



## An approach to singular modules by indeterminacy concept

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### Abstract

In this article, we characterize a singular module and present several new strong relationships with neutrosophic (NC) properties. Some properties and characterizations neutrosophic of singular modules are given. Also, different basic results about these modules are considered. Moreover; for a  $(R \cup I)$ -module; if  $NC(MUI)$  over  $NC(RUI)$ , then  $NC(Z(MUI)) \leq NC(MUI)$ . Any neutrosophic simple module is either nonsingular or singular. On the other hand, if  $NC(RUI)$  has no zero divisors then  $NC(Z(MUI)) = NC(T(MUI))$  where  $NC(T(MUI))$  is a neutrosophic torsion module. Finally, some definitions and properties of neutrosophic singular module have been presented in this article.

**Keywords:** Singular module; Free module Annihilator module; Neutrosophic ring; Neutrosophic submodule.

### 1. Introduction

Smarandache [1] in 1999 introduced a new mathematical idea called neutrosophic set as an extension of fuzzy set [2]. This idea aroused the attention of numerous researchers around the world, by merging it with other branches of mathematics. For example, Salama and Alblawi [3] offered the notion of neutrosophic topological spaces utilizing idea of neutrosophic sets and many of their applications see [4,5]. Kandasamy and Smarandache [6] first established algebraic structures with neutrosophic ideas, following them Abed et al. [7,8] merged neutrosophic with module theory. Al-Sharqi et al. [9,10] applied the idea of neutrosophic with complex values and used this idea to solve some real-life applications. On the other hand, algebraic structures is a fertile environment that has attracted the attention of many researchers around the world, Al-Jumaili et al. [11] investigated a new class of mappings with strongly closed graphs in topological algebraic structures. Al-Hamido [12] discussed a new approach to a neutrosophic algebraic structure called a neutrosophic groupoid. Zail et al. [13] studied some new results of a generalization of BCK-algebra ( $\Omega$ -BCK-algebra). and other contributions, see [14-16].

Throughout this paper will be assumed any ring is a commutative with identity and every module is unitary. Any submodule  $A$  of an  $R$ -module  $M$  is called essential [17] if there exists  $B$  is a submodule of  $M$  such that  $A \cap B \neq 0$ . An element  $m$  belong to module  $M$  over the ring  $R$  [18] is called singular element if the right of annihilator  $m$ :  $\text{ann}(m) = \{r \in R: mr = 0\}$  is essential in  $R$ . We denote  $Z(M)$  to the set of all singular elements:

$$Z(M) = \{m \in M: \text{ann}(m) \leq_{\text{ess}} R\}.$$

A module  $M$  is called singular [19] if  $Z(M) = M$  and is called nonsingular if  $Z(M) = 0$ . On the other hand, Kasch [20] defined torsion free module by; if  $T(M) = M$  and is called torsion free module if  $T(M) = 0$  where  $T(M)$  refer to all torsion element in  $M$ .

This paper is organized as follows: In Section 2 recall different notions and fundamental results which play vital role in this study. In Section 3, we display several characterizations and essential properties concerning of

Indeterminacy of one important module namely singular module. Finally, the difficulties and importance of the study were discussed in the conclusion section in Section 4.

## 2. Preliminaries

**Definition 2.1. [1]** We refer to neutrosophic set  $NC(S)$  by

$$B = \{ \langle u, (\mu_B(u), \beta_B(u), \delta_B(u)) \rangle : u \in U \}$$

where  $U$  the universe of discourse and  $u$  is an element of  $U$  with  $\mu, \beta, \delta : \rightarrow [0, 1]$  are define respectively the degree of membership (True), the degree of indeterminacy and the degree of non-membership (False) of  $u$  in  $U$  such that

$$\{0 \leq (\mu_B(u) + \beta_B(u) + \delta_B(u)) \leq 3\}$$

**Note:** We denote  $NC(R \cup I)$  to neutrosophic ring and  $NC(M \cup I)$  neutrosophic module.

**Definition 2.2. [6]** Let  $I$  be neutrosophic of element with  $I^2=I$  and let  $(G, *)$  be a group such that  $\langle G \cup I \rangle = \{x+yI : x, y \in G\}$ . The set  $NC(G) = \{ \langle G \cup I \rangle, * \}$  is called neutrosophic group and generate by  $G$  and  $I$ .

