ε, δ – Filters of Lattice Wajsberg Algebras

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Abstract

In this paper, we defined the ε, δ – filters and (φ, ρ)- filters of Lattice Wajsberg algebras and related assets are discussed. We prove that every (ε, ε)– filters are (ε, ε V δ)– filters and provide the condition for (ε, ε V δ)– filters are (ε, ε)– filters. Further conditions for ε V δ –filters are discussed.

Keywords: Lattice Wajsberg Algebras; Neutrosophic sets; ε, δ – filters and (φ, ρ)- filters.

1. Introduction

In 1935, Wajsberg [10] introduced the concept of Wajsberg algebra. In 1984, Front, Antonio and Torrens [3] led the lattice wajsberg algebra and define filters and obtain some properties of filters.Smarandache [6] introduced the concept of neutrosophic sets.After that Monoranjan and Madhumangal [5] recall some definitions and introduced the truth value based neutrosophic sets and neutrosophic sets and define new operations with examples.T.Anitha, V.Amarendra Babu, G. Bhanu Vinolia[7,8,9] introduced the NW- filters ,MBJ- filters and BMBJ –filters of lattice wajsberg algebras and proved the some properties.

In this paper, we define the ε, δ – filters and (φ, ρ)- filters of Lattice wajsberg algebras and related assets are discussed. We prove that every $(\varepsilon, \varepsilon)$ – filters are $(\varepsilon, \varepsilon \vee \delta)$ – filters and provide the condition for $(\varepsilon, \varepsilon \vee \delta)$ – filters are $(\varepsilon, \varepsilon)$ – filters. Further conditions for $\varepsilon \vee \delta$ –filters are discussed.

2. Preliminaries

Definition 2.1[3]: Let $(\mathcal{W}, \curvearrowright, ', 1_m)$ be a wajsberg algebra if it satisfies the following axioms for all $\mathfrak{x}_m, \mathfrak{y}_m, \mathfrak{z}_m \in \mathcal{W}$

1. $1_m \curvearrowright \mathfrak{x}_m = \mathfrak{x}_m$
2. $(\mathfrak{x}_m \curvearrowright \mathfrak{y}_m) \curvearrowright ((\mathfrak{y}_m \curvearrowright \mathfrak{z}_m) \curvearrowright (\mathfrak{x}_m \curvearrowright \mathfrak{z}_m)) = 1_m$
3. $(\mathfrak{x}_m \curvearrowright \mathfrak{y}_m) \curvearrowright \mathfrak{y}_m = (\mathfrak{y}_m \curvearrowright \mathfrak{x}_m) \curvearrowright \mathfrak{x}_m$
4. $(\mathfrak{x}'_m \curvearrowright \mathfrak{y}'_m) \curvearrowright (\mathfrak{y}_m \curvearrowright \mathfrak{x}_m) = 1_m$

Definition 2.2[3]: The wajsberg algebra $\mathcal{W}$ is called a lattice wajsberg algebra with the bounds$0_m, 1_m$ if it satisfies the following axioms for all $\mathfrak{x}_m, \mathfrak{y}_m \in \mathcal{W}$

A partial ordering $\leq$ on $\mathcal{W}$, such that $\mathfrak{x}_m \leq \mathfrak{y}_m$ if and only if $\mathfrak{x}_m \curvearrowright \mathfrak{y}_m = 1_m$, $(\mathfrak{x}_m \vee \mathfrak{y}_m) = (\mathfrak{x}_m \curvearrowright \mathfrak{y}_m) \curvearrowright \mathfrak{y}_m$and $(\mathfrak{x}_m \wedge \mathfrak{y}_m) = ((\mathfrak{x}'_m \curvearrowright \mathfrak{y}'_m) \curvearrowright \mathfrak{y}'_m)'$

Definition2.3[6]: Aneutrosophic set($\mathcal{N}^s$), if the structure

$A_m = \{< \mathfrak{y}_m, \mathcal{W}_T^A(\mathfrak{y}_m), \mathcal{W}_I^A(\mathfrak{y}_m), \mathcal{W}_F^A(\mathfrak{y}_m)>, \mathfrak{y}_m \in \mathbb{X}\}$where$\mathcal{W}_T^A$is truth membershipfunction, $\mathcal{W}_I^A$is an indeterminate membership function and $\mathcal{W}_F^A$is false membership function, on a nonempty set $\mathbb{X}$.

Throughout this paper' $\mathcal{W}$' refers the lattice wajsberg algebra

3. ε, δ –filters of Lattice Wajsberg Algebras

For the neutrosophic set $A_w = (\mathcal{W}_T^A, \mathcal{W}_I^A, \mathcal{W}_F^A)$ on $\mathcal{W}$, define the cut sets as follows:

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$\mathcal{w}_{T_\varepsilon}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_T^A(\mathfrak{x}_w) \geq \alpha \}$,

$\mathcal{w}_{I_\varepsilon}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_I^A(\mathfrak{x}_w) \geq \beta \}$,

$\mathcal{w}_{F_\varepsilon}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_F^A(\mathfrak{x}_w) \leq \gamma \}$,

$\mathcal{w}_{T_\delta}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_T^A(\mathfrak{x}_w) + \alpha > 1 \}$,

$\mathcal{w}_{I_\delta}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_I^A(\mathfrak{x}_w) + \beta > 1 \}$

$\mathcal{w}_{F_\delta}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_F^A(\mathfrak{x}_w) + \gamma < 1 \}$,

$\mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_T^A(\mathfrak{x}_w) \geq \alpha \text{ or } \mathcal{w}_T^A(\mathfrak{x}_w) + \alpha > 1 \}$,

$\mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_I^A(\mathfrak{x}_w) \geq \beta \text{ or } \mathcal{w}_I^A(\mathfrak{x}_w) + \beta > 1 \}$,

$\mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\alpha} = \{ \mathfrak{x}_w \in \mathcal{w} \ / \ \mathcal{w}_F^A(\mathfrak{x}_w) \leq \gamma \text{ or } \mathcal{w}_F^A(\mathfrak{x}_w) + \gamma < 1 \}$,

where $\alpha, \beta \in (0,1]$ and $\gamma \in [0,1)$. The sets $\mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathcal{w}_{F_\varepsilon}^{A\gamma}$ are called ε – cut sets, $\mathcal{w}_{T_\delta}^{A\alpha}$, $\mathcal{w}_{I_\delta}^{A\beta}$ and $\mathcal{w}_{F_\delta}^{A\gamma}$ are called δ –cut sets and $\mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha}$, $\mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta}$ and $\mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$ are called $\varepsilon \vee \delta$ – cut sets.

Theorem3.1: The nonempty ε – cut sets of a neutrosophic set $A_w = (\mathcal{w}_T^A, \mathcal{w}_I^A, \mathcal{w}_F^A)$ on $\mathcal{w}$ are filters of $\mathcal{w}$ if and only if A_w satisfies the following inequalities for all $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$

(3.1) $\mathcal{w}_T^A(1_w) \vee 0.5 \geq \mathcal{w}_T^A(\mathfrak{x}_w)$,

$\mathcal{w}_I^A(1_w) \vee 0.5 \geq \mathcal{w}_I^A(\mathfrak{x}_w)$,

$\mathcal{w}_F^A(1_w) \wedge 0.5 \leq \mathcal{w}_F^A(\mathfrak{x}_w) \forall \mathfrak{x}_w \in \mathcal{w}$.

(3.2) $\mathcal{w}_T^A(\mathfrak{y}_w) \vee 0.5 \geq \min \{\mathcal{w}_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_T^A(\mathfrak{x}_w)\}$

$\mathcal{w}_I^A(\mathfrak{y}_w) \vee 0.5 \geq \min \{\mathcal{w}_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_I^A(\mathfrak{x}_w)\}$

$\mathcal{w}_F^A(\mathfrak{y}_w) \wedge 0.5 \leq \max \{\mathcal{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_F^A(\mathfrak{x}_w)\} \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathcal{w}$.

Proof: Suppose that the nonempty ε – cut sets are filters of $\mathcal{w}$ for all $\alpha, \beta \in (0.5, 1]$ and $\gamma \in [0,0.5)$.

If $\mathcal{w}_T^A(1_w) \vee 0.5 < \mathcal{w}_T^A(\mathbb{a})$, $\mathcal{w}_I^A(1_w) \vee 0.5 < \mathcal{w}_I^A(\mathbb{b})$ and $\mathcal{w}_F^A(1_w) \wedge 0.5 > \mathcal{w}_F^A(\mathbb{c})$ for some $\mathbb{a}, \mathbb{b}$ and $\mathbb{c} \in \mathcal{w}$ --------------- (3.3)

Then $\alpha_{\mathbb{a}} = \mathcal{w}_T^A(\mathbb{a}) \in (0.5,1]$, $\beta_{\mathbb{b}} = \mathcal{w}_I^A(\mathbb{b}) \in (0.5,1]$ and $\gamma_{\mathbb{c}} = \mathcal{w}_T^A(\mathbb{c}) \in [0,0.5)$.

So $\mathbb{a} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{c} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$.

From (3.3), $\mathcal{w}_T^A(1_w) < \mathcal{w}_T^A(\mathbb{a})$, $\mathcal{w}_I^A(1_w) < \mathcal{w}_I^A(\mathbb{b})$ and $\mathcal{w}_F^A(1_w) > \mathcal{w}_F^A(\mathbb{c})$, then clearly $1_w \notin \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $1_w \notin \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $1_w \notin \mathcal{w}_{F_\varepsilon}^{A\gamma}$. This is contradiction,

so we have $\mathcal{w}_T^A(1_w) \vee 0.5 \geq \mathcal{w}_T^A(\mathfrak{x}_w)$, $\mathcal{w}_I^A(1_w) \vee 0.5 \geq \mathcal{w}_I^A(\mathfrak{x}_w)$ and $\mathcal{w}_F^A(1_w) \wedge 0.5 \leq \mathcal{w}_F^A(\mathfrak{x}_w) \ \forall \mathfrak{x}_w \in \mathcal{w}$.

