



Somewhat Neutrosophic δ -Continuous Functions in Neutrosophic Topological Spaces

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Abstract

In this paper the concepts of somewhat neutrosophic δ -continuous functions, somewhat neutrosophic δ -open functions between neutrosophic topological spaces are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are studied. The concepts of neutrosophic δ -resolvable spaces and neutrosophic δ -irresolvable spaces are also introduced and studied in this paper. These functions can be extended to a somewhat neutrosophic δ -irresolute continuous functions in neutrosophic topological spaces.

Keywords: Neutrosophic open set; Neutrosophic regular open set; Neutrosophic δ -open set; Neutrosophic δ -resolvable space; Neutrosophic δ -irresolvable space; somewhat Neutrosophic continuous function; somewhat Neutrosophic open function

AMS (2000) subject classification: 03E72, 54A05, 54A40, 54C05, 54C08.

1 Introduction

In Mathematics, the concept of fuzzy set between the real standard intervals was first introduced by Zadeh²⁶ in discipline of logic and set theory. The general topology has been framework with fuzzy set was undertaken by Chang⁴ as fuzzy topological space. In 1983, Atanassov³ initiated intuitionistic fuzzy set which is a combination of membership and non-membership values. Coker⁵ created intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological space. In classical topology, the class of somewhat continuous functions was introduced and studied by Karl. R. Gentry and Hughes B. Hoyle.⁶ Later, the concept of somewhat in classical topology has been extended to fuzzy topological spaces. Besides giving characterizations of these functions, several interesting properties of these functions are studied. Hewitt⁷ introduced the concepts of resolvability and irresolvability in topological spaces. The concepts of neutrosophy and neutrosophic set was introduced Smarandache^{11,16,17} at the beginning of 20th century. After the introduction of neutrosophic set, there are many fields of mathematics and various applications.^{1,9,10,15} Salama and Alblowi^{12,13} in 2012, originated neutrosophic set in a neutrosophic topological space. Saha¹⁸ defined δ -open sets in fuzzy topological spaces. Recently, Vadivel et al.²² introduced the neutrosophic regular open sets, neutrosophic δ open sets and neutrosophic δ continuous functions. In his paper,¹⁹⁻²¹ neutrosophic δ -open mappings, neutrosophic δ -closed mappings, somewhat neutrosophic δ -irresolute continuous mappings and some operators using neutrosophic δ open sets are introduced. Also, Vadivel et al. in^{23,24} introduced e -open sets and their maps in a neutrosophic topological space.

In this paper, we develop the concept of somewhat neutrosophic δ -continuous functions and somewhat neutrosophic δ -open sets are introduced and studied. Also, neutrosophic δ -resolvability and neutrosophic δ -irresolvability in a neutrosophic topological spaces. Some results concerning these functions that preserve the neutrosophic δ -resolvable and neutrosophic δ -irresolvable spaces in the context of images and pre images are also obtained.

2 Preliminaries

Definition 2.1.¹² Let X be a non-empty set. A neutrosophic set (briefly, $N_s s$) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ where $\mu_A \rightarrow [0, 1]$ denote the degree of membership function, $\sigma_A \rightarrow [0, 1]$ denote the degree of indeterminacy function and $\nu_A \rightarrow [0, 1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set A and $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ for each $x \in X$.

Remark 2.2.¹² A $N_s s$ $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ can be identified to an ordered triple $\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ in $[0, 1]$ on X .

Definition 2.3.¹² Let X be a non-empty set and the $N_s s$'s A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$, then

- (i) $0_N = \langle x, 0, 0, 1 \rangle$ and $1_N = \langle x, 1, 1, 0 \rangle$,
- (ii) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ & $\nu_A(x) \geq \nu_B(x) : x \in X$,
- (iii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$,
- (iv) $1_N - A = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X\} = A^c$,
- (v) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \max(\sigma_A(x), \sigma_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$,
- (vi) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \min(\sigma_A(x), \sigma_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle : x \in X\}$.

Definition 2.4.¹² A neutrosophic topology (briefly, $N_s t$) on a non-empty set X is a family Ψ_N of neutrosophic subsets of X satisfying

- (i) $0_N, 1_N \in \Psi_N$.
- (ii) $A_1 \cap A_2 \in \Psi_N$ for any $A_1, A_2 \in \Psi_N$.
- (iii) $\bigcup A_x \in \Psi_N, \forall A_x : x \in X \subseteq \Psi_N$.

Then (X, Ψ_N) is called a neutrosophic topological space (briefly, $N_s ts$) in X . The Ψ_N elements are called neutrosophic open sets (briefly, $N_s os$) in X . A $N_s s$ C is called a neutrosophic closed sets (briefly, $N_s cs$) iff its complement C^c is $N_s os$.

Definition 2.5.¹² Let (X, Ψ_N) be $N_s ts$ on X and A be a $N_s s$ on X , then the neutrosophic interior of A (briefly, $N_s int(A)$) and the neutrosophic closure of A (briefly, $N_s cl(A)$) are defined as

$$N_s int(A) = \bigcup \{I : I \subseteq A \text{ and } I \text{ is a } N_s os \text{ in } X\}$$

$$N_s cl(A) = \bigcap \{J : A \subseteq J \text{ and } J \text{ is a } N_s cs \text{ in } X\}.$$

Definition 2.6.² Let (X, Ψ_N) be $N_s ts$ on X and A be a $N_s s$ on X . Then A is said to be a neutrosophic regular open set (briefly, $N_s ros$) if $A = N_s int(N_s cl(A))$.

The complement of a $N_s ros$ is called a neutrosophic regular closed set (briefly, $N_s rcs$) in X .

Definition 2.7.²² A set A is said to be a neutrosophic

(i) δ interior of A (briefly, $N_s\delta int(A)$) is defined by $N_s\delta int(A) = \bigcup\{I : I \subseteq A \text{ and } I \text{ is a } N_sros \text{ in } X\}$.

(ii) δ closure of A (briefly, $N_s\delta cl(A)$) is defined by $N_s\delta cl(A) = \bigcap\{J : A \subseteq J \text{ and } J \text{ is a } N_srcs \text{ in } X\}$.

Definition 2.8. ²² A set A is said to be a neutrosophic δ -open set (briefly, $N_s\delta os$) if $A = N_s\delta int(A)$. The complement of a $N_s\delta os$ is called a neutrosophic δ closed set (briefly, $N_s\delta cs$) in X .

Definition 2.9. ^{14,22} Let (X, τ_N) and (Y, σ_N) be any two Nts 's. A map $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is said to be a neutrosophic (resp. δ) continuous (briefly, N_sCts (resp. $N_s\delta Cts$)) iff inverse image of every N_sos in (Y, σ_N) is a N_sos (resp. $N_s\delta os$) in (X, τ_N) .