**Definition 2.4. [6]** The set  $NC(R) = \{ \langle R \cup I \rangle, +, * \} = \{x+yI : x, y \in R\}$  is called neutrosophic ring and generate by  $R$  and  $I$  where  $R$  is a ring with two binary operation  $+$  and  $*$ .

**Examples 2.5. [6]** The ring of integer number  $Z$ , the ring of integer modulo  $n$   $Z_n$  and the ring of rational numbers  $Q$  ( $\langle Z \cup I \rangle = x+yI$   $x, y \in Z$ ,  $\langle Z_n \cup I \rangle = x+yI$   $x, y \in Z_n$  and  $\langle Q \cup I \rangle = x+yI$   $x, y \in Q$ ) are called  $NC(Z)$ ,  $NC(Z_n)$  and  $NC(Q)$  respectively.

**Definition 2.6. [13]** Any  $R$ -module  $(M, +, \cdot)$  or  $(I(M), +, \cdot)$  is called weak neutrosophic of  $M$  ( $NC(M)$ ) if  $M(I) = \langle M \cup I \rangle$  is  $NC(S)$  and generate by  $M$  and  $I$ .

**Remark 2.7. [13]** If  $M(I)$  is neutrosophic module over  $R = \langle R \cup I \rangle$ , then  $M(I) = \langle M \cup I \rangle$  is called strong neutrosophic  $(R \cup I)$ -module.

**Example 2.8.**  $(R \cup I)$  is very important example of  $NC(R \cup I)$ . Also,  $NC(G \cup I)$  is another example.

**Definition 2.9. [14]** If we have  $NC(M \cup I)$  over  $NC(R \cup I)$ , then  $NC((N \cup I))$  is a strong  $NC(N \cup I)$  submodule of  $NC(M \cup I)$  if  $NC(N \cup I)$  is itself strong too.

## 3. The main Results

In this section we study the Indeterminacy of one important module namely singular module. Also, some new results have been presented in this paper.

**Definition 3.1.**  $(Im)$  in the  $NC(M \cup I)$  over  $NC(R \cup I)$  means the right of annihilator  $(Im)$ :  $\text{ann}(Im) = \{I \in NC(R \cup I) : (Im)I = I0\}$  is essential in  $(R \cup I)$ .

**Remark 3.2.** We denote  $NC(Z(M \cup I))$  to the set of all neutrosophic singular module:

$$NC(Z(M \cup I)) = \{Im \in NC((M \cup I)) : \text{ann}(M \cup I) \leq_{\text{ess}} NC((R \cup I))\}.$$

**Proposition 3.3.** Suppose that we have  $NC(M \cup I)$  over  $NC(R \cup I)$ . Then  $NC(Z(M \cup I)) \leq NC(M \cup I)$ .

**Proof:**

To prove  $NC(Z(M \cup I))$  is a submodule of  $NC((M \cup I))$ , we need to show that  $NC(Z(M \cup I)) \neq \emptyset$ . It is clear that  $(I0) \in NC(Z(M \cup I))$  and hence  $NC(Z(M \cup I)) \neq \emptyset$ . Assume that  $Ix, Iy$  are two elements in  $Z(M \cup I)$ . So

$$\text{ann}(Ix) \leq_{\text{ess}} NC((R \cup I)) \text{ and } \text{ann}(Iy) \leq_{\text{ess}} NC((R \cup I)).$$

But

$$\text{ann}(I(x-y)) \supset \text{ann}(Ix) \cap \text{ann}(Iy) \leq_{\text{ess}} NC((R \cup I)).$$

