The Geometrical Characterization for The Solutions of a Vectorial Equation By Using Weak Fuzzy Complex Numbers and Other Generalizations Of Real Numbers

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Abstract

The main goal of this paper is to study the geometrical characterization of the solutions for a vectorial equation defined in the two/three dimensional Euclidean spaces. The geometrical characterization of the solutions for the desired vectorial equation is obtained for many different values of t based on the circles and spheres in some generalizations of the real field, especially dual numbers, weak fuzzy complex numbers split-complex numbers, and complex numbers.

Keywords: A-curve; dual numbers; weak fuzzy complex numbers; split-complex numbers

1. Introduction

Expansions of real numbers have occupied many researchers around the world, where we can find many extensions built over some ideas that are very similar to complex numbers.
For example, we can find the following extended sets:
Dual numbers $\boldsymbol{D}=\{\boldsymbol{a}+\boldsymbol{bJ};\ \boldsymbol{J}^{2}=\boldsymbol{0}\}$, neutrosophic numbers $\boldsymbol{N}=\{\boldsymbol{a}+\boldsymbol{bJ};\ \boldsymbol{J}^{2}=\boldsymbol{J}\}$, split-complex numbers $\boldsymbol{S}(\boldsymbol{J})=\{\boldsymbol{a}+\boldsymbol{bJ};\boldsymbol{J}^{2}=\boldsymbol{1}\}$, and weak fuzzy complex numbers $\boldsymbol{W}=\{\boldsymbol{a}+\boldsymbol{bJ};\ \boldsymbol{J}^{2}\in]\boldsymbol{0},\boldsymbol{1}[\}$. [1-4]. It is clear that all these extensions have two dimensions, and they are all modules by the ordinary algebraic meaning over themselves.
On the other hand, these extended sets were used in the study of many algebraic structures, such as linear spaces, mathematical cryptography, and Diophantine equations [5-9].
Also, some logical extensions of algebraic structures were defined and studied by many authors, such as neutrosophic structures [11-14, 17], and plithogenic structures [10, 15-16].
In this work, we study the following vectorial equation is some Euclidean spaces:

$$\begin{cases}\|X\|^2+\|Y\|^2=r_1\\ \langle x,y\rangle=r_2\end{cases};\ r_1,r_2\geq 0, t\in R, \langle,\rangle \textit{ is the Euclidean inner product on } R^2 \textit{ or } R^3 .$$

And we use the extensions of the real field to describe their solutions as an algebraic curve.

2. Main discussion.

Definition:

Let $X=(x_1,y_1), Y=(x_2,y_2)$ be two vectors of R^2, we say that X,Y have the property (A) if and only if:

$$\begin{cases}\|X\|^2+\|Y\|^2=r_1\\ \langle x,y\rangle=r_2\end{cases};\ r_1,r_2\geq 0, t\in R, \langle,\rangle \textit{ is the Euclidean inner product on } R^2$$

