An S-Original Algebraic Property Of Lie Algebras

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Abstract
This paper is dedicated to answer the following question:If L is a Lie algebra over a field with characteristic zero, and if S is an original property in Lie algebras class, then does S is a characteristic ideal of L?.For this goal, we have proved many related theorems, and illustrated some examples.

Keywords: S-Original Algebraic Property; Lie Algebras; characteristic zero field

1. Introduction

A subspace I of a Lie algebra L is called an ideal of L, if $x \in L$, $y \in I$**,together imply** $[x, y] \in I$.

A derivation D in L is a linear mapping of L into L satisfying the following condition:

D (x y) = D(x) y + x D(y) for every $x, y \in L$. denote by Der(L) the set of derivations in L Ideal I of L is said to be characteristic ideal of L, denote it by $I \lhd L$, if I is sub vector space of L and $D(I) \subset L$ for every $D \in Der(L)$.The ideal I of Lie algebra L is said to be D-invariant , if $D(I) \subseteq I$, D is derivation of L .Define a sequence of ideal of L by

$L^{(0)} = L, L^{(1)} = [L, L], L^{(2)} = [L^{(1)}, L^{(1)}], \dots L^{(i)} = [L^{(i-1)}, L^{(i-1)}]$

Called L solvable if $L^{(n)} = 0$ for som n

Define a sequence of ideal of L by $L^0 = L, L^1 = [L, L], L^2 = [L, L^1] \dots \dots L^n = [L, L^{n-1}]$

L is called nilpotent if $L^n = 0$ for some $n \geq 1$. [see 1].

A Lie algebra L is called a complete Lie algebra if its center C (L) is zero and its derivation are all inner [see 2] . Let L be finite-dimensional Lie algebra over a field of characteristic zero, then L has Levi decomposition i.e. L = S + R , where S is a maximal semi simple sub algebra of L and is called the Levi sub algebra of L and R is maximal solvable ideal of L and its called the radical of L , the ideal

$I_0 = [L, L] \cap R = [L, R]$ is called the nilpotent radical of L [see 2].

DEFINITION 1-1: The class S of S-algebra is said to be S-radical property in class Lie algebra, if the class S satisfy the following conditions:

1)- Class S is closed homomorphic,

2)- Every algebra L contain a maximal ideal belongs to class S, denote it by S(L) ,

3)- For every algebra L, quotient algebra L/S(L) no contain ideals difference of zero belongs to class S .

S-radical property is called over solvable, if solvable algebra's belongs to class S.

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II - D-INVARIANT RADICAL PROPERTY

All considered algebra in this part are Lie algebra over a field of characteristic zero

DEFINITION 2-1: S-radical property is said to be D-invariance if for every algebra L, its radical is characteristic ideal in L.

LEMMA 2-1: If I is an ideal of algebra L, such that $I^2 = I$, then I is characteristic ideal of L .

Proof its clearly.

LEMMA 2-2: Let S is an S-radical property, if there exists non zero S radical of abelian algebra B, then S is a solvable property.

Proof: Let L be an arbitrary nonzero abelian algebra, denote by < a > the ideal which generated by a in algebra L. Because L and B are abelian algebra's, so every linear mapping between L and B is homomorphism algebra, that is ideal < a > is homomorphism image nonzero algebra S radical B.

Hence, ideal is S-radical its true for every element algebra L, then L is S radical algebra. Further proof is induction on grade solvability algebra.

LEMMA 2-3: If I is S-radical ideal of algebra L, such that L / I is S-radical algebra, then algebra L is S-radical.

Proof: [see 1,3] .

THEOREM 2-1: If S-radical property is not invariance, then S is radical property over solvable.

Proof: Let S be a radical property which is not invariance on account derivation, then there exists so algebra L and so it derivation D, that S (L) not contain D(S(L)). By lemma 2-1, we get $(S(L))^2 \neq S(L)$. Because algebra S(L) is S-radical, then

$B = S(L)/(S(L))^2$ is nonzero S-algebra. Algebra B is abelian, hence since lemma 2-1 we have S-radical property is over solvable.

LEMMA 2-3: If I is ideal of algebra L, D it derivation, then I + D(I) is also ideal of algebra L [see 1].

LEMMA 2-4: If I is solvable ideal of algebra L, D it derivation, then I + D(I) is solvable ideal of algebra L [see 1].