Definition 2.10. ¹⁹ Let (X, τ_N) and (Y, σ_N) be any two Nts 's. A map $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is said to be neutrosophic (resp. δ) open map (briefly, N_sO (resp. $N_s\delta O$)) if the image of every N_sos in (X, τ_N) is a N_sos (resp. $N_s\delta os$) in (Y, σ_N) .

3 Somewhat neutrosophic δ -continuous functions

Definition 3.1. A function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ from a $Nsts$ (X, τ_N) into another $Nsts$ (Y, σ_N) is called somewhat neutrosophic (resp. δ) continuous (briefly, swN_sCts (resp. $swN_s\delta Cts$)) if $A \in \sigma_N$ and $h^{-1}(A) \neq 0_N$ implies that there exist a N_sos (resp. $N_s\delta os$) B in (X, τ_N) such that $B \neq 0_N$ and $B \subseteq h^{-1}(A)$. That is, $N_sint[h^{-1}(A)] \neq 0_N$ (resp. $N_s\delta int[h^{-1}(A)] \neq 0_N$).

Proposition 3.2. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta Cts$ function from a $Nsts$ (X, τ_N) into a $Nsts$ (Y, σ_N) , then h is a swN_sCts .

Proof. Let A be a non-zero N_sos in (Y, σ_N) such that $h^{-1}(A) \neq 0_N$. Since h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) , there exist a $N_s\delta os$ B in (X, τ_N) such that $B \neq 0_N$ and $B \subseteq h^{-1}(A)$. That is, $N_s\delta int[h^{-1}(A)] \neq 0_N$. But $N_s\delta int[h^{-1}(A)] \subseteq N_sint[h^{-1}(A)]$, implies that $N_sint[h^{-1}(A)] \neq 0_N$. Therefore h is a swN_sCts . □

Definition 3.3. Let (X, τ_N) and (Y, σ_N) be any two $Nsts$. A function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is called a somewhat neutrosophic nearly continuous (briefly, swN_sN_eCts) function if $A \in \sigma_N$ and $h^{-1}(A) \neq 0_N$, there exists a non-zero N_sos B of (X, τ_N) such that $B \subseteq N_scl[h^{-1}(A)]$. That is, $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a swN_sN_eCts function if $N_sint(N_scl(h^{-1}(A))) \neq 0_N$, for any non-zero N_sos A in (Y, σ_N) .

Remark 3.4. The implications contained in the following diagram are true and the reverse implications need not be true.

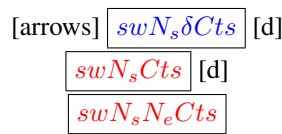


Figure 1: $swN_s\delta Cts$ function in Nts .

Proposition 3.5. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta Cts$ function from a $Nsts$ (X, τ_N) into a $Nsts$ (Y, σ_N) and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ is a N_sCts function from (Y, σ_N) into a $Nsts$ (Z, ρ_N) , then $g \circ h : (X, \tau_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta Cts$ function from (X, τ_N) into (Z, ρ_N) .

Proof. Let A be a non-zero N_sos in (Z, ρ_N) . Since g is a N_sCts function from (Y, σ_N) into (Z, ρ_N) , $g^{-1}(A)$ is a N_sos in (Y, σ_N) . Since h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) and $g^{-1}(A) \in \sigma_N$ and $g^{-1}(A) \neq 0_N$, there exists a non-zero $N_s\delta os$ B of (X, τ_N) such that $B \subseteq h^{-1}(g^{-1}(A))$. That is, $B \subseteq (g \circ h)^{-1}(A)$. Hence $N_s\delta int(g \circ h)^{-1}(A) \neq 0_N$. Therefore, $g \circ h$ is a $swN_s\delta Cts$ function from (X, τ_N) into (Z, ρ_N) . □

Proposition 3.6. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $N_s\delta Cts$ function from a $Nsts$ (X, τ_N) into a $Nsts$ (Y, σ_N) and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta Cts$ function from (Y, σ_N) into a $Nsts$ (Z, ρ_N) , then $g \circ h : (X, \tau_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta Cts$ function from (X, τ_N) into (Z, ρ_N) .

Proof. Let A be a non-zero N_sos in (Z, ρ_N) . Since g is a $swN_s\delta Cts$ function from (Y, σ_N) into (Z, ρ_N) , there exists a non-zero $N_s\delta os$ B of (Y, σ_N) such that $B \subseteq g^{-1}(A)$. Then, $h^{-1}(B) \subseteq h^{-1}(g^{-1}(A))$. That is, $h^{-1}(B) \subseteq (g \circ h)^{-1}(A)$. Again since h is a $N_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) and B is a $N_s\delta os$ in (Y, σ_N) , $h^{-1}(B)$ is $N_s\delta os$ in (X, τ_N) . Hence $N_s\delta int(g \circ h)^{-1}(A) \neq 0_N$. Therefore, $g \circ h$ is a $swN_s\delta Cts$ function from (X, τ_N) into (Z, ρ_N) . \square

Proposition 3.7. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta Cts$ function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta Cts$ function from (Y, σ_N) into a $N_s ts$ (Z, ρ_N) , then $g \circ h : (X, \tau_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta Cts$ function from (X, τ_N) into (Z, ρ_N) .

Proof. Let A be a non-zero N_sos in (Z, ρ_N) . Since g is a $swN_s\delta Cts$ function from (Y, σ_N) into (Z, ρ_N) , there exists a non-zero $N_s\delta os$ B of (Y, σ_N) such that $B \subseteq g^{-1}(A)$. Then, $h^{-1}(B) \subseteq h^{-1}(g^{-1}(A))$. That is, $h^{-1}(B) \subseteq (g \circ h)^{-1}(A)$. Since any $N_s\delta os$ is N_sos in a $N_s ts$, B is a N_sos in (Y, σ_N) . Again since h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) and B is a N_sos in (Y, σ_N) , there exists a $N_s\delta os$ η in (X, τ_N) such that $\eta \subseteq h^{-1}(B)$. This implies that $\eta \subseteq h^{-1}(B) \subseteq (g \circ h)^{-1}(A)$. Hence $N_s\delta int(g \circ h)^{-1}(A) \neq 0_N$. Therefore, $g \circ h$ is a $swN_s\delta Cts$ function from (X, τ_N) into (Z, ρ_N) . \square

Proposition 3.8. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) and the graph function $g : X \rightarrow X \times Y$ of h is $swN_s\delta Cts$, then h is $swN_s\delta Cts$.

