New certain classes of Generalized Neutrosophic Mappings and Their Applications

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Abstract

The main objective of the present this article is to apply another generalized form of neutrosophic open sets namely,neutrosophic *δ-ß*-open sets to introduce and study a completely description for another new concept of generalized neutrosophic continuous maps namely, neutrosophic *δ-ß*-continuous maps in neutrosophic topological spaces. Several of the fundamental properties related to this kind of neutrosophic continuous maps have been investigated. In addition, the interrelationships between this kind of maps and other well-known related neutrosophic maps have been discussed. On the other hand, new class of generalized neutrosophic maps namely, neutrosophic *δ-ß*-irresolute maps has been studied, and some of the essential properties concerning of this class of generalized neutrosophic maps have been introduced. Moreover, various suitable examples to support of these results have been provided.

Keywords: Neutrosophic *δ-ß*-open sets; neutrosophic *δ-ß*-continuous maps; neutrosophic *δ-ß*- irresolute maps; neutrosophic *δ-ß*- $U_{1/2}$ space

1. Introduction

Smarandache [1] in 1999 introduced a new mathematical idea called neutrosophic set as an extension of fuzzy set [2]. This idea aroused the attention of numerous researchers around the world, by merging it with other branches of mathematics. For example, Salama and Alblowi [3] offered the notion of neutrosophic topological spaces utilizing idea of neutrosophic sets and many of their applications see [4-6], as well the notion of continuous mappings have been studied through Salama et al. [7]. Kandasamy and Smarandache [8] first established algebraic structures with neutrosophic ideas, following them Abed et al. [9,10] merged neutrosophic with module theory. Al-Sharqi et al. [11,12] applied the idea of neutrosophic with complex values and used this idea to solve some real-life applications. On the other hand, topological space is a fertile environment that has attracted the attention of many researchers around the world, Hatir and Noiri [13]. studied a new class of generalized open sets, called *δ-ß*-open sets (e^*-open sets). Al-Jumaili et al. [14,15] investigated a new class of mappings with strongly closed graphs in topological spaces. Iswarya and Bageerathi [16] studied neutrosophic semi-open sets in neutrosophic topological spaces. In addition to a number of other research works we refer the reader to see [17-20].

The goal of presenting this work is to introduce and investigate other new concepts of generalized neutrosophic maps namely, neutrosophic δ-ß-continuous and neutrosophic δ-ß-irresolute maps in neutrosophic topological spaces. Several characterizations and basic properties related to these types of generalized neutrosophic maps have been discussed. In addition, some appropriate examples have been illustrated to support our main results".

This paper is organized as follows: In Section 2 recall different notions and fundamental results which play vital role in this study. In Section 3, we display several characterizations and essential properties concerning of neutrosophic δ-ß-continuous mappings utilizing δ-ß-neutrosophic open sets. Several characterizations and fundamental properties concerning of neutrosophic δ-ß-irresolute mappings utilizing δ-ß- neutrosophic open sets have been obtained in Section 4. Finally, the difficulties and importance of the study were discussed in the conclusion section in Section 5.

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2. Materials and Methods

In this section, recall different notions and fundamental results which play vital role in this study and helpful for verifying our major results .

Definition 2.1: [3] Let $\mathcal{Y} \neq \emptyset$. A neutrosophic set (Concisely, $\mathcal{N}_{\mathcal{S}}S$) $\mathbb{L}$ is object having the shape $\mathbb{L} = \{\langle \mathfrak{F}y, \Gamma_{\mathbb{L}}(y), \lambda_{\mathbb{L}}(y), q_{\mathbb{L}}(y) i \rangle : y \in \mathcal{Y}\}$ wherever $\Gamma_{\mathbb{L}} \longrightarrow [0,1]$ indicate to degree of membership map, $\lambda_{\mathbb{L}} \longrightarrow [0,1]$ indicate to degree of indeterminacy map with $q_{\mathbb{L}} \longrightarrow [0,1]$ indicate to degree of non-membership map resp, $\forall\, y \in \mathcal{Y}$ to set $\mathbb{L}$ & $0 \leq \Gamma_{\mathbb{L}}(y) + \lambda_{\mathbb{L}}(y) + q_{\mathbb{L}}(y) \leq 3, \forall y \in \mathcal{Y}$.

Remark 2.2: [3] A $\mathcal{N}_{\mathcal{S}}S$ $\mathbb{L} = \{\langle \mathfrak{F}y, \Gamma_{\mathbb{L}}(y), \lambda_{\mathbb{L}}(y), q_{\mathbb{L}}(y) i \rangle : y \in \mathcal{Y}\}$ can be specified to arranged as $\mathfrak{F}y, \Gamma_{\mathbb{L}}(y), \lambda_{\mathbb{L}}(y), q_{\mathbb{L}}(y) i$ in [0,1] on $\mathcal{Y}$.

Definition2.3: [3] Let $\mathcal{Y} \neq \emptyset$ with $\mathcal{N}_{\mathcal{S}}S'$ S $\mathbb{L}$ & $\mathcal{H}$ in the shape

$\mathbb{L} = \{\langle \mathfrak{F}y, \Gamma_{\mathbb{L}}(y), \lambda_{\mathbb{L}}(y), q_{\mathbb{L}}(y) i \rangle : y \in \mathcal{Y}\}$, $\mathcal{H} = \langle \mathfrak{F}y, \Gamma_{\mathcal{H}}(y), \lambda_{\mathcal{H}}(y), q_{\mathcal{H}}(y) i \rangle : y \in \mathcal{Y}\}$, then

$(\boldsymbol{a})$ $0_{\mathcal{N}} = \mathfrak{F}y, 0,0,1 i$ & $1_{\mathcal{N}} = \mathfrak{F}y, 1,1,0 i$;

$(\boldsymbol{b})$ $\mathbb{L} \subseteq \mathcal{H} \Leftrightarrow \Gamma_{\mathbb{L}}(y) \leq \Gamma_{\mathcal{H}}(y), \lambda_{\mathbb{L}}(y) \leq \lambda_{\mathcal{H}}(y)$ & $q_{\mathbb{L}}(y) \geq q_{\mathcal{H}}(y) : y \in \mathcal{Y}$;