So

$$I_x - I_y = I(x - y) \in NC(Z(MUI)).$$

Let  $I_x \in NC(Z(MUI))$  and  $I_r \in NC(Z(RUI))$ . We must prove that  $I(xr) \in NC(Z(MUI))$ , that is to show  $\text{ann}(I_x) \leq_{\text{ess}} NC((RUI))$ . Given any element  $I_a \in NC((RUI)) - \text{ann}(I(xr))$ . So,  $I(xr)I_a \neq I_0$ . But  $\text{ann}(I_x) \leq_{\text{ess}} (RUI)$ . Then

$$\exists I_b \in NC((RUI)) \ni I(ra)I_b \in \text{ann}(I_x) \text{ and } I(ra)I_b \neq I_0.$$

So

$$I(x(rab)) = I_0, I_b \in NC((RUI)) \ni I(rab) \neq I_0.$$

We have  $I(ab) \neq I_0$  and  $I(ab) \in \text{ann}(I(xr))$ . Thus,  $\text{ann}(I(xr)) \leq_{\text{ess}} NC((RUI))$ .

#### Remarks 3.4.

a) We say  $NC(Z(MUI)) = \{I_m \in NC(MUI) : (I_m)A = (I_0) \text{ for some } NC(AUI) \leq_{\text{ess}} NC(RUI)\}$ ; because: Let  $NC(HUI) = \{I_m \in NC(MUI) : (I_m)A = (I_0), NC(AUI) \leq_{\text{ess}} NC(RUI)\}$ . We must prove that  $NC(HUI) = NC(Z(MUI))$ .

Let  $I_m \in NC(Z(MUI))$ . Then  $\text{ann}(I_m) \leq_{\text{ess}} NC(RUI)$ . So  $(I_m) \cdot (\text{ann}(I_m)) = 0$ . Hence

$$I_m \in NC(HUI). \text{ Therefore } NC(HUI) \supseteq NC(Z(MUI)).$$

Now, the other direction, suppose that  $I_m \in NC(HUI)$ . So,

$$(I_m)A = 0, NC(AUI) \leq_{\text{ess}} NC(RUI).$$

But  $NC(AUI) \subseteq \text{ann}(I_m)$ . Then  $\text{ann}(I_m) \leq_{\text{ess}} NC(RUI)$ . Therefore  $I_m \in NC(Z(MUI))$ . Hence,

$$NC(HUI) \subseteq NC(Z(MUI)).$$

Then

$$NC(Z(MUI)) = NC(HUI).$$

Thus

$$NC(HUI) = NC(Z(MUI)).$$

a)  $Z(I_m)$ .  $\text{Soc}(NC(RUI)) = 0$ , because: let  $I_m \in NC(Z(MUI))$ . Then  $\text{ann}(I_m) \leq_{\text{ess}} NC(RUI)$ . But  $\text{Soc}(NC(RUI))$  equal the intersection of all neutrosophic of essential ideals  $NC(AUI)$ . Hence  $\text{Soc}(NC(RUI)) \subseteq \text{ann}(I_m)$ . So,

$$(I_m) \cdot (\text{Soc}(NC(RUI))) = 0 \quad \forall I_m \in NC(Z(MUI))$$

Thus

$$NC(Z(MUI)) \cdot (\text{Soc}(NC(RUI))) = 0.$$

**Corollary 3.5.** Let  $f: NC(MUI) \rightarrow NC(NUI)$  be a neutrosophic (RUI)-homomorphism. Then  $f(NC(Z(MUI))) \subseteq NC(Z(NUI))$ .

#### Proof:

Suppose that  $f: NC(MUI) \rightarrow NC(NUI)$  be a neutrosophic (RUI)-homomorphism. Let  $I_y \in f(NC(Z(MUI)))$  such that  $I_y = f(I_m)$ ;  $I_m \in NC(Z(MUI))$ . Hence  $\text{ann}(I_m) \leq_{\text{ess}} NC(RUI)$ . But

$\text{ann}(I_m) \subseteq \text{ann}(f(I_m))$ . Since  $\forall I_r \in \text{ann}(I_m)$ , so  $I(mr) = 0$ . Then  $f(I_mr) = 0$ . Hence  $f(I_m)(I_r) = I_0$ . Therefore  $I_r \in \text{ann}(f(I_m))$ . So  $\text{ann}(f(I_m)) \leq_{\text{ess}} NC(RUI)$ . Thus  $I_y = f(I_m) \in NC(Z(NUI))$ .