Suppose that $\mathcal{w}_T^A(\mathbb{b}) \vee 0.5 < \min \{\mathcal{w}_T^A(\mathbb{a} \curvearrowright \mathbb{b}), \mathcal{w}_T^A(\mathbb{a})\}$ for some $\mathbb{a}, \mathbb{b} \in \mathcal{w}$ and take $\alpha = \min \{\mathcal{w}_T^A(\mathbb{a} \curvearrowright \mathbb{b}), \mathcal{w}_T^A(\mathbb{a})\}$ implies $\alpha \in (0.5,1]$ and $\mathbb{a}, \mathbb{a} \curvearrowright \mathbb{b} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$. But $\mathbb{b} \notin \mathcal{w}_{T_\varepsilon}^{A\alpha}$ since $\mathcal{w}_T^A(\mathbb{b}) < \alpha$, which is a contradiction. So $\mathcal{w}_T^A(\mathfrak{y}_w) \vee 0.5 \geq \min\{\mathcal{w}_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_T^A(\mathfrak{x}_w)\} \ \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathcal{w}$. Similarly we can prove that $\mathcal{w}_I^A(\mathfrak{y}_w) \vee 0.5 \geq \min \{\mathcal{w}_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_I^A(\mathfrak{x}_w)\} \ \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathcal{w}$. Suppose there exist $\mathbb{c}, \mathbb{d} \in \mathcal{w}$ such that $\mathcal{w}_F^A(\mathbb{d}) \wedge 0.5 > \max \{\mathcal{w}_F^A(\mathbb{c} \curvearrowright \mathbb{d}), \mathcal{w}_F^A(\mathbb{c})\}$ and take $\gamma = \max\{\mathcal{w}_T^A(\mathbb{c} \curvearrowright \mathbb{d}), \mathcal{w}_T^A(\mathbb{c})\}$ implies $\gamma \in [0,0.5)$ and $\mathbb{c}, \mathbb{c} \curvearrowright \mathbb{d} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$. But $\notin \mathcal{w}_{T_\varepsilon}^{A\alpha}$, which is a contradiction. So $\mathcal{w}_F^A(\mathfrak{y}_w) \wedge 0.5 \leq \max \{\mathcal{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathcal{w}_F^A(\mathfrak{x}_w)\} \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathcal{w}$. Consequently A_w satisfies (3.1) and (3.2).

Conversely, the neutrosophic set A_w on $\mathcal{w}$ satisfies (3.1) and (3.2).

Let $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$ such that ε – cut sets are nonempty. For any $\mathbb{a} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{c} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$, we have

$\mathcal{w}_T^A(1_w) \vee 0.5 \geq \mathcal{w}_T^A(\mathbb{a}) \geq \alpha > 0.5$,

$\mathcal{w}_I^A(1_w) \vee 0.5 \geq \mathcal{w}_I^A(\mathbb{b}) \geq \beta > 0.5$,

$\mathcal{w}_F^A(1_w) \wedge 0.5 \leq \mathcal{w}_F^A(\mathbb{c}) \leq \gamma < 0.5$.

Then clearly $1_w \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $1_w \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $1_w \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$. Let $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in \mathcal{w}$ such that $\mathbb{x} \curvearrowright \mathbb{y}$, $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{a} \curvearrowright \mathbb{b}$, $\mathbb{a} \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{u} \curvearrowright \mathbb{v}$, $\mathbb{u} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$. It follows from (3.2) that

$\mathcal{w}_T^A(\mathbb{y}) \vee 0.5 \geq \min \{\mathcal{w}_T^A(\mathbb{x} \curvearrowright \mathbb{y}), \mathcal{w}_T^A(\mathbb{x})\} \geq \alpha > 0.5$,

$\mathcal{w}_I^A(\mathbb{b}) \vee 0.5 \geq \min \{\mathcal{w}_I^A(\mathbb{a} \curvearrowright \mathbb{b}), \mathcal{w}_I^A(\mathbb{a})\} \geq \beta > 0.5$,

$\mathcal{w}_F^A(\mathbb{v}) \wedge 0.5 \leq \max \{\mathcal{w}_F^A(\mathbb{u} \curvearrowright \mathbb{v}), \mathcal{w}_F^A(\mathbb{u})\} \leq \gamma < 0.5$.

So clearly $\mathbb{y} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{v} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$.

Therefore the nonempty ε – cut sets of a neutrosophic set A_w are filters of $\mathcal{w}$ for all $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$.

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Definition3.2: Let cut sets$\varphi, \rho \in \{\varepsilon, \delta, \varepsilon \vee \delta\}$then neutrosophic set A_w on $\mathscr{w}$ is called a (φ, ρ)- filter of $\mathscr{w}$ if $\mathbb{x} \in \mathscr{w}_{T_\varphi}^{A\alpha_x} \Rightarrow 1_w \in \mathscr{w}_{T_\rho}^{A\alpha_x}$, $\mathbb{x} \in \mathscr{w}_{I_\varphi}^{A\beta_x} \Rightarrow 1_w \in \mathscr{w}_{I_\rho}^{A\beta_x}$, $\mathbb{x} \in \mathscr{w}_{F_\varphi}^{A\gamma_x} \Rightarrow 1_w \in \mathscr{w}_{F_\rho}^{A\gamma_x}$ for all $\mathbb{x} \in \mathscr{w}$ and
$\mathbb{x} \curvearrowright \mathbb{y} \in \mathscr{w}_{T_\varphi}^{A\alpha_y}$, $\mathbb{x} \in \mathscr{w}_{T_\varphi}^{A\alpha_x} \Rightarrow \mathbb{y} \in \mathscr{w}_{T_\rho}^{A(\alpha_x \wedge \alpha_y)}$,
$\mathbb{x} \curvearrowright \mathbb{y} \in \mathscr{w}_{I_\varphi}^{A\beta_y}$, $\mathbb{x} \in \mathscr{w}_{I_\varphi}^{A\beta_x} \Rightarrow \mathbb{y} \in \mathscr{w}_{I_\rho}^{A(\beta_x \wedge \beta_y)}$ and
$\mathbb{x} \curvearrowright \mathbb{y} \in \mathscr{w}_{F_\varphi}^{A\gamma_y}$, $\mathbb{x} \in \mathscr{w}_{F_\varphi}^{A\gamma_x} \Rightarrow \mathbb{y} \in \mathscr{w}_{F_\rho}^{A(\gamma_x \wedge \gamma_y)}$ for all $\mathbb{x}, \mathbb{y} \in \mathscr{w}$ and
$\alpha_x, \alpha_y, \beta_x, \beta_y \in (0,1]$ and $\gamma_x, \gamma_y \in [0,1)$.

Theorem3.3: The neutrosophic set $A_w = (\mathscr{w}_T^A, \mathscr{w}_I^A, \mathscr{w}_F^A)$ on $\mathscr{w}$ is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathscr{w}$ if and only if A_w satisfies the following inequalities

(3.6) $\mathscr{w}_T^A(1_w) \geq \mathscr{w}_T^A(\mathfrak{x}_w) \wedge 0.5$,
$\mathscr{w}_I^A(1_w) \geq \mathscr{w}_I^A(\mathfrak{x}_w) \wedge 0.5$,
$\mathscr{w}_F^A(1_w) \leq \mathscr{w}_F^A(\mathfrak{x}_w) \vee 0.5 \forall \mathfrak{x}_w \in \mathscr{w}$.

(3.7) $\mathscr{w}_T^A(\mathfrak{y}_w) \geq \min\{\mathscr{w}_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_T^A(\mathfrak{x}_w), 0.5\}$
$\mathscr{w}_I^A(\mathfrak{y}_w) \geq \min\{\mathscr{w}_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_I^A(\mathfrak{x}_w), 0.5\}$
$\mathscr{w}_F^A(\mathfrak{y}_w) \leq \max\{\mathscr{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_F^A(\mathfrak{x}_w), 0.5\} \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathscr{w}$.