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Definition.
Let $r_1, r_2, t \geq 0$ be real numbers, the set $\{X, Y; X, Y \in R^2\}$ that has the property (A) with respect r_1, r_2, t is called A-Curve, and it is denoted by C_{A_t}.
Theorem.
Let $r_1, r_2, t \geq 0$ be real numbers, and C_{A_t} is the corresponding A-Curve, then it can be represented by the following system of equations:
$$\begin{cases} {x_1}^2 + {y_1}^2 + t({x_2}^2 + {y_2}^2) = r_1 \\ \quad x_1x_2 + y_1y_2 = r_2 \end{cases}$$
The proof is easy and clear.
Definition.
The ring of dual numbers is defined as follows:
$D = \{a + b\varepsilon;\ \varepsilon^2 = 0, a, b \in R\}$
The ring of split-complex numbers is defined as follows:
$S = \{a + b\varepsilon;\ \varepsilon^2 = 1, a, b \in R\}$
The ring of weak fuzzy complex numbers is defined as follows:
$C_w = \{a + b\varepsilon;\ \varepsilon^2 \in]0,1[, a, b \in R\}$
The field of complex numbers is defined as follows:
$C = \{a + bi;\ i^2 = -1, a, b \in R\}$
Definition.
The complex circle of radius $r_1 + r_2 i$ with center at the origin point is defined as follows:
$X^2 + Y^2 = r_1 + r_2 i; X = x_1 + y_1 i, Y = x_2 + y_2 i, i^2 = -1.$
The dual circle of radius $r_1 + r_2\varepsilon$ with center at the origin point is defined as follows:
$X^2 + Y^2 = r_1 + r_2\varepsilon; X = x_1 + y_1\varepsilon, Y = x_2 + y_2\varepsilon, \varepsilon^2 = 0.$
The split-complex circle of radius $r_1 + r_2\varepsilon$ with center at the origin point is defined as follows:
$X^2 + Y^2 = r_1 + r_2\varepsilon; X = x_1 + y_1\varepsilon, Y = x_2 + y_2\varepsilon, \varepsilon^2 = 1.$
The weak fuzzy complex circle of radius $r_1 + r_2\varepsilon$ with center at the origin point is defined as follows:
$X^2 + Y^2 = r_1 + r_2\varepsilon; X = x_1 + y_1\varepsilon, Y = x_2 + y_2\varepsilon, \varepsilon^2 \in]0,1[.$
Theorem.
The A-Curve C_{A_0} is equivalent to the dual circle of radius $r_1 + 2r_2\varepsilon$ with center at the origin point:
$$\begin{cases} \|X\|^2 = r_1 \\ \langle x, y \rangle = r_2 \end{cases}$$
Which is equivalent to:
$$\begin{cases} {x_1}^2 + {y_1}^2 = r_1 \\ x_1x_2 + y_1y_2 = r_2 \end{cases}$$
On the other hand, consider the dual circle:
$X^2 + Y^2 = r_1 + r_2\varepsilon; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon, \varepsilon^2 = 0$
$(x_1 + x_2\varepsilon)^2 + (y_1 + y_2\varepsilon)^2 = r_1 + 2r_2\varepsilon$, hence:
$$\begin{cases} {x_1}^2 + {y_1}^2 = r_1 \\ x_1x_2 + y_1y_2 = r_2 \end{cases}$$
Thus, the proof is complete.
Result.
The A-Curve C_{A_0} is a circle in the dual space.
Theorem.
The A-Curve C_{A_1} is equivalent to the split-complex circle of radius $r_1 + 2r_2\varepsilon$ with center at the origin.
Proof.
C_{A_1} has the following equation:
$$\begin{cases} \|X\|^2 + \|Y\|^2 = r_1 \\ \quad \langle x, y \rangle = r_2 \end{cases}$$
So,
$$\begin{cases} {x_1}^2 + {x_2}^2 + {y_1}^2 + {y_2}^2 = r_1 \\ \quad x_1x_2 + y_1y_2 = r_2 \end{cases}$$
On the other hand, consider the split-complex circle
$X^2 + Y^2 = r_1 + r_2\varepsilon; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon, \varepsilon^2 = 1.$
$$\begin{cases} {x_1}^2 + {x_2}^2 + {y_1}^2 + {y_2}^2 = r_1 \\ \quad 2(x_1x_2 + y_1y_2) = 2r_2 \end{cases}$$
Hence, the proof is complete.

