More on Open Maps and Closed Maps in Fuzzy Hypersoft Topological Spaces and Application in Covid-19 Diagnosis using Cotangent Similarity Measure

S. Aranganayagi[1], M. Saraswathi[2], K. Chitirakala[3]
[1]Department of Mathematics, Government Arts College, Dharmapuri, Tamil Nadu - 636 705, India
[1,2]Department of Mathematics, Kandaswami Kandar's College P-velur, Tamil Nadu - 638 182, India
[3]Department of Mathematics, M.Kumarasamy College of Engineering, Karur - 639 113; Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India
Emails: aranganayagi19@gmail.com[1]; msmathsnkl@gmail.com[2]; chitrakalalaksana@gmail.com[3]
Corresponding authors: M. Saraswathi and K. Chitirakala

Abstract

The purpose of this paper is to introduce and study fuzzy hypersoft open maps, fuzzy hypersoft semi open maps, fuzzy hypersoft pre open maps, fuzzy hypersoft δ open maps, fuzzy hypersoft δ pre open maps, fuzzy hypersoft δ semi open maps, fuzzy hypersoft e open maps, fuzzy hypersoft $\delta\alpha$ open maps, fuzzy hypersoft e^* open maps and their respective closed maps in fuzzy hypersoft topological spaces. Also, we have discussed the properties of various forms of fuzzy hypersoft open and closed maps. Moreover, a new cotangent similarity measure for fuzzy hypersoft sets is introduced and an application in Covid-19 diagnosis is explained with an example.
Keywords: fuzzy hypersoft δ open maps; fuzzy hypersoft e open maps; fuzzy hypersoft δ closed maps; fuzzy hypersoft e closed maps; cotangent similarity measure

AMS (2000) subject classification: 03E72, 54A10, 54A40, 54C05, 54C10.

1 Introduction

The real world decision making problems in medical diagnosis, management, computer science, engineering, artificial intelligence, economics,social sciences, environmental science and sociology contains more uncertain and inadequate data. The traditional mathematical methods cannot deal with these kind of problems due to the imprecise data. The fuzzy set was introduced by Zadeh[24] in 1965 to deal the real world decision making problems with uncertainty which contains the membership value in [0,1]. In fuzzy set, every element of the universe is a member of the set but with some value or degree of belongingness called as membership value of an element which lies between 0 and 1. The fuzzy topological space was developed by Chang.[8] In 1999, the soft set theory was introduced by Molodstov.[11] Soft set is a collection of parameters which describe the characteristics, properties or attributes of the objects. The soft set theory have several applications in different fields such as decision making, optimization, forecasting, data analysis etc. Consequently, the soft topological spaces were developed by Shabir and Naz.[17]

By replacing function with the cartesian product of a multi-argument function with a different set of attributes, Smarandache[18] extended the notion of a soft set to a hypersoft set and then to plithogenic set. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Abbas et al.[2] defined the basic operations on hypersoft sets and hypersoft point

https://doi.org/10.54216/IJNS.210203
Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

in all the universe of discourses. The topological structures of fuzzy hypersoft set, intuitionistic hypersoft set and neutrosophic hypersoft set were developed by Ajay and Charisma.[4] Fuzzy hypersoft topology and intuitionistic hypersoft topology are generalized by the general framework neutrosophic hypersoft topology. Fuzzy hypersoft semi-open sets were defined and an application in multiattribute group decision making were developed by Ajay et al.[5]

Saha[14] defined δ-open sets in fuzzy topological spaces. Vadivel et al.[19–21] introduced δ-open sets in neutrosophic topological spaces. In 2019, Acikgoz and Esenbel[1] defined neutrosophic soft δ-topology. The notion of e-open sets were introduced by Ekici[9] in a general topology, Seenivasan et al.[16] in fuzzy topological space, Chandrasekar et al.[7] in intuitionistic fuzzy topological space, Vadivel et al.[22,23] in neutrosophic topological spaces and Revathi et al.[12,13] in neutrosophic soft topological spaces. Aras and Bayramov[6] introduced neutrosophic soft continuity in neutrosophic soft topological spaces. The concepts of e-continuity, e-irresolute maps, e-open maps, e-closed maps and e-homeomorphisms were developed by Vadivel et al.[22,23] in neutrosophic topological spaces and Revathi et al.[13] in neutrosophic soft topological spaces. Ahsan et al.[3] studied a theoretical and analytical approach for fundamental framework of composite mappings on fuzzy hypersoft classes.

