Extended Uncertainty Principle for Inventory Control: An Updated Review of Environments and Applications

Ankit Dubey, Ranjan Kumar*
School of Advanced Sciences, VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India
Emails: ankitd.vit22@gmail.com, *ranjank.nit52@gmail.com

Abstract

This paper provides a comprehensive evaluation and categorization of the various uncertain environment employed by researchers and scientists to model and analyze inventory management systems in diverse sectors, including healthcare, supply chain, and routing issues. Additionally, it examines the challenges associated with the classical inventory model and introduces the concepts of fuzzy theory and the extended fuzzy principle in inventory management. The article presents important definitions related to fuzzy theory, including the fuzzy inventory model and its challenges. It also explores the applications of the extended fuzzy principle in real-life problems. The study focuses on inventory management under the extended fuzzy principle (Intuitionistic, Neutrosophic, Pythagorean, and so on), considering uncertain demand and imprecise data. The research contributes to the field by providing insights into the potential of fuzzy theory in overcoming the challenges of classical models and improving decision-making in inventory management.

Keywords: Triangular fuzzy number(TFN); Triangular neutrosophic number(TNN); Neutrosophic inventory management(NIM); Supply chain; Economic order quantity (EOQ).

1 Introduction

Operational research (OR) was developed during World War II [1939-1944] by Watt and Rowe when there was a dire need to manage scarce resources. The goal of OR is to help decision-makers make informed choices based on quantitative analysis and logical reasoning. According to Russell and Taylor,[1] "Operational research is the application of the scientific method to the study of problems involving the control and management of organized systems in business, industry, government, and other organizations." For example, mathematical models can be used to determine the optimal order quantity and reorder point based on the cost of holding inventory, ordering costs, and demand variability. Simulation can be used to test the performance of different inventory policies under different scenarios, while optimization techniques can be used to find the best possible solution given a set of constraints. Effective inventory management is crucial for businesses and organizations seeking to control costs and enhance customer satisfaction. Through the application of operations research techniques, inventory models provide a powerful tool for optimizing inventory levels and reducing costs while maintaining high levels of service quality.

An inventory control model is a mathematical formula or algorithm used to determine the optimal inventory levels of a business or organization. It takes into account factors such as demand, lead time, ordering costs, carrying costs, and stockout costs to determine the appropriate inventory levels that balance the cost of holding inventory with the cost of running out of inventory. By using an inventory control model, businesses can improve their inventory management practices, minimize stockouts and overstocking, reduce costs, and enhance customer service. There are various types of inventory control models, including deterministic

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models, probabilistic models, and simulation models, each with its own strengths and limitations. According to Ronald H. Ballou, "Inventory models are quantitative models that seek to determine how much to order, when to order, and how much safety stock to hold for a particular inventory item or set of items". To enhance inventory control systems, several researchers have developed several optimisation strategies. Taking into account stochastic replenishment intervals and discounts, Taleizadeh et al. (2010),[2] recommend employing a genetic algorithm to optimise inventory control systems that manage various items and limitations. A harmony search algorithm was presented in 2012 by Taleizadeh et al.[3] to address stochastic inventory control issues with dynamic demand, partial back-ordering, and multiple products, and chance constraints. Additionally, Maihami and Kamalabadi (2012)[4] present a model that considers non-instantaneous deterioration, partial backlogging, and time- and price-dependent demand in joint pricing and inventory control. Sivashankari and Panayappan (2015)[5] propose a production inventory model that accounts for deteriorative items and shortages in a two-level production system. Finally, Mousavi et al. (2014)[6] introduce a parameter-tuned particle swarm optimization algorithm to solve a multi-product multi-period inventory control problem that considers inflation and discounts. These algorithms offer unique solutions to different aspects of inventory control, highlighting the diverse approaches researchers have taken to improve inventory management systems.

After examining the introduction, it has become evident that there are significant gaps in the study of classical inventory and fuzzy inventory problems. Consequently, we are compelled to investigate the application of the Fuzzy extension principle to inventory control and provide an updated analysis of methodologies and applications. Our main objective is to offer valuable insights into evolutionary methods commonly associated with inventory theory, with a focus on essential aspects of management such as inventory and supply chain management in fuzzy extension scenarios. Through this comprehensive review paper, our aim is to assist researchers and students in developing a profound understanding of Extended Fuzzy Inventory Models (EFIM), including Neutrosophic, Intuitionistic to promote advancements in the field of EFIM. To accomplish this goal, our review article comprehensively surveys crucial aspects of EFIM. Specifically, we explore the existence of inventory theory and its sub-areas in real-life problems, addressing current challenges associated with uncertainty in inventory management. These challenges encompass routing problems, customer service management, supply chain management, production management, and blood bank management.

The paper is organized into several sections. Section 2 discusses the drawbacks of the classical inventory model, while Section 3 introduces the fuzzy inventory model. Section 4 analyzes the limitations of the fuzzy inventory model, and Section 5 provides an introduction to the fuzzy extension principle. In Section **??**, we present an updated review of methodologies and applications of the fuzzy extension principle for inventory control, highlighting the latest trends and advancements. Finally, we summarize our conclusions regarding the extension of fuzzy in inventory management.