LEMMA 2-5: If I is ideal of algebra L, then for any derivation D of algebra L we get:

1)- $D(I^{(n+1)}) \subseteq I^{(n)}$,

2)- $D(I^{n+1}) \subseteq I^{n}$.

Proof: It is known [5], if I is ideal of algebra L, then for every natural number k>0, $I^{(k)}$, I^{k} are ideals in L. So we have

$D\left(I^{(n+1)}\right) = D\left[I^{(n)}, I^{(n)}\right] \subseteq I^{(n)}D\left(I^{(n)}\right) + D\left(I^{(n)}\right)I^{(n)} \subseteq I^{(n)}$

Proof to condition 2 is similar.

THEOREM 2-2: Let S be a radical property in class lie algebra, if D is any derivation of lie algebra L, that $D((S(L)^{n}) \subseteq S(L)^{n}$ for some $n \geq 1$ then $D((S(L)) \subseteq S(L)$.

Proof: Let $D(S(L)^{(n)}) \subseteq S(L)^{(n)}$, $n \geq 1$. Since $(L)^{(n)} \subseteq S(L)$, then $S(L_1) = S(L)/S(L)^{(n)}$, where $L_1 = L/S(L)^{(n)}$, from this we have that $S(L_1)$ is solvable ideal of algebra L, hence by lemma (2-4) $S(L_1) + D_1(S(L_1))$ is solvable ideal in Algebra L; where D_1 is derivation D-algebra L .

If S property is not solvable, then by theorem 2-1 we get $D(S(L)) \subseteq S(L)$. Assume that the Sproperty is over solvable, then $S(L_1) + D_1(S(L_1)) \subseteq S(L_1)$ hence $D_1(S(L_1)) \subseteq S(L_1)$ thus $D(S(L)) \subseteq S(L)$.

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Corollary 2-1: If radical S(L) algebra L satisfy condition $S(L_1)^{(n)}) = S(L)^{(n+1)}$ for some natural number n, then S(L) is characteristic ideal in algebra L .

Proof: According to the lemma 2-5, $D(S(L)^{(n+1)}) \subseteq S(L)^{(n+1)}$ for any derivation D of algebra L, from this and from theorem 2-2 we have $D(S(L)) \subseteq S(L)$.

Theorem 2-3 : Let S be a S-radical property, then for every Artinian algebra L radical S(L) is characteristic ideal in L.

Proof: If L is Artinian algebra, then there exists a natural number k such, that $S(L)^{(k)} = S(L)^{(k+1)}$ from this and according corollary 2-1 from theorem 2-1 we have $D(S(L)) \subseteq S(L)$ for any derivation D of algebra L.

Corollary 2-2 : Let L be an algebra ,S-radical property and D-derivation algebra L if linear space D(S(L)) is finite dimensional, then $D(S(L)) \subseteq S(L)$.

Proof: Since the linear space D(S(L)) has finite dimensional, therefore there exists a natural number n such that $D\big(S(L)^{(n)}\big) = S(L)^{(n+1)}$ from lemma 2-5 and corollary 2-1 from theorem 2-2 we get $D(S(L)) \subseteq S(L)$.

Now, we reflect over the following question. If radical algebra L precipitate as simple component in L the is it characteristic ideal in L ?

To this effect we consider the following situation.

Let φ be a any homomorphism algebra L in algebra B , we denote by L φ B the simple sum L⊕B to vector spaces L, B which defined in following way:

The multiplication operation, if a ∈ L, b ∈ B, then a.b = φ(a).b. Clearly show that L φ B with above operation is Lie algebra over the same field .

In algebras L and B easy check, that B is ideal in algebra L φ B.

Lemma 2-6: Let D be a derivation of algebra L φ B such, that D(B) non contain in algebra B, then C(L), (the center of algebra L) is different of zero.

Proof: From assumption there exists an element b ∈ B such, that $D(b) \notin B$. Hence D(b) = b' + b* where $b' \in B$, $b^* \in L$, $b^* \neq 0$. Now let a be a arbitrary element of algebra L, then $a.b = \varphi(a).b \in B^2$, consequently $D(a.b) = D(a).b + a.d(b) \in B$. Because $b.b' \in B$, that $B'D(a.b) = D(a).b + a.D(b) = D(a).b + ab' + ab^*$, since $D(a).b + a.b' \in B$ then $ab^* \in B$. On the other hand $ab^* \in L$ hence $ab^* = 0$ that is the Centrum C (L) of algebra L contain element $b^* \neq 0$ as desired.

