Shortest Path Problem using Pythagorean Fuzzy Triangular Number

M. Asim Basha[1], M. Mohammed Jabarulla[1], Broumi said[2]

[1] PG and Research Department of Mathematics, Jamal Mohamed College (Autonomous), Tiruchirappalli – 620020, Tamil Nadu, India

[2] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University of Hassan II, Casablanca, Morocco

Emails: asimbasha36@gmail.com; m.md.jabarulla@gmail.com; broumisaid78@gmail.com

Abstract

This paper, they developed a new path to compromise with Pythagorean Shortest Path Problem (PSPP) in a network location each edge weight is expressed an Triangular Fuzzy Pythagorean Numbers (TFPNs). Then the Proposed Algorithm (PA) further provide the SP length from the source node (SN) to destination node (DN) from applying a Ranking Function for Pythagorean Fuzzy Numbers (PFN). Certainly, a descriptive example is also included.

Keywords: Pythagorean fuzzy numbers (PFN); Triangular Pythagorean fuzzy numbers (TPFN); Score function (SF); Shortest path problem (SPP).

1. Introduction

The "shortest path issue" is a problem in graph theory that involves finding a path between two vertices (or nodes) in a network that reduces the sum of the weights of all of its constituent edges. To simulate the difficulty of discovering the shortest path between two crossings on a road map, a specific variant of the shortest path (SP) problem in graphs may be employed, with the vertices symbolising crossings and the edges denoting road segments, each weighted by the segment's lengths.

Then the Shortest path problems (SPP) are considerate along with mostly tested in the different areas of scientific and engineering's departments. For example, application of road networks, transportation, communication of channels and problem solving. L.A. Zadeh (1965), ⟨6⟩ initiated the notation of fuzzy set (FS), an increase of the classical set. The FS is specified by a mem functionμ. This assigns a$[0,1]$, this indicates the grade for membership of an object of extensions. Atanassov (1983), ⟨5⟩, imported the concept of intuitionistic fuzzy set (IFS). Then the IFS initiate the membership μ and non-membershipλ.

That assigns $\mu + \lambda \leq 1$ and has a non-zero indeterminacy part of $\pi = 1 - \mu - \lambda$. For modesty, Xu (2007), ⟨13⟩ initiate $A = (\mu_A, \lambda_A)$ as IF number. Yager (2013, 2014), ⟨10,11⟩ introduced the new area for PF set the observation of IFS, where degree of belonging & degree of not belonging associate the element of $\mu^2 + \lambda^2 \leq 1$. For modesty, Zhang and Xu (2014), ⟨12⟩ introduced the PFN is $P = (\mu_P, \lambda_P)$. Then the distinctive of PF is $\mu^2 + \lambda^2 \leq 1$. For example, $(0.8)^2 + (0.6)^2 \leq 1$.

Xinfan Wang (2008), ⟨13⟩, initiate SF and accuracy functions from fuzzy aggregation in IF information, to develop in to the triangular IF value for SF, S is $s(a) = \frac{a+2b+c}{4} - \frac{e+2f+g}{4}, s(a) \in [-1,1]$ and the Accuracy Function (AF) H is $H(a)\frac{a+2b+c}{4} + \frac{e+2f+g}{4}, H(a) \in [0,1]$. This paper discuss the PTFSPP in network. Then the ranking

Doi: https://doi.org/10.54216/JNFS.060205
Received: December 17, 2022 Revised: April 18, 2023 Accepted: May 08, 2023

terminology for PF SF and AF. Calculate, two fuzzy values from the PA for to define in the minimum value from the SF and AF for SP values. The proposed way for solving SPP under generalized PF environments. Finally, to solve the example for SPP in a fuzzy environments.

2. PREFACE

Definition 2.1 [8]

Let U' be a universe of discussion. Then, the Pythagorean fuzzy set (PFS) $\widetilde{P}'$ on U' is given, $\widetilde{P}' = \{< u', \mu_{\tilde{P}}(u'), \lambda_{\tilde{P}}(u') | u' \in U' >\}$, where the function $\mu_{\tilde{P}}: U' \to [0,1]$ and $\lambda_{\tilde{P}}: U' \to [0,1]$ define degree of (mem) & degree of (non-mem) functions the fundamental $u' \in U'$ the set $\tilde{P}$, properly, $0 \leq \left(\mu_{\tilde{P}}(u')\right)^2 + \left(\lambda_{\tilde{P}}(u')\right)^2 \leq 1, \forall\, u' \in U'$.