1.1 List of Abbreviations used throughout this paper

TFN stands for "Triangular fuzzy number".
TNN stands for "Triangular Neutrosophic number".
MILP stands for "Mixed integer linear programming".
EOQ stands for "Economic order quantity".
FEOQ stands for "Fuzzy economic order quantity".
EPQ stands for "Economic production quantity".
PPADMM stands for "Proximal alternating direction method of multipliers".
EFIM stands for "Extended fuzzy inventory management".
IFN stands for the "Intuitionistic fuzzy number".
TIFN stands for "Triangular intuitionistic fuzzy number".
FIM stands for "Fuzzy inventory management".
NIM stands for "Neutrosophic inventory management".
MCGDM stands for "Multi-criteria group, decision-making problems".
MOFIM stands for "Multi-Objective Fuzzy Inventory Model".
MIMPFIC stands for "Multi-Item Multi-Periodic Fuzzy Inventory Control".
MPMPIC stands for "Multi-product multi-period inventory control".
PSO stands for "Particle swarm optimization".
FINP stands for "Fuzzy integer non-linear programming".

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IFLPP stands for "Intuitionistic fuzzy linear programming problem".
IVIF stands for "Interval-valued intuitionistic fuzzy".
IFLPP stands for "Intuitionistic fuzzy linear programming problems".
IFMOLPP stands for "Intuitionistic fuzzy multi-objective linear programming problems".

2 Understanding Challenges of the Classical Inventory Model: A Thoughtful Discussion

The classical inventory model is a widely used approach in supply chain management and inventory control. While it provides a structured framework for managing inventory levels, it also poses several challenges such as demand variability, lead time variability, holding and ordering costs, inventory inaccuracy, seasonality and trends, lack of flexibility, and data availability and accuracy. Addressing these challenges requires advanced inventory management techniques, such as demand forecasting models that consider variability, safety stock optimization, dynamic ordering policies, and real-time data integration. Additionally, technology solutions like advanced analytics, machine learning, and integrated supply chain systems can help overcome these challenges by providing better visibility, automation, and decision-support capabilities. It's important for businesses to recognize the limitations of the classical inventory model and explore more sophisticated approaches to inventory management to ensure optimal inventory levels, minimize costs, and meet customer demands effectively.

Finally, we observe that the Inventory Models are mathematical tools that aim to optimize stock on hand, purchase orders, and reorder points to satisfy consumer demands at minimum cost. However, in decision-making processes where multiple factors are involved, some of which are challenging to quantify, Fuzzy Logic offers a more flexible and nuanced approach compared to Inventory Models. In order to comprehend and effectively implement fuzzy logic in the context of inventory management, it is imperative to provide a comprehensive explanation of key definitions associated with this concept. Section 3 of this research article is dedicated to the discussion of these crucial definitions.

3 Some Important Definitions Related to Fuzzy Theory

Definition 3.1. [7] Fuzzy Set : Zadeh introduced the fuzzy set idea in 1965. The framework provides the system boundaries that are ill-defined or vaguely described, or even incomplete information. As per the Zedah's definition: Fuzzy sets (FS) are defined as $P = \{\langle p, \mu_P(p)\rangle : p \in P\}$, where $\mu_P(p) : S \to [0, 1]$ is a membership function of p or degree of belonging to the set S, where S is a non-empty set of discourse.

Definition 3.2. [8] Traingular fuzzy : A triangular fuzzy number $\tilde{Q}_d = (\Theta_{t_{1d}}, \Theta_{t_{2d}}, \Theta_{t_{3d}})$ by satisfying the following condition:

I. $\mu_{\tilde{Q}_d}(P)$, a strictly decreasing and continuous function between the intervals $[\Theta_{t_{2d}}, \Theta_{t_{3d}}]$.

II. $\mu_{\tilde{Q}_d}(P)$, a strictly increasing and continuous function between the intervals $[\Theta_{t_{1d}}, \Theta_{t_{2d}}]$.

III. $\mu_{\tilde{Q}_d}(P)$, a continuous function under the interval [0,1].

A TFN can be written as $\tilde{Q}_d = (\Theta_{t_{1d}}, \Theta_{t_{2d}}, \Theta_{t_{3d}})$ whose membership function is:

$$\mu_{\tilde{Q}_{d_{TFN}}}(P) = \begin{Bmatrix} \dfrac{p - \Theta_{t_{1d}}}{\Theta_{t_{2d}} - \Theta_{t_{1d}}}, & \text{if } \Theta_{t_{1d}} \le p \le \Theta_{t_{2d}} \\ 1, & \text{if p} = \Theta_{t_{2d}} \\ \dfrac{\Theta_{t_{3d}} - p}{\Theta_{t_{3d}} - \Theta_{t_{2d}}}, & \text{if } \Theta_{t_{2d}} \le p \le \Theta_{t_{3d}} \\ 0, & Elsewhere \end{Bmatrix}$$