$(\boldsymbol{c})$ $\mathbb{L} = \mathcal{H} \Leftrightarrow \mathbb{L} \subseteq \mathcal{H}$ and $\mathcal{H} \subseteq \mathbb{L}$;

$(\boldsymbol{d})$ $1_{\mathcal{N}} - \mathbb{L} = \{\langle y, q_{\mathbb{L}}(y), 1 - \lambda_{\mathbb{L}}(y), \Gamma_{\mathbb{L}}(y) i \rangle : y \in \mathcal{Y}\} = \mathbb{L}^c$;

(**e**) $\mathbb{L} \cap \mathcal{H} = \{\mathfrak{F}y, min(\Gamma_{\mathbb{L}}(y), \Gamma_{\mathcal{H}}(y)), min\,(\lambda_{\mathbb{L}}(y), \lambda_{\mathcal{H}}(y)), max(q_{\mathbb{L}}(y), q_{\mathcal{H}}(y))\, i : y \in \mathcal{Y}\}$.

$(\boldsymbol{f})$ $\mathbb{L} \cup \mathcal{H} = \{\mathfrak{F}y, max(\Gamma_{\mathbb{L}}(y), \Gamma_{\mathcal{H}}(y)), max\,(\lambda_{\mathbb{L}}(y), \lambda_{\mathcal{H}}(y)), min\,(q_{\mathbb{L}}(y), q_{\mathcal{H}}(y))\, i : y \in \mathcal{Y}\}$.

Definition2.4: [3] On $\mathcal{Y} \neq \emptyset$ a neutrosophic Topology(Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{T}$) is a collection $\Phi_{\mathcal{N}}$ of $\mathcal{N}_{\mathcal{S}}S$ of $\mathcal{Y}$ satisfactory:

$(\boldsymbol{i})$ $0_{\mathcal{N}}, 1_{\mathcal{N}} \in \Phi_{\mathcal{N}}$;

$(\boldsymbol{ii})$ $\mathbb{L}_1 \cap \mathbb{L}_2 \in \Phi_{\mathcal{N}}, \forall\, \mathbb{L}_1, \mathbb{L}_2 \in \Phi_{\mathcal{N}}$;

$(\boldsymbol{iii})$ $\cup \mathbb{L}_x \in \Phi_{\mathcal{N}}, \forall\, \mathbb{L}_x : x \in \mathcal{X} \subseteq \Phi_{\mathcal{N}}$.

In that case, $(\mathcal{Y}, \Phi_{\mathcal{N}})$ is said to neutrosiphic Topological-Space(Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{TS}$) in $\mathcal{Y}$. A $\Phi_{\mathcal{N}}$ elements are said to neutrosiphic open-set (Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{OS}$) in $\mathcal{Y}$. A $\mathcal{N}_{\mathcal{S}}S$ $\mathcal{C}$ is neutrosiphic closed-set (Concisely, $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$) *iff* $\mathcal{C}^c$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$.

Definition2.5: [3] If$(\mathcal{Y}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$ on $\mathcal{Y}$ & $\mathbb{L}$ is $\mathcal{N}_{\mathcal{S}}S$ of $\mathcal{Y}$, so a neutrosophic interior of $\mathbb{L}$ (Concisely, $\mathcal{N}_{\mathcal{S}}Int(\mathbb{L})$) & neutrosophic closure of($\mathbb{L}$)(Concisely, $\mathcal{N}_{\mathcal{S}}Cl(\mathbb{L})$)are describe:

$$\mathcal{N}_{\mathcal{S}}Int(\mathbb{L}) = \cup\{\mathcal{J} : \mathcal{J} \subseteq \mathbb{L}\ \&\ \mathcal{J}\ is\ \mathcal{N}_{\mathcal{S}}\mathcal{OS}\ in\ \mathcal{Y}\}$$

$$\mathcal{N}_{\mathcal{S}}Cl(\mathbb{L}) = \cap\{\mathcal{J} = \mathcal{J} \subseteq \mathbb{L}\ \&\ \mathcal{J}\ is\ \mathcal{N}_{\mathcal{S}}\mathcal{CS}\ in\ \mathcal{Y}\}.$$

Definition2.6: [20] If $(\mathcal{Y}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$ on $\mathcal{Y}$ & $\mathbb{L}$ is $\mathcal{N}_{\mathcal{S}}S$. In that case,($\mathbb{L}$) is neutrosophic regular open set (Concisely, $\mathcal{N}_{\mathcal{S}}\mathcal{ROS}$) *if* $\mathbb{L} = \mathcal{N}_{\mathcal{S}}Int(\mathcal{N}_{\mathcal{S}}Cl(\mathbb{L}))$.

The$(\mathcal{N}_{\mathcal{S}}\mathcal{ROS})^C$is neutrosophic regular closed-set(Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{RCS}$) in $\mathcal{Y}$.

Definition2.7: [16]A subset $\mathcal{K}$ is:

$(\boldsymbol{i})$ $\mathcal{N}_{\mathcal{S}}\delta$-interior of $\mathcal{K}$ (Concisely,$\mathcal{N}_{\mathcal{S}}\delta Int(\mathcal{K})$) describe via $\mathcal{N}_{\mathcal{S}}\delta Int(\mathcal{K}) = \cup\{\mathcal{B} : \mathcal{B} \subseteq \mathcal{K}$ & B is $\mathcal{N}_{\mathcal{S}}\mathcal{ROS}$ in $\mathcal{Y}\}$;

$(\boldsymbol{ii})$ $\mathcal{N}_{\mathcal{S}}\delta$-closure of $\mathcal{K}$ (Concisely,$\mathcal{N}_{\mathcal{S}}\delta Cl(\mathcal{K})$) describe via $\mathcal{N}_{\mathcal{S}}\delta Cl(\mathcal{K}) = \cap\{\mathcal{A} : \mathcal{K} \subseteq \mathcal{A}$ & A is $\mathcal{N}_{\mathcal{S}}\mathcal{RCS}$ in $\mathcal{Y}\}$.