Theorem 2-4 : Let S-radical algebra L φ B be equal to B, then it is characteristic ideal in algebra L φ B.

Proof: According to the theorem 2-1 we can assume, that S-radical property is over solvable.

Suppose that for certain derivation D of algebra L φ B, $D(B) \not\subset B$ from this and according to lemma 2-6 that in algebra L there exists nonzero ideal I Centrum C(L) algebra L. Algebra I + B is S-radical as homomorphic image (L φ B)/ B, this is not possible because S(L φ B) = B.

COROLLARY 2-3 : Let S be a S-radical property of algebra L, if the radical S(L) Algebra L distribute as simple component, then it is characteristic ideal algebra L .

Proof: From assumption there exists ideal I algebra L, such that L = I + S(L).

Let j be a zero homomorphism algebra I in S(L), then

$L = I + S(L) = I_{\varphi=0}S(L)$. Now by theorem 2-4 $D(S(L)) \subseteq S(L)$ for any derivation D algebra L .

DEFINITION 2-2: Derivation d algebra L is said to be nil-derivation, if for every element a ∈ L there exists as natural number n>0, such that $D^n(a) = 0$

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It is known [4,5] that if L is nil-derivation algebra L invariance relative on automorphism algebra L, $D(L) \subseteq L$. From this we have the following lemma.

LEMMA 2-7: If D is nil-derivation algebra L, then $D(S(L)) \subseteq S(L)$whereas S is S-radical property.

Let $a_1, a_2, a_3, \dots, a_n$ are elements of algebra L. Denote by $(a_1, a_2, a_3, \dots, a_n)_\rho$ the multiplication n-elements of part bricks ρ, whereas ρ part bracket of word $a_1, a_2, a_3, \dots, a_n$.

LEMMA 2- 8: Let $a_1, a_2, a_3, \dots, a_n$are elements of ideal I of algebra L, then for every derivation D algebra L we have

$(D(a_1), D(a_2), \dots, D(a_n))_\rho \in I + 1/n!\, D^n(a_1, a_2, \dots, a_n)_\rho$

Proof: From inequality which we can fined in [6] we have

$$D^n(a_1, a_2, \dots, a_n) = \sum \binom{n}{\alpha_1} . \binom{n-\alpha_1}{\alpha_2} \dots \binom{n-\alpha_1-\alpha_2-\dots-\alpha_{n-2}}{\alpha_{n-1}} (D^{\alpha_1}(a_1), \dots, D^{\alpha_n}(a_n))_\rho \ (*)$$

Where the sum over all $\alpha_1, \alpha_2, \dots, \alpha_{n-1}$ such that $0 \le \alpha \le n$, $0 \le \alpha_i \le n - \alpha_1 - .. - \alpha_{n-1}$.

$\alpha_n = n - \alpha_1 - .. - \alpha_{n-1}$, $1 \le I \le n-1$, $D^0(a) = a$, as $\alpha_1 + .. + \alpha_n + \alpha$ and all exponent α_i are non negative, then $\alpha_1 = \alpha_2 = .. = \alpha_n = 1$ or $\alpha_I = 0$ for some I. If exponent $\alpha_I = 0$, then element $(D^{\alpha_1}(\alpha_1), \dots, D^{\alpha_n}(\alpha_n))_\rho \in I$.

Because $\alpha_I \in I$ and $D(\alpha_I) = \alpha_I$ so one expression which all exponent α_I are non equal to zero, therefore $(D(\alpha_1), \dots, D(\alpha_n))_\rho$ is manoeuvrig factor is equal to n! hence from (*) we have

$(D^n(\alpha_1, \dots, \alpha_n)_\rho = n!\,(D(\alpha_1), \dots, D(\alpha_n))_\rho$

Let us recall that an element x of Lie algebra L over a field K is called algebraic if there exists a polynomial f(t) ∈ K[t] depending on x such that f(adx) = 0 [see 7].

