets and systems*, Vol 36.
- [25] Smarandache, F. (1998). *A unifying field of logics. Neutrosophy: neutrosophic probability, set and logic.* Rehoboth: American Research Press.
- [26] Smarandache, F. & Pramanik, S. (Eds). (2016). *New trends in neutrosophic theory and applications.* Brussels: Pons Editions.
- [27] Mehmet Merkepci, Mohammad Abobala, Ali Allouf, The Applications of Fusion Neutrosophic Number Theory in Public Key Cryptography and the Improvement of RSA Algorithm, *Fusion: Practice and Applications*, Vol. 10, No. 2, (2023) : 69-74 (Doi : <https://doi.org/10.54216/FPA.100206>)
- [28] Suman Das, Rakhal Das and Surapati Pramanik (2022). Single Valued Bipolar Pentapartitioned Neutrosophic Set and Its Application in MADM Strategy, *Neutrosophic sets and systems*, Vol 49.
- [29] Wang, H., Smarandache, F., Sunderraman, R., & Zhang, Y.Q. (2010). Single valued neutrosophic sets. *Multi-space and Multi-structure*, 4, 410-413.
- [30] Ye, J. (2017). Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Computing*, 21(3), 817-825.
- [31] Zadeh, L.A., "Fuzzy sets", *Information and control*, Vol. 8, (1965), 338-353.