Critical Path Method and Project Evaluation and Review Technique under Uncertainty:A State-of-Art Review

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Abstract

The research article offers an extensive examination of extended fuzzy principles and their practical applications for addressing networking problems. Extended fuzzy principles have gained a significant impact that serve as an act of expansion from crisp and fuzzy logic, in addressing various uncertain environmental conditions. The Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT) have emerged as valuable tools for tackling complex applications of network problems. The main emphasis lies on the discussion of three important extended fuzzy principles: Intuitionistic, Pythagorean, and Neutrosophic, specifically in the context of CPM/PERT. Overall, this research article aims to provide valuable insights into the application of extended fuzzy principles, enabling better decision-making under different environmental conditions. The findings contribute to the ongoing development of fuzzy extensions and their potential to overcome challenges in networking.

Keywords: Project Management; Network Analysis; Critical Path Method; Project Evaluation and Review Technique; Fuzzy Critical Path Method; Fuzzy Project Evaluation and Review Technique; Uncertainty; Extended Fuzzy Principle

1 Introduction

1.1 Critical Path Method/Project Evaluation and Review Technique

Project management, extensively employs a range of techniques for scheduling and planning tasks within prescribed time-limit. Emphasis is placed on network techniques like those developed by Fulkerson (1961)[1] and Prager (1963),[2] which use a network flow formulation. Implementing these models involves defining a project's scope and deliverable, then breaking them down into tasks using a work breakdown structure (WBS). The WBS, a hierarchical decomposition of the total scope of work to meet the objectives. Thus, network planning methods are key tools for project management, assisting in planning, controlling, and scheduling through graphical representations of project activities.

This paper reviews network planning methods, specifically the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT), a strategic solutions to complex scheduling challenges. Both CPM and PERT originated from a need to address task dependencies that cannot be handled by traditional bar charts, such that CPM/PERT have been extensively used in large-scale project management. CPM, a deterministic tool, that identifies the critical path of dependent tasks that sets the project's minimum completion time under optimal resource allocation, task prioritization, and risk mitigation. PERT, a probabilistic tool, that especially beneficial for highly uncertain projects. It uses three-time estimates for each task to ensure the critical tasks, manage potential delays proactively, and estimate completion times based on statistical distributions. Both

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CPM and PERT, acts as an integral parts of network analysis, which significantly contribute to efficient and effective project management. Building on previous points, various studies and real-world applications have demonstrated the efficacy of CPM and PERT in crisp environments. For instance, a study by Dolabi et.al., (2014)[3] noted that even minor changes can disrupt the schedule, underscoring the unrealistic assumption of precise activity durations in CPM. To address these shortcomings and embrace uncertainty, the PERT technique was introduced into the field. Most practical issues, such as resource allocation, integrated planning and scheduling, time-cost trade-offs, and risk management, increase the perception of total project duration and are amenable to a variety of approaches and algorithms.

CPM and PERT techniques have long been integral to classical project management, offering a clear strategy for managing complex projects. While they retain their fundamental characteristics, the adoption of new technologies and evolving project environments have broadened their application. Their integration with simulation, heuristic and meta-heuristic methods, and mathematical modelling tools has improved scheduling solutions for large projects, rendering these techniques indispensable to modern projects. By the increasing complexity and uncertainty in today's project environments, there has been a shift from the traditional crisp approach of CPM and PERT to a more flexible, fuzzy approach. This shift acknowledges that factors like changing requirements, unforeseen risks, and resource constraints can render task durations more dynamic than fixed. Therefore, fuzzy logic and fuzzy set theory have been incorporated into CPM and PERT methodologies to allow a more probabilistic and adaptable project management approach.

1.2 Survey of Uncertainty

One of the most common day-to-day phenomena in daily life is decision-making (DM). Traditionally, it has been observed that the decision information used to access in the form of crisp numbers. Later, DM impacts the significant amount of study in the fields of engineering, social science, medical science, mathematical science, and economics. Several theoretical advancements have been done and still going on concerning different characteristics and features. However, most decisions in real-life circumstances are frequently ill-defined or ambiguous, by necessitating several stages to reach the destination, fuzzy set theory came into existence. Zadeh (1965)[4] interpreted fuzzy information analysis as a highly valuable decision-making and finds applications in various domains. However, a limitation of this tool is that it can only accommodate a single membership function and cannot represent non-membership functions.