Proof. Let A be a non-zero N_sos in (Y, σ_N) . Then $1_N \times A$ is a N_sos in $X \times Y$. Since g is a $swN_s\delta Cts$ from X into $X \times Y$ we have $N_s\delta int[g^{-1}(1_N \times A)] \neq 0_N$. But $h^{-1}(A) = 1_N \wedge h^{-1}(A) = g^{-1}(1_N \times A)$. This implies that $N_s\delta int[h^{-1}(A)] \neq 0_N$. Hence h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) . \square

Definition 3.9. A $N_s s$ A in a $N_s ts$ (X, τ_N) is called neutrosophic δ -dense (briefly, $N_s\delta D$) set if there exists no $N_s\delta cs$ B in (X, τ_N) such that $A \subset B \subset 1_N$. That is, $N_s\delta cl(A) = 1_N$.

Proposition 3.10. If h is a $swN_s\delta Cts$ function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) , then $h^{-1}(1_N - A)$ is not a $N_s\delta D$ set in (X, τ_N) , for a N_sos A in (Y, σ_N) .

Proof. Let h be a $swN_s\delta Cts$ function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) . Then, for each $A \in \sigma_N$ and $h^{-1}(A) \neq 0_N$, there exists a $N_s\delta os$ B in (X, τ_N) such that $B \neq 0_N$ and $B \subseteq h^{-1}(A)$. That is, $N_s\delta int[h^{-1}(A)] \neq 0_N$. Now $1_N - N_s\delta int[h^{-1}(A)] \neq 1_N$. This implies that $N_s\delta cl[1_N - h^{-1}(A)] \neq 1_N$ and hence $N_s\delta cl[h^{-1}(1_N - A)] \neq 1_N$. Therefore, $h^{-1}(1_N - A)$ is not a $N_s\delta D$ set in (X, τ_N) . \square

Proposition 3.11. If h is a $swN_s\delta Cts$ function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) , then $h^{-1}(1_N - A)$ is not a neutrosophic dense set in (X, τ_N) , for a N_sos A in (Y, σ_N) .

Proof. Let A be a non-zero N_sos in (Y, σ_N) . Since h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) , by Proposition 3.10, $h^{-1}(1_N - A)$ is not a $N_s\delta D$ set in (X, τ_N) . Now $N_s cl(B) \subseteq N_s\delta cl(B)$, for a $N_s s$ B in (X, τ_N) , implies that $N_s cl(h^{-1}(1_N - A)) \subseteq N_s\delta cl(h^{-1}(1_N - A))$. Then, $N_s cl(h^{-1}(1_N - A)) \neq 1_N$. [Otherwise $N_s cl(h^{-1}(1_N - A)) = 1_N$, will imply that $N_s\delta cl(h^{-1}(1_N - A)) = 1_N$, a contradiction]. Therefore, $h^{-1}(1_N - A)$ is not a neutrosophic dense set in (X, τ_N) . \square

Proposition 3.12. Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$'s. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a 1 - 1 and onto function. Then the following are equivalent:

- (i) h is $swN_s\delta Cts$.
- (ii) If A is a $N_s cs$ of (Y, σ_N) such that $h^{-1}(A) \neq 1_N$, then there exists a $N_s\delta cs$ $B \neq 1_N$ of (X, τ_N) such that $B \supseteq h^{-1}(A)$.
- (iii) If A is a $N_s\delta D$ set in (X, τ_N) , then $h(A)$ is a $N_s\delta D$ set in (Y, σ_N) .

Proof. (i) \Rightarrow (ii) Let h be a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) and A is a $N_s cs$ in (Y, σ_N) such that $h^{-1}(A) \neq 1_N$. Clearly $1_N - A \in S$ and $h^{-1}(1_N - A) = 1_N - h^{-1}(A) \neq 0_N$ (Since $h^{-1}(A) \neq 1_N$). By (i), there exists a $N_s\delta os$ γ in (X, τ_N) such that $\gamma \subseteq h^{-1}(1_N - A)$. Then $\gamma \subseteq 1_N - h^{-1}(A)$ and hence $h^{-1}(A) \subseteq 1_N - \gamma$. Clearly $1_N - \gamma$ is a $N_s\delta cs$ in (X, τ_N) and by taking $1_N - \gamma = B$, we find that (i) \Rightarrow (ii) is proved.

(ii) \Rightarrow (iii) Let A be a $N_s\delta D$ set in (X, τ_N) and suppose that $h(A)$ is not a $N_s\delta D$ set in (Y, σ_N) . Then there exists a non-zero $N_s\delta cs$ γ of (Y, σ_N) such that

$$h(A) \subset \gamma \subset 1_N. \quad (1)$$

Since $\gamma \subset 1_N$, $h^{-1}(\gamma) \neq 1_N$ and so by (ii), there exists a $N_s\delta cs$ $B \neq 1_N$ of (X, τ_N) such that $B \supseteq h^{-1}(\gamma)$. Then, $B \supseteq h^{-1}(\gamma) \supseteq h^{-1}h(A) \supseteq A$. [from (1)]. That is, there exists a $N_s\delta cs$ B in (X, τ_N) such that $B \supset A$, which is a contradiction to the assumption on A . Therefore (ii) \Rightarrow (iii) is proved.

(iii) \Rightarrow (i) Let $A (\neq 0_N) \in \sigma_N$ and $h^{-1}(A) \neq 0_N$. Suppose that there exists no $N_s\delta os$ B in (X, τ_N) such that $B \neq 0_N$ and $B \subseteq h^{-1}(A)$. That is, $N_s\delta int[h^{-1}(A)] = 0_N$. Then, $1_N - N_s\delta int[h^{-1}(A)] = 1_N$. This will imply that $N_s\delta cl[1_N - h^{-1}(A)] = 1_N$. Then, by (iii), $h[1_N - h^{-1}(A)]$ will be a $N_s\delta D$ set in (Y, σ_N) . That is, $N_s\delta cl(h[1_N - h^{-1}(A)]) = 1_N$. But $h[1_N - h^{-1}(A)] = h[h^{-1}(1_N - A)] \subseteq 1_N - A \subset 1_N$. Now $h[1_N - h^{-1}(A)] \subseteq 1_N - A$, implies that $N_s\delta cl(h[1_N - h^{-1}(A)]) \subseteq N_s\delta cl(1_N - A) \subset N_s\delta cl(1_N) = 1_N$ [$N_s\delta cl(1_N) \subseteq N_s\delta cl(1_N)$, implies that $1_N \subseteq N_s\delta cl(1_N)$] and then we will have $1_N \subset 1_N$, a contradiction. Hence we must have $N_s\delta int[h^{-1}(A)] \neq 0_N$. Therefore h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) . \square

Proposition 3.13. Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$'s. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta Cts$, $1 - 1$ and onto function and if $N_s\delta int(A) = 0_N$, for any non-zero $N_s s$ A in (X, τ_N) , then $N_s\delta int[h(A)] = 0_N$ in (Y, σ_N) .