Saqlain et al.[15] studied single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft set. Jafar et al.[10] studied trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection.

In this paper, we develop the concept of fuzzy hypersoft open maps, open maps, δ open maps, δ open maps, δ open maps, e open maps, δ α open maps, e^* open maps and their respective closed maps in fuzzy hypersoft topological spaces and some of their basic properties are analyzed with examples. Also, an application in Covid-19 diagnosis is explained with the algorithm and example using cotangent similarity measure for fuzzy hypersoft sets.

2 Preliminaries

Definition 2.1. [24] Let Θ be an initial universe. A function λ from Θ into the unit interval I is called a fuzzy set in Θ. For every $\chi \in \Theta,\ \lambda(\chi) \in I$ is called the grade of membership of χ in λ. Some authors say that λ is a fuzzy subset of Θ instead of saying that λ is a fuzzy set in Θ. The class of all fuzzy sets from Θ into the closed unit interval I will be denoted by I^{Θ}.

Definition 2.2. [11] Let Θ be an initial universe, Υ be a set of parameters and $\mathcal{P}(\Theta)$ be the power set of Θ. A pair $(\tilde{\theta}, \zeta)$ is called the a soft set over Θ where $\tilde{\theta}$ is a mapping $\tilde{\theta} : \Upsilon \to \mathcal{P}(\Theta)$. In other words, the soft set is a parametrized family of subsets of the set Θ.

Definition 2.3. [18] Let Θ be an initial universe and $\mathcal{P}(\Theta)$ be the power set of Θ. Consider $\upsilon_1, \upsilon_2, \upsilon_3, ..., \upsilon_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $\Upsilon_1,\ \Upsilon_2,\ ...,\ \Upsilon_n$ with $\Upsilon_i \cap \Upsilon_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$. Then the pair $(\tilde{\theta},\ \Upsilon_1 \times \Upsilon_2 \times ... \times \Upsilon_n)$ where $\tilde{\theta} : \Upsilon_1 \times \Upsilon_2 \times ... \times \Upsilon_n \to \mathcal{P}(\Theta)$ is called a hypersoft set over Θ.

Definition 2.4. [2] Let Θ be an initial universal set and $\Upsilon_1, \Upsilon_2, ..., \Upsilon_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\Theta)$ be the set of all fuzzy sets of Θ. Let E_i be the nonempty subset of the pair Υ_i for each $i = 1, 2, ..., n$. A fuzzy hypersoft set (briefly, FH_ySs) over Θ is defined as the pair $(\tilde{\theta}, E_1 \times E_2 \times ... \times E_n)$ where $\tilde{\theta} : E_1 \times E_2 \times ... \times E_n \to \mathcal{P}(\Theta)$ and $\tilde{\theta}(E_1 \times E_2 \times ... \times E_n) = \{(\upsilon, \langle \chi, \mu_{\tilde{\theta}(\upsilon)}(\chi)\rangle : \chi \in \Theta) : \upsilon \in E_1 \times E_2 \times ... \times E_n \subseteq \Upsilon_1 \times \Upsilon_2 \times ... \times \Upsilon_n\}$ where $\mu_{\tilde{\theta}(\upsilon)}(\chi)$ is the membership value such that $\mu_{\tilde{\theta}(\upsilon)}(\chi) \in [0, 1]$.

Definition 2.5. [2] Let Θ be an universal set and $(\tilde{\theta_1}, \zeta_1)$ and $(\tilde{\theta_2}, \zeta_2)$ be two FH_ySs's over Θ. Then $(\tilde{\theta_1}, \zeta_1)$ is the fuzzy hypersoft subset of $(\tilde{\theta_2}, \zeta_2)$ if $\mu_{\tilde{H}(\upsilon)}(\chi) \leq \mu_{\tilde{G}(\upsilon)}(\chi)$.

It is denoted by $(\tilde{\theta_1}, \zeta_1) \subseteq (\tilde{\theta_2}, \zeta_2)$.

Definition 2.6. [2] Let Θ be an universal set and $(\tilde{\theta_1}, \zeta_1)$ and $(\tilde{\theta_2}, \zeta_2)$ be FH_ySs's over Θ. $(\tilde{\theta_1}, \zeta_1)$ is equal to $(\tilde{\theta_2}, \zeta_2)$ if $\mu_{\tilde{\theta_1}(\upsilon)}(\chi) = \mu_{\tilde{\theta_2}(\upsilon)}(\chi)$.