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Example 3.3. Traingular fuzzy :If A TFN can be written as $\tilde{Q}_d = (5, 8, 15)$ then;

$$\mu_{\tilde{Q}_{d_{TFN}}}(P) = \begin{cases} \dfrac{p-5}{3}, & \text{if } 5 \leq p \leq 8 \\ 1, & \text{if p = 8} \\ \dfrac{15-p}{7}, & \text{if } 8 \leq p \leq 15 \\ 0, & Elsewhere \end{cases}$$

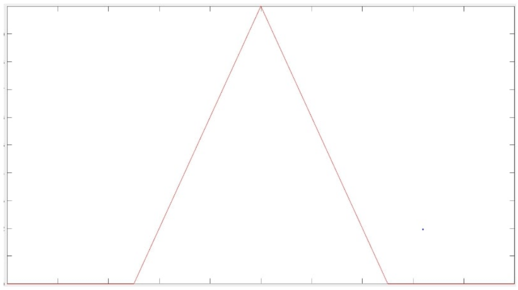

Figure 1: Graphical Representation of triangular fuzzy membership function

Hence, Fuzzy Logic can accommodate a broader range of factors and incorporate subjective or qualitative information, which makes it a preferred choice over Inventory Models in certain scenarios. In Section 2, we have comprehended that classical inventory management inadequately addresses uncertainty, leading to diminished accuracy in identifying optimal solutions. Consequently, Dubois and Parade introduced fuzzy inventory management in 1988 as a response to this issue, which we will delve into in Section 3.1.

3.1 The Introduction of the Fuzzy Inventory Model

In 1988, Dubois and Parade[9] introduced a new inventory model that incorporates fuzzy logic to address the inherent imprecision and uncertainty in inventory decision-making. The model considers factors such as demand and lead time variability, as well as cost and service level constraints, to determine the optimal level of inventory that minimizes expenses and meets service standards. Since then, the fuzzy inventory model has been widely used in economics, engineering, and management to effectively manage inventory under uncertain and imprecise conditions which is shown in above Section 3.

The FIM (Fuzzy Inventory Model) is a crucial function for any business dealing with physical products. Recent advancements in uncertain inventory management aim to improve accuracy and efficiency in forecasting demand, managing inventory levels, and minimizing stockouts. Machine learning algorithms have emerged as a popular trend to analyze historical sales data and relevant factors to provide more precise demand forecasts. Additionally, cloud-based inventory management systems are gaining popularity, offering scalability, flexibility, and real-time visibility into inventory levels across multiple locations. Advancements in RFID technology have enabled more efficient tracking and management of inventory, optimizing supply chain operations and reducing waste. Table 1 provides a brief overview of some of the significant contributions to the FIM, which are further discussed in conjunction with Figure 2 , visually representing key concepts associated with the FIM. Together, Table 1 and Figure 2 offer a valuable resource for a comprehensive understanding of the FIM and its importance in vendor problems and manufacturing settings amidst uncertainty.

Table 1 presents a comprehensive summary of the major contributions to Triangular Fuzzy Inventory Control Models, providing a brief overview of their varied applications in different fields. The table highlights the key features and benefits of each approach for easy reference.

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Table 1: Presents a short summary of the major contributions to Triangular Fuzzy Inventory Control Models

Authors	**Year**	**Application and Environment**	**Contribution**
Mandal et al.[10]	2005	Application: MOFIM Environment: TFN	To introduce a MOFIM with three constraints using a geometric programming approach..
Jana et al.[11]	2013	Application: Production Management Environment: TFN	To propose a novel approach to modeling an imprecise production inventory model with volume flexibility using a fuzzy simulation technique combined with a contractive mapping genetic algorithm.
Samal and Pratihar[12]	2014	Application: EOQ Environment: TFN	To present an optimization approach for FEOQ inventory models with variable demand, considering both scenarios with and without backordering.
Saha and Chakrabarti[13]	2017	Application: Supply Chain Environment: TFN	Degenerate items in a supply chain can benefit from a better inventory model that accounts for price-dependent demand and the absence of backorders.

Figure 2, featured in this article, offers a graphical illustration of the various components and processes that are integral to Fuzzy Inventory Control. This visual aid helps readers to gain a better understanding of the discussed concepts and provides a practical insight into how Fuzzy Inventory Control operates. Table 1 is also included in Figure 2 for easy reference.