Definition 2.2 [7]

A Pythagorean fuzzy graph (PFG) on a nonempty set X' is a combination in $G = (\widetilde{P}', \widetilde{Q}')$ with $\widetilde{P}'$ a PFSet on X' and $\widetilde{Q}'$ a PF Relation on X' is,

$$\mu_{\widetilde{Q'}}(uv) \leq \mu_{\widetilde{P'}}(u) \wedge \mu_{\widetilde{P'}}(v)$$

$$\lambda_{\widetilde{Q'}}(uv) \leq \lambda_{\widetilde{P'}}(u) \vee \lambda_{\tilde{P}'}(v)$$

where $\mu_{\widetilde{Q'}}: X' \times X' \to [0,1]$ and $\lambda_{\widetilde{Q'}}: X' \times X' \to [0,1]$ represent the mem and non-mem functions of $\widetilde{Q}'$, respectively, with $0 \leq \left(\mu_{\widetilde{Q'}}(u)\right)^2 + \left(\lambda_{\widetilde{Q'}}(u)\right)^2 \leq 1,\ \forall\, u, v \in E$.

Definition 2.3:- [7]

The degree of general x to $\widetilde{P}'$ is given, $\pi_{\widetilde{P'}}(x) = \sqrt{1 - \mu^2_{\widetilde{P'}}(x) - \lambda^2_{\widetilde{P'}}(x)}$. If $\widetilde{P}' = (\mu_{\tilde{P}'}, \lambda_{\widetilde{P'}})$ is a PFN, then the degree of general element $x \in X$ is given $\pi_{\tilde{P}} = \sqrt{1 - \mu^2_{\tilde{P}} - \lambda^2_{\tilde{P}}}$, where $\mu_{\tilde{P}}, \lambda_{\tilde{P}} \in [0,1]$ and $0 \leq (\mu_{\tilde{P}})^2 + (\lambda_{\widetilde{P'}})^2 \leq 1$.

3. Triangular Pythagorean Fuzzy Number and Algebraic Operators:

A documentation convenience to the TPF value

$$A' = \langle (a', b', c'), (e', f', g') \rangle$$

where, $\left(\mu_{\tilde{P}_1}(u')\right)^2, \left(\mu_{\tilde{P}_2}(u')\right)^2, \left(\mu_{\tilde{P}_3}(u')\right)^2 = (a', b', c')$,

$$\left(\lambda_{\tilde{P}_1}(u)\right)^2, \left(\lambda_{\tilde{P}_2}(u)\right)^2, \left(\lambda_{\tilde{P}_3}(u)\right)^2 = (e', f', g').$$

Definition 3.1:-

A TPFN $A' = \langle (a', b', c'), (e', f', g') \rangle$ is said to zero TPFN iff,

$$(a', b', c') = (\_,\_,\_);\ (e', f', g') = (1,1,1);$$

Definition 3.2:- [7]

Let $A' = A_1', A_2'$, then $A_1' = \langle (a_1', b_1', c_1'), (e_1', f_1', g_1') \rangle$ and $A_2' = \langle (a_2', b_2', c_2'), (e_2', f_2', g_2') \rangle$ be two TPFV in a set of $\mathcal{R}$, and $\lambda > 0$. then, operations is follows,

$$(i)\ A_1' \oplus A_2' = \left\langle \sqrt{(a_1^2 + a_2^2 - a_1^2 a_2^2)}, \sqrt{(b_1^2 + b_2^2 - b_1^2 b_2^2)}, \sqrt{(c_1^2 + c_2^2 - c_1^2 c_2^2)}, (e_1 e_2, f_1 f_2, g_1 g_2) \right\rangle$$

$$(ii)\ A_1' \otimes A_2' = \left\langle \sqrt{(e_1^2 + e_2^2 - e_1^2 e_2^2)}, \sqrt{(f_1^2 + f_2^2 - f_1^2 f_2^2)}, \sqrt{(g_1^2 + g_2^2 - g_1^2 g_2^2)}, (e_1 e_2, f_1 f_2, g_1 g_2) \right\rangle$$

Doi: https://doi.org/10.54216/JNFS.060205
Received: December 17, 2022 Revised: April 18, 2023 Accepted: May 08, 2023