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Result.
The A-Curve C_{A_1} is a circle in split-complex space.
Theorem.
The A-Curve $C_{A_{-1}}$ is equivalent to the complex circle of radius $r_1 + 2r_2 i$ and center at the origin.
Proof.
$C_{A_{-1}}$ has the following equation:
$$\begin{cases} \|X\|^2 - \|Y\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}$$
S0,
$$\begin{cases} {x_1}^2 + {x_2}^2 + {y_1}^2 + {y_2}^2 = r_1 \\ x_1x_2 + y_1y_2 = r_2 \end{cases}$$
On the other hand, the complex circle of radius $r_1 + 2r_2 i$ and center at the origin is:
$X^2 + Y^2 = r_1 + 2r_2 i; X = x_1 + x_2 i, Y = y_1 + y_2 i, i^2 = -1.$
$$\begin{cases} {x_1}^2 - {y_1}^2 + {x_2}^2 - {y_2}^2 = r_1 \\ 2(x_1x_2 + y_1y_2) = 2r_2 \end{cases}$$
Hence, the proof is complete.
Result.
The A-Curve $C_{A_{-1}}$ is a circle in the complex space.
Theorem.
The A-Curve $C_{A_t}; t \in]0,1[$ is equivalent to the weak fuzzy complex circle with radius $r_1 + 2r_2\varepsilon$ and center at the origin.
Proof.
C_{A_t} has the following equation:
$$\begin{cases} \|X\|^2 + t\|Y\|^2 = r_1 \\ x_1x_2 + y_1y_2 = r_2 \end{cases}; t \in]0,1[$$
S0,
$$\begin{cases} {x_1}^2 + {x_2}^2 + t({y_1}^2 + {y_2}^2) = r_1 \\ 2x_1x_2 + 2y_1y_2 = 2r_2 \end{cases}$$
Hence, the proof is complete.
Result.
The A-Curve $C_{A_t}; t \in]0,1[$ is a circle in the weak fuzzy complex space.
The A-Curves in R^3.
Definition.
The A_t-Curve in R^3 is defined as the solution of system:
$$\begin{cases} \|X\|^2 + t\|Y\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}; r_1, r_2 \geq 0, t \in R, X = (x_1y_1, z_1), Y = (x_2, y_2, z_2)$$
Remark.
A_t-Curve in R^3 equivalent:
$$\begin{cases} {x_1}^2 + {y_1}^2 + {z_1}^2 + t({x_2}^2 + {y_2}^2 + {z_2}^2) = r_1 \\ x_1x_2 + y_1y_2 + z_1z_2 = r_2 \end{cases}$$
Definition.
The complex sphere with radius $r_1 + r_2 i$ with center at the origin point is defined as follows:
$X^2 + Y^2 + Z^2 = r_1 + r_2 i; X = x_1 + x_2 i, Y = y_1 + y_2 i,, Z = z_1 + z_2 i, i^2 = -1.$
The dual sphere with radius $r_1 + r_2\varepsilon$ with center at the origin point is defined as follows:
$X^2 + Y^2 + Z^2 + Z^2 = r_1 + r_2\varepsilon; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon,, Z = z_1 + z_2\varepsilon, \varepsilon^2 = 0.$
The split-complex sphere with radius $r_1 + r_2\varepsilon$ and center at the origin point is defined as follows:
$X^2 + Y^2 + Z^2 = r_1 + r_2\varepsilon; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon,, Z = z_1 + z_2\varepsilon, \varepsilon^2 = 1.$
The weak fuzzy complex sphere with radius $r_1 + r_2\varepsilon$ and center at the origin point is defined as follows:
$X^2 + Y^2 + Z^2 = r_1 + r_2\varepsilon; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon,, Z = z_1 + z_2\varepsilon, \varepsilon^2 = t \in]0,1[.$
Theorem.
Let $t \in R$ and C_{A_t}be the corresponding A-Curve in R^3, then:
1). C_{A_0} the dual sphere with radius $r_1 + 2r_2\varepsilon$ and center the origin.
2). $C_{A_{-1}}$ the complex sphere with radius $r_1 + 2r_2 i$ and center the origin.
3). C_{A_1} the split-complex sphere with radius $r_1 + 2r_2\varepsilon$ and center the origin.
4). $C_{A_t}; t \in]0,1[$ the weak fuzzy complex sphere with radius $r_1 + 2r_2\varepsilon$ and center the origin.
Proof.
1). C_{A_0} has the equation:

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$$\begin{cases} \|X\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}$$

So:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 = r_1 \\ x_1x_2 + y_1y_2 + z_1z_2 = r_2 \end{cases}$$

The dual sphere is $X^2 + Y^2 + Z^2 + Z^2 = r_1 + 2r_2\varepsilon; \varepsilon^2 = 0$, which implies that:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 = r_1 \\ 2(x_1x_2 + y_1y_2 + z_1z_2) = 2r_2 \end{cases}$$

and (1) holds.

2). $C_{A_{-1}}$ has the equation:

$$\begin{cases} \|X\|^2 - \|Y\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}$$

So:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 - x_2{}^2 - y_2{}^2 - z_2{}^2 = r_1 \\ x_1x_2 + y_1y_2 + z_1z_2 = r_2 \end{cases}$$

The complex sphere is $X^2 + Y^2 + Z^2 + Z^2 = r_1 + 2r_2 i; i^2 = -1$, which implies that:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 - x_2{}^2 - y_2{}^2 - z_2{}^2 = r_1 \\ 2(x_1x_2 + y_1y_2 + z_1z_2) = 2r_2 \end{cases}$$

and (2) holds.

