Interval – Valued Pythagorean Fuzzy Soft Graphs

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Abstract

Interval-valued Pythagorean fuzzy soft sets and graph theory are combined in this essay. Then, we present notations for Pythagorean fuzzy soft graphs with interval values. On interval-valued Pythagorean fuzzy soft graphs, we also provide a variety of operations, such as Cartesian product and composition, and we look at some of their characteristics. Finally, we consider the application of I-VPFSGs for the selection of suitable houses and got the appropriate result by using score function.

Keywords: Interval-valued fuzzy graph (I-VFG); Interval-valued fuzzy soft graph (I-VFSG); Interval-valued Pythagorean fuzzy soft graphs (I-VPFSG).

1. Introduction

Fuzzy sets (FS) theory Zadeh [14] is a mathematical model to handle the imprecise and incomplete data. For distinguishing the hesitancy more individually, FS was extended to intuitionistic fuzzy set (IFS) by Atanassov [3], which designates a membership value μ and a non membership value v to the objects, satisflying the condition $\mu + v \leq 1$ and hesitancy part $\pi = 1 - \mu - v$. To overcome this situation, the notion of Pythagorean fuzzy set (PFS) was introduced by Yager [10][11] satisfying the condition $\mu^2 + v^2 \leq 1$. A PFS has more potential as compared to IFS is solving decision-making problems. The Pythagorean fuzzy number (PFG) was established by Zhang and Xu [13]. Zhang [12] conferred the Pythagorean fuzzy weighted averaging operator. In decision- making problems, Garg [4][5] considered the applications the applications of PFSs. The notion of interval-valued fuzzy set was introduced by Akram and Dudek [1] as a continuation of fuzzy sets. Because they present more adequate description for uncertainty, interval-valued fuzzy sets more useful than traditional fuzzy sets. Yahya Mohamed and Mohamed Ali [6] established by interval-valued Pythagorean fuzzy graph. Soft set theory was originated by Molodstov [7] for the parameterized point of view for uncertainty modeling and soft computing. The idea of IFSGs was given by Shahzadi and Akram [8]. The concept of novel intuitionistic fuzzy soft multiple–attribute decision- making methods was handled by Akram and Shahzadi [2]. Pythagorean fuzzy soft graphs with applications were introduced by Shahzadi and Akram [9]. In this paper work is to develop the interval-valued Pythagorean fuzzy soft graph is defined and some operations on interval-valued Pythagorean fuzzy soft graph are studied. Also investigate of their properties.

2. Preliminaries

Definition 2.1. A FSG $\tilde{\xi} = (\xi^*, \tilde{X}, \tilde{Y}, \delta)$ is a 4- tuple such that

$i.$ δ is non empty set of parameters,

$ii.$ $(\tilde{X}, \delta)$ is a fuzzy soft set over V,

$iii.$ $(\tilde{Y}, \delta)$ is a fuzzy soft set over E,

$iv.$ $(\tilde{X}(e), \tilde{Y}(e))$ is a fuzzy of ξ^* for all $u \in \delta$. That is

$\tilde{Y}(e)\,(\phi\psi) \leq \min\{\tilde{X}(e)(\phi), \tilde{X}(e)(\psi)\}\ \forall\, e \in \delta.$ and $x, y \in V$.The FG $\tilde{X}(e), \tilde{Y}(e)$ is denoted by $\tilde{H}(e)$ for convenience. A FSG is a parameterized family of fuzzy graphs. The class of all fuzzy soft graphs of ξ^* is denoted by $F(\xi^*)$.

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Definition 2.2. An interval-valued fuzzy soft graph over the set V is given by ordered 4 tuple $\tilde{\xi} = (\xi^*, X, Y, \delta)$ such that
$i.$ δ is non empty set of parameters,
$ii.$ (X, δ) is an I-VPFSs over V,
$iii.$ (Y, δ) is an I-VPFSs over E,
$iv.$ $(X(e), Y(e))$ is an I-VPFSG for all $e \in A$.
That is,
$\alpha^-_{Y(e)}((\phi\psi)) \leq \min(\alpha^-_{X(e)}(\phi), \alpha^+_{X(e)}(\psi))$ and
$\alpha^+_{Y(e)}((\phi\psi)) \leq \min(\alpha^+_{X(e)}(\phi), \alpha^+_{X(e)}(\psi))$ for all $\{\phi\psi\} \in E$.We denote $\xi^* = (V, E)$ a crisp graph $H(e) = (X(e), Y(e))$ an interval-valued fuzzy graph and $\tilde{\xi} = (\xi^*, X, Y, \delta)$ an I-VPFSG.

3. Interval- valued Pythagorean Fuzzy Soft Graphs

Definition 3.1. An I-VPFSG over the set V is given by $\tilde{\xi} = (\xi^*, X, Y, \delta)$ such that
$i.$ δ is non empty set of parameters,
$ii.$ (X, δ) is an I-VPFSs over V,
$iii.$ (Y, δ) is an I-VPFSs over E,
$iv.$ $(X(e), Y(e))$ is an I-VPFSG for all $e \in A$.
The functions $\widetilde{\alpha_Y}: E \subseteq V \times V \to D[0,1]$ and $\widetilde{\beta_Y}: E \subseteq V \times V \to D[0,1]$ such that
$\alpha^+_{YU}((\phi, \psi)) \leq \min(\alpha^+_{XU}(\phi), \alpha^+_{XU}(\psi))$ and
$\beta^+_{YU}((\phi, \psi)) \geq \max(\beta^+_{XU}(\phi), \beta^+_{XU}(\psi))$,
$\alpha^-_{YL}((\phi, \psi)) \leq \min(\alpha^-_{XL}(\phi), \alpha^-_{XL}(\psi))$, and
$\beta^-_{YL}((\phi, \psi)) \geq \max(\beta^-_{XL}(\phi), \beta^-_{XL}(\psi))$,
$0 \leq \alpha^2_{YU}(\phi, \psi) + \beta^2_{YU}(\phi, \psi) \leq 1 \ \forall (\phi, \psi) \in E$.