Additionally, Chang and Zadeh proposed the idea of fuzzy mapping and control in 1972.[5]Later, several researchers around the world studied fuzzy numbers in significant ways to refine and develop the tenets of uncertainty theory, which ultimately led to many intriguing results in various fields. Our work's main contribution is to review the CPM and PERT, which are frequently employed to resolve difficult scheduling issues in the context of uncertain environments.

1.3 Organization of the paper

In this research article, Section 1.1 presents the introduction of CPM/PERT in a crisp environment and later Section 1.2 discusses the basic uncertainty's origin; Section 2 contains preliminaries of the reviewed literature work presented in the further sections; Section 3 presents the advancement of the fuzzy environment under CPM/PERT; Section 4 addresses the introduction of the extended fuzzy principle; and Section 5 reviews the extended fuzzy working principle under CPM/PERT. Lastly, the paper concludes, followed by the references.

2 Preliminaries

The preliminary section discusses the essential definitions that are reviewed in the literature survey such as fuzzy sets, fuzzy numbers of Trapezoidal and LR fuzzy that depicted in Figure.1.

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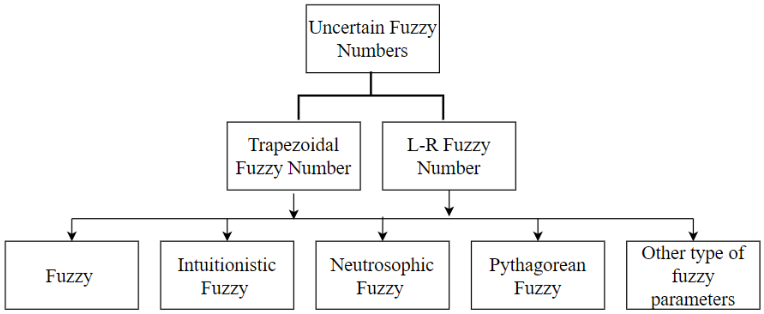

Figure 1: A concise flowchart depicting different uncertain parameters

Definition 2.1. [4] Fuzzy set: A set $\widetilde{f}$ is illustrated as $\widetilde{f} = \{(\psi, \mu_{\widetilde{f}}(\psi)) : \psi \in f, \mu_{\widetilde{f}}(\psi) \in [0,1]\}$ and generally denoted by the ordered pair $(\psi, \mu_{\widetilde{f}}(\psi))$, here $\psi \in f$ be the crisp set and $\mu_{\widetilde{f}}(\psi) \in [0,1]$; such that $0 \leq \mu_{\widetilde{f}}(\psi) \leq 1$, where $\widetilde{f}$ is termed as the fuzzy set.

Definition 2.2. [6] Trapezoidal Fuzzy Number (TFN): A trapezoidal fuzzy number $\widetilde{TF}$ can be illustrated as (tf_1, tf_2, tf_3, tf_4) with the membership function $\mu_{\widetilde{TF}}$ as follows:

$$\mu_{\widetilde{TF}}(\psi) = \begin{cases} \dfrac{\psi - tf_1}{tf_2 - tf_1}, & tf_1 \leq \psi \leq tf_2; \\ 1, & tf_2 \leq \psi \leq tf_3; \\ \dfrac{tf_4 - \psi}{tf_4 - tf_3}, & tf_3 \leq \psi \leq tf_4; \\ 0, & Otherwise. \end{cases}$$
$$where \quad tf_1, tf_2, tf_3, tf_4 \in \mathbb{R}$$

The graphical representation of TFN is shown in below Figure.2.