(iii) $\left\langle \sqrt{(1-(1-a_1^2)^\lambda)}, \sqrt{(1-(1-b_1^2)^\lambda)}, \sqrt{(1-(1-c_1^2)^\lambda)}, (e_1^\lambda, f_1^\lambda, g_1^\lambda) \right\rangle$

(iv) $\left\langle (a_1^\lambda, b_1^\lambda, c_1^\lambda), \left(\sqrt{(1-(1-e_1^2)^\lambda)}, \sqrt{(1-(1-f_1^2)^\lambda)}, \sqrt{(1-(1-g_1^2)^\lambda)} \right) \right\rangle$

where, $\lambda > 0$.

The approach of SF is S & the AF is H are tested to analyse the TPFS. These TPFS and using these approch paths to be ranked.

Definition 3.3:- [3]

Let $A_1' = \langle (a_1', b_1', c_1'), (e_1', f_1', g_1') \rangle$ be the Triangular Pythagorean fuzzy value (TPFV) then, the SF is $S(A_1')$ and an AF is $H(A_1')$ of TPFV are follows.

$$(i)\ S(A'_1) = \frac{1}{4}[(a_1^2 + 2b_1^2 + c_1^2) - (e_1^2 + 2f_1^2 + g_1^2)]$$

$$(ii)\ H(A'_1) = \frac{1}{4}[(a_1^2 + 2b_1^2 + c_1^2) + (e_1^2 + 2f_1^2 + g_1^2)]$$

Then the order to make a connections between two TPFV, the relations order between two TPFVs.

Definition 3.4

Let $A_1' = \langle (a_1', b_1', c_1'), (e_1', f_1', g_1') \rangle$ and $A_2' = \langle (a_2', b_2', c_2'), (e_2', f_2', g_2') \rangle$ be two TPFVs in the set of $\mathcal{R}$. Then, define a ranking method is given by,

(i) If $S(A'_1) > S(A'_2)$, then $A'_1 > A'_2$, that is, A'_1 is sup to A'_2, denoted by $A'_1 > A_{\prime 2}$.

(ii) If $S(A'_1) = S(A'_2)$, and $H(A'_1) > H(A'_2)$ then $A'_1 > A'_2$, that is A'_1 is sup to A'_2, denoted by $A'_1 > A'_2$.

Network Terminology: -

Let us treated a directed Network $G(V', E')$. Completing a finite set of nodes $V' = \{1,2,3, \dots, n\}$ and a Set of m directed edges $E' \subseteq V' \times V'$. All edge is denoted by an ordered pair (i, j) where $i, j \in V'$ and $i \neq j$. In that network, we consider 2 - nodes, stand for s & t, representing the Source node (SN) & Destination node (DN), respectively. We determine a path $P_{ij} = \{i = i, (i_1, i_2), i_2, \dots, (i_{l-1}, i_l), i_l = j\}$ of developing nodes and edges. The presence of partially one path P_{s_i} in $G(V', E')$ is affected for each $i \in V' - \{S\}$.

d_{ij} stand for TPFN identical the edge (i, j), comparable to the length required to traverse (i, j) in distinction to $i \rightarrow j$ the Pythagorean Distance (PD) onward the path P is stand for $d(P)$ is defined, $d(P) = \sum_{(i,j \in P)} d_{ij}$.

Remark: - A node i is called a predecessor node (PN) j if,

(i) node i, is precisely connected to node j.

(ii) The direction of the path connecting nodes i & j in distinction to $i \rightarrow j$.

4. Triangular Fuzzy Pythagorean Path Problem (TFPPP)

This work, the length edge in a network is studied as PN, especially TPFN. The Algorithm for SP proceeds in (**6-steps).**

STEP – I: Consider $d_1 = \langle (\_, \_, \_), (1,1,1) \rangle$ and the SN (e.g. nod 1) as $[d_1 = \langle (0,0,0), (1,1,1) \rangle, -]$.

STEP – II: Find $d_j = \min[\{d_i \oplus d_{ij}\}, j = 2,3,4, \dots, n]$.