**Corollary 3.6.** Let  $NC(MUI)$  be neutrosophic module and  $NC(NUI) \leq NC(MUI)$ . Then  $NC(Z(NUI)) = NC(Z(MUI)) \cap NC(NUI)$ .

Since any  $I_m \in NC(Z(NUI))$ . So,  $I_m \in NC(NUI)$  with  $\text{ann}(I_m) \leq_{\text{ess}} NC(RUI)$ . Hence

$$NC(Z(NUI)) \leq NC(Z(MUI)) \cap NC(NUI).$$

Clear that;  $NC(Z(MUI)) \cap NC(NUI) \leq NC(Z(NUI))$ . Thus, the result is obtained.

**Examples 3.7.**

a) Let  $NC(ZUI)$  be neutrosophic as a  $Z$ -module. The neutrosophic singular submodule of  $NC(ZUI)$  equal  $(I_0)$ . Since  $\forall I_x \in NC(ZUI)$  and  $I_x \neq I_0$ , so  $\text{ann}(I_x) \not\subseteq_{\text{ess}} NC(ZUI)$ .

b) Let  $NC(Z_{12}UI)$  be neutrosophic as  $(ZUI)$ -module.  $\forall I_x \in NC(Z_{12}UI)$  with  $\text{ann}_{(ZUI)}(I_x) \neq 0$  and so  $\text{ann}(I_x) \subseteq_{\text{ess}} NC(ZUI)$ . Hence  $NC(Z(Z_{12}UI)) = NC(Z_{12}UI)$  and then  $\forall I_n \in NC(ZUI); I_n > 1$ ,  $NC(Z(Z_nUI)) = NC(Z_nUI)$ .

c) Let  $NC(Z_4UI)$  be neutrosophic as  $NC(Z_4UI)$ -modul. So  $nc(Z_4UI) = \{\overline{I_0}, \overline{I_2}\} < NC(Z_4UI)$ .

d) For the  $(Z_4UI)$ -module  $(QUI)$ ,  $NC(Z(QUI)) = 0$

Now we study the neutrosophic of singular module and non-singular module, but before that we need to present the following definition.

**Definition 3.8.** An  $NC(RUI)$ -module  $NC(MUI)$  is called neutrosophic singular if  $NC(Z(MUI)) = NC(MUI)$  and it is called neutrosophic non-singular if  $NC(Z(MUI)) = I_0$ .

**Remarks 3.9 .**

a)  $NC(RUI)$  is called neutrosophic singular(non-singular) if  $NC(RUI)$  as  $NC(RUI)$ -module is neutrosophic singular (neutrosophic non-singular) .

b)  $\forall (I_n) \in NC(ZUI); (I_n) > (I_1)$ , imply that  $NC(Z_nUI)$  as  $(ZUI)$ -module is neutrosophic singular, while each of  $(ZUI)$ -module  $NC(ZUI)$ ,  $NC(QUI)$  are neutrosophic non-singular and  $NC(Z_nUI)$  as  $(ZUI)$ -module is neutrosophic non-singular.

c) Let  $NC(NUI)$  be neutrosophic submodule of  $NC(MUI)$ . If  $NC(MUI)$  is singular implies  $NC(NUI)$  is also singular and if  $NC(MUI)$  is nonsingular, so  $NC(NUI)$  is also nonsingular.

d) any neutrosophic simple module is either nonsingular or singular. On the other hand, if  $NC(RUI)$  has no zero divisors then  $NC(Z(MUI)) = NC(T(MUI))$  where  $NC(T(MUI))$  is a neutrosophic torsion module.