Example 3.2. Consider a graph $\xi^*= (X, Y)$ be a simple graph with $X = \{a, b, c, d\}$ and $Y = \{ab, bc, cd, ad\}$.Let $\delta = \{e_1, e_2\}$ be a parameter set and (X, δ) be an interval-valued Pythagorean fuzzy set V defined by

$$X(e_1) = \left\{\begin{matrix}\langle a\ [0.3,0.4]\ [0.2,0.7]\rangle, \langle b\ [0.2, 0.5][0.3,0.7]\ \rangle, \langle c, [0.1,0.6][0.2,0.5]\ \rangle\ and \\ \langle D\ [0.2,0.7][0.3,0.5]\ \rangle\end{matrix}\right\},$$

$$X(e_2) = \{\langle a, [0.2,0.7]\ [0.3,0.5]\ \rangle, \langle b\ [0.1, 0.6][0.2,0.5]\ \rangle, \langle c, [0.3,0.4][0.2,0.7]\ \rangle\}.$$

Now let (Y, δ) be an interval-valued Pythagorean fuzzy soft set E defined by

$$Y(e_1) = \left\{\begin{matrix}\langle ab\ [0.1,0.3][0.4,0.8]\ \rangle, \langle bc\ [0.1,0.4][0.4,0.8]\ \rangle, \langle ad\ [0.1,0.3][0.4,0.6]\ \rangle \\ and \langle cd\ [0.1,0.5][0.4,0.6]\ \rangle\end{matrix}\right\},$$

$$Y(e_2) = \{\langle ab[0.1,0.5][0.4,0.6]\ \rangle, \langle bc[0.1,04][0.4,0.8]\rangle, \langle ac[0.1,0.3][0.4,0.8]\ \rangle\}.$$

It is clearly seen that $H(e_1) = (X(e_1), Y(e_1))$ and $H(e_2) = (X(e_2), Y(e_2))$ are I-VPFSGs corresponding to the parameters e_1 and e_2 respectively, as shown Figure1. Hence $\tilde{\xi} = (\xi^*, X, Y, \delta)$ I-VPFSGs.

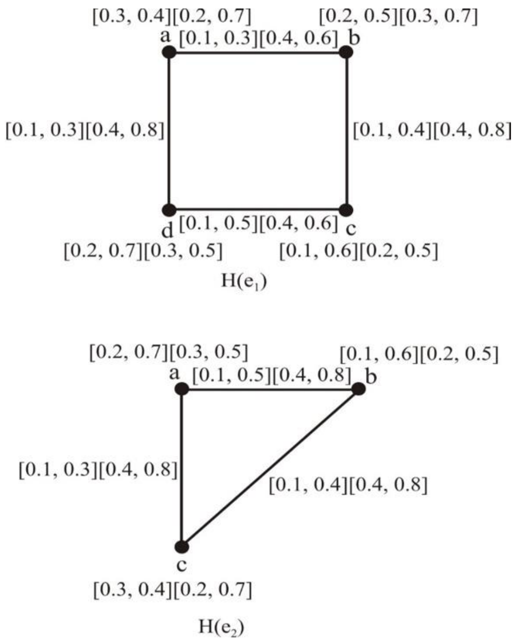

Figure 1: Interval-valued Pythagorean fuzzy soft graphs $\tilde{\xi}$

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Definition 3.3. Let $\widetilde{\xi_1} = (\xi_1^*, X_1, Y_1, \delta)$ and $\widetilde{\xi_2} = (\xi_1^*, X_2, Y_2, \eta)$ be two I-VPFSGs of simple graphs $\xi_1^* = (\alpha_1, \beta_1)$ and $\xi_2^* = (\alpha_2, \beta_2)$ respectively. The Cartesian product of $\widetilde{\xi_1}$and $\widetilde{\xi_2}$is denoted by

$\widetilde{\xi_1} \times \widetilde{\xi_2} = (\xi^*, X, Y, \delta \times \eta)$ where $\xi^* = (X_1 \times X_2, Y_1 \times Y_2)$ and is defined by

1) $(\alpha_{X_1L} \times \alpha_{X_2L})(\phi_1, \phi_2) = \min\left(\alpha_{X_1L}(\phi_1),\ \alpha_{X_2L}(\phi_2)\right)$,

$(\alpha_{X_1U} \times \alpha_{X_2U})(\phi_1, \phi_2) = \min\left(\alpha_{X_1U}(\phi_1),\ \alpha_{X_2U}(\phi_2)\right)$,

$(\beta_{X_1L} \times \beta_{X_2L})(\phi_1, \phi_2) = \max\left(\beta_{X_1L}(\phi_1),\ \beta_{X_2L}(\phi_2)\right)$,

$(\beta_{X_1U} \times \beta_{X_2U})(\phi_1, \phi_2) = \max\left(\beta_{X_1U}(\phi_1),\ \beta_{X_2U}(\phi_2)\right),\ \forall\, \phi_1 \in V_1, \phi_2 \in V_2$.