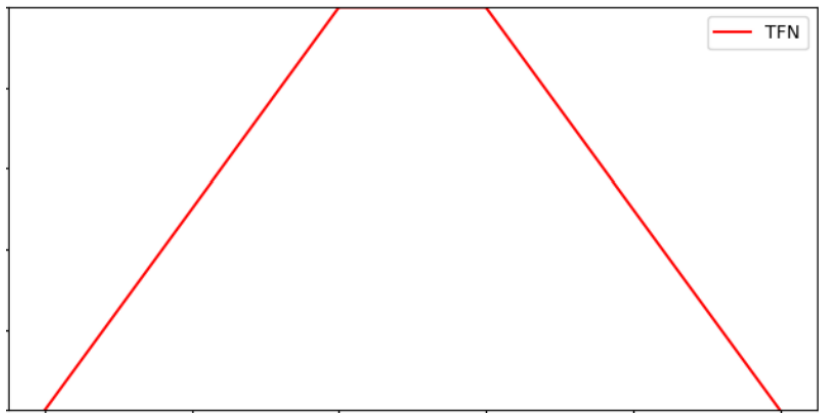

Figure 2: Trapezoidal fuzzy number

Definition 2.3. [7] LR-type Fuzzy Number: LR-Form Fuzzy set is characterized by the presence of reference functions as left (L) and right (R) , along with scalars $p > 0$, $q > 0$, which together define its membership

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function as:

$$\mu_{\widetilde{TF}}(\psi) = \begin{cases} L\left(\dfrac{\sigma-\psi}{p}\right), & \psi \le \sigma; \\ 1, & \sigma \le \psi \le \gamma; \\ R\left(\dfrac{\psi-\gamma}{q}\right), & \psi \ge \gamma. \end{cases}$$

In an LR-Form Fuzzy set, the left and right mean values, denoted by σ and γ respectively, which represents the central tendencies. The left and right spreads, represented by p & q,indicate the extent or range of the fuzzy set. The membership function of the fuzzy set is determined by $\widetilde{TF} = (\sigma, \gamma, p, q)_{LR}$ the combination of the parameters.

Definition 2.4. [8] Intuitionistic Fuzzy set (IFS): A set $\widetilde{IFS}$, denoted as $\widetilde{IFS} = \{\langle \alpha; [\tau(\alpha), \gamma(\alpha)] \rangle : \alpha \in \psi\}$ can be represented graphically as a membership function where $\tau(\alpha), \gamma(\alpha) : \psi \to [0,1]$, the truth membership function is denoted by $\tau(\alpha)$ and the false membership function is denoted by $\gamma(\alpha)$. The condition for the set $\tau(\alpha), \gamma(\alpha)$ to satisfy the following condition

$$0 \le \tau(\alpha) + \gamma(\alpha) \le 1$$

Definition 2.5. [9] Trapezoidal Intuitionistic Fuzzy Number (TrIFN): Trapezoidal Intuitionistic fuzzy number denoted as $\widetilde{ITF}$, which possesses a membership $\tau_{\widetilde{ITS}}(\psi)$, and a non-membership $\gamma_{\widetilde{ITF}}(\psi)$ functions as follows:

$$\tau_{\widetilde{ITS}}(\psi) = \begin{cases} \dfrac{(\psi-q)}{(n-q)}\tau_{\widetilde{ITF}}, & \text{q} \le \psi \le r; \\ \tau_{\widetilde{ITF}}, & \text{r} \le \psi \le s; \\ \dfrac{(t-\psi)}{(t-s)}\tau_{\widetilde{ITF}}, & \text{s} < \psi \le t; \\ 0, & \text{Otherwise.} \end{cases} \quad \gamma_{\widetilde{ITF}}(\psi) = \begin{cases} \dfrac{(r-\psi)+\nu_{\widetilde{ITF}}(\psi-q_1)}{(r-m_1)}\gamma_{\widetilde{ITF}}, & \text{q} \le \psi \le r; \\ \gamma_{\widetilde{ITF}}, & \text{n} \le \psi \le s; \\ \dfrac{(\psi-s)+\nu_{\widetilde{ITF}}(t_1-\psi)}{(t_1-o)}\gamma_{\widetilde{ITF}}, & \text{s} < \psi \le t; \\ 0, & \text{Otherwise.} \end{cases}$$

The graphical representation of TFN is shown in below Figure.3.