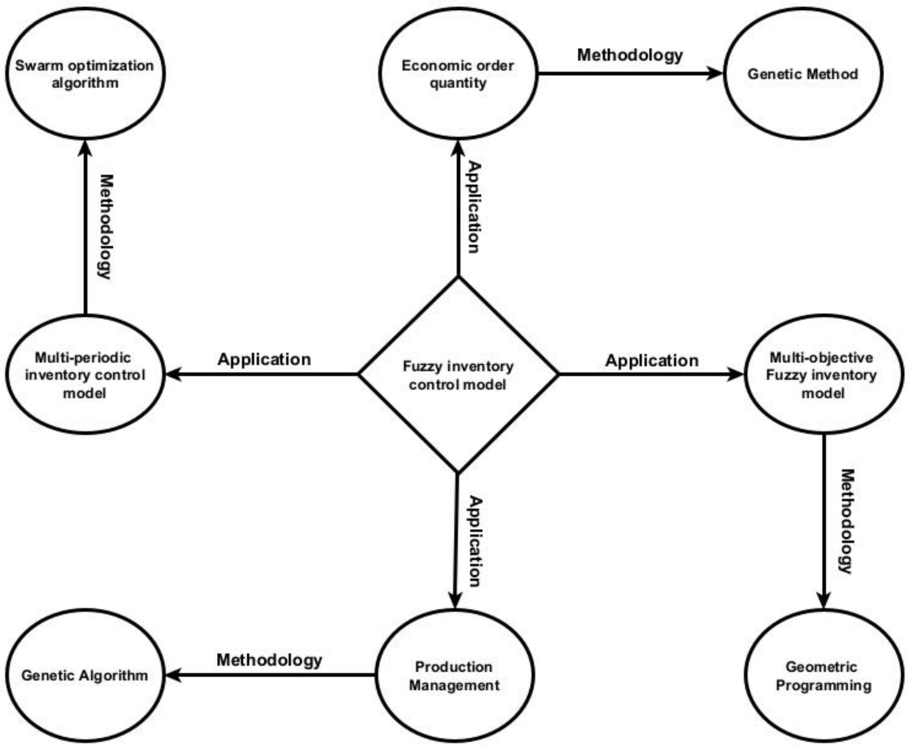

Figure 2: This figure depicts the different components and processes involved in fuzzy inventory control

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In our current investigation, we have thus far explored the distinct attributes of fuzzy logic. However, it is imperative to acknowledge that there exist certain challenges inherent to this approach, which necessitate a comprehensive discussion in the subsequent section.

3.2 Understanding Challenges of the fuzzy Inventory Model: A Thoughtful Discussion

The fuzzy inventory model is a technique used to manage inventory levels by considering the uncertainty and variability in demand and supply. It takes into account the imprecise nature of forecasting and aims to strike a balance between stockouts and excess inventory. While the fuzzy inventory model offers certain advantages, it also poses some challenges such as that need to be carefully addressed. Here are a few challenges associated with the fuzzy inventory model: Data accuracy, forecasting accuracy, defining membership functions, optimization complexity, implementation and integration, and cost considerations. Addressing these challenges requires a thoughtful approach that involves continuous monitoring and refinement of the model, collaboration between different departments, and leveraging advanced analytics and optimization techniques. It is essential to regularly evaluate the performance of the fuzzy inventory model and adapt it to changing market conditions and business requirements. . Furthermore, after conducting an extensive literature survey, it becomes evident that fuzzy theory alone is insufficient in handling uncertainty. Consequently, several scientists have introduced extended fuzzy theories such as Intuitionistic (developed by Atanassov 1983[14]), neutrosophic (developed by Samarandche in 1990[15]), Pythagorean (developed by Yager in 2013), and others. The subsequent paragraph will delve deeper into the discussion of these extended principles.

4 Some Important Defintions, Introduction Related to Fuzzy Extened Theory

The introductory section of the Extended Fuzzy Principle (ref. Figure 3) offers a comprehensive overview of the fundamental concepts and principles of fuzzy logic. Fuzzy logic is a popular computational framework that addresses uncertainty and imprecision in decision-making processes. This extended version of the fuzzy principle builds upon the traditional approach by incorporating additional features and techniques, thereby increasing its applicability and effectiveness across diverse domains. By establishing a solid foundation of fuzzy logic, this introduction lays the groundwork for comprehending the advancements and contributions of the extended fuzzy principle. Through a blend of theoretical discussions and practical examples, this paragraph serves as a gateway to the realm of extended fuzzy logic, emphasizing its significance and potential applications in solving complex real-world problems. Some of the new extended theory has been developed as follows:

a. Intuitionistic

b. Neutrosophic

c. Pythagorean and so on

Intuitionistic fuzzy sets (IFS)[16] : IFS are a extension of classical sets that adds information on the degree of uncertainty by the non-membership and membership values. Intuitionistic fuzzy sets, first proposed by Atanassov[14] in 1986, aim to represent the uncertainty and fuzziness of actual decision-making. Each element in classical fuzzy sets is given a membership value between 0 and 1, denoting the degree of the set. A Non-membership value, a new parameter introduced by intuitionistic fuzzy sets, quantifies the extent to which an element does not belong to the set.