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Definition2.8: A subset $\mathbb{L}$ is: [16,20]

$(\boldsymbol{a})$ A $\mathcal{N}_{\mathcal{S}}$ Pre-open-set(Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{POS}$) *if* $\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Int(\mathcal{N}_{\mathcal{S}}Cl(\mathbb{L}))$

$(\boldsymbol{b})$ A $\mathcal{N}_{\mathcal{S}}$ Semi-open-set(Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{SOS}$) *if* $\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Cl(\mathcal{N}_{\mathcal{S}}Int(\mathbb{L}))$

$(\boldsymbol{c})$ A $\mathcal{N}_{\mathcal{S}}$ α-open-set(Concisely,$\mathcal{N}_{\mathcal{S}}\alpha\mathcal{OS}$) *if* $\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Int(\mathcal{N}_{\mathcal{S}}Cl(\mathcal{N}_{\mathcal{S}}Int(\mathbb{L})))$

$(\boldsymbol{d})$ A $\mathcal{N}_{\mathcal{S}}$ δ-open-set(Concisely,$\mathcal{N}_{\mathcal{S}}\delta\mathcal{OS}$) *if* $\mathbb{L} = \mathcal{N}_{\mathcal{S}}\delta Int(\mathbb{L})$;

$(\boldsymbol{e})$ A $\mathcal{N}_{\mathcal{S}}$ δ-Pre open-set (Concisely,$\mathcal{N}_{\mathcal{S}}\delta\mathcal{POS}$) *if* $\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Int(\mathcal{N}_{\mathcal{S}}\delta Cl(\mathbb{L}))$;

$(\boldsymbol{f})$ A $\mathcal{N}_{\mathcal{S}}$ δ-Semi open-set(Concisely,$\mathcal{N}_{\mathcal{S}}\delta\mathcal{SOS}$) *if* $\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Cl(N_{\mathcal{S}}\delta Int(\mathbb{L}))$;

$(\boldsymbol{g})$ A $\mathcal{N}_{\mathcal{S}}$ E-open-set(Concisely,$\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$) *if*

$\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Cl(\mathcal{N}_{\mathcal{S}}\delta Int(\mathbb{L})) \cup \mathcal{N}_{\mathcal{S}}Int(\mathcal{N}_{\mathcal{S}}\delta Cl(\mathbb{L}))$;

$(\boldsymbol{h})$ A $\mathcal{N}_{\mathcal{S}}$ E^{*}-open-set(Concisely,$\mathcal{N}_{\mathcal{S}}E^{*}\mathcal{OS}$) *if* $\mathbb{L} \subseteq \mathcal{N}_{\mathcal{S}}Cl\left(\mathcal{N}_{\mathcal{S}}Int(\mathcal{N}_{\mathcal{S}}\delta Cl(\mathbb{L}))\right)$.

Remark 2.9: The complement of an $\mathcal{N}_{\mathcal{S}}\mathcal{POS}$

(*resp.*$\mathcal{N}_{\mathcal{S}}\mathcal{SOS}, \mathcal{N}_{\mathcal{S}}\alpha\mathcal{OS}, \mathcal{N}_{\mathcal{S}}\delta\mathcal{OS}, \mathcal{N}_{\mathcal{S}}\delta\mathcal{POS}, \mathcal{N}_{\mathcal{S}}\delta\mathcal{SOS}, \mathcal{N}_{\mathcal{S}}E\mathcal{OS}, \mathcal{N}_{\mathcal{S}}E^{*}\mathcal{OS}$) is called a neutrosophic closed sets, and denoted by $\mathcal{N}_{\mathcal{S}}\mathcal{PCS}$

(*resp*. $\mathcal{N}_{\mathcal{S}}\mathcal{SCS}, \mathcal{N}_{\mathcal{S}}\alpha\mathcal{CS}, \mathcal{N}_{\mathcal{S}}\delta\mathcal{CS}, \mathcal{N}_{\mathcal{S}}\delta\mathcal{PCS}, \mathcal{N}_{\mathcal{S}}\delta\mathcal{SCS}, \mathcal{N}_{\mathcal{S}}E\mathcal{CS}, \mathcal{N}_{\mathcal{S}}E^{*}\mathcal{CS}$) in $\mathcal{Y}$.

Remark 2.10: The following diagram describes the relationships among some well-known generalized neutrosophic open sets in neutrosophic topological spaces . None of these implications is reversible as shown via examples in [20].

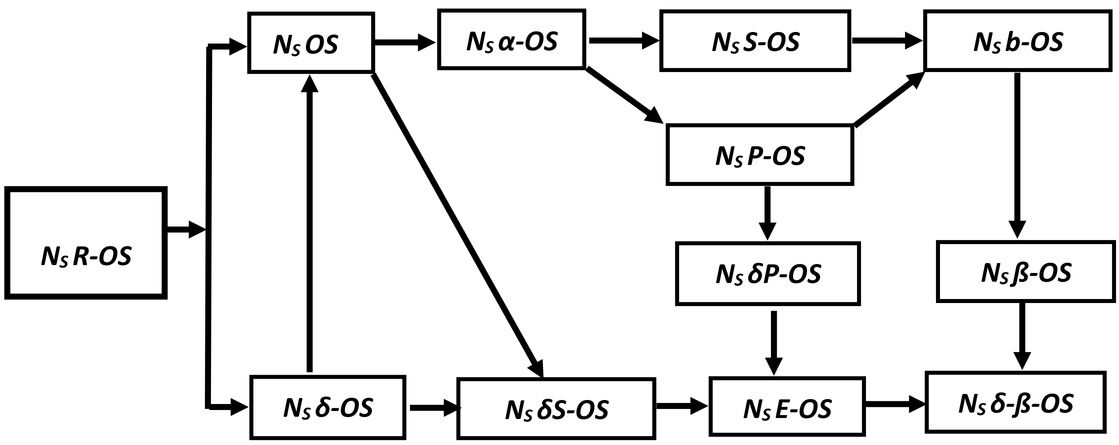

Figure 1: The relationships among some well-known generalized Neutrosophic open sets

3. Results and Discussion

Various Characterizations of Neutrosophic δ-ß-Continuous Map in $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$

This section is devoted to display several characterizations and essential properties concerning of neutrosophic δ-ß-continuous mappings utilizing δ-ß-neutrosophic open sets .