DEFINITION 2-3: Derivation D algebra L is called algebraic derivation bounded index, if there exists a natural number n > 1, such that for every element a ∈ L, $D^n(a) \in \langle a, D(a), \dots, D^{n-1}(a) \rangle$, where

$\langle a, D(a), \dots, D^{n-1}(a) \rangle$ is sub algebra generated by elements $a, D(a), \dots, D^{n-1}(a)$

THEOREM 2-5: Let S be a radical property, D derivation algebraic bounded index of algebra L, then $D(S(L)) \subseteq S(L)$.

Proof: By virtue of theorem 2.1, we can assume, that S-radical property is over solvable, from definition of D there is a natural number n, such that

$D^n(\alpha) \in \prec \alpha, D(\alpha), \dots, D^n(\alpha) \succ$ for some element $\alpha \in L$

Let $\alpha_1, \alpha_2, \dots, \alpha_n$ are any elements from radical S(L) and let $a = ((..(a_1 a_2) a_3)..) a_n)$, by virtue of lemma 2.5 we:

$D^i(\alpha) \in D^i(S(L)^n) \subseteq S(A)^{n-i}$, i=1,2,…,n-1

So element $D^{n-1}(\alpha)$ belongs to radical S(L) algebra L. By virtue of lemma 2.3 S(L) + D (S(L)) is ideal of algebra L , hence S(L) + D (S(L)) / S(L) is solvable ideal of algebra L/S(L) this is , in the presence of over solvable radical S, that

$S(L) + D(S(L)) \subseteq S(L)$, from this we have $D(S(L)) \subseteq S(L)$

III-NORMAL RADICAL PROPERTY

Let L be a Lie algebra over a field K and let B always ,associative, commutative with unit element 1 (over also K) as known in [8] that $L \underset{K}{\otimes} B$ is Lie algebra and algebra L may be imbedding in algebra $L \underset{K}{\otimes} B$.

LEMMA 3-1 : If D is derivation of algebra B ,then the mapping

$id \otimes : dL \underset{K}{\otimes} B \to L \underset{K}{\otimes} B$

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Defined at below:

$\left(\text{id} \underset{K}{\otimes} \text{d}\right)(\sum a_i \otimes p_i) = \sum_{i=1}^{n} a_i \otimes d(p_i)$

is derivation of algebra $L \underset{K}{\otimes} B$. If however D is nil-derivation, then derivation $\text{id} \underset{K}{\otimes} d$ is also nil-derivation.

Proof : see N. JACOBSON. Lie algebra. Intersciance New York 1962 page 158.

DEFINITION 3-1 : We say that S-radical property is B-normal if for every algebra L, $S\left(L \underset{K}{\otimes} B\right) = I \underset{K}{\otimes} B$, where I is reliable ideal of algebra A. [see5]

THEOREM 3-1 : S-radical property is B-normal iff for every algebra L, from condition $S\left(L \underset{K}{\otimes} B\right) \neq 0$ result consequence $S\left(L \underset{K}{\otimes} B\right) \cap A = 0$.

Now, we show following lemma

LEMMA 3-2 : If for every algebra L there exists nil-derivations $d_1, d_2, \dots, d_n$ algebra B such that from condition $S\left(L \underset{K}{\otimes} B\right) \cap L \neq 0$ consequence

$(\text{id} \otimes d_1)(\text{id} \otimes d_2) \dots (\text{id} \otimes d_n) S\left(L \underset{K}{\otimes} B\right) \cap L \neq 0$

then S-radical property is B-normal.

Proof: Suppose that $S\left(L \underset{K}{\otimes} B\right) \neq 0$ then there is element $0 \neq a \in S\left(L \underset{K}{\otimes} B\right)$ such that $d_1, d_2, \dots, d_n$ are nil-derivation, hence from lemma 2-7 and lemma 3-1 we have for i=1,2,3.. $(\text{id} \otimes d_n)(S\left(L \underset{K}{\otimes} B\right)) \subseteq S\left(L \underset{K}{\otimes} B\right)$, therefore

$0 \neq (\text{id} \otimes d_1)(\text{id} \otimes d_2) \dots (\text{id} \otimes d_n)(a) \in S\left(A \underset{K}{\otimes} B\right) \cap A$

Thus we have complete the proof.

THEOREM 3-2: Let L be a finite dimension algebra , then for every S radical property, B-normal radical $S\left(L \underset{K}{\otimes} B\right)$ is invariance respect to derivation algebra $L \underset{K}{\otimes} B$.