Proof. Let A be a non-zero $N_s s$ in (X, τ_N) such that $N_s\delta int(A) = 0_N$. Now $1_N - N_s\delta int(A) = 1_N$, implies that $N_s\delta cl(1_N - A) = 1_N$. Since h is a $swN_s\delta Cts$ function from (X, τ_N) into (Y, σ_N) and $1_N - A$ is a $N_s\delta D$ set in (X, τ_N) , by Proposition 3.12, $h(1_N - A)$ is a $N_s\delta D$ set in (Y, σ_N) . That is, $N_s\delta cl[h(1_N - A)] = 1_N$. Since h is a $1 - 1$ and onto function, $h(1_N - A) = 1_N - h(A)$. Then we have $N_s\delta cl[1_N - h(A)] = 1_N$ and hence $N_s\delta int[h(A)] = 0_N$ in (Y, σ_N) . \square

Definition 3.14. A $N_s s$ A in a $N_s ts$ (X, τ_N) is called neutrosophic δ -nowhere dense (briefly, $N_s\delta ND$) if there exists no non-zero $N_s\delta os$ B in (X, τ_N) such that $B \subset N_s\delta cl(A)$. That is, $N_s\delta int(N_s\delta cl(A)) = 0_N$.

Proposition 3.15. Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$'s. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta Cts$, $1 - 1$ and onto function and if A is a $N_s\delta ND$ set in (X, τ_N) , then $N_s\delta int(h[N_s\delta cl(A)]) = 0_N$ in (Y, σ_N) .

Proof. Let $A (\neq 0_N)$ be a $N_s\delta ND$ set in (X, τ_N) . Then, $N_s\delta int(N_s\delta cl(A)) = 0_N$. Then, by Proposition 3.13, $N_s\delta int(h[N_s\delta cl(A)]) = 0_N$ in (Y, σ_N) . \square

Proposition 3.16. Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$'s. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta Cts$, $1 - 1$ and onto function and if A is a $N_s\delta ND$ set in (X, τ_N) , then $N_s\delta int(h[N_s\delta cl(A)]) = 0_N$ in (Y, σ_N) .

Proof. Let $A (\neq 0_N)$ be a $N_s\delta ND$ set in (X, τ_N) . Then, $N_s\delta int(N_s\delta cl(A)) = 0_N$. Then, by Proposition 3.13, $N_s\delta int(h[N_s\delta cl(A)]) = 0_N$ in (Y, σ_N) . Now $N_s\delta cl(A) \subseteq N_s\delta cl(A)$, implies that $h[N_s\delta cl(A)] \subseteq h[N_s\delta cl(A)]$ and hence we have $N_s\delta int(h[cl(A)]) \subseteq N_s\delta int(h[N_s\delta cl(A)])$. Therefore $N_s\delta int(h[N_s\delta cl(A)]) = 0_N$ in (Y, σ_N) . \square

Proposition 3.17. Let (X, τ_N) , (X_1, τ_{N_1}) and (X_2, τ_{N_2}) be $N_s ts$'s and $p_i : X_1 \times X_2 \rightarrow X_i$ ($i = 1, 2$) be any $N_s Cts$ function. If $h : (X, \tau_N) \rightarrow (X_1, \tau_{N_1}) \times (X_2, \tau_{N_2})$ is a $swN_s\delta Cts$ function, then $p_i \circ h$ is also a $swN_s\delta Cts$ function for $i = 1, 2$.

Proof. For any non-zero N_sos A of X_i , we have $(p_i \circ h)^{-1}(A) = h^{-1}(p_i^{-1}(A))$. Now $p_i^{-1}(A) \neq 0_N$ (since $A \neq 0_N$). Since p_i is N_sCts , $p_i^{-1}(A)$ is N_sos and since h is a $swN_s\delta Cts$ function, there exist a $N_s\delta os$ B in (X, τ_N) such that $B \neq 0_N$ of X and $B \subseteq h^{-1}(p_i^{-1}(A))$. That is, $B \subseteq (p_i \circ h)^{-1}(A)$. Hence $p_i \circ h$ is a $swN_s\delta Cts$ function for $i = 1, 2$. \square

Definition 3.18. A $N_s ts$ (X, τ_N) is called a neutrosophic δ - $D \setminus S$ space (briefly, $N_s\delta$ - $D \setminus S$) if every non-zero N_sos ξ of X , is $N_s\delta D$ set in X .

Proposition 3.19. If h is a $swN_s\delta Cts$ function from a $N_s ts$ (X, τ_N) onto a $N_s ts$ (Y, σ_N) and if (X, τ_N) is a $N_s\delta$ - $D \setminus S$, then (Y, σ_N) is a $N_s\delta$ - $D \setminus S$.

Proof. Let ξ be a non-zero N_sos in (Y, σ_N) . We wish to prove that ξ is a $N_s\delta D$ set in (Y, σ_N) . Suppose not. Then there exists a $N_s\delta cs$ ς in (Y, σ_N) such that $\xi < \varsigma < 1$ and $h^{-1}(\xi) < h^{-1}(\varsigma) < h^{-1}(1) = 1$. Since $\xi \neq 0$, $h^{-1}(\xi) \neq 0$ and since h is $swN_s\delta Cts$, there exists a $N_s\delta os$ γ in (X, τ_N) such that $\gamma \leq h^{-1}(\xi)$ and $\gamma \leq h^{-1}(\xi) < h^{-1}(\varsigma) < N_s\delta cl[h^{-1}(\varsigma)] < 1$. That is, $\gamma < N_s\delta cl[h^{-1}(\varsigma)] < 1$. This contradicts the fact that (X, τ_N) is a $N_s\delta$ - $D \setminus S$, in which the $N_s\delta os$ γ in (X, τ_N) must be a $N_s\delta D$ in (X, τ_N) and it proves that (Y, σ_N) is a $N_s\delta$ - $D \setminus S$. \square

4 Somewhat neutrosophic δ -open functions

Definition 4.1. A function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ from a $N_s ts$ (X, τ_N) into another $N_s ts$ (Y, σ_N) is called somewhat neutrosophic (resp. δ) open (briefly, swN_sO (resp. $swN_s\delta O$)) function if $A \in \tau_N$ and $A \neq 0_N$ implies that there exists a N_sos (resp. $N_s\delta os$) η in (Y, σ_N) such that $\eta \neq 0_N$ and $\eta \subseteq h(A)$. That is, $N_s\delta int[h(A)] \neq 0_N$.