2) $(\alpha_{Y_1L} \times \alpha_{Y_2L})(\phi, \phi_2)(\phi, \psi_2) = \min\left(\alpha_{Y_1L}(\phi),\ \alpha_{Y_2L}(\phi_2, \psi_2)\right)$,

$(\alpha_{Y_1U} \times \alpha_{Y_2U})(\phi, \phi_2)(\phi, \psi_2) = \min\left(\alpha_{Y_1U}(\phi),\ \alpha_{Y_2U}(\phi_2, \psi_2)\right)$,

$(\beta_{Y_1L} \times \beta_{Y_2L})(\phi, \phi_2)(\phi, \psi_2) = \max\left(\beta_{Y_1L}(\phi),\ \beta_{Y_2L}(\phi_2, \psi_2)\right)$,

$(\beta_{Y_1U} \times \beta_{Y_2U})(\phi, \phi_2)(\phi, \psi_2) = \max\left(\beta_{Y_1U}(\phi), \beta_{Y_2U}(\phi_2\psi_2)\right),\ \forall\, \phi \in V_1, \phi_2\psi_2 \in E_2$.

3) $(\alpha_{Q_1L} \times \alpha_{Q_2L})(\phi_1, \rho)(\psi_1, \rho) = \min\left(\alpha_{Y_1L}(\phi, \psi_1),\ \alpha_{Y_2L}(\rho)\right)$,

$(\alpha_{Y_1U} \times \alpha_{Y_2U})(\phi_1\ {}_{,}\rho)(\psi_1, \rho) = \min\left(\alpha_{Y_1U}(\phi, \psi_1),\ \alpha_{Y_2U}(\rho)\right)$,

$(\beta_{Y_1L} \times \beta_{Y_2L})(\phi_1, \rho)(\psi_1, \rho) = \min\left(\beta_{Y_1L}(\phi, \psi_1),\ \beta_{Y_2L}(\rho)\right)$,

$(\beta_{Y_1U} \times \beta_{Y_2U})(\phi_1, \rho)(\psi_1,) = \min\left(\beta_{Y_1U}(\phi, \psi_1), \beta_{Y_2U}(\rho)\right), \forall\, \rho \in V_2, \phi_1\psi_1 \in E_1$.

Definition 3.5. Let $\widetilde{\xi_1} = (\xi_1^*, X_1, Y_1, \delta)$ and $\widetilde{\xi_2} = (\xi_2^*, X_2, Y_2, \eta)$ be two interval- valued Pythagorean fuzzy soft graphs of simple graphs $\xi_1^* = (X_1, Y_1)$ and $\xi_2^* = (X_2, Y_2)$ respectively.The composition of $\widetilde{\xi}_1$and $\widetilde{\xi}_2$ is denoted by $\widetilde{\xi_1} \circ \widetilde{\xi_2} = (X_1 \circ X_2, Y_1 \circ Y_2)$ and is defined by

1) $(\alpha_{X_1L} \circ \alpha_{X_2L})(\phi_1, \phi_2) = \min\left(\alpha_{X_1L}(\phi_1),\ \beta_{X_2L}(\phi_2)\right)$,

$(\alpha_{X_1U} \circ \alpha_{X_2U})(\phi_1, \phi_2) = \min\left(\alpha_{X_1U}(\phi_1),\ \alpha_{X_2U}(\phi_2)\right)$,

$(\beta_{X_1L} \circ \beta_{X_2L})(\phi_1, \phi_2) = \max\left(\beta_{X_1L}(\phi_1),\ \beta_{X_2L}(\phi_2)\right)$,

$(\beta_{X_1U} \circ \beta_{X_2U})(\phi_1, \phi_2) = \max\left(\beta_{X_1U}(\phi_1),\ \beta_{X_2U}(\phi_2)\right),\ \forall\, \phi_1 \in V_1, \phi_2 \in V_2$.

2) $(\alpha_{Y_1L} \circ \alpha_{Y_2L})(\phi, \phi_2)(\phi, \psi_2) = \min\left(\alpha_{Y_1L}(\phi),\ \alpha_{Y_2L}(\phi_2, \psi_2)\right)$,

$(\alpha_{Y_1U} \circ \alpha_{Y_2U})(\phi, \phi_2)(\phi, \psi_2) = \min\left(\alpha_{Y_1U}(\phi),\ \alpha_{Y_2U}(\phi_2, \psi_2)\right)$,

$(\beta_{Y_1L} \circ \beta_{Y_2L})(\phi, \phi_2)(\phi, \psi_2) = \max\left(\beta_{Y_1L}(\phi),\ \beta_{Y_2L}(\phi_2, \psi_2)\right)$,

$(\beta_{Y_1U} \circ \beta_{Y_2U})(\phi, \phi_2)(\phi, \psi_2) = \max\left(\beta_{Y_1U}(\phi), \beta_{Y_2U}(\phi_2\psi_2)\right),\ \forall\, \phi \in V_1, \phi_2\psi_2 \in E_2$.

3) $(\alpha_{Y_1L} \circ \alpha_{Y_2L})(\phi_1, \rho)(\psi_1, \rho) = \min\left(\alpha_{Y_1L}(\phi_1\psi_1),\ \alpha_{Y_2L}(\rho)\right)$,

$(\alpha_{Y_1U} \circ \alpha_{Y_2U})(\phi_{1,}\rho)(\psi_1, \rho) = \min\left(\alpha_{Y_1U}(\phi_1\psi_1),\ \alpha_{Y_2U}(\rho)\right)$,

$(\beta_{Y_1L} \circ \beta_{Y_2L})(\phi_1, \rho)(\psi_1, \rho) = \max\left(\beta_{Y_1L}(, \phi_1\psi_1),\ \beta_{Y_2L}(\rho)\right)$,

$(\beta_{Y_1U} \circ \beta_{Y_2U})(\phi_1, \rho)(\psi_1, \rho) = \max\left(\beta_{Y_1U}(\phi_1\psi_1), \beta_{Y_2U}(\rho)\right), \forall\, \rho \in V_2, u_1v_1 \in E_1$.