Doi: https://doi.org/10.54216/JNFS.060205
Received: December 17, 2022 Revised: April 18, 2023 Accepted: May 08, 2023

STEP – III: If min occurs with a single value of i, i.e.., $i = r$, then node j is denoted as $[d_j, r]$. If min occurs with extra one value of i than it means that capable are extra one value of i, then it means there is extra one Interval-Valued Pythagorean Path (IVPP) between SN & nod j, but TFPD along the path is d_j, so take any value of i.

STEP – IV: Let DN (nod n) be labelled as $[d_n, l]$, then the TFPS distance between SN is d_n.

STEP – V: Since DN is labelled as $[d_n, l]$. So, find the TPFSP between SN and DN, check the label of nod 1. Assume that it is $[d_l, P]$ then check the label of nod P, and so on. Repeat the same method until you get the nod 1.

STEP – VI: Now the TPFSP can be collected by combining all the nodes collected in Step – V.

Note :- Let $A_i = (1,2, \dots, n)$ be a set is TPFNs, if $S(A'_K) < S(A'_i)$, $\forall\, i$, the TPFN is the min of A'_K.

Illustration 4.1:-

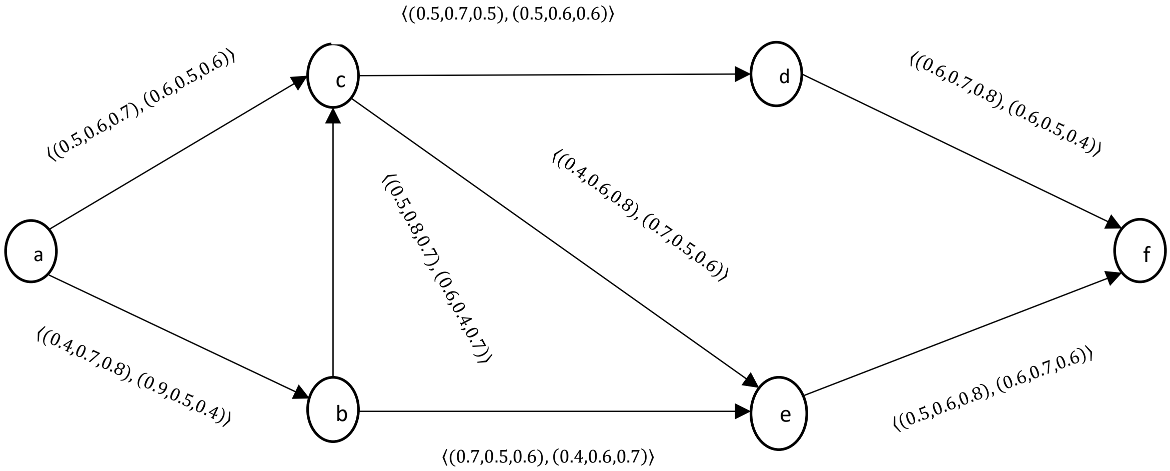

Figure 1: PTF Directed Graph.

The above network each edge has been defined to TPFNs:

Table 1: Weights of the TPFNs.

Node	TPFNs
a – b	$\langle(.4,.7,.8),(.9,.5,.4)\rangle$
a – c	$\langle(.5,.6,.7),(.6,.5,.6)\rangle$
b – c	$\langle(.5,.8,.7),(.6,.4,.7)\rangle$
c – d	$\langle(.5,.7,.5),(.5,.6,.6)\rangle$
c – e	$\langle(.4,.6,.8),(.7,.5,.6)\rangle$
b – e	$\langle(.7,.5,.6),(.4,.6,.7)\rangle$
d – f	$\langle(.6,.7,.8),(.6,.5,.4)\rangle$
e – f	$\langle(.5,.6,.8),(.6,.7,.6)\rangle$

Above the table are defined as edges and TPFD at network (Fig.1).

Explanation:

Now the node 6 is DN, so $n = \mathbf{6}$.

Consider $d_1 = \langle(\_,\_,\_),(1,1,1)\rangle$ and the SN (say nod a) as $[\langle(\_,\_,\_),(1,1,1)\rangle, \_]$, the value of $d_j; j = b, c, d, e, f$ can be captured as given below:

Itr 1: From the FIG.1 PN b is node a, so put the values of $i = a$ and $j = b$ respectively in Step – II for the above algorithm.