Definition 3.1: A map $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is said to be neutrosophic δ-ß-continuous (Shortly, $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-cont.) if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß$\mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$ $\forall$ $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{D}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$.

Definition 3.2: [19] Let $(\mathcal{X}, \Phi_{\mathcal{N}})$ and $(\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$' S. A map$\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is:

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$(\boldsymbol{a})$ A neutrosophic continuous (Concisely,$\mathcal{N}_{\mathcal{S}}$-cont.) if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{X},\Phi_{\mathcal{N}})$ ∀ $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{D}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

$(\boldsymbol{b})$ A $\mathcal{N}_{\mathcal{S}}$-δ-cont. if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta\mathcal{OS}$ in $(\mathcal{X},\Phi_{\mathcal{N}})$ for each $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{D}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

$(\boldsymbol{c})$ A $\mathcal{N}_{\mathcal{S}}$-δP-cont. if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta\mathcal{POS}$ in$(\mathcal{X},\Phi_{\mathcal{N}})$ for each $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{D}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

$(\boldsymbol{d})$ A $\mathcal{N}_{\mathcal{S}}$-δS-cont. if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta\mathcal{SOS}$ in $(\mathcal{X},\Phi_{\mathcal{N}})$ for each$\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{D}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

$(\boldsymbol{e})$ A $\mathcal{N}_{\mathcal{S}}$-$E$-cont. if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$ in$(\mathcal{X},\Phi_{\mathcal{N}})$ for each$\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ $\mathcal{D}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

Example 3.3: Assume, $\mathcal{X} = \mathcal{Y} = \{r, p, z\}$ & describe $\mathcal{N}_{\mathcal{S}}\mathcal{S}$' $\mathcal{S}$ $\mathcal{X}_1, \mathcal{X}_2$ & $\mathcal{X}_3$ in $\mathcal{X}$ & $\mathcal{Y}_1$ in $\mathcal{Y}$ as follows:

$$\mathcal{X}_1 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.7}, \frac{q_z}{0.6}\right) i;$$

$$\mathcal{X}_2 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.1}, \frac{\Gamma_p}{0.1}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.9}, \frac{q_p}{0.9}, \frac{q_z}{0.6}\right) i;$$

$$\mathcal{X}_3 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.4}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.6}, \frac{q_z}{0.6}\right) i;$$

$$\mathcal{Y}_1 = \mathfrak{F}\mathcal{Y}, \left(\frac{\Gamma_r}{0.2}, \frac{\Gamma_p}{0.4}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.8}, \frac{q_p}{0.6}, \frac{q_z}{0.6}\right) i.$$

In that case, obtain $\Phi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{X}_1, \mathcal{X}_2, 1_{\mathcal{N}}\}$ & $\Psi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{Y}_1, 1_{\mathcal{N}}\}$. Suppose $\mathfrak{F}: (\mathcal{X},\Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y},\Psi_{\mathcal{N}})$ is the identity map, so $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-*δ*-*ß*-cont. map.

Proposition 3.4: For a map$\mathfrak{F}: (\mathcal{X},\Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y},\Psi_{\mathcal{N}})$ the next properties are hold, but the converse need not to be true.

$(\boldsymbol{a})$ Every $\mathcal{N}_{\mathcal{S}}$-δ-Cont. is a $\mathcal{N}_{\mathcal{S}}$-Cont.

$(\boldsymbol{b})$ Every $\mathcal{N}_{\mathcal{S}}$-Cont. is a $\mathcal{N}_{\mathcal{S}}$-δS-Cont.

$(\boldsymbol{c})$ Every $\mathcal{N}_{\mathcal{S}}$-Cont. is a $\mathcal{N}_{\mathcal{S}}$-δP-Cont.

$(\boldsymbol{d})$ Every $\mathcal{N}_{\mathcal{S}}$-δS-Cont. is a $\mathcal{N}_{\mathcal{S}}$-*E*-Cont.

$(\boldsymbol{e})$ Every $\mathcal{N}_{\mathcal{S}}$-δP-Cont. is a $\mathcal{N}_{\mathcal{S}}$-*E*-Cont.

$(\boldsymbol{f})$ Every $\mathcal{N}_{\mathcal{S}}$-*E*-Cont. is a $\mathcal{N}_{\mathcal{S}}$-*δ*-*ß*-Cont.

$(\boldsymbol{g})$ Every $\mathcal{N}_{\mathcal{S}}$-δS-Cont. is a $\mathcal{N}_{\mathcal{S}}$-*δ*-*ß*-Cont.

$(\boldsymbol{h})$ Every $\mathcal{N}_{\mathcal{S}}$-δP-Cont. is a $\mathcal{N}_{\mathcal{S}}$-*δ*-*ß*-Cont.

Proof: The proof of the cases(a), (b) & (c) are presented and established in [20].

$(\boldsymbol{d})$- Presume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $\mathcal{Y}$. Because, $\mathfrak{F}$ is$\mathcal{N}_{\mathcal{S}}$-δS-Cont., $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta\mathcal{SOS}$ in $\mathcal{X}$. As, each $\mathcal{N}_{\mathcal{S}}\delta\mathcal{SOS}$ is $\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$ [21], so $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$ in $\mathcal{X}$. Consequently, $\mathfrak{F}$ is$\mathcal{N}_{\mathcal{S}}$-*E*-Cont.

$(\mathbf{e})$-This ***proof*** is similar to that of case (d).

$(\boldsymbol{f})$- Presume that $\mathcal{D}$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $\mathcal{Y}$. Because $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-*E*-Cont., $\mathfrak{F}^{-1}(\mathcal{D})$ is a $\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$ in$\mathcal{X}$. As each $\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$ is $\mathcal{N}_{\mathcal{S}} - \delta - ß - \mathcal{OS}$ [21], so $\mathfrak{F}^{-1}(\mathcal{D})$ is$\mathcal{N}_{\mathcal{S}} - \delta - ß - \mathcal{OS}$ in $\mathcal{X}$. Consequently, $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-*δ*-*ß*-Cont.