4) $(\alpha_{Y_1L} \circ \alpha_{Y_2L})(\phi_1, \phi_2)(\psi_1, \psi_2) = \min\left(\alpha_{X_2L}(\phi_2),\ \alpha_{X_2L}(\psi_2), \alpha_{X_1L}(\phi_1, \psi_1)\right)$,

$(\alpha_{Y_1U} \circ \alpha_{Y_2U})(\phi_1\ {}_{,}\phi_2)(\psi_1, \psi_2) = \min\left(\alpha_{X_2U}(\phi_2),\ \alpha_{X_2U}(\psi_2), \alpha_{Y_1U}(\phi_1, \psi_1)\right)$,

$(\beta_{Y_1L} \circ \beta_{Y_2L})(\phi_1, \phi_2)(\psi_1, \psi_2) = \max\left(\beta_{X_2L}(\phi_2),\ \beta_{X_2L}(\psi_2), \beta_{Y_1L}(\phi_1, \psi_1)\right)$,

$(\beta_{Y_1U} \circ \beta_{Y_2U})(\phi_1, \phi_2)(\psi_1, \psi_2) = \max\,(\beta_{X_2U}(\phi_2), \beta_{X_2U}(\psi_2),\ \beta_{Y_1U}(\phi_1, \psi_1)), \forall\, (\phi_1, \phi_2)(\psi_1, \psi_2) \in E^\circ - E$

Where $E^\circ = E \cup \{(\phi_1, \phi_2)(\psi_1, \psi_2) | \phi_1\psi_1 \in E_1, \phi_2 \neq \psi_2\}$.

Definition 3.7. Let $\widetilde{\xi_1} = (\xi_1^*, X_1, Y_1, \delta)$ and $\widetilde{\xi_2} = (\xi_2^*, X_2, Y_2, \eta)$ be two I-VPFSGs of simple graphs $\xi_1^* = (V_1, E_1)$ and $\xi_2^* = (V_2, E_2)$ respectively. The union of $\widetilde{\xi}_1$and $\widetilde{\xi}_2$is denoted by

$\widetilde{\xi_1} \cup \widetilde{\xi_2} = (\xi^*, X, Y, \delta \cup \eta)$, where $(X_1 \cup X_2, Y_1 \cup Y_2)$ and is defined as follows

1)

i. $(\alpha_{X_1L} \cup \alpha_{X_2L})(\phi) = \max\left(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\phi)\right)$ $if\ \phi \in V_1 \cap V_2$,

$(\alpha_{X_1U} \cup \alpha_{X_2U})(\phi) = \max\left(\alpha_{X_1U}(\phi), \alpha_{X_2U}(\phi)\right)$ $if\ \phi \in V_1 \cap V_2$,

ii. $(\beta_{X_1L} \cup \beta_{X_2L})(\phi) = \min\left(\beta_{X_1L}(\phi), \beta_{X_2L}(\phi)\right)$ $if\ \phi \in V_1 \cap V_2$,

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$\left(\beta_{X_1U} \cup \beta_{X_2U}\right)(\phi) = \min\left(\beta_{X_1U}(\phi), \beta_{X_2U}(\phi)\right)$ $if\ \phi \in V_1 \cap V_2$.

2)

$i.$ $\left(\alpha_{Y_1L} \cup \alpha_{Y_2L}\right)(\phi, \psi) = \max\left(\alpha_{X_1L}(\phi, v), \alpha_{X_2L}(\phi, \psi)\right)$ $if\ \phi\psi \in E_1 \cap E_2$,

$\left(\alpha_{Y_1U} \cup \alpha_{Y_2U}\right)(\phi, \psi) = \max\left(\alpha_{X_1U}(\phi, v), \alpha_{X_2U}(\phi, \psi)\right)$ $if\ \phi\psi E_1 \cap E_2$,

$ii.$ $\left(\beta_{Y_1L} \cup \beta_{Y_2L}\right)(\phi, \psi) = \min\left(\beta_{X_1L}(\phi), \beta_{X_2L}(\phi)\right)$ $if\ \phi\psi \in E_1 \cap E_2$,

$\left(\beta_{Y_1U} \cup \beta_{Y_2U}\right)(\phi, \psi) = \min\left(\beta_{Y_1U}(\phi), \beta_{Y_2U}(\phi)\right)$ $if\ \phi\psi \in E_1 \cap E_2$.

Definition 3.8. Let $\widetilde{\xi_1} = (\xi_1^*, X_1, Y_1, \delta)$ and $\widetilde{\xi_2} = (\xi_2^*, X_2, Y_2, \eta)$ be two interval - valued Pythagorean fuzzy soft graphs of simple graphs $\xi_1^* = (X_1, Y_1)$ and $\xi_2^* = (X_2, Y_2)$ respectively. The join of $\widetilde{\xi_1}$and $\widetilde{\xi_2}$is denoted by $\widetilde{\xi_1} + \widetilde{\xi_2} = (\xi^*, X, Y, \delta + \eta)$ Where $G^* = (X_1 + X_2, Y_1 + Y_2)$ and is defined by

1) $\left(\alpha_{X_1L} + \alpha_{X_2L}\right)(\phi) = \left(\alpha_{X_1L} \cup \alpha_{X_2L}\right)(\phi)$,

$\left(\alpha_{X_1U} + \alpha_{X_2U}\right)(\phi) = \left(\alpha_{X_1U} \cup \alpha_{X_2U}\right)(\phi)$ if $\phi \in V_1 \cup V_2$,

$\left(\beta_{X_1L} + \beta_{X_2L}\right)(\phi) = (\beta_{X_1L} \cup \beta_{X_2L})(\phi)$,

$\left(\beta_{X_1U} + \beta_{X_2U}\right)(\phi) = (\beta_{X_1U} \cup \beta_{X_2U})(\phi)$ if $\phi \in V_1 \cup V_2$.