**Proof (d)**

Since  $NC(T(MUI)) = \{I_x \in NC(T(MUI)): \text{ann}(I_x) \neq I_0\}$  and  $NC(RUI)$  has no zero divisors, then  $\forall I_x \in NC(T(MUI))$ . Hence  $\text{ann}(I_x) \subseteq_{\text{ess}} NC(RUI)$ . Thus

$$NC(T(MUI)) \subseteq NC(Z(MUI)) \forall I_x \in NC(Z(MUI)).$$

So,

$$\text{ann}(I_x) \neq I_0 \text{ and hence } I_x \in NC(T(MUI)).$$

Therefore,

$$NC(Z(MUI)) \subseteq NC(T(MUI)).$$

Thus

$$NC(Z(MUI)) = NC(T(MUI)).$$

**Proposition 3.10.** Let  $NC(MUI)$  be a module over  $NC(RUI)$ . Then  $NC(MUI)$  is nonsingular if and only if  $\text{Hom}[NC(M_1UI), NC(MUI)] = 0$  where  $NC(M_1UI)$  is a singular module.

**Proof:**

Suppose that  $NC(MUI)$  is nonsingular. Then  $NC(Z(MUI)) = I_0$ . Let  $NC(M_1UI)$  be a singular module. So  $NC(Z(MUI)) = NC(M_1UI)$ .

Take  $NC(M_1UI) \rightarrow NC(MUI)$  as  $NC(RUI)$ -homomorphism. Then

$$f(NC(Z(M_1UI)) \subseteq NC(Z(MUI)).$$

Hence

$$f(\text{NC}(M_1 \cup I)) \subseteq (I_0).$$

Thus,

$$f(\text{NC}(M_1 \cup I)) = I_0 \quad (f=i_0).$$

Conversely: to prove  $\text{NC}(M \cup I)$  is a nonsingular. Since

$$\begin{aligned} \text{NC}(Z(\text{NC}(Z(M \cup I)))) &= \text{NC}(Z(M \cup I)) \\ &= \text{NC}(Z(M \cup I)) \cap \text{NC}(M \cup I) \\ &= \text{NC}(Z(M \cup I)) \end{aligned}$$

with  $\text{NC}(Z(\text{NC}(Z(M \cup I)))) = \text{NC}(Z(M \cup I))$ , then  $\text{NC}(Z(M \cup I))$  is a singular module. Hence

$$\text{Hom}(\text{NC}(Z(M \cup I)), \text{NC}(M \cup I)) = I_0.$$

Also,  $\text{NC}(Z(M \cup I)) \leq \text{NC}(M \cup I)$ . Therefore,  $i \in \text{Hom}(\text{NC}(Z(M \cup I)), \text{NC}(M \cup I)) = I_0$ ,  $i$  is inclusion mapping. Hence  $i = I_0$  with  $\text{NC}(Z(M \cup I)) = I_0$ . Thus  $\text{NC}(M \cup I)$  is a nonsingular.

**Proposition 3.11.** Let  $\text{NC}(M \cup I)$  be neutrosophic  $\text{NC}(R \cup I)$ -module. Then  $\text{NC}(M \cup I)$  is singular if and only if there exists  $I_0 \rightarrow \text{NC}(M_1 \cup I) \xrightarrow{f} \text{NC}(M_2 \cup I) \xrightarrow{g} I_0$ , where  $f$  is a neutrosophic essential monomorphism.

**Proof:**

Suppose that  $\text{NC}(M \cup I)$  is a singular. Take an exact sequence

$$I_0 \rightarrow \text{NC}(M_1 \cup I) \xrightarrow{i} \text{NC}(M_2 \cup I) \xrightarrow{g} I_0$$

with  $\text{NC}(M_1 \cup I) \subseteq \text{NC}(M_2 \cup I)$  and  $\text{NC}(M_2 \cup I)$  is a neutrosophic free module.