3). C_{A_1} has the equation:

$$\begin{cases} \|X\|^2 + \|Y\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}$$

So:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 + x_2{}^2 + y_2{}^2 + z_2{}^2 = r_1 \\ x_1x_2 + y_1y_2 + z_1z_2 = r_2 \end{cases}$$

The split-complex sphere is $X^2 + Y^2 + Z^2 + Z^2 = r_1 + 2r_2\varepsilon; \varepsilon^2 = 1$, which implies that:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 + x_2{}^2 + y_2{}^2 + z_2{}^2 = r_1 \\ 2(x_1x_2 + y_1y_2 + z_1z_2) = 2r_2 \end{cases}$$

and (3) holds.

4). $C_{A_t}; t \in]0,1[$ has the equation:

$$\begin{cases} \|X\|^2 + t\|Y\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}$$

Hence:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 + t(x_2{}^2 + y_2{}^2 + z_2{}^2) = r_1 \\ x_1x_2 + y_1y_2 + z_1z_2 = r_2 \end{cases}$$

The weak fuzzy complex sphere is $X^2 + Y^2 + Z^2 + Z^2 = r_1 + 2r_2\varepsilon; \varepsilon^2 = 1$, which implies that:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 + t(x_2{}^2 + y_2{}^2 + z_2{}^2) = r_1 \\ 2(x_1x_2 + y_1y_2 + z_1z_2) = 2r_2 \end{cases}$$

and (4) holds.

3. Conclusion and open problems

In this paper, we have defined for the first time the concept of A-Curve in R^2 and R^3 by the following vectorial system of equation:

$$\begin{cases} \|X\|^2 + t\|Y\|^2 = r_1 \\ \langle X, Y \rangle = r_2 \end{cases}$$

Where $r_1, r_2 \geq 0, t \in R, \langle , \rangle$ I the Euclidean inner product defined over R^2, R^3 respectively.

Also, we have proved that A-Curve $C_{A_0}, C_{A_1}, C_{A_{-1}}, C_{A_t}; t \in]0,1[$ can be characterized as circle in dual, split-complex, complex, and weak fuzzy complex planes if $X, Y \in R^2$ and as sphere if $X, Y \in R^3$.

We show some open research problems that emerge from our study.

Problem (1). find the tangent surface of C_{A_t} in R^2.

Problem (2). Find an algorithm to find the tangent surface of C_{A_t} in R^3.

Problem (3). Characterize the general A-Curve defined with:

$$\begin{cases} f(\|X\|, \|Y\|) = c_1 \\ g(X, Y) = c_2 \end{cases}; \ c_1, c_2 \geq 0, f, g \text{ are two functions.}$$

For problem (3), we recommend to study special different cases of f, g (consider some of famous functions).

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References

[1] Deckelman, S.; Robson, B. Split-Complex Numbers and Dirac Bra-kets. Communications In Information and Systems **2014**, 14, 135-159.
[2] Akar, M.; Yuce, S.; Sahin, S. On The Dual Hyperbolic Numbers and The Complex Hyperbolic Numbers. Jcscm **2018,** 8, DOI: 10.20967/jcscm.2018.01.001.
[3] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.
[4] Hatip, A., "An Introduction To Weak Fuzzy Complex Numbers ", Galoitica Journal Of Mathematical Structures and Applications, Vol.3, 2023.
[5] Khaldi, A., " A Study On Split-Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.
[6] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.
[7] Ahmad, K., " On Some Split-Complex Diophantine Equations", Neoma Journal Of Mathematics and Computer Science, 2023.
[8] Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.
[9] Merkepci, M., and Abobala, M., " On Some Novel Results About Split-Complex Numbers, The Diagonalization Problem And Applications To Public Key Asymmetric Cryptography", Journal of Mathematics, Hindawi, 2023.
[10] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
[11] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
[12] Sarkis, M., "On The Solutions Of Fermat's Diophantine Equation In 3-refined Neutrosophic Ring of Integers", Neoma Journal of Mathematics and Computer Science, 2023.
[13] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021
[14] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021.
[15] Ali, R., and Hasan, Z., " An Introduction To The Symbolic 3-Plithogenic Modules ", Galoitica Journal Of Mathematical Structures and Applications, vol. 6, 2023.
[16] Ali, R., and Hasan, Z., "An Introduction To The Symbolic 3-Plithogenic Vector Spaces", Galoitica Journal Of Mathematical Structures and Applications, vol. 6, 2023.
[17] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic sets and systems, Vol. 45, 2021.

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