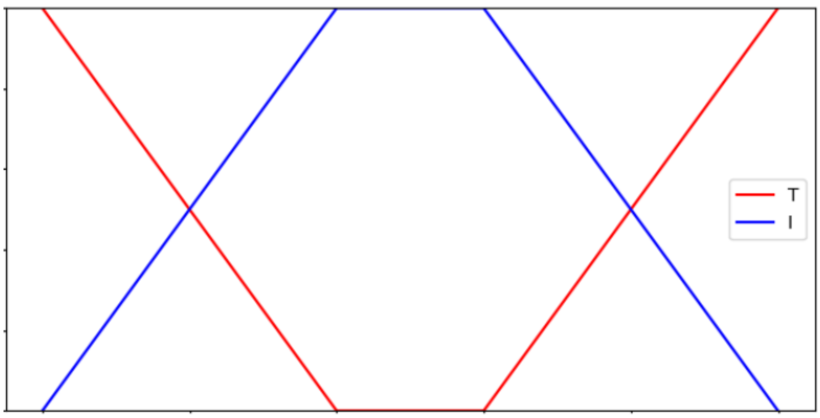

Figure 3: Trapezoidal Intuitionistic Fuzzy

Definition 2.6. [10] Pythagorean Fuzzy set (PFS): Let $\widetilde{PF}$ represent the universal discourse of ψ, a pythagorean fuzzy set denoted by α. This fuzzy set is defined as $\widetilde{PF} = \{\alpha; [\tau(\alpha), \gamma(\alpha)] : \alpha \in \psi\}$,where the truth membership is represented by $\tau(x) : \psi \to [0,1]$ and the indeterminacy membership is represented by $\gamma(\alpha) : \psi \to [0,1]$. Notably, the hesitant degree lies between the non-membership and membership functions $\alpha \in \psi$, expressed as $\pi(\alpha) = \sqrt{1 - (\tau(\alpha))^2 - (\gamma(\alpha))^2}$. In the case of PFS, the restriction corresponds to the degrees of membership $\tau(\alpha)$ and non-membership $\gamma(\alpha)$. Figure.4 illustrates the geometric form of the IFS and PFS.

$$0 \le \left(\tau^2(\alpha)\right) + \left(\gamma^2(\alpha)\right) \le 1$$

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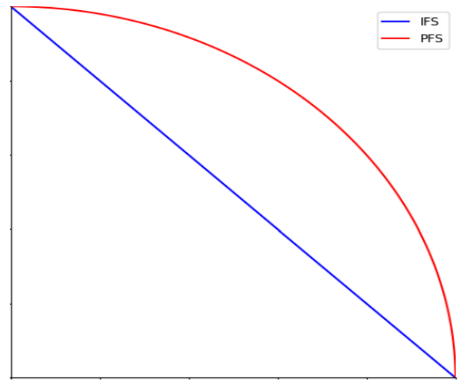

Figure 4: Comparision representation of IFS and PFS

Definition 2.7. [11] Neutrosophic set (NS): In the context of the universal discourse ψ , the set $(neu)^N$ is classified as a neutrosophic number if it satisfies the condition

$$(neu)^N = \left\{ \left\langle \alpha; [\lambda_{(neu)^N}(\alpha), \beta_{(neu)^N}(\alpha), \gamma_{(neu)^N}(\alpha)] \right\rangle : \alpha \in \psi \right\}$$

where $\lambda_{(neu)^N}(\alpha) : \psi \to [0,1]$,$\beta_{(neu)^N}(\alpha) : \psi \to [0,1]$ and $\gamma_{(neu)^N}(\alpha) : \psi \to [0,1]$ represent the truth, indeterminacy, and falsity membership functions, respectively. Additionally, this neutrosophic number exhibits the following relation

$$0 \leq \lambda_{(neu)^N}(\alpha) + \beta_{(neu)^N}(\alpha) + \gamma_{(neu)^N}(\alpha) \leq 3$$