Definition 4.1. [16] :Intuitionistic fuzzy Set A set $\tilde{\omega}$ on Q is defined as $\tilde{\omega} = \{(q, [(\eta(q), \psi(q))]) : q \in Q\}$ Where $\eta(q) : Q \to [0, 1]$ is named as truth function which indicate the degree of assurance and $\psi(q) : Q \to [0, 1]$ is named as falsity function. And $\eta(x), \psi(x)$ satisfies the following relation $0 \leq \eta(x) + \psi(x) \leq 1$

Definition 4.2. [16] :Intutionistic Fuzzy Number : An intuitionistic fuzzy number $\tilde{A}_d$ with the membership function $\sigma_{\tilde{A}_d}(q)$ and non-membership function $\eta_{\tilde{A}_d}(q)$ is:

I. Concave for the non-membership function, i.e., for all $q_1, q_2 \in q, \eta_{\tilde{A}_d}(\lambda_d q_1 + (1-\lambda_d)q_2) \leq \max\{\eta_{\tilde{A}_d}(q_1), \eta_{\tilde{A}_d}(q_2)\}$, where $\lambda_d \in [0, 1]$.

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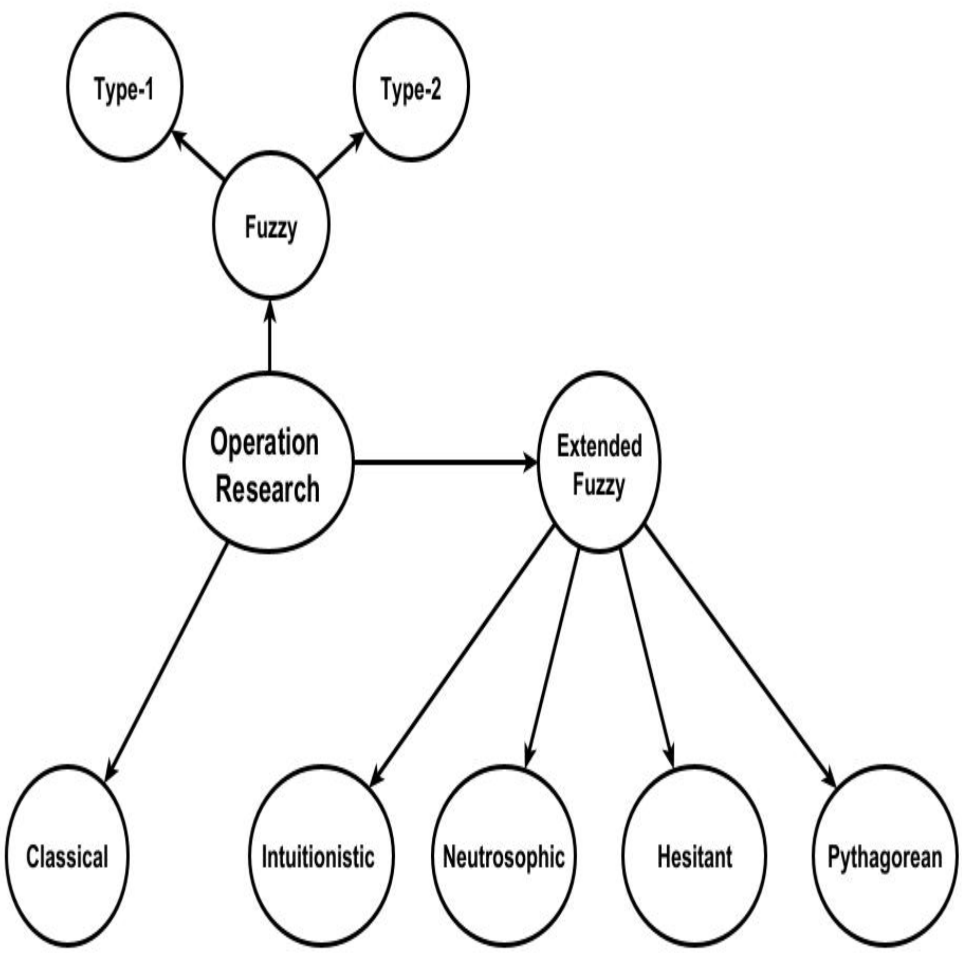

Figure 3: This figure depicts the different types of fuzzy extended principle

II. Convex for the membership function, i.e., for all $q_1, q_2 \in Q, \sigma_{\tilde{A}_d}(\lambda_d q_1 + (1-\lambda_d)q_2) \geq \min\{\sigma_{\tilde{A}_d}(q_1), \sigma_{\tilde{A}_d}(q_2)\}$, where $\lambda_d \in [0,1]$.

III. Normal, i.e., there is a $q_0 \in Q$ such that $\sigma_{\tilde{A}_d}(q_0) = 1$ and $\eta_{\tilde{A}_d}(q_0) = 0$

IV. An intuitionistic fuzzy subset of the real line.

Definition 4.3. [16] :Triangular Intutionistic Fuzzy Number(TIFN) : If A TIFN $\tilde{A}_d = (\delta_1, \delta_2, \delta_3; \delta_1', \delta_2, \delta_3')$.is an intuitionistic fuzzy set in Q with following non-membership function $(\eta_{\tilde{A}_d}(q))$ and membership function $(\sigma_{\tilde{A}_d}(q))$: $\sigma_{\tilde{A}_d}(q) = \begin{cases} \dfrac{q-\delta_1}{\delta_2-\delta_1}, & \text{for} \delta_1 \leq q \leq \delta_2 \\ \dfrac{\delta_3-q}{\delta_3-\delta_2}, & \text{for} \delta_2 \leq q \leq \delta_3 \\ 0, & \text{otherwise} \end{cases}$, and $\eta_{\tilde{A}_d}(q) = \begin{cases} \dfrac{\delta_2-q}{\delta_2-\delta_1'}, & \text{for} \delta_1' \leq q \leq \delta_2 \\ \dfrac{q-\delta_2}{\delta_3'-\delta_2}, & \text{for} \delta_2 \leq q \leq \delta_3' \\ 1, & \text{otherwise} \end{cases}$