Proof: Suppose that neutrosophic set A_w on $\mathscr{w}$ is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathscr{w}$.
Case (i): Let $\mathbb{x} \in \mathscr{w}$ and assume that $\mathscr{w}_T^A(\mathbb{x}) < 0.5$. If $\mathscr{w}_T^A(1_w) < \mathscr{w}_T^A(\mathbb{x})$ then $\mathscr{w}_T^A(1_w) < \alpha_x < \mathscr{w}_T^A(\mathbb{x})$ for some $\alpha_x \in (0,0.5]$. It follows that $\mathbb{x} \in \mathscr{w}_{T_\varepsilon}^{A\alpha_x}$ and $1_w \notin \mathscr{w}_{T_\varepsilon}^{A\alpha_x}$. Also $\mathscr{w}_T^A(1_w) + \alpha_x < 1$, so $1_w \notin \mathscr{w}_{T_\delta}^{A\alpha_x}$ and hence $1_w \notin \mathscr{w}_{T_{\varepsilon\vee\delta}}^{A\alpha_x}$. Which is contradiction, and so $\mathscr{w}_T^A(1_w) \geq \mathscr{w}_T^A(\mathfrak{x}_w)$ $\forall \mathfrak{x}_w \in \mathscr{w}$.
Case (ii): If $\mathscr{w}_T^A(\mathbb{x}) \geq 0.5$ then $\mathbb{x} \in \mathscr{w}_{T_\varepsilon}^{A_{0.5}}$ and then $1_w \in \mathscr{w}_{T_{\varepsilon\vee\delta}}^{A_{0.5}}$.
Case (iii): If $\mathscr{w}_T^A(1_w) < 0.5$ then $\mathscr{w}_T^A(1_w) + \alpha_x < 1$, so $1_w \notin \mathscr{w}_{T_\delta}^{A\alpha_x}$. Which is contradiction and thus $\mathscr{w}_T^A(1_w) \geq 0.5$. Consequently, $\mathscr{w}_T^A(1_w) \geq \mathscr{w}_T^A(\mathfrak{x}_w) \wedge 0.5$ $\forall \mathfrak{x}_w \in \mathscr{w}$.
Similarly we can prove that $\mathscr{w}_I^A(1_w) \geq \mathscr{w}_I^A(\mathfrak{x}_w) \wedge 0.5$ $\forall \mathfrak{x}_w \in \mathscr{w}$.
Assume that $\mathbb{y} \in \mathscr{w}$ such that $\mathscr{w}_F^A(1_w) > \mathscr{w}_F^A(\mathbb{y}) \vee 0.5$. Then $\mathscr{w}_F^A(1_w) > \gamma_y > \mathscr{w}_F^A(\mathbb{y}) \vee 0.5$ for some $\gamma_y \in (0,1)$, which implies that $\gamma_y \geq 0.5$, $\mathbb{y} \in \mathscr{w}_{F_\varepsilon}^{A\gamma_y}$ and $1_w \notin \mathscr{w}_{F_\varepsilon}^{A\gamma_y}$. Since $1_w \notin \mathscr{w}_{F_\delta}^{A\gamma_y}$.
So $\mathscr{w}_F^A(1_w) \leq \mathscr{w}_F^A(\mathfrak{x}_w) \vee 0.5 \forall \mathfrak{x}_w \in \mathscr{w}$.
Let $\mathbb{a}$ *and* $\mathbb{b} \in \mathscr{w}$ such that $\mathscr{w}_T^A(\mathbb{b}) < \min\{\mathscr{w}_T^A(\mathbb{a} \curvearrowright \mathbb{b}), \mathscr{w}_T^A(\mathbb{a}), 0.5\}$ then $\mathscr{w}_T^A(\mathbb{b}) < \alpha < \min\{\mathscr{w}_T^A(\mathbb{a} \curvearrowright \mathbb{b}), \mathscr{w}_T^A(\mathbb{a}). 0.5\}$ for some $\alpha \in (0,1]$. It follows that $\mathscr{w}_T^A(\mathbb{a} \curvearrowright \mathbb{b}), \mathscr{w}_T^A(\mathbb{a}) \in \mathscr{w}_{T_\varepsilon}^{A\alpha}$ and $\notin \mathscr{w}_{T_\varepsilon}^{A\alpha}$. Since $\alpha \leq 0.5$ we have$\mathscr{w}_T^A(\mathbb{b}) + \alpha < 2\alpha \leq 1$ and $\mathbb{b} \notin \mathscr{w}_{T_\delta}^{A\alpha}$. This is a contradiction, so

$$\mathscr{w}_T^A(\mathfrak{y}_w) \geq \min\{\mathscr{w}_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_T^A(\mathfrak{x}_w), 0.5\} \ \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathscr{w}.$$

Similarly we can prove that $\mathscr{w}_I^A(\mathfrak{y}_w) \geq \min\{\mathscr{w}_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_I^A(\mathfrak{x}_w), 0.5\} \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathscr{w}$.
Now suppose that$\mathscr{w}_F^A(\mathbb{y}) > \max\{\mathscr{w}_F^A(\mathbb{x} \curvearrowright \mathbb{y}), \mathscr{w}_F^A(\mathbb{x}), 0.5\}$ for some $\mathbb{x}, \mathbb{y} \in \mathscr{w}$.Then there exist $\gamma \in (0,1)$ such that $\mathscr{w}_F^A(\mathbb{y}) > \gamma > \max\{\mathscr{w}_F^A(\mathbb{x} \curvearrowright \mathbb{y}), \mathscr{w}_F^A(\mathbb{x}), 0.5\}$. Thus $\gamma > 0.5$, $\mathbb{x} \curvearrowright \mathbb{y}$, $\mathbb{x} \in \mathscr{w}_{F_\varepsilon}^{A\gamma}$ then $\mathbb{y} \in \mathscr{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$. Since $\mathscr{w}_F^A(\mathbb{y}) > \gamma$ and $\mathscr{w}_F^A(\mathbb{y}) + \gamma > 2\gamma > 1$, we have $\mathbb{y} \notin \mathscr{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$ a contradiction, so $\mathscr{w}_F^A(\mathfrak{y}_w) \leq \max\{\mathscr{w}_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), \mathscr{w}_F^A(\mathfrak{x}_w), 0.5\} \forall \mathfrak{x}_w, \mathfrak{y}_w \in \mathscr{w}$.

Conversely, let the neutrosophic set $A_w = (\mathscr{w}_T^A, \mathscr{w}_I^A, \mathscr{w}_F^A)$ on $\mathscr{w}$ satisfies the conditions (3.6) and (3.7). For any $\mathbb{x}, \mathbb{y}$ *and* $\mathbb{z} \in \mathscr{w}$, let $\alpha, \beta \in (0,1]$ and $\gamma \in [0, 1)$ such that $\mathbb{x} \in \mathscr{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{y} \in \mathscr{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{z} \in \mathscr{w}_{F_\varepsilon}^{A\gamma}$. Then $\mathscr{w}_T^A(\mathbb{x}) \geq \alpha$, $\mathscr{w}_I^A(\mathbb{y}) \geq \beta$ and $\mathscr{w}_F^A(\mathbb{z}) \leq \gamma$.
Suppose that $\mathscr{w}_T^A(1_w) < \alpha$, $\mathscr{w}_I^A(1_w) < \beta$ and$\mathscr{w}_F^A(1_w) > \gamma$.
If$\mathscr{w}_T^A(\mathbb{x}) < 0.5$, then $\mathscr{w}_T^A(1_w) \geq \mathscr{w}_T^A(\mathbb{x}) \wedge 0.5 = \mathscr{w}_T^A(\mathbb{x})) \geq \alpha$ a contradiction. So we know that $\mathscr{w}_T^A(\mathbb{x}) \geq 0.5$ and so
$\mathscr{w}_T^A(1_w) + \alpha > 2\,\mathscr{w}_T^A(1_w) \geq 2(\mathscr{w}_T^A(\mathbb{x}) \wedge 0.5) = 1$.
Hence$1_w \in \mathscr{w}_{T_\delta}^{A\alpha} \subseteq \mathscr{w}_{T_{\varepsilon\vee\delta}}^{A\alpha}$. Similarly we can prove that $1_w \in \mathscr{w}_{I_{\varepsilon\vee\delta}}^{A\beta}$.
If $\mathscr{w}_F^A(\mathbb{x}) > 0.5$, then $\mathscr{w}_F^A(1_w) \leq \mathscr{w}_F^A(\mathbb{x}) \vee 0.5 = \mathscr{w}_F^A(\mathbb{x})) \geq \gamma$, which is contradiction. Thus $\mathscr{w}_F^A(\mathbb{x}) \leq 0.5$ and so $\mathscr{w}_F^A(1_w) + \gamma < 2\,\mathscr{w}_F^A(1_w) \geq 2(\mathscr{w}_F^A(\mathbb{x}) \vee 0.5) = 1$.
Hence $1_w \in \mathscr{w}_{F_\delta}^{A\gamma} \subseteq \mathscr{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$.
For any $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in \mathscr{w}$, let $\alpha_x, \alpha_y, \beta_{\mathbb{a}}, \beta_{\mathbb{b}} \in (0,1]$ and $\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}} \in [0,1)$ such that

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$x \curvearrowright y \in w_{T_\varepsilon}^{A_{\alpha_y}}$, $x \in w_{T_\varepsilon}^{A_{\alpha_x}}$, $a \curvearrowright b \in w_{I_\varepsilon}^{A_{\beta_b}}$, $a \in w_{I_\varepsilon}^{A_{\beta_a}}$ and $u \curvearrowright v \in w_{F_\varepsilon}^{A_{\gamma_v}}$, $u \in w_{F_\varepsilon}^{A_{\gamma_u}}$. Then $w_T^A(x \curvearrowright y) \geq \alpha_y$, $w_T^A(x) \geq \alpha_x$, $w_I^A(a \curvearrowright b) \geq \beta_b$, $w_I^A(a) \geq \beta_a$, $w_F^A(u \curvearrowright v) \leq \gamma_v$ and $w_F^A(u) \leq \gamma_u$.
Suppose that $w_T^A(y) < \alpha_x \wedge \alpha_y$. If $w_T^A(x \curvearrowright y) \wedge w_T^A(x) < 0.5$, then
$w_T^A(y) \geq \min\{w_T^A(x \curvearrowright y), w_T^A(x), 0.5\} = \min\{w_T^A(x \curvearrowright y), w_T^A(x)\} \geq \alpha_x \wedge \alpha_y$, which is a contradiction. Hence $w_T^A(x \curvearrowright y) \wedge w_T^A(x) \geq 0.5$ and so
$w_T^A(y) + \alpha_x \wedge \alpha_y > 2\, w_T^A(y) \geq 2 \min\{w_T^A(x \curvearrowright y), w_T^A(x), 0.5\} = 1$
This implies $y \in w_{T_\delta}^{A_{\alpha_x \wedge \alpha_y}} \subseteq w_{T_{\varepsilon \vee \delta}}^{A_{\alpha_x \wedge \alpha_y}}$.
Similarly we can prove that $b \in w_{I_\delta}^{A_{\beta_b \wedge \beta_a}} \subseteq w_{I_{\varepsilon \vee \delta}}^{A_{\beta_b \wedge \beta_a}}$.
Assume that $w_F^A(v) > \gamma_u \vee \gamma_v$, then $v \notin w_{F_\varepsilon}^{A_{\gamma_u \vee \gamma_v}}$. If $w_F^A(u \curvearrowright v) \vee w_F^A(u) > 0.5$, then
$w_F^A(v) \leq \max\{w_T^A(u \curvearrowright v), w_T^A(u), 0.5\} = \min\{w_T^A(u \curvearrowright v), w_T^A(u)\} \leq \gamma_u \vee \gamma_v$
which is contradiction. Hence $w_F^A(u \curvearrowright v) \vee w_F^A(u) \leq 0.5$ and so
$w_F^A(v) + \gamma_u \vee \gamma_v < 2 w_F^A(v) \leq 2\max\{w_T^A(u \curvearrowright v), w_T^A(u), 0.5\} = 1$
So $v \in w_{F_\delta}^{A_{\gamma_u \vee \gamma_v}} \subseteq w_{F_{\varepsilon \vee \delta}}^{A_{\gamma_u \vee \gamma_v}}$. Consequently A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w.