Proposition 4.2. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta O$ function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) , then h is a swN_sO function.

Proof. Let A be a non-zero N_sos in (X, τ_N) such that $h(A) \neq 0_N$. Since h is a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) , there exist a $N_s\delta os$ B in (Y, σ_N) such that $B \neq 0_N$ and $B \subseteq h(A)$. That is, $N_s\delta int[h(A)] \neq 0_N$. But $N_s\delta int[h(A)] \subseteq N_s int[h(A)]$, implies that $N_s int[h(A)] \neq 0_N$. Therefore h is a swN_sO function. \square

Definition 4.3. Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$'s. A function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is called a somewhat neutrosophic nearly open (briefly, swN_sN_eO) function if for all $A \in \tau_N$ and $h(A) \neq 0_N$, there exists a non-zero N_sos B of (Y, σ_N) such that $B \subseteq N_s cl[h(A)]$. That is, a function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a swN_sN_eO function if $N_s int(N_s cl(h(A))) \neq 0_N$, for every non-zero N_sos A of (X, τ_N) .

Remark 4.4. The implications contained in the following diagram are true and the reverse implications need not be true.

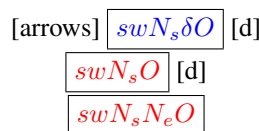


Figure 2: $swN_s\delta O$ function in $N_s ts$.

Proposition 4.5. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta O$ function from a $N_s ts$ (X, τ_N) into a $N_s ts$ (Y, σ_N) and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ is a $N_s\delta O$ function from (Y, σ_N) into a $N_s ts$ (Z, ρ_N) , then $g \circ h : (X, \tau_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta O$ function from (X, τ_N) into (Z, ρ_N) .

Proof. Let A be a non-zero N_sos in (X, τ_N) . Since h is a N_sO function from (X, τ_N) into (Y, σ_N) , $h(A)$ is a N_sos in (Y, σ_N) . Since g is a $swN_s\delta O$ function from (Y, σ_N) into (Z, ρ_N) and $h(A) \in \sigma_N$ and $h(A) \neq 0_N$, there exists a non-zero $N_s\delta os$ B of (Z, ρ_N) such that $B \subseteq g(h(A))$. That is, $B \subseteq (g \circ h)(A)$. Hence $N_s\delta int(g \circ h)(A) \neq 0_N$. Therefore, $g \circ h$ is a $swN_s\delta O$ function from (X, τ_N) into (Z, ρ_N) . \square

Proposition 4.6. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta O$ function from a $N_s ts (X, \tau_N)$ into a $N_s ts (Y, \sigma_N)$ and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta O$ function from (Y, σ_N) into a $N_s ts (Z, \rho_N)$, then $g \circ h : (X, \tau_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta O$ function from (X, τ_N) into (Z, ρ_N) .

Proof. Let A be a non-zero $N_s os$ in (X, τ_N) . Since h is a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) , there exists a non-zero $N_s\delta os$ B of (Y, σ_N) such that $B \subseteq h(A)$. Then, $g(B) \subseteq g(h(A))$. That is, $g(B) \subseteq (g \circ h)(A)$. Since any $N_s\delta os$ is $N_s os$ in a $N_s ts$, B is a $N_s os$ in (Y, σ_N) . Since g is a $swN_s\delta O$ function from (Y, σ_N) into (Z, ρ_N) and $B \in \sigma_N$ and $g(B) \neq 0_N$, there exists a non-zero $N_s\delta os$ η of (Z, ρ_N) such that $\eta \subseteq g(B)$. This implies that $\eta \subseteq g(B) \subseteq (g \circ h)(A)$. Hence $N_s\delta int(g \circ h)(A) \neq 0_N$. Therefore, $g \circ h$ is a $swN_s\delta O$ function from (X, τ_N) into (Z, ρ_N) . \square

Proposition 4.7. If $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $sw\delta O$ function from a $N_s ts (X, \tau_N)$ into a $N_s ts (Y, \sigma_N)$ and $g : (Y, \sigma_N) \rightarrow (Z, \rho_N)$ is a $N_s\delta O$ function from (Y, σ_N) into a $N_s ts (Z, \rho_N)$, then $g \circ h : (X, \tau_N) \rightarrow (Z, \rho_N)$ is a $swN_s\delta O$ function from (X, τ_N) into (Z, ρ_N) .

Proof. Let A be a non-zero $N_s os$ in (Z, ρ_N) . Since h is a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) , there exists a non-zero $N_s\delta os$ B of (Y, σ_N) such that $B \subseteq h(A)$. Then $g(B) \subseteq g(h(A))$. That is, $g(B) \subseteq (g \circ h)(A)$. Since g is a $N_s\delta O$ function from (Y, σ_N) into (Z, ρ_N) and B is a $N_s\delta os$ in (Y, σ_N) , $g(B)$ is $N_s\delta O$ in (X, τ_N) . Hence $N_s\delta int(g \circ h)(A) \neq 0_N$. Therefore, $g \circ h$ is a $swN_s\delta O$ function from (X, τ_N) into (Z, ρ_N) . \square

Proposition 4.8. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a function from a $N_s ts (X, \tau_N)$ into another $N_s ts (Y, \sigma_N)$. Then the following conditions are equivalent:

- (i) h is $swN_s\delta O$.
- (ii) If A is a $N_s\delta D$ set in (Y, σ_N) , then $h^{-1}(A)$ is a $N_s\delta D$ set in (X, τ_N) .

Proof. (i) \Rightarrow (ii) Assume (i). Let A be a $N_s\delta D$ set in (Y, σ_N) and suppose that $h^{-1}(A)$ is not a $N_s\delta D$ set in (X, τ_N) . Then there exists a non-zero $N_s\delta cs$ γ of (X, τ_N) such that $h^{-1}(A) \subset \gamma \subset 1_N$. Then, $1_N - \gamma \subset 1_N - h^{-1}(A) = h^{-1}(1_N - A)$. Now $1_N - \gamma$ is a $N_s\delta os$ in (X, τ_N) . Since any $N_s\delta os$ is $N_s o$ in a $N_s ts$, $1_N - \gamma$ is a $N_s os$ in (X, τ_N) . Since $\gamma \subset 1_N$, $1_N - A \neq 0_N$. Since h is a $swN_s\delta O$ function, there exists a $N_s\delta os$ $B \neq 0_N$ in (Y, σ_N) such that $B \subseteq h(1_N - \gamma)$ and hence $B \subseteq h(h^{-1}(1_N - A)) \subseteq 1_N - A$. That is, $A \subset 1_N - B \subset 1_N$ and $1_N - B$ is a $N_s\delta cs$ in (Y, σ_N) , implies that A is not a $N_s\delta D$ set in (X, τ_N) , which is a contradiction to the assumption on A . Therefore (i) \Rightarrow (ii) is proved.