The proofs of the cases(g)&(h) are similar to that of cases (d)&(e)**,** respectively**.**

Remark 3.5: The following diagram describe the interrelations between Neutrosophic-*δ*-*ß* continuous map and other existing generalized of Neutrosophic continuous maps.

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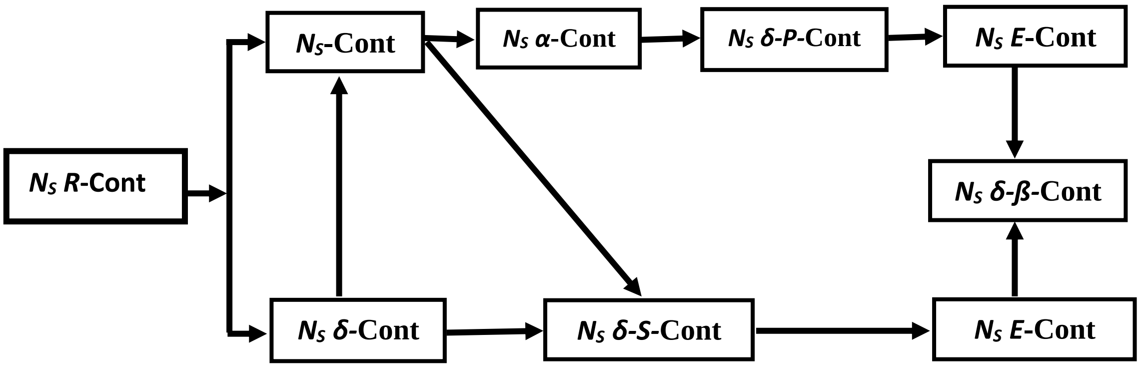

Figure 2: The relationships between Neutrosophic δ-ß-continuous maps and other well-known types of generalized Neutrosophic continuous maps

None of these implications are reversible as shown via the examples in [18] & [20], and utilizing the examples which are given below :

Example 3.6: Assume, $\mathcal{X} = \mathcal{Y} = \{r, p, z\}$ & describe $\mathcal{N}_{\mathcal{S}}S' S, \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3$ & $\mathcal{X}_4$ in $\mathcal{X}$ & $\mathcal{Y}_1$ in $\mathcal{Y}$ as follows:

$$\mathcal{X}_1 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.5}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.7}, \frac{q_p}{0.5}, \frac{q_z}{0.5}\right) i;$$

$$\mathcal{X}_2 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.4}, \frac{\Gamma_p}{0.2}, \frac{\Gamma_z}{0.6}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.6}, \frac{q_p}{0.8}, \frac{q_z}{0.4}\right) i;$$

$$\mathcal{X}_3 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.4}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.6}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.6}, \frac{q_p}{0.5}, \frac{q_z}{0.4}\right) i;$$

$$\mathcal{X}_4 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.7}, \frac{q_p}{0.5}, \frac{q_z}{0.6}\right) i;$$

$$\mathcal{Y}_1 = \mathfrak{F}\mathcal{Y}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.7}, \frac{q_p}{0.5}, \frac{q_z}{0.6}\right) i.$$

In that case, obtain $\Phi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \mathcal{X}_1 \cap \mathcal{X}_2, 1_{\mathcal{N}}\}$ & $\Psi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{Y}_1, 1_{\mathcal{N}}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is the identity map, so $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. but not $\mathcal{N}_{\mathcal{S}}$-δS-Cont., because of the set $\mathfrak{F}^{-1}(\mathcal{Y}_1) = \mathcal{X}_4$ is $\mathcal{N}_{\mathcal{S}}\delta -$ ß $- \mathcal{OS}$ but not $\mathcal{N}_{\mathcal{S}}\delta\mathcal{SOS}$.

Example 3.7: Utilizing Example (3.3). We obtain, a map $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont., but not $\mathcal{N}_{\mathcal{S}}$-δP-Cont., because of the set $\mathfrak{F}^{-1}(\mathcal{Y}_1) = \mathcal{X}_3$ is a $\mathcal{N}_{\mathcal{S}}\delta -$ ß $- \mathcal{OS}$ but not $\mathcal{N}_{\mathcal{S}}\delta\mathcal{POS}$.

Remark 3.8: It is obvious that Every $\mathcal{N}_{\mathcal{S}}$-$E$-Cont. is a $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. Utilizing Example (3.4) in [20] below, to show that the converse not necessarily to be true.

Example 3.9: Let $\mathcal{X} = \mathcal{Y} = \{r, p\}$ & describe $\mathcal{N}_{\mathcal{S}}S' S, \mathcal{X}_1$ & $\mathcal{X}_2$ in$\mathcal{X}$ & $\mathcal{Y}_1$ in $\mathcal{Y}$ as follows:

$$\mathcal{X}_1 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.2}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}\right), \left(\frac{q_r}{0.5}, \frac{q_p}{0.5}\right) i;$$

$$\mathcal{X}_2 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.5}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}\right), \left(\frac{q_r}{0.7}, \frac{q_p}{0.6}\right) i;$$

$$\mathcal{Y}_1 = \mathfrak{F}\mathcal{Y}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.5}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}\right), \left(\frac{q_r}{0.7}, \frac{q_p}{0.6}\right) i.$$

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In that case, get $\Phi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{X}_1, \mathcal{N}\}$ and $\Psi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{Y}_1, 1_{\mathcal{N}}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is the identity map, so $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. but not $\mathcal{N}_{\mathcal{S}}$-$E$-Cont., because of the set $\mathfrak{F}^{-1}(\mathcal{Y}_1) = \mathcal{X}_2$ is a $\mathcal{N}_{\mathcal{S}} - \delta - ß - \mathcal{OS}$ but not $\mathcal{N}_{\mathcal{S}}E\mathcal{OS}$.

Theorem 3.10: A map $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. *iff* the inverse image of every $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$.