Definition 2.8. [12] Single-Valued Neutrosophic set (SVNS):According to Definition 2.7, a set $(neu)^N$ is classified as a single-valued neutrosophic $(Sneu)^N$ if α is associated with a single-valued independent variable. In this case,

$$(Sneu)^N = \left\{ \left\langle \alpha; [\lambda_{(Sneu)^N}(\alpha), \beta_{(Sneu)^N}(\alpha), \gamma_{(Sneu)^N}(\alpha)] \right\rangle : \alpha \in \psi \right\}$$

where $\lambda_{(Sneu)^N}(\psi), \beta_{(Sneu)^N}(\psi), \gamma_{(Sneu)^N}(\psi)$ denotes the truth, indeterminacy, and falsity memberships, respectively. If there exists the points $p_0, q_0 and r_0$ for which $\lambda_{(Sneu)^N}(p_0) = 1$,$\beta_{(Sneu)^N}(q_0) = 1$, and $\gamma_{(Sneu)^N}(r_0) = 1$, then $(Sneu)^N$ is referred as neut-normal. Furthermore, if the set satisfies certain conditions and can be represented as a subset of a real line, $(SneuS)^N$ is termed neut-convex.

$$\lambda_{(Sneu)^N} \langle \delta p_1 + (1-\delta) p_2 \rangle \geq \min \left\langle \lambda_{(Sneu)^N}(p_1), \lambda_{(Sneu)^N}(p_2) \right\rangle,$$

$$\beta_{(Sneu)^N} \langle \delta p_1 + (1-\delta) p_2 \rangle \leq \max \left\langle \beta_{(Sneu)^N}(p_1), \beta_{(Sneu)^N}(p_2) \right\rangle,$$

$$\gamma_{(Sneu)^N} \langle \delta p_1 + (1-\delta) p_2 \rangle \leq \max \left\langle \gamma_{(Sneu)^N}(p_1), \gamma_{(Sneu)^N}(p_2) \right\rangle.$$

Where, $p_1, p_2 \in \mathbb{R}$ and $\delta \in [0,1]$

Definition 2.9. [13] Single-valued trapezoidal neutrosophic number (SVTN): The set $\widetilde{a}$, interpreting SVTN

$$\widetilde{a} = \langle (p_1, q_1, r_1, s_1)\, ; x_{\widetilde{a}}, y_{\widetilde{a}}, z_{\widetilde{a}} \rangle$$

on the real number R , is a unique neutrosophic representation set that is distinguished by truth, indeterminacy, and falsity membership functions.

$$\lambda_{\widetilde{a}}(\alpha) = \begin{cases} (\alpha - p_1)\alpha_{\widetilde{a}}/(q_1 - p_1), & p_1 \leq \alpha < q_1; \\ \alpha_{\widetilde{a}}, & q_1 \leq \alpha \leq r_1; \\ (r_1 - \alpha)\alpha_{\widetilde{a}}/(s_1 - r_1), & r_1 < \alpha \leq s_1; \\ 0, & \text{Otherwise.} \end{cases}$$

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$$\beta_{\tilde{a}}(\alpha) = \begin{cases} (q_1 - \alpha + y_{\tilde{a}}(\alpha - p_1))/(q_1 - p_1), & p_1 \leq \alpha < q_1; \\ y_{\tilde{a}}, & q_1 \leq \alpha \leq r_1; \\ (\alpha - r_1 + y_{\tilde{a}}(s_1 - \alpha))/(s_1 - r_1), & r_1 < \alpha \leq s_1; \\ 1, & \text{Otherwise.} \end{cases}$$

$$\gamma_{\tilde{a}}(\alpha) = \begin{cases} (q_1 - x + z_{\tilde{a}}(x - p_1))/(q_1 - p_1), & p_1 \leq x < q_1; \\ z_{\tilde{a}}, & q_1 \leq x \leq r_1; \\ (x - r_1 + z_{\tilde{a}}(s_1 - x))/(s_1 - r_1), & r_1 < x \leq s_1; \\ 1, & \text{Otherwise.} \end{cases}$$

The graphical representation of SVTN is shown in below Figure.5.