The path for d_b is, $d_b = \min\{d_a \oplus d_{ab}\}$

$$= \min\{\langle(\_,\_,\_),(1,1,1)\rangle \oplus \langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle\}$$

Doi: https://doi.org/10.54216/JNFS.060205
Received: December 17, 2022 Revised: April 18, 2023 Accepted: May 08, 2023

$d_b = \min\{\langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle\}$

$d_b = \langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle$

The above calculation, min occurs comparable to the nod a & then nod b as $[\langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle, a]$.

Itr 2: The PN for node c are node a and node b, so put $i = a, b$ and $j = c$ in Step – II for the above algorithm.

The path for d_c is, $d_c = \min\{d_a \oplus d_{ac}, d_b \oplus d_{bc}\}$

$$= \min\left\{\begin{matrix}\langle(0,0,0),(1,1,1)\rangle \oplus \langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle,\\ \langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle \oplus \langle(0.5,0.8,0.7),(0.6,0.4,0.7)\rangle\end{matrix}\right\}$$

$d_c = \min\{\langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle, \langle(0.61,0.904,0.904),(0.54,0.2,0.28)\rangle\}$

by using definition 3.3, (based on SF), the value d_c is $d_c = \langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle$

The above calculation, min occurs comparable to the nod a & then nod c as $[\langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle, a]$.

Itr 3: The PN d is nod c, put the values of $i = c$ and $j = d$ respectively in Step – II for the above algorithm.

The path for d_d is, $d_d = \min\{d_c \oplus d_{cd}\}$

$$= \min\{\langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle \oplus \langle(0.5,0.7,0.5),(0.5,0.6,0.6)\rangle\}$$

$d_d = \min\{\langle(0.688,0.914,0.89),(0.3,0.3,0.36)\rangle\}$

$d_d = \langle(0.688,0.914,0.89),(0.3,0.3,0.36)\rangle$

The above calculation, min occurs comparable to the nod c & then nod d as $[\langle(0.688,0.914,0.89),(0.3,0.3,0.36)\rangle, c]$.

Itr 4: The PN for node e are nod b & nod c, so put $i = b, c$ and $j = e$ in Step – II for the above algorithm.

The path for d_e is, $d_e = \min\{d_b \oplus d_{be}, d_c \oplus d_{ce}\}$

$$= \min\left\{\begin{matrix}\langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle \oplus \langle(0.7,0.5,0.5),(0.4,0.6,0.6)\rangle,\\ \langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle \oplus \langle(0.4,0.6,0.8),(0.7,0.5,0.6)\rangle\end{matrix}\right\}$$

$$d_e = \min\left\{\begin{matrix}\langle(0.782,0.828,0.9296),(0.36,0.3,0.24)\rangle,\\ \langle(0.62,0.8304,1.0264),(0.42,0.25,0.36)\rangle\end{matrix}\right\}$$

by using definition 3.3, (based on SF), the value d_c is $d_e = \langle(0.62,0.8304,1.0264),(0.42,0.25,0.36)\rangle$

The above calculation, min occurs comparable to the node c & then nod e as $[\langle(0.62,0.8304,1.0264),(0.42,0.25,0.36)\rangle, c]$.

Itr 5 : The PN for node f are nod d & nod e, so put $i = d, e$ and $j = f$ in Step – 2 of the above algorithm.

The path for d_f is, $d_f = \min\{d_d \oplus d_{df}, d_e \oplus d_{ef}\}$

$$= \min\left\{\begin{matrix}\langle(0.688,0.914,0.89),(0.3,0.3,0.36)\rangle \oplus \langle(0.6,0.7,0.8),(0.6,0.5,0.4)\rangle,\\ \langle(0.62,0.8304,1.0264),(0.42,0.25,0.36)\rangle \oplus \langle(0.5,0.6,0.8),(0.6,0.7,0.6)\rangle\end{matrix}\right\}$$

$$d_f = \min\left\{\begin{matrix}\langle(0.8776,0.9947,0.1.0231),(0.18,0.15,0.144)\rangle,\\ \langle(0.774,0.9422,0.9909),(0.252,0.175,0.216)\rangle\end{matrix}\right\}$$

by using definition 3.3, (based on SF), the value d_c is

$$d_f = \langle(0.774,0.9422,0.9909),(0.252,0.175,0.216)\rangle$$

The above calculation, min occurs comparable to the nod e & then nod f as $[\langle(0.774,0.9422,0.9909),(0.252,0.175,0.216)\rangle, e]$.