Where, $\delta_1' \leq \delta_1 \leq \delta_2 \leq \delta_3 \leq \delta_3'$ and $\eta_{\tilde{A}_d}(q) \leq 1, 0 \leq \sigma_{\tilde{A}_d}(q)$, for $\sigma_{\tilde{A}_d}(q) = 1 - \eta_{\tilde{A}_d}(q), \forall q \in Q$

Neutrosophic logic is a type of logic that incorporates features from both classical and fuzzy logic to handle uncertain or missing data. Mathematicians, computer scientists, A researchers, decision makers, and fusion researchers have all taken notice since its introduction in the 1990s by Florentin Smarandache[15] Each proposition in neutrosophic logic can be either true (T), false(F), and Indeterminancy(I). Neutrophilic logic allows for more leeway and expression when reasoning with insufficient or ambiguous data by the use of indeterminacy. This extends the concepts introduced in[7]"fuzzy set" and[17]"intuitional fuzzy set," respectively. It's an improvement above prior ways for representing ambiguous data used in making decisions. Some extensions of NSs, including interval NS,[18] single-valued NS,[19] neutrosophic linguistic set,[20] interval rough neutrosophic,[21] and bipolar neutrosophic set[22] article have been proposed and applied to solve various problems such as Neutrosophic soft set traffic signal control model,[23] Covid-19 decision-making model,[24] medical diagnosis,[25] and so on in the context of the neutrosophic domain.

Definition 4.4. [26] :Neutrosophic Set :A set $\widetilde{neuS}$ in the universal discourse N, symbolically denoted by $n$it is called Neutrosophic Set if $\widetilde{neuS} = (n, [t_{\widetilde{neuS}}(n), i_{\widetilde{neuS}}(n), f_{\widetilde{neuS}}(n)]) : n \in N$ Where truth, Falsity, indeterminacy , membership function which has the degree of belongingness $t_{\widetilde{neuS}}(n) : N \to [0,1]$,$f_{\widetilde{neuS}}(n) : N \to [0,1]$,and $i_{\widetilde{neuS}}(n) : N \to [0,1]$ of the decision maker. $T_{\widetilde{neuS}}(n)$, , $F_{\widetilde{neuS}}(n)$, $I_{\widetilde{neuS}}(n)$ exhibits the following relation.

$$0 \le SupT_{\widetilde{neuS}}(n) + SupF_{\widetilde{neuS}}(n) + SupI_{\widetilde{neuS}}(n) \le 3$$

.

Definition 4.5. [26] :Triangular neutrosophic number (TNNs) : If A TNNs is denoted by

$$\zeta^N = < (\gamma^l, \gamma^m, \gamma^u), (\tau, l, \nu) >$$

who's membership function represents for the truth, indeterminacy, and falsity of x can be defined as follows:

$$\tau_{\zeta^N}(x) = \begin{Bmatrix} \frac{(x-\gamma)}{(\gamma^m-\gamma^l)}\mu, & \gamma^l \le x \le \gamma^m, \\ \mu, & x = \gamma^m \\ \frac{(\gamma^u - x)}{(\gamma^u-\gamma^m)}\mu, & \gamma^m \le x \le \gamma^u \\ 0, & otherwise \end{Bmatrix}, l_{\zeta^N}(x) = \begin{Bmatrix} \frac{(\gamma^m-x)}{(\gamma^m-\gamma^l)}i, & \gamma^l \le x \le \gamma^m, \\ i, & x = \gamma^m \\ \frac{(x-\gamma^u)}{(\gamma^u-\gamma^m)}i, & \gamma^m \le x \le \gamma^u \\ 1, & otherwise \end{Bmatrix}$$

$$\nu_{\zeta^N}(x) = \begin{Bmatrix} \frac{(\gamma^m-x)}{(\gamma^m-\gamma^l)}\omega, & \gamma^l \le x \le \gamma^m, \\ \omega, & x = \gamma^m \\ \frac{(x-\gamma^u)}{(\gamma^u-\gamma^m)}\omega, & \gamma^m \le x \le \gamma^u \\ 1, & otherwise \end{Bmatrix}$$

Where, $0 \le \tau_{\zeta^N}(x) + l_{\zeta^N}(x) + \nu_{\zeta^N}(x) \le 3, x \in \zeta^N$.

4.1 The Application of the Extended fuzzy Principle in different real life problems

Extended fuzzy principles have a wide range of sectors across various domains. Here are some common applications: Table 2 provides a detailed overview of the significant advancements in Extended Fuzzy principle, showcasing their diverse applications across various fields. The Table 2 emphasizes the essential characteristics and advantages of each approach for convenient reference.