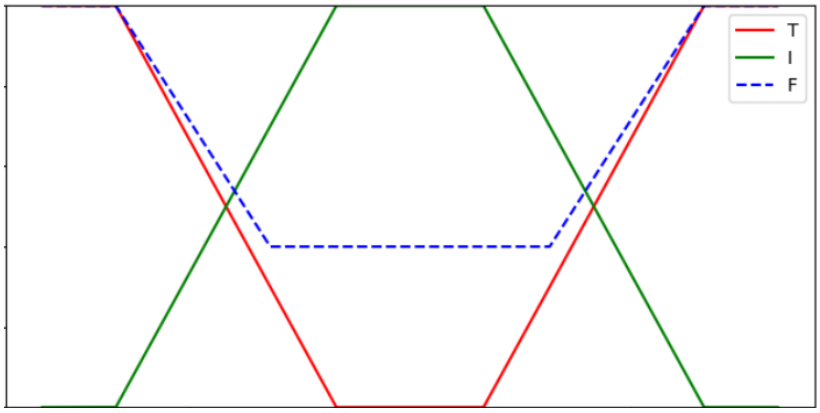

Figure 5: Single-valued trapezoidal neutrosophic

3 Advancements under Fuzzy CPM/ Fuzzy PERT

A significant portion of the data cannot be predicted accurately for most of the real-world problems. Due to this, fuzzy numbers are one of the methods that are used to deal with ambiguity and vagueness. Optimization of project schedule activity problems represents challenges with multiple potential solutions, one of which is the optimal option. Linear, differentiable, convex, and continuous optimization problems are well suited to deterministic techniques. The drawback of these approaches is their inability to solve non-convex, nonlinear, non-differentiable, and high-dimensional problems. Most disciplines use fuzzy numbers, including engineering, statistics, computer sciences, experimental sciences, and physics. It becomes challenging to manipulate fuzzy numbers in uncertain situations. Due to today's challenging and complex circumstances, new approaches are quick and provide better solutions. These approaches can be categorized into analytical, heuristic, and meta-heuristic models that are employed to solve issues in challenging and complex situations. In such cases of simplifying, the paper reviews especially researchers work that have applied in developing algorithms to find optimal and sub-optimal solutions under fuzzy CPM/fuzzy PERT using fuzzy numbers alike L-R fuzzy number and the trapezoidal fuzzy number that is previewed in Table 1.

Table 1: Recent Advancements of applied algorithms & Methods under the working model of fuzzy CPM/PERT using different fuzzy numbers

Main Contribution	**Type of Fuzzy Numbers & Applied Methods**	**Author(s) information & Year**
Construction-related research employing fuzzy CPM/PERT methods	LR-type Fuzzy	Rahman et.al., (2017)[14]

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Table 1 – Continued from previous page

Main Contribution	Types of Fuzzy Numbers & Applied Methods	Author(s) information & Year
Analyzing the proposed method for issues with numerous resource types in project scheduling	Heuristic Parallel Scheduling Method + Trapezoidal Fuzzy + CPM	Khalilzadeh et.al., (2017)[15]
To calculate project time and cost accurately	Trapezoidal Fuzzy	Habibi et.al., (2018)[16]
Outline a simulation approach to the fuzzy PERT problem	Different combination of Fuzzy Numbers + Simulation	Tenjo-García and Figueroa-García (2019)[17]
To assess the degree of criticality in a project network where activity durations are uncertain in determining late-time activities	Trapezoidal Fuzzy	Ammar and Abd-ElKhalek (2022)[18]
To determine the time to treat patients at the hospital's emergency room in Aminu Kano	Trapezoidal Fuzzy	Shuaibu and Muhammad (2022)[19]

The relationship between fuzzy and crisp numbers is an area of interest for researchers in many fields, as described in the literature from Table 1 above. Additionally, several tweaks and additions to the traditional CPM/PERT and fuzzy CPM/PERT arise to address certain flaws and offer greater accuracy for decision-making in diverse circumstances. Depending on the characteristics of the problem and the sort of uncertainty involved, each of these frameworks has certain advantages and applicability. In such cases, the extended principal theory of fuzzy discussion has been reviewed in further sections.