Theorem 3.4: Every $(\varepsilon, \varepsilon \vee \delta)$ -filter of w fulfills the following result:
(2.5) If $x_w \curvearrowright (y \curvearrowright z) = 1_w$ then
$w_T^A(z) \geq \min\{w_T^A(x), w_T^A\, y).0.5\}$,
$w_I^A(z) \geq \min\{w_I^A(x), w_I^A(y), 0.5\}$ and
$w_F^A(z) \leq \max\{w_F^A(x)), w_F^A(y), 0.5\}$ $\forall x, y, z \in w$.
Proof: Suppose A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w and $x \curvearrowright(\curvearrowright z_w) = 1_w$ for all x, y and $z \in w$.
We get $w_T^A(z) \geq \min\{w_T^A(y \curvearrowright z), w_T^A(y), 0.5\}$
$\geq \min\{\min\{w_T^A(x), w_T^A(x \curvearrowright (y \curvearrowright z))\}, w_T^A(y), 0.5\}$
$\geq \min\{\min\{w_T^A(x), w_T^A(1_w)\}, w_T^A(y), 0.5\}$
$\geq \min\{w_T^A(x), w_T^A(y), 0.5\}$
$w_I^A(z) \geq \min\{w_I^A(y \curvearrowright z), w_I^A(y, 0.5\}$
$\geq \min\{\min\{w_I^A(x), w_I^A(x \curvearrowright (y \curvearrowright z))\}, w_I^A(y), 0.5\}$
$\geq \min\{\min\{w_I^A(x), w_I^A(1_w)\}, w_I^A(y), 0.5\}$
$\geq \min\{w_I^A(x), w_I^A(y), 0.5\}$, and
$w_F^A(z) \leq \max\{w_F^A(y \curvearrowright z), w_F^A(y), 0.5\}$
$\leq \max\{\max\{w_F^A(x), w_F^A(x \curvearrowright (y \curvearrowright z))\}, w_F^A(y), 0.5\}$
$\leq \max\{\max\{w_F^A(x), w_F^A(1_w)\}, w_F^A(y), 0.5\}$
$\leq \max\{w_F^A(x), w_F^A(y), 0.5\}$.

Lemma 3.5: Every $(\varepsilon, \varepsilon \vee \delta)$ -filter of w fulfills the following result:
If $x_{n_w} \curvearrowright (x_{(n-1)_w} \curvearrowright$--------- $(x_{1_w} \curvearrowright y_w)) = 1_w$ then
$w_T^A(y_w) \geq \min\{w_T^A(x_{n_w})$,-----------, $w_T^A(x_{1_w})\}$,
$w_I^A(y_w) \geq \min\{w_I^A(x_{n_w})$, ------------, $w_I^A(x_{1_w})\}$, and
$w_F^A(y_w) \leq \max\{w_F^A(x_{n_w})$,----------, $w_F^A(x_{1_w})\}$ $\forall x_{n_w}$, ----------, $x_{1_w} y_w \in w$.
Theorem3.6: The neutrosophic set $A_w = (w_T^A, w_I^A, w_F^A)$ on w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w if and only if the nonempty ε – cut sets are filters of w for all $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$.
Proof: Suppose that A_w on w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w.
Let $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$ such that ε – cut sets are nonempty.
Using (3.6), we have $w_T^A(1_w) \geq w_T^A(x) \wedge 0.5$ $\forall x \in w_{T_\varepsilon}^{A_\alpha}$,
$w_I^A(1_w) \geq w_I^A(y) \wedge 0.5$ $\forall y \in w_{I_\varepsilon}^{A_\beta}$,
$w_F^A(1_w) \leq w_F^A(z)) \vee 0.5 \forall z \in w_{F_\varepsilon}^{A_\gamma}$.
It follows that $w_T^A(1_w) \geq \alpha \wedge 0.5 = \alpha$, $w_I^A(1_w) \geq \beta \wedge 0.5 = \beta$ and $w_F^A(1_w) \leq \gamma \vee 0.5 = \gamma$
That is $1_w \in w_{T_\varepsilon}^{A_\alpha}$, $1_w \in w_{I_\varepsilon}^{A_\beta}$ and $1_w \in w_{F_\varepsilon}^{A_\gamma}$. Let x, y, a, b, u and $v \in w$ such that $x \curvearrowright y$, $x \in w_{T_\varepsilon}^{A_\alpha}$, $a \curvearrowright b$, $a \in w_{I_\varepsilon}^{A_\beta}$ and $u \curvearrowright v$, $u \in w_{F_\varepsilon}^{A_\gamma}$ for $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$.
Then $w_T^A(x \curvearrowright y) \geq \alpha$, $w_T^A(x) \geq \alpha$,
$w_I^A(a \curvearrowright b) \geq \beta$, $w_I^A(a) \geq \beta$ and
$w_F^A(u \curvearrowright v) \leq \gamma$, $w_F^A(u) \leq \gamma$.

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It follows from (3.7) that

$w_T^A(y) \geq \min\{w_T^A(x \curvearrowright y), w_T^A(x), 0.5\} \geq \alpha \wedge 0.5 = \alpha$

$w_I^A(b) \geq \min\{w_I^A(a \curvearrowright b), w_I^A(a), 0.5\} \geq \beta \wedge 0.5 = \beta$

$w_F^A(v) \leq \max\{w_F^A(u \curvearrowright v), w_F^A(u), 0.5 \leq \gamma \vee 0.5 = \gamma$

and so $y \in w_{T_\varepsilon}^{A\alpha}$, $b \in w_{I_\varepsilon}^{A\beta}$, $v \in w_{F_\varepsilon}^{A\gamma}$.

Hence the nonempty ε – cut sets are filters of w for all $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$.

Conversely let A_w be a neutrosophic set on w such that the nonempty ε – cut sets are filters of w for all $\alpha, \beta \in (0.5,1]$ and $\gamma \in [0,0.5)$. If there x, y *and* $z \in w$ such that

$w_T^A(1_w) < w_T^A(x) \wedge 0.5$,

$w_I^A(1_w) < w_I^A(y) \wedge 0.5$,

$w_F^A(1_w) > w_F^A(z)) \vee 0.5$.

Then $w_T^A(1_w) < \alpha_x \leq w_T^A(x) \wedge 0.5$,

$w_I^A(1_w) < \beta_y \leq w_I^A(y) \wedge 0.5$,

$w_F^A(1_w) > \gamma_z \geq w_F^A(z)) \vee 0.5$ for some $\alpha_x, \beta_y \in (0.5,1]$ and $\gamma_z \in [0,0.5)$.

So $1_w \notin w_{T_\varepsilon}^{A\alpha}$, $1_w \notin w_{I_\varepsilon}^{A\beta}$ and $1_w \notin w_{F_\varepsilon}^{A\gamma}$, which is contradiction.

Therefore $w_T^A(1_w) \geq w_T^A(\mathfrak{x}_w) \wedge 0.5$,

$w_I^A(1_w) \geq w_I^A(\mathfrak{x}_w) \wedge 0.5$,

$w_F^A(1_w) \leq w_F^A(\mathfrak{x}_w) \vee 0.5 \,\forall\, \mathfrak{x}_w \in w$.

Assume that x, y, a, b, u *and* $v \in w$ such that

$w_T^A(y) < \min\{w_T^A(x \curvearrowright y), w_T^A(x), 0.5\}$

$w_I^A(b) < \min\{w_I^A(a \curvearrowright b), w_I^A(a), 0.5\}$

$w_F^A(v) > \max\{w_F^A(u \curvearrowright v), w_F^A(u), 0.5\}$.

Taking $\alpha = \frac{1}{2}(w_T^A(y), \min\{w_T^A(x \curvearrowright y), w_T^A(x), 0.5\})$, implies $\alpha \in (0.5,1]$ and

$w_T^A(y) < \alpha < \min\{w_T^A(x \curvearrowright y), w_T^A(x), 0.5\}$. Then $x \curvearrowright y$, $x \in w_{T_\varepsilon}^{A\alpha}$ but $y \notin w_{T_\varepsilon}^{A\alpha}$,

which is contradiction. So $w_T^A(\mathfrak{y}_w) \geq \min\{w_T^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_T^A(\mathfrak{x}_w), 0.5\}\} \forall\, \mathfrak{x}_w, \mathfrak{y}_w \in w$.