Table 2: Literature survey on extended fuzzy principle

Authors	Year	Environment	Application	Contribution
Biswas et al.[27]	2016	TNN	Multi-Attribute Decision Making	Utilizing TNN information aggregation for multi-attribute decision making.
Wang et al.[28]	2018	TNN	Group Decision Making with Multiple Criteria	Enhanced VIKOR approach for group decision making with multiple criteria using TNN.

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Table 2 – Continued the Literature survey on extended fuzzy principle

Authors	Year	Environment	Application	Contribution
Abdel-Basset et al.[29]	2019	TNN	NLPP	A new approach to address the challenges of solving fully neutrosophic linear programming problems.
Qiuping et al.[30]	2023	TNN	Solid Transportation Problem	The solid transportation problem addressed by a parametric neutrosophic model.

This article includes Figure 4, which presents a visual representation of the essential components and processes involved in Extended Fuzzy Principle. This graphical illustration enhances readers' comprehension of the discussed concepts and provides a practical glimpse into the functioning of Extended Fuzzy Principle. Additionally, Figure 4 incorporates Table 2 for convenient referencing purposes.

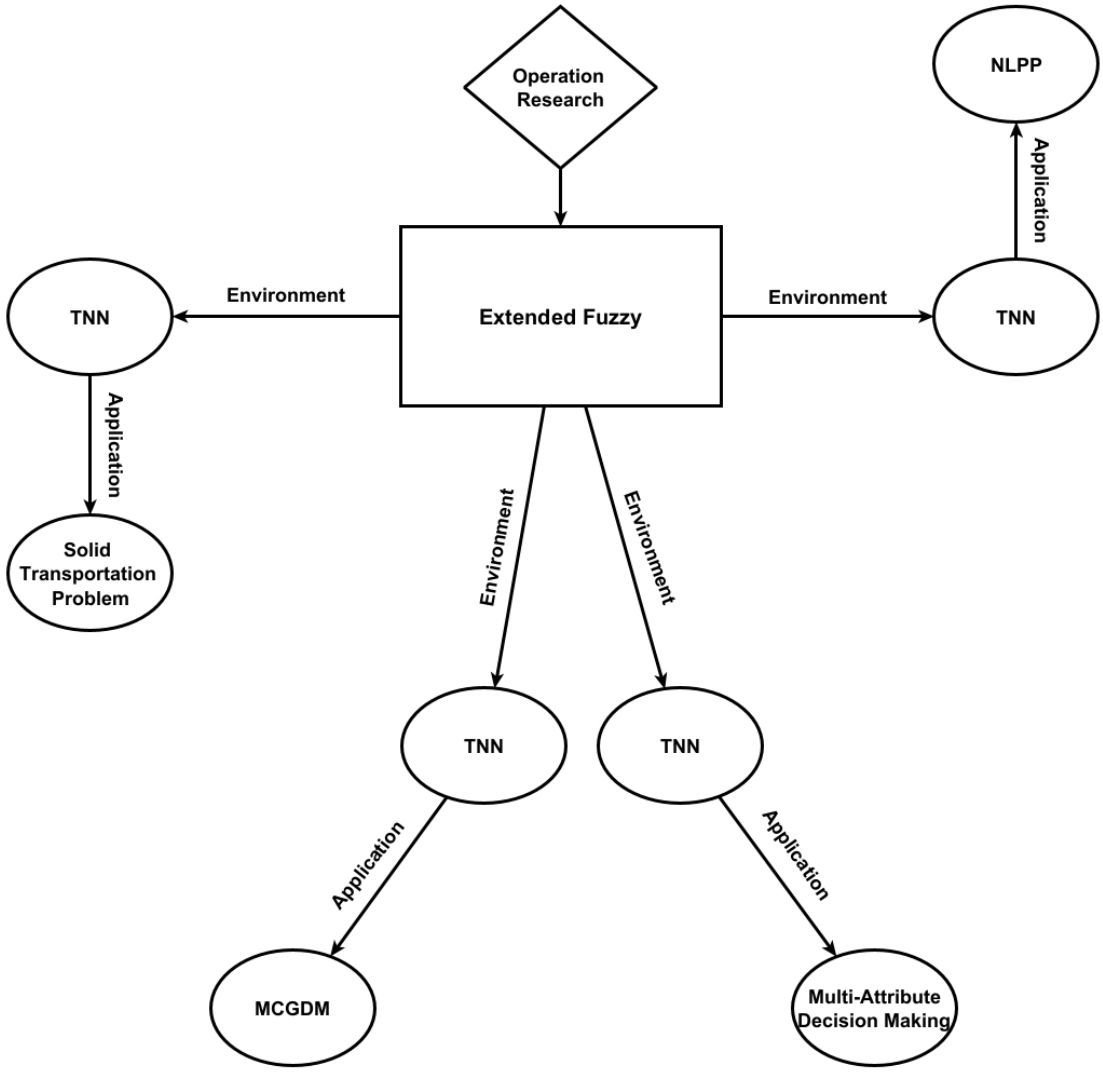

Figure 4: The depicted figure illustrates the various environments and applications associated with Extended Fuzzy Principle

These are just a few examples of the applications of extended fuzzy principles. The flexibility and ability to handle uncertainties make them applicable in various domains where imprecise and vague information needs to be considered for analysis and decision-making.