Let  $\{\text{Im}_\beta\}$  be a neutrosophic basis for  $\text{NC}(M_2 \cup I)$ . Then  $g(\text{Im}_\beta) (A_\beta \cup I) = I_0 \ni (A_\beta \cup I)$  is a neutrosophic essential ideal ( $\text{NC}(M_2 \cup I)$  is a singular). Hence

$$g(\text{Im}_\beta A_\beta) = I_0, \text{ so } m_\beta A_\beta \subseteq \text{Ker}(g).$$

But  $\text{ker}(g) = \text{Im}(i) = \text{NC}(M_1 \cup I)$ . So,

$$I(m_\beta A_\beta) \subseteq \text{NC}(M_1 \cup I) \quad \forall I_\beta.$$

Since  $I A_\beta \leq_{\text{ess}} \text{NC}(R \cup I)$ , we obtain  $I(m_\beta A_\beta) \leq_{\text{ess}} \text{NC}((R \cup I))$ .

Therefore

$$\bigoplus (\text{Im}_\beta A_\beta) \leq_{\text{ess}} \bigoplus (\text{Im}_\beta \text{NC}(R \cup I)) = \text{NC}(M_2 \cup I).$$

But  $I(m_\beta A_\beta) \leq \text{NC}((M_1 \cup I) \subseteq \text{NC}((M_2 \cup I))$ . Then,  $\text{NC}(M_1 \cup I) \leq_{\text{ess}} \text{NC}(M_2 \cup I)$ . Thus  $f: \text{NC}(M_1 \cup I) \rightarrow \text{NC}(M_2 \cup I)$  is essential monomorphism.

Conversely: Assume that we have neutrosophic exact sequence. Let  $\text{Im}_2 \in (M_2 \cup I)$  and let  $h: \text{NC}(R \cup I) \rightarrow \text{NC}(M_2 \cup I) \ni h(\text{Ir}) = I(m_2 r)$ ;  $\text{Ir} \in \text{NC}(R \cup I)$ .

Since  $f(M_1 \cup I) \leq_{\text{ess}} \text{NC}((M_2 \cup I))$ , then  $h^{-1}(f(M_1 \cup I)) \leq_{\text{ess}} \text{NC}((R \cup I))$ .

But

$$\begin{aligned} h^{-1}(f(M_1 \cup I)) &= \{\text{Ir} \in \text{NC}(R \cup I) : h(\text{Ir}) \in f(M_1 \cup I)\} \\ &= \{\text{Ir} \in \text{NC}(R \cup I) : I(m_2 r) \in f(M_1 \cup I)\}. \end{aligned}$$

Let  $\text{NC}(A \cup I) = h^{-1}(f(M_1 \cup I))$ . So,

$$\text{NC}(A \cup I) \leq_{\text{ess}} \text{NC}((R \cup I)) \text{ and } I(m_2 A) \subseteq f(M_1 \cup I) = \text{Ker}(g).$$

Then  $g(\text{Im}_2) A = I_0$  and hence  $g(\text{Im}_2) \in \text{NC}(Z(M \cup I))$ . But  $g$  is an epi. Hence

$$\forall \text{Im} \in \text{NC}(\text{MU I}) \ni \text{Im}_2 \in \text{NC}(\text{MUI}) \text{ with } g(\text{Im}_2) = \text{Im}.$$

Hence  $\text{NC}(\text{Z}(\text{MU I}))$ . So,  $\forall \text{Im} \in \text{NC}(\text{MUI}) \ni \text{Im}_2 \in \text{NC}(\text{RUI})$  with  $g(\text{Im}_2) = \text{Im}$ , we obtain the following

$$\text{NC}(\text{MUI}) = \text{NC}(\text{Z}(\text{MU I})).$$

Thus  $\text{NC}(\text{MU I})$  is a singular.

## 2. Conclusion

The concepts of neutrosophy and neutrosophic sets have putted the foundation for a whole family of novel mathematical theories to generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. The main target of our manuscript is to display and study a new idea of neutrosophic of singular modules. Also, different basic results about these modules are considered. Moreover; for a  $(R \cup I)$ -module; if  $\text{NC}(\text{MUI})$  over  $\text{NC}(\text{RUI})$ , then  $\text{NC}(\text{Z}(\text{MUI})) \leq \text{NC}(\text{MUI})$ . Any neutrosophic simple module is either nonsingular or singular. Finally, some definitions and properties of neutrosophic singular module have been investigated in this article.

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