Proof: Assume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}}) \Longrightarrow \mathcal{D}^c$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Since $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont., then $\mathfrak{F}^{-1}(\mathcal{D}^c)$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Because, $\mathfrak{F}^{-1}(\mathcal{D}^c) = (\mathfrak{F}^{-1}(\mathcal{D}))^c$, so $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$.

Conversely: Suppose, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. In that case $\mathcal{D}^c$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Via supposition $\mathfrak{F}^{-1}(\mathcal{D}^c)$ is a $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$.

Since, $\mathfrak{F}^{-1}(\mathcal{D}^c) = (\mathfrak{F}^{-1}(\mathcal{D}))^c$, so $(\mathfrak{F}^{-1}(\mathcal{D}))^c$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Consequently $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}} - \delta - ß - \mathcal{CS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Thus, $\mathfrak{F}^{-1}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont.

Theorem3.11: Let $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map and $\mathcal{F}: (\mathcal{Y}, \Psi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ be a $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont., then $\mathcal{F}o\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ is a $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont.

Proof: Assume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$. Then, via supposition, $\mathcal{F}^{-1}(\mathcal{D})$ is a $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Since, $\mathfrak{F}$ is a $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map, so $\mathfrak{F}^{-1}(\mathcal{F}^{-1}(\mathcal{D}))$ $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Consequently, $\mathcal{F}o\mathfrak{F}$ is a $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont.

Remark 3.12: it's important to show by the next example the composition of two $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map not necessarily to be $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont.

Example 3.13: Assume that, $\mathcal{X} = \mathcal{Y} = \mathcal{Z} = \{r, p, z\}$ & describe $\mathcal{N}_{\mathcal{S}}\mathcal{S}'\mathcal{S}, \mathcal{X}_1$ & $\mathcal{X}_2$ in $\mathcal{X}$ and $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3$ & $\mathcal{Y}_4$ in $\mathcal{Y}$ in $\mathcal{Y}$ and $\mathcal{Z}_1$ in $\mathcal{Z}$ as follows:

$$\mathcal{X}_1 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.1}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.3}, \frac{q_p}{0.4}, \frac{q_z}{0.4}\right) i;$$

$$\mathcal{X}_2 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.2}, \frac{\Gamma_z}{0.5}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.2}, \frac{q_p}{0.2}, \frac{q_z}{0.4}\right) i;$$

$$\mathcal{Y}_1 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.4}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.4}, \frac{q_p}{0.5}, \frac{q_z}{0.5}\right) i;$$

$$\mathcal{Y}_2 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.5}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.5}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.5}, \frac{q_p}{0.5}, \frac{q_z}{0.5}\right) i;$$

$$\mathcal{Y}_3 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.5}, \frac{\Gamma_p}{0.5}, \frac{\Gamma_z}{0.5}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.4}, \frac{q_p}{0.5}, \frac{q_z}{0.5}\right) i;$$

$$\mathcal{Y}_4 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.4}, \frac{\Gamma_p}{0.3}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.5}, \frac{q_p}{0.5}, \frac{q_z}{0.5}\right) i;$$

$$\mathcal{Z}_1 = \mathfrak{F}\mathcal{X}, \left(\frac{\Gamma_r}{0.3}, \frac{\Gamma_p}{0.4}, \frac{\Gamma_z}{0.4}\right), \left(\frac{\lambda_r}{0.5}, \frac{\lambda_p}{0.5}, \frac{\lambda_z}{0.5}\right), \left(\frac{q_r}{0.4}, \frac{q_p}{0.4}, \frac{q_z}{0.5}\right) i.$$

In that case, obtain $\Phi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{X}_1, \mathcal{X}_2, 1_{\mathcal{N}}\}$, $\Psi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \mathcal{Y}_4, 1_{\mathcal{N}}\}$ & $\mathfrak{T}_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{Z}_1, 1_{\mathcal{N}}\}$.

Presume $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ & $\mathcal{F}: (\mathcal{Y}, \Psi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ are the identity maps, so $\mathfrak{F}$ & $\mathcal{F}$ are $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. maps, but $\mathcal{F}o\mathfrak{F}$ is not $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map.

Definition 3.14: A neutrosophic topological space $(\mathcal{X}, \Phi_{\mathcal{N}})$ is called a δ-ß-$U_{\frac{1}{2}}$ (Briefly, $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$\mathcal{T}$-$U_{\frac{1}{2}}$) space if every $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$.

Theorem3.15: Let $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map, then $\mathfrak{F}$ is a $\mathcal{N}_{\mathcal{S}}$-Cont. if $(\mathcal{X}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$\mathcal{T}$-$U_{\frac{1}{2}}$) space.

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Proof: Assume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$. In that case by supposition $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta-ß-\mathcal{OS}$ in $(\mathcal{X},\Phi_{\mathcal{N}})$. Since $(\mathcal{X},\Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$\mathcal{T}$-$U_{\frac{1}{2}}$) space, so by Definition (3.14) $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{X},\Phi_{\mathcal{N}})$. Consequently $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-Cont.

Theorem3.16: For $\mathfrak{F}:(\mathcal{X},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$.The next statements are equivalent:

(***a***) $\mathfrak{F}\big(\mathcal{N}_{\mathcal{S}}\delta-ßCl(\mathcal{D})\big)\subseteq\mathcal{N}_{\mathcal{S}}\delta Cl\big(\mathfrak{F}(\mathcal{D})\big),\ \forall\ \mathcal{N}_{\mathcal{S}}\mathcal{CS}\ \mathcal{D}$ in $(\mathcal{X},\Phi_{\mathcal{N}})$.

(***b***) $\mathcal{N}_{\mathcal{S}}\delta-ßCl\big(\mathfrak{F}^{-1}(\Gamma)\big)\subseteq\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}\delta Cl(\Gamma)),\ \forall\ \mathcal{N}_{\mathcal{S}}\mathcal{CS}\ \Gamma$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

Proof: The proof of these cases are studied and established in [20].

Remark 3.17: If $\mathfrak{F}:(\mathcal{X},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map, then

(***a***) $\mathfrak{F}\big(\mathcal{N}_{\mathcal{S}}\delta-ß\mathrm{Cl}(\mathcal{D})\big)$ need not be equal to $\mathcal{N}_{\mathcal{S}}\delta\mathrm{Cl}\big(\mathfrak{F}(\mathcal{D})\big)$ where$\mathcal{D}\in(\mathcal{X},\Phi_{\mathcal{N}})$..