Similarly we can prove that $w_I^A(\mathfrak{y}_w) \geq \min\{w_I^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_I^A(\mathfrak{x}_w), 0.5\} \forall\, \mathfrak{x}_w, \mathfrak{y}_w \in w$

Taking $\gamma = \max\{w_F^A(u \curvearrowright v), w_F^A(u), 0.5\}$, implies $\gamma \in [0,0.5)$ and then $u \curvearrowright v$, $u \in w_{F_\varepsilon}^{A\gamma}$, clearly $v \notin w_{F_\varepsilon}^{A\gamma}$.

This is a contradiction. Therefore

$w_F^A(\mathfrak{y}_w) \leq \max\{w_F^A(\mathfrak{x}_w \curvearrowright \mathfrak{y}_w), w_F^A(\mathfrak{x}_w), 0.5\} \forall\, \mathfrak{x}_w, \mathfrak{y}_w \in w$.

Then by (3.3) neutrosophic set A_w on w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w.

Clearly every $(\varepsilon, \varepsilon)$ -filter of w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w. But $(\varepsilon, \varepsilon \vee \delta)$ -filter of w need not to be a $(\varepsilon, \varepsilon)$ -filter of w as shown in the following example:

Example 3.7:: Let $w = \{0_w, \mathfrak{x}_w, \mathfrak{y}_w, \mathfrak{z}_w, \mathfrak{v}_w, 1_w\}$ is lattice wajsberg algebra with the binary operation $\curvearrowright$ as follows:

$\curvearrowright$	0_w	$\mathfrak{x}_w$	$\mathfrak{y}_w$	$\mathfrak{z}_w$	$\mathfrak{v}_w$	1_w
0_w	1_w	1_w	1_w	1_w	1_w	1_w
$\mathfrak{x}_w$	$\mathfrak{z}_w$	1_w	$\mathfrak{y}_w$	$\mathfrak{z}_w$	$\mathfrak{y}_w$	1_w
$\mathfrak{y}_w$	$\mathfrak{v}_w$	$\mathfrak{x}_w$	1_w	$\mathfrak{y}_w$	$\mathfrak{x}_w$	1_w
$\mathfrak{z}_w$	$\mathfrak{x}_w$	$\mathfrak{x}_w$	1_w	1_w	$\mathfrak{x}_w$	1_w
$\mathfrak{v}_w$	$\mathfrak{y}_w$	1_w	1_w	$\mathfrak{y}_w$	1_w	1_w
1_w	0_w	$\mathfrak{x}_w$	$\mathfrak{y}_w$	$\mathfrak{z}_w$	$\mathfrak{v}_w$	1_w

The neutrosophic set $A_w = (w_T^A, w_I^A, w_F^A)$ defined on w as follows :

$\mathfrak{z}$	$w_T^A(\mathfrak{z})$	$w_I^A(\mathfrak{z})$	$w_F^A(\mathfrak{z})$
0_w	.1	.2	. 7
$\mathfrak{x}_w$	.1	.6	. 65
$\mathfrak{y}_w$	.4	.2	. 7
$\mathfrak{z}_w$	.4	.2	. 7
$\mathfrak{v}_w$	.1	.2	. 7
1_w	.6	.5	. 4

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$\mathcal{w}_{T_\varepsilon}^{A\alpha}$ = $\mathcal{w}$ if $\alpha \in (0, 0.1]$
= {1_w, $\mathfrak{y}_w$, $\mathfrak{z}_w$ }if $\alpha \in (0.1,0.4]$
= {1_w} if $\alpha \in (0.4,0.5]$
$\mathcal{w}_{I_\varepsilon}^{A\beta}$ = $\mathcal{w}$ if $\beta \in (0, 0.2]$
= {1_w, $\mathfrak{x}_w$ }if $\beta \in (0.2,0.5]$
$\mathcal{w}_{F_\varepsilon}^{A\gamma}$ = $\mathcal{w}$ if $\gamma \in [0.7,1)$
= {1_w, $\mathfrak{x}_w$ }if $\gamma \in [0.7,0.7)$
= {1_w} if $\gamma \in [0.5,0.6)$
which are filters of $\mathcal{w}$ for all $\alpha, \beta \in (0,0.5]$ and $\gamma \in [0.5,1)$. Hence A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$. But it is not a $(\varepsilon, \varepsilon)$ -filter of $\mathcal{w}$ since $\mathfrak{x}_w \in \mathcal{w}_{I_\varepsilon}^{A_{0.6}}$ but $1_w \notin \mathcal{w}_{I_\varepsilon}^{A_{0.6}}$.

Theorem 3.8: If the neutrosophic set $A_w = (\mathcal{w}_T^A, \mathcal{w}_I^A, \mathcal{w}_F^A)$ on $\mathcal{w}$ is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$ such that $\mathcal{w}_T^A(\mathfrak{x}_w) < 0.5$, $\mathcal{w}_I^A(\mathfrak{x}_w) < 0.5$ and $\mathcal{w}_F^A(\mathfrak{x}_w) > 0.5$ $\forall\, \mathfrak{x}_w \in \mathcal{w}$ then it is a $(\varepsilon, \varepsilon)$ -filter of $\mathcal{w}$.

Proof: Let $\mathbb{x}, \mathbb{y}$ *and* $\mathbb{z} \in \mathcal{w}$, $\alpha, \beta \in (0,1]$ and $\gamma \in [0,1)$ such that $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{y} \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{z} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$. Then $\mathcal{w}_T^A(\mathbb{x}) \geq \alpha$, $\mathcal{w}_I^A(\mathbb{y}) \geq \beta$ and $\mathcal{w}_F^A(\mathbb{z}) \leq \gamma$,
Which implies from (3.6) $\mathcal{w}_T^A(1_w) \geq \mathcal{w}_T^A(\mathbb{x}) \wedge 0.5 = \mathcal{w}_T^A(\mathbb{x}) \geq \alpha$,

$$\mathcal{w}_I^A(1_w) \geq \mathcal{w}_I^A(\mathbb{y}) \wedge 0.5 = \mathcal{w}_I^A(\mathbb{y}) \geq \beta,$$
$$\mathcal{w}_F^A(1_w) \leq \mathcal{w}_F^A(\mathbb{z})) \vee 0.5 = \mathcal{w}_F^A(\mathbb{z}) \leq \gamma.$$

It follows that $1_w \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $1_w \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $1_w \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$.
For any $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in \mathcal{w}$, $\alpha_x, \alpha_y, \beta_{\mathbb{a}}, \beta_{\mathbb{b}} \in (0,1]$ and $\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}} \in [0,1)$ such that
$\mathbb{x} \curvearrowright \mathbb{y} \in \mathcal{w}_{T_\varepsilon}^{A\alpha_y}$, $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A\alpha_x}$, $\mathbb{a} \curvearrowright \mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A\beta_{\mathbb{b}}}$, $\mathbb{a} \in \mathcal{w}_{I_\varepsilon}^{A\beta_{\mathbb{a}}}$ and $\mathbb{u} \curvearrowright \mathbb{v} \in \mathcal{w}_{F_\varepsilon}^{A\gamma_{\mathbb{v}}}$, $\mathbb{u} \in \mathcal{w}_{F_\varepsilon}^{A\gamma_{\mathbb{u}}}$.
Then $\mathcal{w}_T^A(\mathbb{x} \curvearrowright \mathbb{y}) \geq \alpha_y$, $\mathcal{w}_T^A(\mathbb{x}) \geq \alpha_x$, $\mathcal{w}_I^A(\mathbb{a} \curvearrowright \mathbb{b}) \geq \beta_{\mathbb{b}}$, $\mathcal{w}_I^A(\mathbb{a}) \geq \beta_{\mathbb{a}}$,
$\mathcal{w}_F^A(\mathbb{u} \curvearrowright \mathbb{v}) \leq \gamma_{\mathbb{v}}$ *and* $\mathcal{w}_F^A(\mathbb{u}) \leq \gamma_{\mathbb{u}}$. By (3.7), we have
$\mathcal{w}_F^A(\mathbb{y}) \geq \max\{\mathcal{w}_F^A(\mathbb{x} \curvearrowright \mathbb{y}), \mathcal{w}_F^A(\mathbb{x}), 0.5\} = \max\{\mathcal{w}_F^A(\mathbb{x} \curvearrowright \mathbb{y}), \mathcal{w}_F^A(\mathbb{x})\} \geq \alpha_x \wedge \alpha_y$. So $\mathbb{y} \in \mathcal{w}_{T_\varepsilon}^{A\alpha_x \wedge \alpha_y}$.
Similarly we can prove that $\mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A\beta_{\mathbb{a}} \wedge \beta_{\mathbb{b}}}$ and $\mathbb{v} \in \mathcal{w}_{F_\varepsilon}^{A\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}$.
Therefore A_w is a $(\varepsilon, \varepsilon)$ -filter of $\mathcal{w}$.

Theorem 3.9: Every $(\varepsilon \vee \delta, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$ is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$.