(ii) \Rightarrow (i) Let $A (\neq 0_N) \in \tau_N$ and $h(A) \neq 0_N$. Suppose that there exists no $N_s\delta os$ B in (Y, σ_N) such that $B \neq 0_N$ and $B \subseteq h(A)$. That is, $N_s\delta int[h(A)] = 0_N$. Then, $1_N - N_s\delta int[h(A)] = 1_N$. This will imply that $N_s\delta cl[1_N - h(A)] = 1_N$. Now $h^{-1}[1_N - h(A)] = 1_N - h^{-1}[h(A)] \subseteq 1_N - A \subset 1_N$ (since $A \neq 0_N$). That is, $h^{-1}[1_N - h(A)] \subset 1_N$. Then $N_s\delta cl(h^{-1}(1_N - h(A))) \subset N_s\delta cl(1_N) = 1_N$. This will imply that $N_s\delta cl(h^{-1}(1_N - h(A))) \neq 1_N$, a contradiction. Hence we must have $N_s\delta int[h(A)] \neq 0_N$. Therefore h is a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) . \square

Proposition 4.9. Let (X, τ_N) and (Y, σ_N) be any two $N_s ts$ and $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a one-to-one and onto function. Then the following are equivalent:

- (i) h is $swN_s\delta O$.
- (ii) If A is a $N_s cs$ of (X, τ_N) such that $h(A) \neq 1_N$, then there exists $N_s\delta c$ $B \neq 1_N$ of (X, τ_N) such that $B \supset h(A)$.

Proof. (i) \Rightarrow (ii) Let h be a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) and A is a $N_s cs$ in (X, τ_N) such that $h(A) \neq 1_N$. Clearly $1_N - A \in \sigma_N$ and $h(1_N - A) = 1_N - h(A) \neq 0_N$ (since h is one-to-one and onto). By (i), there exists a $N_s\delta os$ γ in (Y, σ_N) such that $\gamma \subseteq h(1_N - A)$. Then $\gamma \subseteq 1_N - h(A)$ and hence $h(A) \subseteq 1_N - \gamma$. Clearly $1_N - \gamma$ is a $N_s\delta cs$ in (X, τ_N) and by taking $1_N - \gamma = B$, we find that (i) \Rightarrow (ii) is proved.

(ii) \Rightarrow (i) Let A be a non-zero $N_s\delta OS$ in (X, τ_N) such that $h(A) \neq 0_N$. Then, $1_N - A$ is a $N_s\delta CS$ in (X, τ_N) such that $h(1_N - A) = 1_N - h(A) \neq 1_N$ (Since h is one-to-one and onto, $h(1_N - A) = 1_N - h(A)$). By hypothesis, there exists a $N_s\delta CS$ $B \neq 1_N$ of (X, τ_N) such that $B \supset h(1_N - A)$. Then $B \supset 1_N - h(A)$. Hence $1_N - B \subset h(A)$, where $1_N - B$ is a $N_s\delta OS$ in (Y, σ_N) . Therefore h is a $swN_s\delta O$ function and (ii) \Rightarrow (i) is proved. \square

Proposition 4.10. Let (X, τ_N) and (Y, σ_N) be any two $N_s\delta TS$'s. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ is a $swN_s\delta O$ function and if $N_s\delta int(A) = 0_N$, for any non-zero $N_s\delta S$ A in (Y, σ_N) , then $N_s\delta int[h^{-1}(A)] = 0_N$ in (X, τ_N) .

Proof. Let $A (\neq 0_N)$ be a non-zero $N_s\delta S$ in (Y, σ_N) such that $N_s\delta int(A) = 0_N$. Now $1_N - N_s\delta int(A) = 1_N$, implies that $N_s\delta cl(1_N - A) = 1_N$. Since h is a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) and $1_N - A$ is a $N_s\delta D$ set in (Y, σ_N) , by Proposition 4.8, $h^{-1}(1_N - A)$ is a $N_s\delta D$ set in (X, τ_N) . That is, $N_s\delta cl[h^{-1}(1_N - A)] = 1_N$. Since $h^{-1}(1_N - A) = 1_N - h^{-1}(A)$, we have $N_s\delta cl[1_N - h^{-1}(A)] = 1_N$ and hence $N_s\delta int[h^{-1}(A)] = 0_N$ in (X, τ_N) . \square

5 $N_s\delta$ -Resolvable and $N_s\delta$ -Irresolvable Spaces

Definition 5.1. A $N_s\delta TS$ (X, τ_N) is called a neutrosophic δ -resolvable space (briefly, $N_s\delta RS$) if there exists a non-zero $N_s\delta D$ set A in (X, τ_N) such that $N_s\delta cl(1_N - A) = 1_N$. Otherwise (X, τ_N) is called a neutrosophic δ -irresolvable space (briefly, $N_s\delta IS$).

Proposition 5.2. If a $N_s\delta TS$ (X, τ_N) has a pair of $N_s\delta D$ sets A_1 and A_2 such that $A_1 \subseteq 1_N - A_2$, then (X, τ_N) is a $N_s\delta RS$.

Proof. Let the $N_s\delta TS$ (X, τ_N) has a pair of $N_s\delta D$ sets A_1 and A_2 such that $A_1 \subseteq 1_N - A_2$. We wish to prove that (X, τ_N) is a $N_s\delta RS$. Assume the contrary. Then, for all $N_s\delta D$ sets A_i in (X, τ_N) we have $N_s\delta cl(1_N - A_i) \neq 1_N$. In particular $N_s\delta cl(1_N - A_2) \neq 1_N$, implies that there exists a non-zero $N_s\delta CS$ B in (X, τ_N) such that $1_N - A_2 \subset B \subset 1_N$. Then, $A_1 \subseteq 1_N - A_2$, implies that $A_1 \subseteq 1_N - A_2 \subset B \subset 1_N$ and hence $A_1 \subset B \subset 1_N$. This will imply that $N_s\delta cl(A_1) \neq 1_N$, a contradiction. Therefore, (X, τ_N) is a $N_s\delta RS$. \square

Proposition 5.3. If $\bigvee_{i=1}^n (A_i) = 1_N$, where $N_s\delta int(A_i) = 0_N$ in a $N_s\delta TS$ (X, τ_N) , then (X, τ_N) is a $N_s\delta RS$.