(***b***) $\mathcal{N}_{\mathcal{S}}\delta-ß\mathrm{Cl}\big(\mathfrak{F}^{-1}(\Gamma)\big)$ need not be equal to $\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}\delta\mathrm{Cl}(\Gamma))$, where $\Gamma\in(\mathcal{Y},\Psi_{\mathcal{N}})$.

As publicized in example **(5.9)** in [20].

Theorem 3.18: If $\mathfrak{F}:(\mathcal{X},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont., then

$\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}\delta Int(\Gamma))\subseteq\mathcal{N}_{\mathcal{S}}\delta-ßInt(\mathfrak{F}^{-1}(\Gamma)),\forall\ \mathcal{N}_{\mathcal{S}}\mathcal{CS}\ \Gamma$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$.

Proof: The proof of this case is studied in [20].

Remark 3.19: If $\mathfrak{F}:(\mathcal{X},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map, then $\mathcal{N}_{\mathcal{S}}\delta-ßInt(\mathfrak{F}^{-1}(\Gamma))$ need not be equal to $\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}\delta Int(\Gamma))$, where$\Gamma\in\mathcal{Y},\Psi_{\mathcal{N}})$, as publicized in example (5.12) in [20]

4. Some Properties of Neutrosophic δ-ß-Irresolute Map in $\mathcal{N}_{\mathcal{S}}\mathcal{TS}$

In this section, several characterizations and fundamental properties concerning of neutrosophic δ-ß-irresolute mappings utilizing δ-ß- neutrosophic open sets have been obtained .

Definition4.1: A map $\mathfrak{F}:(\mathcal{X},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$ is called δ-ß-irresolute(Shortly,$\mathcal{N}_{\mathcal{S}}\delta-ß-Irr$)map if $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta-ß-\mathcal{OS}$ in $(\mathcal{X},\Phi_{\mathcal{N}}),\forall\ \mathcal{N}_{\mathcal{S}}\delta-ß-\mathcal{OS}\ \mathcal{D}$ of $\mathcal{Y},\Psi_{\mathcal{N}})$.

Theorem4.2: Let $\mathfrak{F}:(\mathcal{X},\Phi_{\mathcal{N}})\longrightarrow(\mathcal{Y},\Psi_{\mathcal{N}})$ be a $\mathcal{N}_{\mathcal{S}}\delta-ß-Irr$ map, then $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont.

Proof: Assume, $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta-ß-Irr$ map and $\mathcal{D}$ be any $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Y},\Psi_{\mathcal{N}})$. Since each $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ is $\mathcal{N}_{\mathcal{S}}\delta-ß-\mathcal{OS}$, so $\mathcal{D}$ is a $\mathcal{N}_{\mathcal{S}}\delta-ß-\mathcal{OS}$ in $\mathcal{Y},\Psi_{\mathcal{N}})$. Via supposition $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta-ß-\mathcal{OS}$ in $\mathcal{Y},\Psi_{\mathcal{N}})$. thus $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. map.

Remark 4.3: The converse of Theorem (4.2) not necessarily to be true in general as shown in the example below:

Example 4.4: Assume, $\mathcal{X}=\mathcal{Y}=\mathcal{Z}=\{r,p,z\}$ & describe $\mathcal{N}_{\mathcal{S}}\mathcal{S}'\ \mathcal{S},\mathcal{X}_1,\mathcal{X}_2$ & $\mathcal{X}_3$ in $\mathcal{X}$ with $\mathcal{Y}_1$ & $\mathcal{Y}_2$ in $\mathcal{Y}$ as follows:

$$\mathcal{X}_1=\mathfrak{F}\mathcal{X},\left(\frac{\Gamma_r}{0.2},\frac{\Gamma_p}{0.3},\frac{\Gamma_z}{0.4}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.8},\frac{q_p}{0.7},\frac{q_z}{0.6}\right)i;$$

$$\mathcal{X}_2=\mathfrak{F}\mathcal{X},\left(\frac{\Gamma_r}{0.1},\frac{\Gamma_p}{0.1},\frac{\Gamma_z}{0.4}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.9},\frac{q_p}{0.9},\frac{q_z}{0.6}\right)i;$$

$$\mathcal{X}_3=\mathfrak{F}\mathcal{X},\left(\frac{\Gamma_r}{0.2},\frac{\Gamma_p}{0.4},\frac{\Gamma_z}{0.4}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.8},\frac{q_p}{0.6},\frac{q_z}{0.6}\right)i;$$

$$\mathcal{Y}_1=\mathfrak{F}\mathcal{X},\left(\frac{\Gamma_r}{0.1},\frac{\Gamma_p}{0.1},\frac{\Gamma_z}{0.4}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.9},\frac{q_p}{0.9},\frac{q_z}{0.6}\right)i;$$

$$\mathcal{Y}_2=\mathfrak{F}\mathcal{X},\left(\frac{\Gamma_r}{0.1},\frac{\Gamma_p}{0.4},\frac{\Gamma_z}{0.5}\right),\left(\frac{\lambda_r}{0.5},\frac{\lambda_p}{0.5},\frac{\lambda_z}{0.5}\right),\left(\frac{q_r}{0.9},\frac{q_p}{0.6},\frac{q_z}{0.5}\right)i.$$

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In that case, get $\Phi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{X}_1, \mathcal{X}_2, 1_{\mathcal{N}}\}$ & $\Psi_{\mathcal{N}} = \{0_{\mathcal{N}}, \mathcal{Y}_1, 1_{\mathcal{N}}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ is the identity map, so $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont. but not $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$, because of the set $\mathcal{Y}_2$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $\mathcal{Y}$ but $\mathfrak{F}^{-1}(\mathcal{Y}_2)$ is not $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$.

Theorem4.5: Let $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$ map,so $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}} - Irr$ if $(\mathcal{X}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$U_{\frac{1}{2}}$ -space.