Doi: https://doi.org/10.54216/JNFS.060205
Received: December 17, 2022 Revised: April 18, 2023 Accepted: May 08, 2023

Then the path $a \to c \to e \to f$ is identified the SP & SD between SN and DN for $\langle(0.774,0.9422,0.9909),(0.252,0.175,0.216)\rangle$

By given the PA, in SP for all nodes in distinction to SN and labelling are given:

Table 2: TPFD and SP.

Node	d_i	TPFSP between j th and 1st node
b	$\langle(0.4,0.7,0.8),(0.9,0.5,0.4)\rangle$	$a \to b$
c	$\langle(0.5,0.6,0.7),(0.6,0.5,0.6)\rangle$	$a \to c$
d	$\langle(0.688,0.914,0.89),(0.3,0.3,0.36)\rangle$	$a \to c \to d$
e	$\langle(0.62,0.8304,1.0264),(0.42,0.25,0.36)\rangle$	$a \to c \to e$
f	$\langle(0.774,0.9422,0.9909),(0.252,0.175,0.216)\rangle$	$a \to c \to e \to f$

5. Conclusion

In this work, the Proposed Algorithm (PA) is used to solve SPP to identify network of Triangular fuzzy Pythagorean Path (TFPP) edges. Then explained the method using an example with the problematic data. Then we continued the following PA of TPFNs, then the SPP in a Trapezoidal Pythagorean fuzzy environment (TrPFE).

References

[1]. A. Nagoor Gani, M. Mohammed Jabarulla, On Searching Intui Fuzzy SP in a Network, Applied Mathematical Sciences, Vol.4, no.69,3447-3454,2010.

[2]. A. Nagoor Gani, M. Mohammed Jabarulla, Multiple labelling Approach For Finding SP with Intui Fuzzy Arc Length, International Journal of Scientific and engineering Research, V3, Issue11, PP.102-106, 2012.

[3]. Guiwu Wei, Xiaofei Zhao, Rui Lin, some induced Aggregating Operators with Fuzzy Number Intui Fuzzy Information and Their Applications to Group Decision Making, International Journal Of Computational Intelligence Systems, Vol.3, N0.1, 84-95, 2010.

[4]. H. Garg (ed.), PFS, Theory and Applications, under exclusive license to Springer Nature Singapore Ptr Ltd. 2021.

[5]. K.T. Atanassov, Intui Fuzzy Sets, Springer, 2019.

[6]. L.A. Zadeh, Fuzzy Sets, Information and Control 8(3), 338-353, 1965.

[7]. Lazim Abdullah, Pinxin Goh, Decision making method based on PFS and its application to solid waste management, Complex and Intelligent System, 5:185-198, 2019.

[8]. L. Sujatha, Hyacinta JD, The SPP on Networks with Intui Fuzzy Edge Weights, Global Journal of Pure and Applied Mathematics, Vol 13, No.7, PP. 3285-3300, 2017.

[9]. M. Muhammad Akram, Farwa Ilyasa and Arsham Borumand Saeid, Certain Notions of PF Graphs, Journal of Intelligent and Fuzzy Systems 36, 5857-5874, 2019.

[10]. S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu, Shortest path problem under triangular fuzzy neutrosophic information, 10th International Conference on software, Knowledge, Information Management & Applications, PP. 169 – 174, 2016.

[11]. R.R. Yager, PF subsets, In: Proc Joint IFSA world congress and NAFIPS Annual Meeting, Edmonton, Canada, PP.57-61, 2019.

[12]. R.R. Yager, Pythagorean membership grades in decision making, IEEE Transactions on Fuzzy Systems 22(4), 958-965, 2019.

[13]. X. Zhang and Z. Xu, Extension of TOPSIS to multiple criteria decision making with PFS, International Journal of Intelligent Systems 29(12), 1061-1078, 2014.

[14]. Xinfan Wang, Fuzzy Number Intui Fuzzy Arithmetic Aggregation Operators, International Journal of Fuzzy Systems, Vol.10, No.2, 2008.

[15]. Z. Xu, Intui fuzzy aggregation operators, IEEE Transactions on Fuzzy Systems 15(6), 1179-1187, 2007.

Doi: https://doi.org/10.54216/JNFS.060205
Received: December 17, 2022 Revised: April 18, 2023 Accepted: May 08, 2023