4 Introduction of the Extended Fuzzy Principle

Zadeh (1965)[4] introduced the fuzzy set theory to address the challenges associated with imprecise or vague information, that is characterized by membership values in the theory of uncertainty. However, classical theory often falls short in solving problems involving different types of uncertainties encountered today. In many cases, the information may not be precise enough to make decisions or produce the desired outcomes. To overcome these challenges, higher-order fuzzy sets have been introduced. One significant advancement is the concept of intuitionistic fuzzy sets (IFS), introduced by Atanassov (1986).[8] IFS is suitable for handling problems with imprecise information and is characterized by its membership and non-membership values. Following this, Ye[20] explained the design of TrIFN and later, Yager and Abbasov[21] and Yager[22] developed the PFS as an extension of IFS. PFS enhances both the flexibility and applicability of IFS by indicating not only the level of agreement among experts but also the degree of fuzziness within that agreement. To summarize, these advancements in fuzzy set theory, including IFS and PFS, address the limitations of classical theory and enable more effective handling of problems with imprecise information and various uncertainties.

Over time, the generalization of fuzzy sets was unable to fully handle problems with ambiguous or uncertain data. To overcome this restriction, in 1998, Smarandache[11] presented neutrosophic sets as a mixture of previously defined fuzzy sets. Neutrosophic sets encompass three components: truth, indeterminacy, and falsity-membership degrees, which are well-suited for representing indeterminacy and inconsistent information. The idea of single-valued neutrosophic sets (SVNS) was introduced by Wang et.al.,[12] in various practical problems. The neutrosophic domain has garnered attention from researchers in multiple fields, including multi-criteria decision-making,[23] graph theory,[24] and optimization techniques.[25] Several other published articles[26] related to the neutrosophic domain have made significant contributions to application-oriented research,but discussing the prior related to the scope of area. Further, these advancements have provided valuable tools for handling

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uncertainty in real-life applications through the efforts of various researchers. A summary of their contributions can be found in Table 2.

Table 2: Various areas working under different extended fuzzy theories

Main Contribution	Environment	Area application	Author(s) information & Year
Addressing the complexity of multi-objective nonlinear transportation problems with fuzzy parameters	Neutrosophic Fuzzy	Transportation Problem	Ahmad and Adhami (2019)[27]
Implementing a solution methodology for the assignment problem involving neutrosophic costs	Neutrosophic	Assignment Problem	Bera and Mahapatra (2018)[28]
To propose a solution approach by incorporating all relevant parameters, to achieve fair outcomes	Single Valued Trapezoidal Neutrosophic	Simplex Method + Linear Programming Problem	Bera and Mahapatra (2020)[29]
An advanced similarity measure for Pythagorean fuzzy sets and illustrate its practical applications in addressing transportation problems	Pythagorean Fuzzy	Transportation Problem	Saikia et.al., (2023)[30]
To bring new insights into queueing systems under the neutrosophic environment and offer an effective tool for managing uncertainties using Maple Code	Single-Valued Trapezoidal Neutrosophic	Queueing Systems	Masri et.al.,(2023)[31]

The literature review described above served as motivation to develop a new technique, namely CPM/PERT in different environmental conditions that aim to improve accuracy and decision-making in the presence of uncertainty. Further section focuses on working model and solving problems of CPM/PERT using the neutrosophic environment.