Proof: Let A_w be a $(\varepsilon \vee \delta, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$. Let $\mathbb{x}, \mathbb{y}$ *and* $\mathbb{z} \in \mathcal{w}$, $\alpha, \beta \in (0,1]$ and $\gamma \in [0,1)$ such that $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $\mathbb{y} \in \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $\mathbb{z} \in \mathcal{w}_{F_\varepsilon}^{A\gamma}$. Then clearly $\mathbb{x} \in \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha}$, $\mathbb{y} \in \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta}$ and $\mathbb{z} \in \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$. It follows from(3.4)that $1_w \in \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha}$, $1_w \in \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta}$ and $1_w \in \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$. For any $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in \mathcal{w}$, $\alpha_x, \alpha_y, \beta_{\mathbb{a}}, \beta_{\mathbb{b}} \in (0,1]$ and $\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}} \in [0,1)$ such that
$\mathbb{x} \curvearrowright \mathbb{y} \in \mathcal{w}_{T_\varepsilon}^{A\alpha_y}$, $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A\alpha_x}$, $\mathbb{a} \curvearrowright \mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A\beta_{\mathbb{b}}}$, $\mathbb{a} \in \mathcal{w}_{I_\varepsilon}^{A\beta_{\mathbb{a}}}$ and $\mathbb{u} \curvearrowright \mathbb{v} \in \mathcal{w}_{F_\varepsilon}^{A\gamma_{\mathbb{v}}}$, $\mathbb{u} \in \mathcal{w}_{F_\varepsilon}^{A\gamma_{\mathbb{u}}}$.
Then $\mathbb{x} \curvearrowright \mathbb{y} \in \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha_y}$, $\mathbb{x} \in \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha_x}$, $\mathbb{a} \curvearrowright \mathbb{b} \in \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta_{\mathbb{b}}}$, $\mathbb{a} \in \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta_{\mathbb{a}}}$ and $\mathbb{u} \curvearrowright \mathbb{v} \in \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma_{\mathbb{v}}}$, $\mathbb{u} \in \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma_{\mathbb{u}}}$. Then clearly $\mathbb{y} \in \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha}$, $\mathbb{b} \in \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta}$, $\mathbb{v} \in \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma}$. Therefore A_w be a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$.
S

Theorem3.10: For a neutrosophic set $A_w = (\mathcal{w}_T^A, \mathcal{w}_I^A, \mathcal{w}_F^A)$ on $\mathcal{w}$, if the $\varepsilon \vee \delta$- cut sets are filters of $\mathcal{w}$ for all $\alpha, \beta \in (0,0.5]$ and $\gamma \in [0.5,1)$, then A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$.

Proof: Let $A_w = (\mathcal{w}_T^A, \mathcal{w}_I^A, \mathcal{w}_F^A)$ is a neutrosophic set on $\mathcal{w}$ such that the nonempty $\varepsilon \vee \delta$- cut sets are filters of $\mathcal{w}$ for all $\alpha, \beta \in (0,0.5]$ and $\gamma \in [0.5,1)$.
Assume that $\mathcal{w}_T^A(1_w) < \mathcal{w}_T^A(\mathbb{x}) \wedge 0.5 = \alpha_x$,

$$\mathcal{w}_I^A(1_w) < \mathcal{w}_I^A(\mathbb{y}) \wedge 0.5 = \beta_y,$$
$$\mathcal{w}_F^A(1_w) > \mathcal{w}_F^A(\mathbb{z})) \vee 0.5 = \gamma_z \text{ for some } \mathbb{x}, \mathbb{y}, \mathbb{z} \in \mathcal{w}$$

Then α_x, $\beta_y \in (0,0.5]$ and $\gamma_z \in [0.5,1)$, $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A\alpha_x} \subseteq \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A\alpha_x}$, $\mathbb{y} \in \mathcal{w}_{I_\varepsilon}^{A\beta_y} \subseteq \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A\beta_y}$ and $\mathbb{z} \in \mathcal{w}_{F_\varepsilon}^{A\gamma_z} \subseteq \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A\gamma_z}$, $1_w \notin \mathcal{w}_{T_\varepsilon}^{A\alpha}$, $1_w \notin \mathcal{w}_{I_\varepsilon}^{A\beta}$ and $1_w \notin \mathcal{w}_{F_\varepsilon}^{A\gamma}$,
since $\mathcal{w}_T^A(1_w) + \alpha_x < 2\alpha_x \leq 1$, that is $1_w \notin \mathcal{w}_{T_\delta}^{A\alpha}$
$\mathcal{w}_I^A(1_w) + \beta_y < 2\beta_y \leq 1$, that is $1_w \notin \mathcal{w}_{I_\delta}^{A\beta}$,
$\mathcal{w}_F^A(1_w) + \gamma_z > 2\gamma_z \geq 1$, tha tis $1_w \notin \mathcal{w}_{F_\delta}^{A\gamma}$.

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Then clearly $1_w \notin w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $1_w \notin w_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $1_w \notin w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$, which is a contradiction.
so (3.6) is valid. Assume that there exist $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in w$ such that

$w_T^A(\mathbb{y}) < \min\{w_T^A(\mathbb{x} \curvearrowright \mathbb{y}), w_T^A(\mathbb{x}), 0.5\} = \alpha$

$w_I^A(\mathbb{b}) < \min\{w_I^A(\mathbb{a} \curvearrowright \mathbb{b}), w_I^A(\mathbb{a}), 0.5\} = \beta$

$w_F^A(\mathbb{v}) > \max\{w_F^A(\mathbb{u} \curvearrowright \mathbb{v}), w_F^A(\mathbb{u}), 0.5\} = \gamma$.

Then $\alpha, \beta \in (0,0.5]$ and $\gamma \in [0.5,1)$, $\mathbb{y} \notin w_{T_\varepsilon}^{A_\alpha}$, $\mathbb{b} \notin w_{I_\varepsilon}^{A_\beta}$ and $\mathbb{v} \notin w_{F_\varepsilon}^{A_\gamma}$, and

$\mathbb{x} \curvearrowright \mathbb{y} \in w_{T_\varepsilon}^{A_\alpha} \subseteq w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $\mathbb{x} \in w_{T_\varepsilon}^{A_\alpha} \subseteq w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$,

$\mathbb{a} \curvearrowright \mathbb{b} \in w_{I_\varepsilon}^{A_\beta} \subseteq w_{I_{\varepsilon\vee\delta}}^{A_\beta}$, $\mathbb{a} \in w_{I_\varepsilon}^{A_\beta} \subseteq w_{I_{\varepsilon\vee\delta}}^{A_\beta}$,

$\mathbb{u} \curvearrowright \mathbb{v} \in w_{F_\varepsilon}^{A_\gamma} \subseteq w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$, $\mathbb{u} \in w_{F_\varepsilon}^{A_\gamma} \subseteq w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$ ---------- (3.9)

Since $w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $w_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$ are filters of w for all $\alpha, \beta \in (0,0.5]$ and $\gamma \in [0.5,1)$, from (3.9), we have $\mathbb{y} \in w_{T_\varepsilon}^{A_\alpha} \subseteq w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $\mathbb{b} \in w_{I_\varepsilon}^{A_\beta} \subseteq w_{I_{\varepsilon\vee\delta}}^{A_\beta}$, $\mathbb{v} \in w_{F_\varepsilon}^{A_\gamma} \subseteq w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$. On the other hand ,
$w_T^A(\mathbb{y}) + \alpha < \alpha \leq 1$, $w_I^A(\mathbb{b}) + \beta < \beta \leq 1$, $w_F^A(\mathbb{v}) + \gamma_z > 2\gamma \geq 1$, that is $\mathbb{y} \notin w_{T_\delta}^{A_\alpha}$, $\mathbb{b} \notin w_{I_\delta}^{A_\beta}$ and $\mathbb{v} \notin w_{F_\delta}^{A_\gamma}$. So $\mathbb{y} \notin w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $\mathbb{b} \notin w_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $\mathbb{v} \notin w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$, which is a contradiction. So (3.7) is valid. By theorem 3.3, A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w.

Theorem3.11: For a subset F of w, let A_w be a newtrosophic set on w suchthat

$w_T^A(1_w) \geq w_T^A(\mathfrak{x}_w)$, $w_I^A(1_w) \geq w_I^A(\mathfrak{x}_w)$and $w_F^A(1_w) \leq w_F^A(\mathfrak{x}_w) \forall\, \mathfrak{x}_w \in w$…… (3.10)

$w_T^A(\mathfrak{x}_w) \geq 0.5$, $w_I^A(\mathfrak{x}_w) \geq 0.5$and $w_F^A(\mathfrak{x}_w) \leq 0.5$if $\mathfrak{x}_w \in$ F…… (3.11)

$w_T^A(\mathfrak{x}_w) = 0$, $w_I^A(\mathfrak{x}_w) = 0$and $w_F^A(\mathfrak{x}_w) = 1$ if $\mathfrak{x}_w \in w$ / F…… (3.12).

If F is a filter of w then A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w.

Proof: Let F is a filter of w. Let $\mathbb{x}, \mathbb{y}$ *and* $\mathbb{z} \in w$, $\alpha, \beta \in (0,1]$ and $\gamma \in [0,1)$ such that $\mathbb{x} \in w_{T_\delta}^{A_\alpha}$, $\mathbb{y} \in w_{I_\delta}^{A_\beta}$ and $\mathbb{z} \in w_{F_\delta}^{A_\gamma}$ then

$w_T^A(\mathbb{x}) + \alpha > 1$, $w_I^A(\mathbb{y}) + \beta > 1$ and $w_F^A(\mathbb{z}) + \gamma < 1$ …….(3.13)

From (3.10) and (3.13), we have

$w_T^A(1_w) + \alpha \geq w_T^A(\mathbb{x}) + \alpha > 1$,

$w_I^A(1_w) + \beta \geq w_I^A(\mathbb{y}) + \beta > 1$,

$w_F^A(1_w) + \gamma \leq w_F^A(\mathbb{z}) + \gamma < 1$.

That is $1_w \in w_{T_\delta}^{A_\alpha} \subseteq w_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $1_w \in w_{I_\delta}^{A_\beta} \subseteq w_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $1_w \in w_{F_\delta}^{A_\gamma} \subseteq w_{F_{\varepsilon\vee\delta}}^{A_\gamma}$.