Proof: Assume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. In that case$\mathcal{D}$ is a $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Consequently $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$, via supposition. Since $(\mathcal{X}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$U_{\frac{1}{2}}$ –space, so $\mathfrak{F}^{-1}(\mathcal{D})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Consequently, $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}} - Irr$ map.

Theorem4.6: Let $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ & $\mathcal{F}: (\mathcal{Y}, \Psi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$ maps, then $\mathcal{F}o\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$.

Proof: Assume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$. In that case $\mathcal{F}^{-1}(\mathcal{D})$ is a $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Since, $\mathfrak{F}$ is a $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$ map, so $\mathfrak{F}^{-1}(\mathcal{F}^{-1}(\mathcal{D}))$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Consequently, $\mathcal{F}o\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$ map.

Theorem 4.7: Let $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$ map and $\mathcal{F}: (\mathcal{Y}, \Psi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ be $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont., then $\mathcal{F}o\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont., map.

Proof: Assume, $\mathcal{D}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{OS}$ in $(\mathcal{Z}, \mathfrak{T}_{\mathcal{N}})$. In that case $\mathcal{F}^{-1}(\mathcal{D})$ is a $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ n $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Since $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr, so$ $\mathfrak{F}^{-1}(\mathcal{F}^{-1}(\mathcal{D}))$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Consequently, $\mathcal{F}o\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-Cont.

Theorem 4.8: Let $(\mathcal{X}, \Phi_{\mathcal{N}})$ and $(\mathcal{Y}, \Psi_{\mathcal{N}})$ are $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$U_{\frac{1}{2}}$ –spaces. Then, for a map $\mathfrak{F}: (\mathcal{X}, \Phi_{\mathcal{N}}) \longrightarrow (\mathcal{Y}, \Psi_{\mathcal{N}})$ the next properties are equivalent:

$(\boldsymbol{a})$ $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$;

$(\boldsymbol{b})$ $\mathfrak{F}^{-1}(\Gamma)$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$ for all $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ Γ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$;

$(\boldsymbol{c})$ $\mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma)) \subseteq \mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))$ for all $\mathcal{N}_{\mathcal{S}}\mathcal{S}$ Γ of $(\mathcal{Y}, \Psi_{\mathcal{N}})$.

Proof:$(\boldsymbol{a}) \longrightarrow (\boldsymbol{b})$: Suppose, Γ is any $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. So, Γ^c is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in$(\mathcal{Y}, \Psi_{\mathcal{N}})$ Since, $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$, so $\mathfrak{F}^{-1}(\Gamma^c)$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$.

However, $\mathfrak{F}^{-1}(\Gamma^c) = (\mathfrak{F}^{-1}(\Gamma))^c$. Consequently, $\mathfrak{F}^{-1}(\Gamma)$ is$\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$.

$(\boldsymbol{b}) \longrightarrow (\boldsymbol{c})$: Assume, Γ is any $\mathcal{N}_{\mathcal{S}}\mathcal{S}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$ & $\Gamma \subseteq \mathcal{N}_{\mathcal{S}}Cl(\Gamma)$. So, $\mathfrak{F}^{-1}(\Gamma) \subseteq \mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))$. Since, $\mathcal{N}_{\mathcal{S}}Cl(\Gamma)$ is $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$,so $\mathcal{N}_{\mathcal{S}}Cl(\Gamma)$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Consequently, $(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))^c$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Via supposition, $\mathfrak{F}^{-1}((\mathcal{N}_{\mathcal{S}}Cl(\Gamma)))^c$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{OS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Because, $\mathfrak{F}^{-1}((\mathcal{N}_{\mathcal{S}}Cl(\Gamma))^c) = (\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma)))^c$, so $\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))$ is$\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Since, $(\mathcal{X}, \Phi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}}$-$\delta$-ß-$U_{\frac{1}{2}}$ –space, so $\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))$ is $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Thus,

$$\mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma)) \subseteq \mathcal{N}_{\mathcal{S}}Cl\left(\mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))\right) = \mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma)).$$

This mean $\mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma)) \subseteq \mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma))$.

$(\boldsymbol{c}) \longrightarrow (\boldsymbol{a})$: Assume, Γ is any $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$. Since $(\mathcal{Y}, \Psi_{\mathcal{N}})$ is $\mathcal{N}_{\mathcal{S}} - \delta - ß - U_{\frac{1}{2}} - \text{space, so}\ \Gamma$ is $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ in $(\mathcal{Y}, \Psi_{\mathcal{N}})$ and $\mathcal{N}_{\mathcal{S}}Cl(\Gamma) = \Gamma$. Therefore,

$\mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma)). \subseteq \mathfrak{F}^{-1}(\mathcal{N}_{\mathcal{S}}Cl(\Gamma)) = \mathfrak{F}^{-1}(\Gamma) \Longrightarrow \mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma)) \subseteq \mathfrak{F}^{-1}(\Gamma)$. But obviously $\mathfrak{F}^{-1}(\Gamma) \subseteq \mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma))$. Consequently, $\mathcal{N}_{\mathcal{S}}Cl(\mathfrak{F}^{-1}(\Gamma)) = \mathfrak{F}^{-1}(\Gamma) \Longrightarrow \mathfrak{F}^{-1}(\Gamma)$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{CS}$ and thus it is $\mathcal{N}_{\mathcal{S}}\delta - ß - \mathcal{CS}$ in $(\mathcal{X}, \Phi_{\mathcal{N}})$. Hence, $\mathfrak{F}$ is $\mathcal{N}_{\mathcal{S}}\delta - ß - Irr$ map.

5. Conclusion

The concepts of neutrosophy and neutrosophic sets have putted the foundation for a whole family of novel mathematical theories to generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory . The main target of our manuscript is to display and study a new idea of generalized neutrosophic continuous maps namely, $\mathcal{N}_{\mathcal{S}}\delta$-ß-continuous map. Also, new type of generalized neutrosophic maps called, neutrosophic δ-ß-irresolute maps has been studied. Some characterizations and substantial properties related to these kinds of generalized neutrosophic maps have been investigated.

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