2) $\left(\alpha_{Y_1L} + \alpha_{Y_2L}\right)(\phi, \psi) = \left(\alpha_{Y_1L} \cup \alpha_{Y_2L}\right)(\phi, \psi)$,

$\left(\alpha_{Y_1U} + \alpha_{Y_2U}\right)(\phi, \psi) = \left(\alpha_{Y_1U} \cup \alpha_{Y_2U}\right)(\phi, \psi)$ if $\phi \in E_1 \cap E_2$.

$\left(\beta_{Y_1L} + \beta_{Y_2L}\right)(\phi, \psi) = (\beta_{Y_1L} \cup \beta_{Y_2L})(\phi, \psi)$.

$\left(\beta_{Y_1U} + \beta_{Y_2U}\right)(\phi, \psi) = (\beta_{Y_1U} \cup \beta_{Y_2U})(\phi, \psi)$ if $(\phi, \psi) \in E_1 \cap E_2$.

3) $\left(\alpha_{Y_1L} + \alpha_{Y_2L}\right)(\phi, \psi) = \min\left(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\psi)\right)$,

$\left(\alpha_{Y_1U} + \alpha_{Y_2U}\right)(\phi, \psi) = \min\left(\alpha_{X_1U}(\phi), \alpha_{X_2U}(\psi)\right)$,

$\left(\beta_{YL} + \beta_{Y_2L}\right)(\phi, \psi) = \max\left(\beta_{X_1L}(\phi), \beta_{X_2L}(\psi)\right)$,

$\left(\beta_{Y_1U} + \beta_{Y_2U}\right)(\phi, \psi) = \max\left(\beta_{X_1U}(\phi), \beta_{X_2U})(\psi\right)$ if $\phi\psi \in E$.

Where E is the set of all edges joining the vertices of V_1 and V_2.

Theorem 3.9. If $\widetilde{\xi_1}$ and $\widetilde{\xi_2}$ are two I-VPFSGs, then so is $\widetilde{\xi_1} \times \widetilde{\xi_2}$.

Proof: Let $\widetilde{\xi_1} = (\xi_1^*, X_1, Y_1, \delta)$ and $\widetilde{\xi_2} = (G_2^*, X_2, Y_2, \eta)$ be two I-VPFSGs of simple graphs $\xi_1^* = (X_1, Y_1)$ and $\xi_2^* = (X_2, Y_2)$ respectively. From definition for all $e_1 \in \delta\ and\ e_2 \in \eta$, there are some cases. Let ξ_1 and ξ_2 be I-VPFSGs.

Let $E = \{(\phi, \phi_2)(\phi, \psi_2)/\phi \in V_1, \phi_2\psi_2 \in E_2\} \cup \{(\phi_1, \rho)(v_1, \rho)/\rho \in V_2, \phi_1\psi_1 \in E_1\}$

Consider $(\phi, \phi_2)(\phi, \psi_2) \in E$, we have

$$\left(\alpha_{Y_{1L}} \times \alpha_{Y_{2L}}\right)(\phi, \phi_2)(\phi, \psi_2) = \min\left(\alpha_{Y_{1L}}(\phi), \alpha_{Y_{2L}}(\phi_2\psi_2)\right),$$

$$\leq \min\left(\alpha_{X_{1L}}(\phi), \alpha_{X_{2L}}(\phi_2). \alpha_{X_{2L}}(\psi_2)\right),$$

$$= \min\left(\min\left(\alpha_{X_{1L}}(\phi), \alpha_{X_{2L}}(\phi_2)\right) min\left(\alpha_{X_{1L}}(\phi), \alpha_{X_{2L}}(\psi_2)\right)\right),$$

$\left(\alpha_{Y_{1L}} \times \alpha_{Y_{2L}}\right)(\phi, \phi_2)(\phi, \psi_2) = \min\left(\left(\alpha_{X_{1L}} \times \alpha_{X_{2L}}\right)(\phi, \phi_2), \left(\alpha_{X_{1L}} \times \alpha_{X_{2L}}\right)(\phi, \psi_2)\right)$.

Similarly,

$\left(\alpha_{Y_{1U}} \times \alpha_{Y_{2U}}\right)(\phi, \phi_2)(\phi, \psi_2) = \min\left(\left(\alpha_{X_{1U}} \times \alpha_{X_{2U}}\right)(\phi, \phi_2), (\alpha_{Y_{1U}} \times \alpha_{Y_{2U}})(\phi, \psi_2)\right)$.

Now,

$\left(\beta_{Y_{1L}} \times \beta_{Y_{2L}}\right)(\phi, \phi_2)(\phi, \psi_2) = \max\left(\left(\beta_{X_{1L}} \times \beta_{X_{2L}}\right)(\phi, \psi_2), \left(\beta_{X_{1L}} \times \beta_{X_{2L}}\right)(\phi, \psi_2)\right)$,

Similarly,

$\left(\beta_{Y_{1U}} \times \beta_{Y_{2U}}\right)(\phi, \phi_2)(\phi, \psi_2) = \max\left(\left(\beta_{X_{1U}} \times \beta_{X_{2U}}\right)(\phi, \psi_2), \left(\beta_{X_{1U}} \times \beta_{X_{2U}}\right)(\phi, \psi_2)\right)$.

Consider, $(u_1, w)(v_1, w) \in E$, we have

$$\left(\alpha_{Y_{1L}} \times \alpha_{Y_{2L}}\right)(\phi_1, \rho)(\psi_1, \rho) = \min(\alpha_{Y_{1L}}(\phi_1\psi_1), \left(\alpha_{Y_{2L}}(\rho)\right),$$

$$\leq \min\left(\alpha_{X_{1L}}(\phi_1), \alpha_{X_{2L}}(\psi_1). \alpha_{X_{2L}}(\rho)\right),$$

$$= \min\left(\min\left(\alpha_{X_{1L}}(\phi_1), \alpha_{X_{2L}}(\rho)\right) \min\left(\alpha_{X_{1L}}(\psi_1), \alpha_{X_{2L}}(\rho)\right)\right),$$

$\left(\alpha_{Y_{1L}} \times \alpha_{Y_{2L}}\right)(\phi_1, \rho)(\psi_1, \rho) = \min\left(\left(\alpha_{X_{1L}} \times \alpha_{X_{2L}}\right)(\phi_1, \rho), \left(\alpha_{X_{1L}} \times \alpha_{X_{2L}}\right)(\psi_1, \rho)\right)$.