For any $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in w$, let $\alpha_x, \alpha_y, \beta_{\mathbb{a}}, \beta_{\mathbb{b}} \in (0,1]$ and $\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}} \in [0,1)$ be such that

$\mathbb{x} \curvearrowright \mathbb{y} \in w_{T_\delta}^{A_{\alpha_y}}$, $\mathbb{x} \in w_{T_\delta}^{A_{\alpha_x}}$, $\mathbb{a} \curvearrowright \mathbb{b} \in w_{I_\delta}^{A_{\beta_{\mathbb{b}}}}$, $\mathbb{a} \in w_{I_\delta}^{A_{\beta_{\mathbb{a}}}}$ and $\mathbb{u} \curvearrowright \mathbb{v} \in w_{F_\delta}^{A_{\gamma_{\mathbb{v}}}}$, $\mathbb{u} \in w_{F_\delta}^{A_{\gamma_{\mathbb{u}}}}$.

Then $w_T^A(\mathbb{x} \curvearrowright \mathbb{y}) + \alpha_y > 1$, $w_T^A(\mathbb{x}) + \alpha_x > 1$, $w_I^A(\mathbb{a} \curvearrowright \mathbb{b}) + \beta_{\mathbb{b}} > 1$, $w_I^A(\mathbb{a}) + \beta_{\mathbb{a}} > 1$,

$w_F^A(\mathbb{u} \curvearrowright \mathbb{v}) + \gamma_{\mathbb{v}} < 1$ and $w_F^A(\mathbb{u}) + \gamma_{\mathbb{u}} < 1$.

If $\mathbb{x} \curvearrowright \mathbb{y} \notin$F or $\mathbb{x} \notin$ F then $w_T^A(\mathbb{x} \curvearrowright \mathbb{y}) = 0$, $w_T^A(\mathbb{x}) = 0$. It follows that

$w_T^A(\mathbb{x} \curvearrowright \mathbb{y}) + \alpha_y = \alpha_y \leq 1$ or $w_T^A(\mathbb{x}) + \alpha_x = \alpha_x \leq 1$.

This a contradiction, and so $\mathbb{x} \curvearrowright \mathbb{y} \in$F or $\mathbb{x} \in$ F. Similarly we can prove that $\mathbb{a} \curvearrowright \mathbb{b} \in$ F, $\mathbb{a} \in$ F. If $\mathbb{u} \curvearrowright \mathbb{v} \notin$F or $\mathbb{u} \notin$F, then $w_F^A(\mathbb{u} \curvearrowright \mathbb{v}) = 1$ and$w_F^A(\mathbb{u}) = 1$. We have

$w_F^A(\mathbb{u} \curvearrowright \mathbb{v}) + \gamma_{\mathbb{v}} = 1 + \gamma_{\mathbb{v}} \geq 1orw_F^A(\mathbb{u}) + \gamma_{\mathbb{u}} = 1 + \gamma_{\mathbb{u}} \geq 1$.

This a contradiction, and so $\mathbb{u} \curvearrowright \mathbb{v} \in$F or $\mathbb{u} \in$F . Since F is a filter of w, we get $\mathbb{y}, \mathbb{b}$ *and* $\mathbb{v} \in$F. So $w_T^A(\mathbb{y}) \geq 0.5$, $w_I^A(\mathbb{b}) \geq 0.5$ and $w_F^A(\mathbb{v}) \leq 0.5$.

If $\alpha_x \leq 0.5$ *or* $\alpha_y \leq 0.5$, then $w_T^A(\mathbb{y}) \geq 0.5 \geq \alpha_x \wedge \alpha_y$, that is $\mathbb{y} \in w_{T_\varepsilon}^{A_{\alpha_x \wedge \alpha_y}} \subseteq w_{T_{\varepsilon\vee\delta}}^{A_{\alpha_x \wedge \alpha_y}}$ and if $\alpha_x > 0.5$ *or* $\alpha_y > 0.5$, then $w_T^A(\mathbb{y}) + \alpha_x \wedge \alpha_y > 1$,that is $\mathbb{y} \in w_{T_\delta}^{A_{\alpha_x \wedge \alpha_y}} \subseteq w_{T_{\varepsilon\vee\delta}}^{A_{\alpha_x \wedge \alpha_y}}$.

So $\mathbb{y} \in w_{T_{\varepsilon\vee\delta}}^{A_{\alpha_x \wedge \alpha_y}}$. Similarly we have $\mathbb{b} \in w_{I_{\varepsilon\vee\delta}}^{A_{\beta_x \wedge \beta_y}}$.

If $\gamma_{\mathbb{u}} \geq 0.5$ or $\gamma_{\mathbb{v}} \geq 0.5$, then $w_F^A(\mathbb{v}) \leq 0.5 \leq \gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}$, so $\mathbb{v} \in w_{F_\varepsilon}^{A_{\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}} \subseteq w_{F_{\varepsilon\vee\delta}}^{A_{\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}}$ and

If $\gamma_{\mathbb{u}} < 0.5$ or $\gamma_{\mathbb{v}} < 0.5$, then $w_F^A(\mathbb{v}) + \gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}} \leq 1$, so $\mathbb{v} \in w_{F_\delta}^{A_{\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}} \subseteq w_{F_{\varepsilon\vee\delta}}^{A_{\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}}$.

Hence A_w is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of w.

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Theorem3.12: Every $(\delta, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$ is a $(\varepsilon, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$.

Proof: Let A_w is a $(\delta, \varepsilon \vee \delta)$ -filter of $\mathcal{w}$.For any $\mathbb{x}, \mathbb{y}, \mathbb{z} \in \mathcal{w}$, let α_x , $\beta_y \in (0,1]$ and $\gamma_z \in [0,1)$ be such that $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A_{\alpha_x}}$, $\mathbb{y} \in \mathcal{w}_{I_\varepsilon}^{A_{\beta_y}}$ and $\mathbb{z} \in \mathcal{w}_{F_\varepsilon}^{A_{\gamma_z}}$. Then $\mathcal{w}_T^A(\mathbb{x}) \geq \alpha_x$, $\mathcal{w}_I^A(\mathbb{y}) \geq \beta_y$ and $\mathcal{w}_F^A(\mathbb{z}) \leq \gamma_z$. Suppose $1_w \notin \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A_{\alpha_x}}$,$1_w \notin \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A_{\beta_y}}$ and$1_w \notin \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A_{\gamma_z}}$, then

$\mathcal{w}_T^A(1_w) < \alpha_x$,$\mathcal{w}_I^A(1_w) < \beta_y$ and $\mathcal{w}_F^A(1_w) > \gamma_z$ ------------ (3.14)

$\mathcal{w}_T^A(1_w) + \alpha_x \leq 1$, $\mathcal{w}_I^A(1_w) + \beta_y \leq 1$ $\mathcal{w}_F^A(1_w) + \gamma_z \geq 1$ --------- (3.15)

It follows that $\mathcal{w}_T^A(1_w) < 0.5$,$\mathcal{w}_I^A(1_w) < 0.5$and $\mathcal{w}_F^A(1_w) > 0.5$ ----------- (3.16)

From (3.14) and (3.16), we have

$\mathcal{w}_T^A(1_w) < \alpha_x \wedge 0.5$,$\mathcal{w}_I^A(1_w) < \beta_y \wedge 0.5$ and $\mathcal{w}_F^A(1_w) > \gamma_z \vee 0.5$, and so

$1-\mathcal{w}_T^A(1_w) > 1-(\alpha_x \wedge 0.5) = (1-\alpha_x) \vee 0.5 \geq (1-\mathcal{w}_T^A(\mathbb{x})) \vee 0.5$,

$1-\mathcal{w}_I^A(1_w) > 1-(\beta_y \wedge 0.5) = (1-\beta_y) \vee 0.5 \geq (1-\mathcal{w}_I^A(\mathbb{y})) \vee 0.5$,

$1-\mathcal{w}_F^A(1_w) < 1-(\gamma_z \vee 0.5) = (1-\gamma_z) \wedge 0.5 \leq (1-\mathcal{w}_F^A(\mathbb{z})) \wedge 0.5$.

Hence there exist $\alpha, \beta \in (0,1]$ and $\gamma \in [0,1)$ such that

$1-\mathcal{w}_T^A(1_w) \geq \alpha > (1-\mathcal{w}_T^A(\mathbb{x})) \vee 0.5$,

$1-\mathcal{w}_I^A(1_w) \geq \beta > (1-\mathcal{w}_I^A(\mathbb{y})) \vee 0.5$,and

$1-\mathcal{w}_F^A(1_w) \leq \gamma < (1-\mathcal{w}_F^A(\mathbb{z})) \wedge 0.5$. ---------------------(3.17)

The right inequalities in(3.17), we have

$\mathcal{w}_T^A(\mathbb{x}) + \alpha > 1$, $\mathcal{w}_I^A(\mathbb{y}) + \beta > 1$ and $\mathcal{w}_F^A(\mathbb{z}) + \gamma < 1$, that is $\mathbb{x} \in \mathcal{w}_{T_\delta}^{A_{\alpha_x}}$, $\mathbb{y} \in \mathcal{w}_{I_\delta}^{A_{\beta_y}}$ and $\mathbb{z} \in \mathcal{w}_{F_\delta}^{A_{\gamma_z}}$. Since A_w is a $(\delta, \varepsilon \vee \delta)$ –filter, we have $1_w \in \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $1_w \in \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $1_w \in \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A_\gamma}$.