The main purpose of discussing the specific Table 2 was to highlight that fuzzy is not the sole principle for handling uncertainty. In addition to fuzzy, there exist several other fuzzy extended principles that offer superior and more precise solutions. Moreover, it is evident that the influence of these principles has been rapidly growing across diverse areas and applications in recent times. However, due to constraints, it is not feasible to delve into a detailed discussion of all these aforementioned applications in this paper. Therefore, our focus will be on examining the impact of the extended theory on inventory management.

5 The Inventory Management under the Extended fuzzy Principle

Kar et al. (2018)[31] and Mondal et al. (2018)[32] make important contributions to the growing subject of neutrosophic inventory management and optimization. An inverse link between production cost, production quantity, and setup cost is taken into account in the inventory model proposed by Kar et al. (2018).[31] In addition, they introduce uncertainty into the warehouse by including holding costs as a function of time. They use the degrees of truth (membership), uncertainty (hesitation), and falsity (non-membership) to create the objective and constraint functions. They use hesitation, non-membership functions , and linear membership in conjunction with the Neutrosophic Geometric Programming Technique to find solutions to these issues. In solving Neutrosophic non-linear programming problems, an approach named additive operator and the Geometric Programming technique.

On the other hand, Mondal et al. (2020)[32] introduced a deterministic single-objective EOQ model that takes into account constrained storage space in a neutrosophic environment. The model includes both fixed and variable production costs, as well as holding costs that vary with time. The major goal is to use neutrosophic geometric programming to minimize the overall average cost of the suggested model. The resulting non-linear optimization's model is solved using an extension of current fuzzy and intuitionistic fuzzy geometric programming methods. Further, the effective neutrosophic geometric programming is in comparison to crisp, fuzzy, and intuitionistic fuzzy geometric programming and demonstrated a numerical example by Mondal et al. (2020)[32] present a numerical example. Results show that the solution found via neutrosophic geometric programming is the best option. They also perform a sensitivity analysis of the parameters and provide valuable managerial insights based on their findings.

Pal and Chakraborty (2020)[33] investigated Neutrosophic triangular numbers in two distinct different circumstances. An EOQ model for non-instantaneous commodities with linearly dependent demand and shortages was the primary emphasis of their first paper. They efficiently dealt with uncertainty by employing particular varieties of linear triangular Neutrosophic numbers and by introducing the notion of deneutrosofication. To optimise the model, the authors used a TNN to represent the holding cost. The model's superiority in dealing with practical problems in neutrosophic analysis was demonstrated when its performance was compared to that of other models in both the crisp and neutrosophic domains.

In further study, a production quantity model was developed by Pal and Chakraborty (2020)[34] within a triangular neutrosophic setting. The team's goal was to develop a time-discounted production strategy for depreciating goods with a ramp-type demand rate and unit production that depends on reliability. Weibull's distribution was used to estimate the decay rate, and shortages due to both backorders and sales were taken into account. Both manufacturing period and production process reliability were taken into account by the authors. They found that the model did better in the triangular neutrosophic arena as compared to the crisp environment. They also ran a sensitivity analysis on the ideal option, which shed light on key aspects that managers should consider. Overall, Pal and Chakraborty's[33,34] innovative approaches to utilizing triangular Neutrosophic numbers offer valuable tools for addressing uncertainty and resolving practical problems in both EOQ and production quantity models. Their findings demonstrate the superiority of the neutrosophic domain in these contexts and highlight the importance of considering such approaches in managerial decision-making processes.

Conclusion

In this paper, we aim to conduct a comprehensive evaluation and categorization of uncertain environments utilized in modeling and analyzing inventory management systems across various sectors. By examining

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the challenges associated with the classical inventory model and introducing fuzzy theory and the extended fuzzy principle, this study has expanded the understanding of inventory management approaches. The article has discussed important definitions related to fuzzy theory, highlighted the challenges of the fuzzy inventory model, and explored the diverse applications of the extended fuzzy principle in real-life problems. Furthermore, the focus on inventory management under the extended fuzzy principle, considering uncertain demand and imprecise data, has shed light on the potential of fuzzy theory in overcoming the challenges of classical models. This research contributes valuable insights for scientist and researchers, emphasizing the importance of embracing fuzzy theory to improve decision-making and optimize inventory control in various industries, including healthcare, supply chain, and routing issues. The upcoming review article will comprehensively evaluate and categorize uncertain environments in inventory management systems, highlighting the challenges of classical models and exploring the applications of fuzzy theory and the extended fuzzy principle in addressing these issues, thereby providing valuable insights for researchers in optimizing inventory control across diverse industries.

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