Similarly,

$\left(\alpha_{Y_{1U}} \times \alpha_{Y_{2U}}\right)(\phi_1, \rho)(\psi_1, \rho) = \min\left(\left(\alpha_{X_{1U}} \times \alpha_{X_{2U}}\right)(\phi_1, \rho), \left(\alpha_{X_{1U}} \times \alpha_{X_{2U}}\right)(\psi_1, \rho)\right)$.

Now,

$\left(\beta_{Y_{1L}} \times \beta_{Y_{1U}}\right)(\phi_1, \rho)(\psi_1, \rho) = \max\left(\left(\beta_{X_{1L}} \times \beta_{X_{2L}}\right)(\phi_1, \rho), \left(\beta_{X_{1L}} \times \beta_{X_{2L}}\right)(\psi_1, \rho)\right)$.

Similarly,

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$(\beta_{Y_{1U}} \times \beta_{Y_{2U}})(\phi_1,\rho)(\psi_1,\rho) = \max((\beta_{X_{1U}} \times \beta_{X_{2U}})(\phi_1,\rho), (\beta_{X_{1U}} \times \beta_{X_{2U}})(\psi_1,\rho))$.
Hence $G_1 \times G_2$ is an I-VPFSGs.
Theorem 3.10. The composition $\widetilde{G_1}[\widetilde{G_2}]$ be I-VPFSGs $\widetilde{\xi_1}$ and $\widetilde{\xi_2}$ of ξ_1^* and ξ_2^* is an I-VPFSGs.
Proof: Consider $(\phi,\phi_2)(\phi,\psi_2) \in E$, we have

$$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((\phi,\phi_2)(\phi,\psi_2)) = \min((\alpha_{X_1L}(\phi), \alpha_{Y_2L})(\phi_2\psi_2),$$
$$\leq \min(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\phi_2), \alpha_{X_2L}(\psi_2)),$$
$$= \min\left(\min\left(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\phi_2)\right), \min\left(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\psi_2)\right)\right),$$
$$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((\phi,\phi_2)(\phi,\psi_2) = \min(\alpha_{X_1L} \circ \alpha_{X_2L})(\phi,\psi_2), (\alpha_{X_1L} \circ \alpha_{X_2L})(\phi,\psi_2)).$$

Similarly,

$$(\alpha_{Y_1U} \circ \alpha_{Y_2U})((\phi,\phi_2)(\phi,\psi_2) = \min(\alpha_{X_1U} \circ \alpha_{X_2U})(\phi,\phi_2), (\alpha_{X_1U} \circ \alpha_{X_2U})(\phi,\psi_2)).$$

Consider $(\phi_1,\rho)(\psi_1,\rho) \in E$,

$$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((\phi_1,\rho)(\psi_1,\rho) = \min(\alpha_{Y_1L}(\phi_1,\psi_1), \alpha_{X_2L}(\rho)),$$
$$\leq \min(\alpha_{X_1L}(\phi_1), \alpha_{X_1L}(\psi_1), \alpha_{X_2L}(\rho)),$$
$$= \min(\min\left(\alpha_{X_1L}(\phi_1), \alpha_{X_2L}(\rho)\right), \min(\alpha_{X_1L}(\psi_1), \alpha_{X_2L}(\rho))),$$
$$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((\phi_1,\rho)(\psi_1,\rho) = \min(\alpha_{X_1L} \circ \alpha_{X_2L})(\phi_1,\rho), (\alpha_{X_1L} \circ \alpha_{X_2L})((\phi_1,\rho)).$$

Similarly,

$$(\alpha_{X_1U} \circ \alpha_{X_2U})((\phi_1,\rho)(\psi_1,\rho) = \min(\alpha_{X_1U} \circ \alpha_{X_2U})(\phi_1,\rho), (\alpha_{X_1U} \circ \alpha_{X_2U})((v_1,\rho)),$$

Consider $(\phi_1,\phi_2)(\psi_1,\psi_2) \in E$

$$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((\phi_1,\phi_2)(\psi_1,\psi_2)) = \min(\alpha_{X_2L}(\phi_2), \alpha_{X_2L}(\psi_2), \alpha_{Y_1L}(\phi_1\psi_1)),$$
$$\leq \min(\alpha_{X_2L}(\phi_2), \alpha_{X_2L}(\psi_2)), \min(\alpha_{X_1L}(\phi_1), \alpha_{X_1L}(\psi_1)),$$
$$= \min(\min(\alpha_{X_1L}(\phi_1), \alpha_{X_2L}(\psi_2)), \min(\alpha_{X_{1L}}(v_1),, \alpha_{X_2L}(\psi_2))),$$
$$(\alpha_{Y_1L} \circ \alpha_{Y_2L})((\phi_1,\phi_2)(\psi_1,\psi_2)) = \min((\alpha_{X_1L} \circ \alpha_{X_2L})(\phi_1,\phi_2), (\alpha_{X_1L} \circ \alpha_{X_2L})(\psi_1,\psi_2)).$$

Similarly,

$$(\alpha_{Y_1U} \circ \alpha_{Y_2U})((\phi_1,\phi_2)(\psi_1,\psi_2)) = \min((\alpha_{X_1U} \circ \alpha_{X_2U})(\phi_1,\phi_2), (\alpha_{X_1U} \circ \alpha_{X_2U})(\psi_1,\psi_2)).$$