But left inequalities in (3.17) implies that

$\mathcal{w}_T^A(1_w) + \alpha \leq 1$implies $\mathcal{w}_T^A(1_w) \leq 1-\alpha < 0.5 < \alpha$,

$\mathcal{w}_I^A(1_w) + \beta \leq 1$ implies $\mathcal{w}_I^A(1_w) \leq 1-\beta < 0.5 < \beta$,,

$\mathcal{w}_F^A(1_w) + \gamma \geq 1$implies $\mathcal{w}_F^A(1_w) \geq 1-\gamma > 0.5 > \gamma$,

that is $1_w \notin \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $1_w \notin \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $1_w \notin \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A_\gamma}$. This is a contradiction since A_w is a $(\delta, \varepsilon \vee \delta)$ –filter. So $1_w \in \mathcal{w}_{T_\delta}^{A_{\alpha_x}}$, $1_w \in \mathcal{w}_{I_\delta}^{A_{\beta_y}}$ and $1_w \in \mathcal{w}_{F_\delta}^{A_{\gamma_z}}$.

For any $\mathbb{x}, \mathbb{y}, \mathbb{a}, \mathbb{b}, \mathbb{u}$ *and* $\mathbb{v} \in \mathcal{w}$, let α_x , α_y, $\beta_{\mathbb{a}}, \beta_{\mathbb{b}} \in (0,1]$ and $\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}} \in [0,1)$ be such that

$\mathbb{x} \curvearrowright \mathbb{y} \in \mathcal{w}_{T_\varepsilon}^{A_{\alpha_y}}$, $\mathbb{x} \in \mathcal{w}_{T_\varepsilon}^{A_{\alpha_x}}$, $\mathbb{a} \curvearrowright \mathbb{b} \in \mathcal{w}_{I_\varepsilon}^{A_{\beta_{\mathbb{b}}}}$, $\mathbb{a} \in \mathcal{w}_{I_\varepsilon}^{A_{\beta_{\mathbb{a}}}}$ and $\mathbb{u} \curvearrowright \mathbb{v} \in \mathcal{w}_{F_\varepsilon}^{A_{\gamma_{\mathbb{v}}}}$, $\mathbb{u} \in \mathcal{w}_{F_\varepsilon}^{A_{\gamma_{\mathbb{u}}}}$.

Then $\mathcal{w}_T^A(\mathbb{x} \curvearrowright \mathbb{y}) \geq \alpha_y$, $\mathcal{w}_T^A(\mathbb{x}) \geq \alpha_x$, $\mathcal{w}_I^A(\mathbb{a} \curvearrowright \mathbb{b}) \geq \beta_{\mathbb{b}}$, $\mathcal{w}_I^A(\mathbb{a}) \geq \beta_{\mathbb{a}}$,

$\mathcal{w}_F^A(\mathbb{u} \curvearrowright \mathbb{v}) \leq \gamma_{\mathbb{v}}$ and$\mathcal{w}_F^A(\mathbb{u}) \leq \gamma_{\mathbb{u}}$.

Suppose $\notin \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A_{\alpha_x \wedge \alpha_y}}$, $\mathbb{b} \notin \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A_{\beta_{\mathbb{a}} \wedge \beta_{\mathbb{b}}}}$ and $\mathbb{v} \notin \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A_{\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}}$. Then

$\mathcal{w}_F^A(\mathbb{y}) < \alpha_x \wedge \alpha_y$, $\mathcal{w}_I^A(\mathbb{b}) < \beta_{\mathbb{a}} \wedge \beta_{\mathbb{b}}$, $\mathcal{w}_F^A(\mathbb{v}) > \gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}$ ------------------- (3.18)

$\mathcal{w}_F^A(\mathbb{y}) + \alpha_x \wedge \alpha_y \leq 1$, $\mathcal{w}_I^A(\mathbb{b}) + \beta_{\mathbb{a}} \wedge \beta_{\mathbb{b}} \leq 1$, $\mathcal{w}_F^A(\mathbb{v}) + \gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}} \geq 1$------------------- (3.19)

It follows that $\mathcal{w}_F^A(\mathbb{y}) < 0.5$, $\mathcal{w}_I^A(\mathbb{b}) < 0.5$, $\mathcal{w}_F^A(\mathbb{v}) > 0.5$ --------------------- (3.20)

From (3.18) and (3.20), we have

$\mathcal{w}_F^A(\mathbb{y}) < \min\{\alpha_x, \alpha_y, 0.5\}$, $\mathcal{w}_I^A(\mathbb{b}) < \min\{\beta_{\mathbb{a}}, \beta_{\mathbb{b}}, 0.5\}$, $\mathcal{w}_F^A(\mathbb{v}) > \max\{\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}}, 0.5\}$and this,

$1-\mathcal{w}_F^A(\mathbb{y}) > 1-\min\{\alpha_x, \alpha_y, 0.5\} = \max\{1-\alpha_x, 1-\alpha_y, 0.5\}$

$\geq \max\{1-\mathcal{w}_T^A(\mathbb{x}), 1-\mathcal{w}_T^A(\mathbb{x} \curvearrowright \mathbb{y}), 0.5\}$.

Similarly we have $1-\mathcal{w}_I^A(\mathbb{b}) \geq \max\{1-\mathcal{w}_I^A(\mathbb{a}), 1-\mathcal{w}_I^A(\mathbb{a} \curvearrowright \mathbb{b}), 0.5\}$.

$1-\mathcal{w}_F^A(\mathbb{v}) < 1-\max\{\gamma_{\mathbb{u}}, \gamma_{\mathbb{v}}, 0.5\} = \min\{1-\gamma_{\mathbb{u}}, 1-\gamma_{\mathbb{v}}, 0.5\}$

$\leq \min\{1-\mathcal{w}_F^A(\mathbb{u}), 1-\mathcal{w}_F^A(\mathbb{u} \curvearrowright \mathbb{v}), 0.5\}$.

Now there exist $\alpha, \beta \in (0,0.5]$ and $\gamma \in [0.5,1)$ such that

$1-\mathcal{w}_F^A(\mathbb{y}) \geq \alpha > \max\{1-\mathcal{w}_T^A(\mathbb{x}), 1-\mathcal{w}_T^A(\mathbb{x} \curvearrowright \mathbb{y}), 0.5\}$,

$1-\mathcal{w}_I^A(\mathbb{b}) \geq \beta > \max\{1-\mathcal{w}_I^A(\mathbb{a}), 1-\mathcal{w}_I^A(\mathbb{a} \curvearrowright \mathbb{b}), 0.5\}$,

$1-\mathcal{w}_F^A(\mathbb{v}) \leq \gamma < \min\{1-\mathcal{w}_F^A(\mathbb{u}), 1-\mathcal{w}_F^A(\mathbb{u} \curvearrowright \mathbb{v}), 0.5\}$.

It follows that $\mathcal{w}_T^A(\mathbb{x} \curvearrowright \mathbb{y}) + \alpha > 1$and $\mathcal{w}_T^A(\mathbb{x}) + \alpha > 1$, that is $\curvearrowright \mathbb{y} \in \mathcal{w}_{T_\delta}^{A_{\alpha_y}}$, $\mathbb{x} \in \mathcal{w}_{T_\delta}^{A_{\alpha_x}}$,

$\mathcal{w}_I^A(\mathbb{a} \curvearrowright \mathbb{b}) + \beta > 1$ and $\mathcal{w}_I^A(\mathbb{a}) + \beta > 1$, that is $\mathbb{a} \curvearrowright \mathbb{b} \in \mathcal{w}_{I_\delta}^{A_{\beta_{\mathbb{b}}}}$, $\mathbb{a} \in \mathcal{w}_{I_\delta}^{A_{\beta_{\mathbb{a}}}}$ and

$\mathcal{w}_F^A(\mathbb{u} \curvearrowright \mathbb{v}) + \gamma < 1$ and $\mathcal{w}_F^A(\mathbb{u}) + \gamma < 1$, that is $\mathbb{u} \curvearrowright \mathbb{v} \in \mathcal{w}_{F_\delta}^{A_{\gamma_{\mathbb{v}}}}$, $\mathbb{u} \in \mathcal{w}_{F_\delta}^{A_{\gamma_{\mathbb{u}}}}$.

So $\mathcal{w}_F^A(\mathbb{y}) + \alpha \leq 1$, implies $\mathcal{w}_F^A(\mathbb{y}) < \alpha$ that is $\notin \mathcal{w}_{T_{\varepsilon\vee\delta}}^{A_\alpha}$, $\mathbb{b} \notin \mathcal{w}_{I_{\varepsilon\vee\delta}}^{A_\beta}$ and $\mathbb{v} \notin \mathcal{w}_{F_{\varepsilon\vee\delta}}^{A_\gamma}$

Which is contradiction since A_w is a $(\delta, \varepsilon \vee \delta)$ –filter. Therefore

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$\mathbb{y} \in w_{T_{\varepsilon\vee\delta}}^{A_{\alpha_x \wedge \alpha_y}}$, $\mathbb{b} \in w_{I_{\varepsilon\vee\delta}}^{A_{\beta_{\mathbb{a}} \wedge \beta_{\mathbb{b}}}}$ and $\in w_{F_{\varepsilon\vee\delta}}^{A_{\gamma_{\mathbb{u}} \vee \gamma_{\mathbb{v}}}}$. So, A_w is a $(\varepsilon, \varepsilon \vee \delta)$ –filter.

5. Conclusion

we defined the ε, δ – filters and (φ, ρ)- filters of Lattice wajsberg algebras and related assets are discussed. We prove that every $(\varepsilon, \varepsilon)$ – filters are $(\varepsilon, \varepsilon \vee \delta)$ – filters and provide the condition for $(\varepsilon, \varepsilon \vee \delta)$ – filters are $(\varepsilon, \varepsilon)$ – filters. Further conditions for $\varepsilon \vee \delta$ –filters are discussed.

Further we plan introduce the general model of ε, δ – filters.

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