Proof. Now $\bigvee_{i=1}^n (A_i) = 1_N$, implies that $1_N - \bigvee_{i=1}^n (A_i) = 0_N$. Then, $\bigwedge_{i=1}^n (1_N - A_i) = 0_N$. Then there must be atleast two non-zero disjoint $N_s\delta S$'s $1_N - A_i$ and $1_N - A_j$ in (X, τ_N) . Hence we have $(1_N - A_i) + (1_N - A_j) \subseteq 1_N$. This implies that $(1_N - A_j) \subseteq A_i$. Then $N_s\delta cl(1_N - A_j) \subseteq N_s\delta cl(A_i)$. But $N_s\delta int(A_j) = 0_N$, implies that $N_s\delta cl(1_N - A_j) = 1_N - N_s\delta int(A_j) = 1_N - 0_N = 1_N$. Then $1_N \subseteq N_s\delta cl(A_i)$. That is, $N_s\delta cl(A_i) = 1_N$. Also $N_s\delta int(A_i) = 0_N$, implies that $N_s\delta cl(1_N - A_i) = 1_N$. Therefore (X, τ_N) has a $N_s\delta D$ set A_i such that $N_s\delta cl(1_N - A_i) = 1_N$. Therefore (X, τ_N) is a $N_s\delta RS$. \square

Proposition 5.4. A $N_s\delta TS$ (X, τ_N) is a $N_s\delta IS$ if and only if $N_s\delta int(A) \neq 0_N$, for each $N_s\delta D$ set A in (X, τ_N) .

Proof. Let (X, τ_N) be a $N_s\delta IS$. Then, for each $N_s\delta D$ set A in (X, τ_N) , we have $N_s\delta cl(1_N - A) \neq 1_N$ and hence $1_N - N_s\delta int(A) = N_s\delta cl(1_N - A) \neq 1_N$. Thus, $N_s\delta int(A) \neq 0_N$, for each $N_s\delta D$ set A in (X, τ_N) .

Conversely, let $N_s\delta int(A) \neq 0_N$, for each $N_s\delta D$ set A in (X, τ_N) . Suppose that (X, τ_N) is a $N_s\delta RS$. Then, there exists a non-zero $N_s\delta D$ set A in (X, τ_N) such that $N_s\delta cl(1_N - A) = 1_N$ and hence $1_N - N_s\delta int(A) = N_s\delta cl(1_N - A) = 1_N$, implies that $N_s\delta int(A) = 0_N$, a contradiction to the hypothesis. Therefore, (X, τ_N) is a $N_s\delta IS$. \square

6 Functions of Neutrosophic δ -Resolvable and Irresolvable Spaces

Lemma 6.1. Let $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a mapping and $\{A_j\}, j \in J$ be a family of N_s 's of X . Then

$$(i) \quad h(\cup_{j \in J} A_j) = \cup_{j \in J} h(A_j);$$

$$(ii) \quad h(\cap_{j \in J} A_j) \subseteq \cap_{j \in J} h(A_j).$$

Proposition 6.2. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ from a N_s ts (X, τ_N) onto another N_s ts (Y, σ_N) is a $swN_s\delta Cts$ and 1 – 1 function and if (X, τ_N) is a $N_s\delta RS$, then (Y, σ_N) is a $N_s\delta RS$.

Proof. Let h be a $swN_s\delta Cts$ function from a $N_s\delta RS$ (X, τ_N) onto a N_s ts (Y, σ_N) . Since (X, τ_N) is a $N_s\delta RS$, by Proposition 5.3, $\bigvee_{i=1}^n (A_i) = 1_N$, where $N_s\delta int(A_i) = 0_N$ in (X, τ_N) . Now $h(\bigvee_{i=1}^n (A_i)) = h(1_N) = 1_N$. (Since h is onto). Then, by Lemma 6.1, $\bigvee_{i=1}^n h(A_i) = 1_N$. Since h is a $swN_s\delta Cts$ and 1 – 1 function from (X, τ_N) onto (Y, σ_N) and $N_s\delta int(A_i) = 0_N$ in (X, τ_N) , by Proposition 3.13, $N_s\delta int[h(A_i)] = 0_N$ in (Y, σ_N) . Thus we have $\bigvee_{i=1}^n h(A_i) = 1_N$, where, $N_s\delta int[h(A_i)] = 0_N$ in (Y, σ_N) . Then, by Proposition 5.3, (Y, σ_N) is a $N_s\delta RS$. \square

Proposition 6.3. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ from a N_s ts (X, τ_N) into another N_s ts (Y, σ_N) is a $swN_s\delta O$ function and if (X, τ_N) is a $N_s\delta IS$, then (Y, σ_N) is a $N_s\delta IS$.

Proof. Let $A \neq 0_N$ be an arbitrary N_s s in (Y, σ_N) such that $N_s\delta cl(A) = 1_N$. We claim that $N_s\delta int(A) \neq 0_N$. Assume the contrary. That is, $N_s\delta int(A) = 0_N$. Since h is a $swN_s\delta O$ function from (X, τ_N) into (Y, σ_N) , we have, by Proposition 4.10, $N_s\delta int[h^{-1}(A)] = 0_N$ in (X, τ_N) and by Proposition 4.8, $h^{-1}(A)$ is a $N_s\delta D$ set in (X, τ_N) , for a $N_s\delta D$ set A in (Y, σ_N) . Thus we have $N_s\delta int[h^{-1}(A)] = 0_N$ for a $N_s\delta D$ set $h^{-1}(A)$ in (X, τ_N) . But this is a contradiction to (X, τ_N) , being a $N_s\delta IS$ in which $N_s\delta int(B) \neq 0_N$, for each $N_s\delta D$ set B in (X, τ_N) (by Proposition 5.4). Hence our assumption that $N_s\delta int(A) = 0_N$, for a $N_s\delta D$ set A in (Y, σ_N) , does not hold. Hence we must have that $N_s\delta int(A) \neq 0_N$, for each $N_s\delta D$ set A in (Y, σ_N) . Therefore (Y, σ_N) is a $N_s\delta is$. \square

Proposition 6.4. If the function $h : (X, \tau_N) \rightarrow (Y, \sigma_N)$ from a N_s ts (X, τ_N) onto another N_s ts (Y, σ_N) is a $swN_s\delta Cts$ and 1 – 1 function and if (Y, σ_N) is a $N_s\delta IS$, then (X, τ_N) is a $N_s\delta IS$.