Hence $\widetilde{\xi_1}[\widetilde{\xi_2}]$ is an I-VPFSGs.
Theorem 3.11. The union $\widetilde{\xi_1} \cup \widetilde{\xi_2}$ be I-VPFSGs $\widetilde{\xi_1}$ and $\widetilde{\xi_2}$ of ξ_1^* and ξ_2^* is an I-VPFSGs.
Proof: Let $\widetilde{\xi_1}$ and $\widetilde{\xi_2}$ be I-VPFSGs of $\widetilde{\xi_1}$and $\widetilde{\xi_2}$ respectively. Since all conditions for $X_1 \cup X_2$ are obviously satisfied. It is enough to verify the conditions for $Y_1 \cup Y_2$, Consider $(\phi,\psi) \in E_1 \cup E_2$. Then

$$(\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi,\psi) = \max\left(\alpha_{Y_1L}(\phi,\psi), \alpha_{Y_2L}(\phi,\psi)\right),$$
$$\leq \max(\min(\alpha_{X_1L}(\phi), \alpha_{X_1L}(\phi)), (\min\left(\alpha_{X_2L}(\phi), \alpha_{X_2L}(\psi)\right),$$
$$= \min\left(\max\left(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\phi)\right), \left(\max\left(\alpha_{X_1L}(\phi), \alpha_{X_2L}(\psi)\right)\right)\right),$$
$$= \min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(\psi)),$$
$$(\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi,\psi) = \min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(\psi)).$$

Similarly,

$$(\alpha_{Y_1U} \cup \alpha_{Y_2U})(\phi,\psi) = \min((\alpha_{Y_1U} \cup \alpha_{Y_2U})(\phi), (\alpha_{Y_1U} \cup \alpha_{Y_2U})(\psi)).$$

If $(\phi,\psi) \in E_1$ Then

$$(\alpha_{Y_1L} \cup \alpha_{YL})(\phi,\psi) \leq min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(\psi)),$$
$$(\alpha_{Y_1U} \cup \alpha_{Y_2U})(\phi,\psi) \leq min((\alpha_{Y_1U} \cup \alpha_{Y_2U})(\phi), (\alpha_{Y_1U} \cup \alpha_{Y_2U})(\psi)).$$

If $(\phi,\psi) \in E_2$ Then

$$(\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi,\psi) \leq min((\alpha_{Y_1L} \cup \alpha_{Y_2L})(\phi), (\alpha_{Y_1L} \cup \alpha_{Y_2L})(\psi)),$$
$$(\alpha_{Y_1U} \cup \alpha_{Y_2U})(\phi,\psi) \leq min((\alpha_{Y_1U} \cup \alpha_{Y_2U})(\phi), (\alpha_{Y_1U} \cup \alpha_{Y_2U})(\psi)).$$

Theorem 3.12. The join $\widetilde{\xi_1} + \widetilde{\xi_2}$ be I-VPFSGs $\widetilde{\xi_1}$ and $\widetilde{\xi_2}$ of ξ_1^* and ξ_2^* is an I-VPFSGs.
Proof: Let $\widetilde{\xi_1} + \widetilde{\xi_2}$ be I-VPFSGs of G_1^* and G_2^* respectively, it is enough to prove that $\widetilde{\xi_1} + \widetilde{\xi_2} = (X_1 + X_2, Y_1 + Y_2)$ is an I-VPFSGs.
Let $(\phi,\psi) \in E$

$$(\alpha_{Y_{1L}} + \alpha_{Y_{2L}})(\phi,\psi) = min\left(\alpha_{X_{1L}}(\phi), \alpha_{X_{2L}}(\psi)\right),$$
$$= min((\alpha_{X_{1L}} \cup \alpha_{X_{2L}})(\phi), ((\alpha_{X_{1L}} \cup \alpha_{X_{2L}})(\psi))),$$
$$(\alpha_{Y_{1L}} + \alpha_{Y_{2L}})(\phi,\psi) = min((\alpha_{X_{1L}} + \alpha_{X_{2L}})(\phi), ((\alpha_{X_{1L}} + \alpha_{X_{2L}})(\psi))).$$

Similarly,

$$(\alpha_{Y_{1U}} + \alpha_{Y_{2U}})(\phi,\psi) = min((\alpha_{X_{1U}} + \alpha_{X_{2U}})(\phi), ((\alpha_{X_{1U}} + \alpha_{X_{2U}})(\psi))).$$

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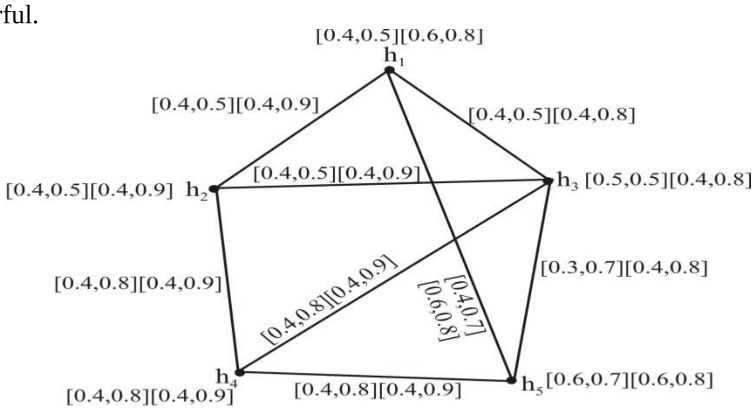
4. Application