5 Implementation of Extended Fuzzy Principle under CPM/PERT

The versatility and capability to deal with uncertainties make fuzzy principles highly applicable in a wide range of domains that involve the consideration of imprecise and vague information for analysis and decision-making. Furthermore, the influence of these principles has been rapidly expanding across diverse areas and applications in recent times. However, due to constraints, it is not feasible to delve into a detailed discussion of all these applications within the scope of this paper. Therefore, the focus of this paper is on discussing the application of CPM and PERT within the framework of fuzzy extensions. Previous researchers have introduced the concept of CPM/PERT in various scenarios.[32] In a related study, Jayagowri and Geetharamani (2014)[33] proposed an innovative analytical method that employed a defuzzification formula for TFN. To aid in the determination of the critical path, this technique was used to compute the float calculation time for each task. To identify the optimal path in an intuitionistic fuzzy weighted graph, the graded mean integration representation procedure was employed, as such an approach assists decision-makers in selecting the most favourable critical path within intuitionistic fuzzy environments.

Additionally, neutrosophic sets offer a thorough framework for describing ambiguous, contradictory, and insufficient data in real-world issues as a development in the subject of fuzziness. Crisp, fuzzy, and intuitionistic are all generalized as neutrosophic sets. The PERT was first mathematically represented in the context of

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neutrosophic sets by Mohamed et.al., (2017)[34] and treated the components of the PERT in three-times estimator under neutrosophic components. To provide a crisp representation of the problem, they used score and accuracy functions. Expanding upon this concept, Mullai and Surya (2019)[35] focused on neutrosophic project evaluation review technique (NPERT) calculations and utilized SVNS to calculate the total predicted neutrosophic task time required for the completion of the project network. Their research significantly contributed to the development of NPERT as an effective method within the neutrosophic framework.

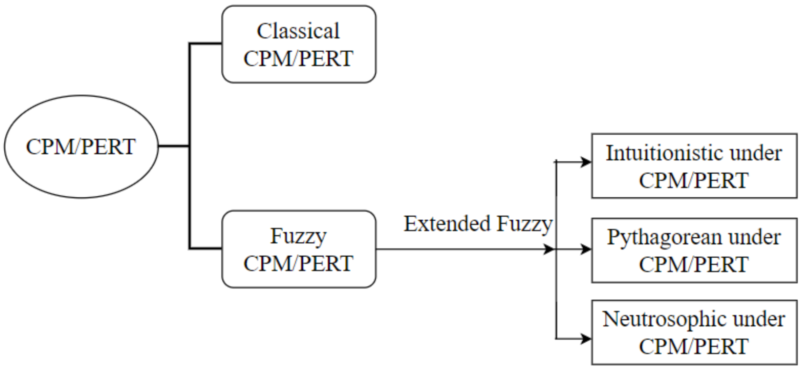

Figure 6: Schematic depiction of the paper flow

The concept of vagueness holds significant importance in constructing mathematical models and addressing real-life problems in various domains, including social and economic contexts. Extensive research has already been conducted in the realm of uncertain environments, covering a wide range of fields. However, there are still many areas that remain unexplored, and ongoing work continues to expand our understanding. Given this context, the main contribution of this paper focuses on reviewing the application of CPM and PERT in an uncertain environment. By examining the existing literature depicted in the Figure 6, this review aims to shed light on the advancements and insights gained in utilizing CPM/PERT under the conditions of uncertainty.

Conclusion

The theory of uncertainty plays a crucial role in real-world scenarios, particularly with the rising popularity in the field of extended fuzzy theory. The formation of corresponding numbers holds great significance for researchers dealing with uncertainty and decision-making problems. This paper focuses on reviewing the concept of CPM/PERT using trapezoidal fuzzy numbers and LR-fuzzy numbers from various perspectives, which have not been extensively explored before. By examining CPM/PERT through the lens of trapezoidal fuzzy numbers and LR-fuzzy numbers, this review sheds light on novel viewpoints and insights that helps us understand the limitations of crisp numbers and explores the conversion from crisp to extended fuzzy numbers with valuable tools in decision-making. The potential to further expand the application-oriented concept of extended fuzzy principles in network analysis, which holds promise for more effective modelling under uncertainty.

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