Proof: S-radical property is B-normal, then $S\left(L \underset{K}{\otimes} B\right) = I \underset{K}{\otimes} B$, because algebra L has finite dimension there exist a natural number n>0 that $I^{(n)} = I^{(n+1)}$, therefore we get $(S\left(L \underset{K}{\otimes} B\right))^{(n+1)} = I^{(n+1)} \underset{K}{\otimes} B^{(n+1)} = (I \underset{K}{\otimes} B)^{(n)} = (S\left(L \underset{K}{\otimes} B\right))^{(n)}$ Now according to corollary1from theorem 2-2 we have demonstration fact.

COROLLARY 3-1: Let L be a finite dimension algebra, then for every S-radical property, ideal $S(L[x_1, x_2, \dots, x_n])$ is characteristic ideal in $L[x_1, x_2, \dots, x_\alpha, \dots, x_n]$ where $L[x_1, x_2, \dots, x_\alpha, \dots, x_n]$ is polynomial algebra with infinite quantity variable, $x_1, x_2, \dots, x_n$,

Proof: It is known that algebra $L[x_1, x_2, \dots, x_\alpha, \dots]$ is isomorphic with algebra $L \underset{K}{\otimes} K[x_1, x_2, \dots, x_\alpha, \dots, x_n]$, we show that every $K[x_1, x_2, \dots, x_\alpha, \dots, x_n]$ radical property is normal. Suppose that $S(L \underset{K}{\otimes} K[x_1, x_2, \dots, x_\alpha, \dots, x_n]) \neq 0$, then there is nonzero polynomial $W(x_{\alpha_1}, x_{\alpha_2}, \dots, x_{\alpha_n}) \in S(A \otimes K[x_1, x_2, \dots, x_\alpha, \dots)$

It is known that derivation $d_\alpha(W)(W/x_\alpha)$ for every valued α is nil-derivation algebra $L[x_1, x_2, \dots, x_\alpha, \dots, x_n]$. Now, we take right derivation $\text{id} \otimes d_\alpha$ we get

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$0 \neq \left(\mathrm{id} \otimes d_{\alpha_{i,j}}\right) \ldots \left(\mathrm{id} \otimes d_{\alpha_{ik}}\right) W(x_{\alpha_1}, x_{\alpha_2}, \ldots, x_{\alpha_n}) \in S(A \otimes K[x_1, x_2, \ldots, x_\alpha, \ldots]) \cap L$

Applying lemma 3.2 we have that S radical property is $K[x_1, x_2, \ldots, x_\alpha, \ldots x_n]$normal hence from theorem 3-2 $S(L[x_1, x_2, \ldots, x_\alpha, \ldots x_n])$ is characteristic ideal in algebra $L[x_1, x_2, \ldots, x_\alpha, \ldots x_n]$.

Now we investigate problem invariable radicals $L \underset{K}{\otimes} B$, where B is a algebra group K[G] over a field K, G is abelian group .We define following symbol. Let $x = a_1 g_1 + a_2 g_2 + \cdots + a_n g_n \in L \otimes K[G]$, where $a_1, a_2, .., a_n \in A$, $g_1, g_2, .., g_n \in G$, denote by Supp x the set of element g_i such that $a_i \neq 0$.

If $\mathrm{Supp}x = \{g_1, g_2, \ldots, g_n\}$,then the number n =l(x) is said to be length element x

LEMMA 3-3: Let S be a radical property, L algebra, G-group, if radical S(L[G]) algebra L[G] contain element x with length n, then S(L) contain element y which also length n and $e \in$ Supp y, where e is unit element of group G .

Proof: Take arbitrary element $z \in S(L[G])$, g any element group G. Because group G is abelian then its elements we can effective with operator of algebra L[G], therefore from [5,3] we get $z\,g \in S(L[G])$.

Now, let element $x \in (L[G])$, l (x) = n and let $g \in$ Supp x because l(x) = l(x/g), $e \in$ Supp (x/g) and $x/g \in S(L[G])$ then be enough take y=x/g

DEFINITION 2-3: Let K is a field. Group G is said to be K-complete if for every element $g \in G$, $g \neq e$, there is a homomorphism f group G in multiplication field K* such that f(g) = 1 , 1 is unit element of field K.