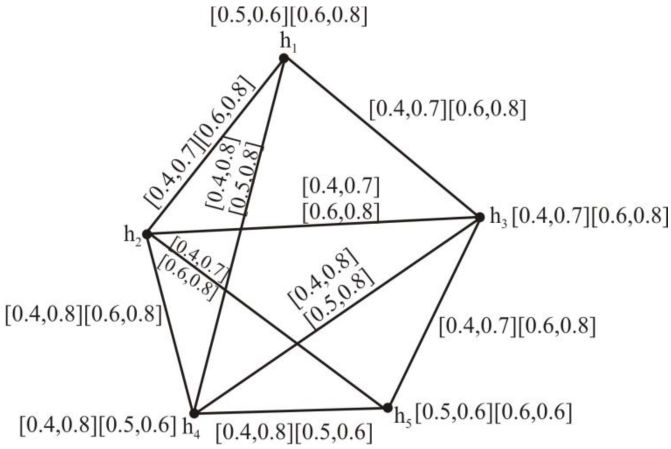
Interval-valued Pythagorean fuzzy soft sets have a variety of uses in decision-making issues and can be applied to handle uncertainty from our various real-world issues. The idea of interval-valued Pythagorean fuzzy soft sets is applied to a decision-making problem in this part, and we then provide an algorithm for choosing the best item based on the available sets of data. Suppose that $V = \{h_1, h_2, h_3, h_4, h_5\}$ is the set of five houses under consideration. Mr. R is going to buy one of houses on the basis of wishing parameters or attributes set $A = \{e_1$ =large, e_2= beautiful, e_3 = colorful$\}$.(X, A) is the Interval-valued Pythagorean fuzzy soft set on V which describes the value of the houses based upon the given Parameters e_1 = large, e_2= beautiful, e_3 =colorful, respectively(Y, A) is the Interval-valued Pythagorean fuzzy soft set on $E = \{h_1h_2,\ h_1h_3,\ h_1h_4,\ h_1h_5,\ h_2h_3, h_2h_4, h_2h_5, h_3h_4, h_4h_5\}$. which describe the value of two houses corresponding to the given parameters e_1, e_2 and e_3. The interval-valued Pythagorean fuzzy soft graphs $H(e_1), H(e_2)$ and $H(e_3)$ of Interval-valued Pythagorean fuzzy soft graphs $H = \{H(e_1), H(e_2), H(e_3)\}$ corresponding to the parameters large, beautiful, colorful.



$H(e_1)$

Table 1: Tabular representation of accuracy function values and choice values of $H(e_1)$.

Houses	h_1	h_2	h_3	h_4	h_5	h_6
h_1	0	0.97	0.8	0	1	2.77
h_2	0.97	0	0.97	0.97	0	2.91
h_3	0.8	0.97	0	0.97	0.8	3.54
h_4	0	0.97	0.97	0	0.97	2.91
h_5	1	0	0.8	0.97	0	2.77



$H(e_2)$

Table 2: Tabular representation of accuracy function values and choice values of $H(e_2)$.

Houses	h_1	h_2	h_3	h_4	h_5	h_6
h_1	0	1	1	0.89	0	2.89
h_2	1	0	1	1	1	4
h_3	1	1	0	0.89	1	3.89
h_4	0.89	1	0.89	0	0.61	3.39
h_5	0	1	1	0.61	0	2.61

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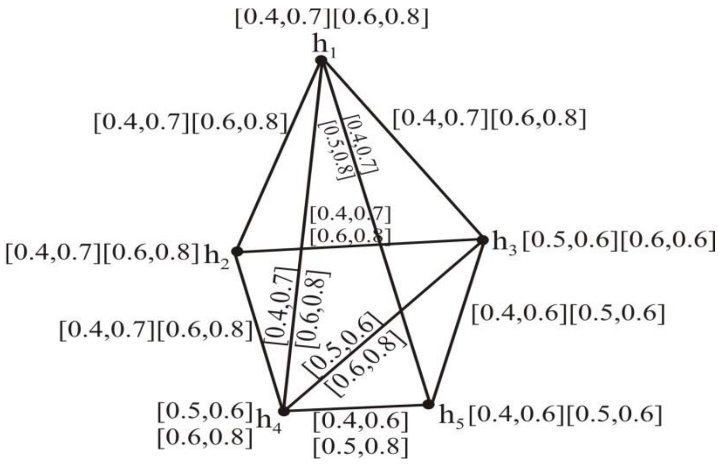

$H(e_3)$

Table 3: Tabular representation of accuracy function values and choice values of $H(e_3)$.

Houses	h_1	h_2	h_3	h_4	h_5	h_6
h_1	0	1	1	1	0.89	3.89
h_2	1	0	1	1	0	3
h_3	1	1	0	1	0.61	3.61
h_4	1	1	1	0	0.89	3.89
h_5	0.89	0	0.61	0.89	0	2.39

Tabular representation accuracy values of $H(e_1)$, $H(e_2)$, and $H(e_3)$ with accuracy function $S_{jk} = \alpha_{YU}^2 + \beta_{YU}^2$ and choice value for each houses $j = 1,2,3,4,5$ is given in tables, The decision is S_j if $S_j = \max\{min\ C_q = \max\{2.77, 2.91, 2.89, 2.61, 2.39\} = 2.91$. Clearly the maximum score value is 2.91, scored by h_2. Mr. R will choose the house h_2.

5. Conclusion

The interval-valued Pythagorean fuzzy soft set model is appropriate for modeling issues involving ambiguity, uncertainty, and inconsistent data when human understanding and evaluation are required. In contrast to interval-valued fuzzy soft and interval-valued intuitionistic fuzzy soft models, interval-valued Pythagorean fuzzy soft models provide systems with sensitivity, flexibility, and conformance. Interval-valued Pythagorean fuzzy soft graphs are a novel idea that is presented in this work. We provided some of the properties of some operations on interval-valued Pythagorean fuzzy soft graphs. Finally we have applied the concept of interval-valued Pythagorean fuzzy soft graphs in real life problems.

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