Proof. Let $A \neq 0_N$ be an arbitrary N_s s in (X, τ_N) such that $N_s\delta cl(A) = 1_N$. We claim that $N_s\delta int(A) \neq 0_N$. Assume the contrary. That is, $N_s\delta int(A) = 0_N$. Since h is a $swN_s\delta Cts$ and 1 – 1 function from (X, τ_N) into (Y, σ_N) , we have, by Proposition 3.13, $N_s\delta int[h(A)] = 0_N$ in (Y, σ_N) and by Proposition 3.12, $h(A)$ is a $N_s\delta D$ set in (Y, σ_N) , for a $N_s\delta D$ set A in (X, τ_N) . Thus, we have $N_s\delta int[h(A)] = 0_N$ for a $N_s\delta D$ set $h(A)$ in (Y, σ_N) . But this is a contradiction to (Y, σ_N) , being a $N_s\delta IS$ in which $N_s\delta int(B) \neq 0_N$, for each $N_s\delta D$ set B in (Y, σ_N) (by Proposition 5.4). Hence our assumption that $N_s\delta int(A) = 0_N$, for a $N_s\delta D$ set A in (X, τ_N) , does not hold. Hence we must have that $N_s\delta int(A) \neq 0_N$, for each $N_s\delta D$ set A in (X, τ_N) . Therefore (X, τ_N) is a $N_s\delta IS$. \square

7 Conclusions

The concepts of $swN_s\delta Cts$ functions, $swN_s\delta O$ functions between N_s ts's are introduced and studied. Also $N_s\delta$ -resolvability and $N_s\delta$ -irresolvability of N_s ts's are introduced. Some results concerning functions that preserve the $N_s\delta$ -resolvable spaces and $N_s\delta$ -irresolvable spaces in the context of images and pre-images are obtained in this paper.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

- [1] R.K. Al-Hamido, "A New Approach Of Neutrosophic Topological space", International Journal of Neutrosophic Science, Vol 7, pp 55-61, 2020.
- [2] I. Arokiarani, R. Dhavaseelan, S. Jafari, and M. Parimala, "On some new notions and functions in neutrosophic topological space", Neutrosophic Sets and Systems, Vol 16, pp 16-19, 2017.
- [3] K. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Syst., Vol 20, pp 87-96, 1986.
- [4] C.L. Chang, "Fuzzy topological space", J. Math. Anal. Appl., Vol 24, pp 182-190, 1968.
- [5] D. Coker, "An introduction to intuitionistic fuzzy topological spaces", Fuzzy sets and syst., Vol 88, pp 81-89, 1997.
- [6] K.R. Gentry and H.B. Hoyle III, "Somewhat continuous functions", Czech. Math. J., Vol 21 (96), pp 5-12, 1971.
- [7] E. Hewitt, "A problem in set theoretic topology", Duke Math. J., Vol 10, pp 309-333, 1943.
- [8] P. Iswarya, and K. Bageerathi, "A Study on Neutrosophic Frontier and Neutrosophic Semi-frontier in Neutrosophic Topological Spaces", Neutrosophic Sets and Systems, Vol 16, pp 6-15, 2017.
- [9] S. Krishna Prabha, Said Broumi, and Selçuk Topal, "A Novel Routing Network Algorithm via Neutrosophic Fuzzy Set Approach", International Journal of Neutrosophic Science, Vol 13, pp 9-22, 2021.
- [10] M. Parimala, M. Karthika, and F. Smarandache, "A Review of Fuzzy Soft Topological Spaces, Intuitionistic Fuzzy Soft Topological Spaces and Neutrosophic Soft Topological Spaces", International Journal of Neutrosophic Science, Vol 10, pp 96-104, 2020.
- [11] A.A. Salama, and F. Smarandache, "Neutrosophic crisp set theory", Educational Publisher, Columbus, Ohio, USA, 2015.
- [12] A.A. Salama, and S.A. Alblowi, "Neutrosophic set and neutrosophic topological spaces", IOSR Journal of Mathematics, Vol 3, pp 31-35, 2012.
- [13] A.A. Salama, F. Smarandache, and S.A. Alblowi, "Characteristic function of neutrosophic set", Neutrosophic Sets and Systems, Vol 3, pp 14-17, 2014.
- [14] A.A. Salama, F. Smarandache, and V. Kromov, "Neutrosophic closed set and neutrosophic continuous functions", Neutrosophic Sets and Systems, Vol 4, pp 4-8, 2014.
- [15] A.A. Salama, H. ElGhawalby, and A.M. Nasr, "Star Neutrosophic Fuzzy Topological Space", International Journal of Neutrosophic Science, Vol 10, pp 77-82, 2019.
- [16] F. Smarandache, "A Unifying field in logics: neutrosophic logic. neutrosophy, neutrosophic set, neutrosophic probability", American Research Press, Rehoboth, NM, 1999.
- [17] F. Smarandache, "Neutrosophy and neutrosophic logic", First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA, 2002.
- [18] S. Saha, "Fuzzy δ -continuous maps", J. Math. Anal. Appl., Vol 126, pp 130-142, 1987.
- [19] A. Vadivel and C. John Sundar, "Neutrosophic δ -Open Maps and Neutrosophic δ -Closed Maps", International Journal of Neutrosophic Science (IJNS), Vol 13 (2), pp 66-74, 2021.
- [20] A. Vadivel and C. John Sundar, "New Operators Using Neutrosophic δ -Open Set", Journal of Neutrosophic and Fuzzy Systems, Vol 1 (2), pp 61-70, 2021.
- [21] A. Vadivel and C. John Sundar, "Somewhat neutrosophic δ -irresolute continuous mappings in neutrosophic topological spaces", TWMS Journal of Applied and Engineering Mathematics, Vol 13 (2), pp 773-781, 2023.

- [22] A. Vadivel, M. Seenivasan, and C. John Sundar, "An Introduction to δ -open sets in a Neutrosophic Topological Spaces", *Journal of Physics: Conference Series*, 1724, 012011, 2021.
- [23] A. Vadivel, P. Thangaraja and C. John Sundar, "Neutrosophic e -continuous maps and neutrosophic e -irresolute maps", *Turkish Journal of Computer and Mathematics Education*, Vol 12 (1S), pp 369-375, 2021.
- [24] A. Vadivel, P. Thangaraja and C. John Sundar, "Neutrosophic e -Open Maps, Neutrosophic e -Closed Maps and Neutrosophic e -Homeomorphisms in Neutrosophic Topological Spaces", *AIP Conference Proceedings*, 2364, 020016, 2021.
- [25] A. Vadivel, P. Thangaraja and C. John Sundar, "Some Spaces in Neutrosophic e -Open Sets", *Algebra, Analysis, and Associated Topics, Trends in Mathematics*, pp 213-25, 2022.
- [26] L.A. Zadeh, "Fuzzy sets", *Inform. Control*, Vol 8, pp 338-353, 1965.