LEMMA 3-4 : Let G be a complete group , then for every S radical property and every K-algebra L , if $S(L[G]) \neq 0$, then $S(L[G]) \cap A \neq 0$.

Proof: Let $a = \sum a_i g_i$ be a nonzero element from radical S (L[G]) with minimal length. By lemma 3-3 we can assume that $e \in$ Supp a. Assumption, that $g_1 = e$ we show that l(a) =1. Assumption $g_2 \neq e$ because group G is K-complete, there exists a homomorphism $f: G \to K^*$ such that $1(g_2) \neq 1$.

We can defined mapping $\bar{f}: G \to L[G]$ at follows [see 6].

$\bar{f}(\sum_i b_i g_i) = \sum_i b_i\, f(g_i) g_i$, $b_i \in L$, $g_i \in G$

f is K-automorphism algebra L[G], $f(g_2) \neq 1$, $f(g_1) = 1$. Hence

$0 \neq \sum_{i=2}^{n} [a_i - a_i f(g_i)] g_i = a - \bar{f}(a) \in S(L(G))$

This is not possible, since $1(a - \bar{f}(a)) < n$ we have l (a) =1 and $S(L[G]) \cap L \neq 0$.

COROLLARY 3-3: If group G is K-complete, algebra L has finite dimension, then for every S-radical property the radical S(L[G]) is characteristic ideal in algebra L[G].

Proof: The proof is result from corollary 3-1 and theorem 3.2.

Now we give some example for S-radical property which is not invariable property.

P. M. Gohn [5], gave example Lie algebra in which equation a x = b has solution for every $a \neq 0$, $a \in L$ and for every $b \in L$.

We consider algebra

$R\left[[x]\right] = \{\sum_{i=1}^{\infty} a_i x_i \,;\, a_i \in L\}$

It is known that algebra of above formal series R[x] with coefficient from Lie algebra and set I formula series from $\sum a\,x$, $a_i \in L$ is ideal in R[[x]].

We fined all image homomorphic of algebra I .

Let $0 \neq a \in L$ and let b be any element from ideal I, then

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$a = a_n x^n + a_{n+1} x^{n+1} + \cdots + a x^m + \cdots,\ \ a_n \neq 0$

$b = b_1 x + b_2 x^2 + b_3 x^3 + \cdots$

$a.b = c = c_{n+1} x^{n+1} + c_{n+2} x^{n+2} + \cdots + c_{n+k} x^{n+k} + \cdots$

$c_{n+1} = a_n b_1$

$c_{n+2} = a_n b_2 + a_{n+1} b_1$

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$c_{n+k} = a_n b_k + a_{n+1} b_{k-1} + \cdots + a_{n+k-1} b_1$

Because in algebra L , inequality ax = b has solution for a ≠ 0, a ∈ L and for every b ∈ L, choose satisfactory accordingly coefficient

$b_1, b_2, \ldots, b_n, ..$ We can get any value $c_{n+1}, c_{n+2}, \ldots, c_{n+k}, ..$ from this result that $I^{n+1} \subseteq < a >$;

< a > is an ideal generated by element a from algebra L.

Now, let I / Y be any homomorphic image of algebra I, let J ≠ 0 then there exists 0 ≠ a ∈ J ∈ I from this result there exists a natural number m > 0 such that $I^m \subseteq < a > \subseteq J$. We showed that every proper homomorphic image of algebra I is nilpotent.

Let S be a minimum S-radical property ,such that algebra I and all its homomorphic image belongs to class S, (by [8] this radical property exists), we show that S(R[x]) = I. Clearly S(I) = I and quotient algebra R[[x]] / I is isomorphic to algebra L.

Algebra L non contain maneuverability ideals and is not nilpotent algebra, also algebras L and I are not isomorphic because $L^2 \neq L$ but $I^2 \neq I$ to mean that algebra L no contain nonzero S-ideals, hence S(R[[x]]) = I.

Now let $a = a_0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$ be an element of algebra R[[x]]. Mapping $D(a_0 + a_1 x + \cdots + a_n x^n) = a_1 + 2a_2 x + \cdots + n a_n x^{n-1}$ is a derivation of algebra R[[x]] Let a ≠ 0 be an element of algebra L, because

S(R[[x]]) = I then ax ∈ I, but for derivation D we have

$D(ax) = a \notin I = S(R[[x]])$.

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