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Definition 2.7. [2] A FH_ySs $(\tilde{\theta_1}, \zeta)$ over the universe set Θ is said to be null fuzzy hypersoft set if $\mu_{\tilde{\theta_1}(\upsilon)}(\chi) = 0, \forall\, \upsilon \in \zeta$ and $\chi \in \Theta$. It is denoted by $\tilde{0}_{(\Theta,\Upsilon)}$.

A FH_ySs $(\tilde{\theta_2}, \zeta)$ over the universal set Θ is said to be absolute fuzzy hypersoft set if $\mu_{\tilde{\theta_1}(\upsilon)}(\chi) = 1\ \forall \upsilon \in \zeta$ and $\chi \in \Theta$. It is denoted by $\tilde{1}_{(\Theta,\Upsilon)}$.

Clearly, $\tilde{0}^c_{(\Theta,\Upsilon)} = \tilde{1}_{(\Theta,\Upsilon)}$ and $\tilde{1}^c_{(\Theta,\Upsilon)} = \tilde{0}_{(\Theta,\Upsilon)}$.

Definition 2.8. [2] Let Θ be an universal set and $(\tilde{\theta_1}, \zeta)$ be FH_ySs over Θ. $(\tilde{\theta_1}, \zeta)^c$ is the complement of $(\tilde{\theta_1}, \zeta)$ if $\mu^c_{\tilde{\theta}(\upsilon)}(\chi) = \tilde{1}_{(\Theta,\Upsilon)} - \mu_{\tilde{\theta}(\upsilon)}(\chi)$ where $\forall\, \upsilon \in \zeta$ and $\forall \chi \in \Theta$. It is clear that $((\tilde{\theta_1}, \zeta)^c)^c = (\tilde{\theta_1}, \zeta)$.

Definition 2.9. [2] Let Θ be the universal set and $(\tilde{\theta_1}, \zeta_1)$ and $(\tilde{\theta_2}, \zeta_2)$ be FH_ySs's over Θ. Extended union $(\tilde{\theta_1}, \zeta_1) \cup (\tilde{\theta_2}, \zeta_2)$ is defined as

$$\mu((\tilde{\theta_1}, \zeta_1) \cup (\tilde{\theta_2}, \zeta_2)) = \begin{cases} \mu_{\tilde{\theta_1}(\upsilon)}(\chi) & \text{if } \upsilon \in \zeta_1 - \zeta_2 \\ \mu_{\tilde{\theta_2}(\upsilon)}(\chi) & \text{if } \upsilon \in \zeta_2 - \zeta_1 \\ max\{\mu_{\tilde{\theta_1}(\upsilon)}(\chi), \mu_{\tilde{\theta_2}(\upsilon)}(\chi)\} & \text{if } \upsilon \in \zeta_1 \cap \zeta_2 \end{cases}$$

Definition 2.10. [2,4] Let Θ be the universal set and $(\tilde{\theta_1}, \zeta_1)$ and $(\tilde{\theta_2}, \zeta_2)$ be FH_ySs's over Θ. Extended intersection $(\tilde{\theta_1}, \zeta_1) \cap (\tilde{\theta_2}, \zeta_2)$ is defined as

$$\mu((\tilde{\theta_1}, \zeta_1) \cap (\tilde{\theta_2}, \zeta_2)) = \begin{cases} \mu_{\tilde{\theta_1}(\upsilon)}(\chi) & if \upsilon \in \zeta_1 - \zeta_2 \\ \mu_{\tilde{\theta_2}(\upsilon)}(\chi) & if \upsilon \in \zeta_2 - \zeta_1 \\ min\{\mu_{\tilde{\theta_1}(\upsilon)}(\chi), \mu_{\tilde{\theta_2}(\upsilon)}(\chi)\} & if \upsilon \in \zeta_1 \cap \zeta_2 \end{cases}$$

Definition 2.11. [4] Let (Θ, Υ) be the family of all FH_ySs's over the universe set Θ and $\tau \subseteq FH_ySs(\Theta, \Upsilon)$. Then τ is said to be a fuzzy hypersoft topology (briefly, FH_ySt) on Θ if

(i) $\tilde{0}_{(\Theta,\Upsilon)}$ and $\tilde{1}_{(\Theta,\Upsilon)}$ belongs to τ

(ii) the union of any number of FH_ySs's in τ belongs to τ

(iii) the intersection of finite number of FH_ySs's in τ belongs to τ.

Then (Θ, Υ, τ) is called a fuzzy hypersoft topological space (briefly, FH_ySts) over Θ. Each member of τ is said to be fuzzy hypersoft open set (briefly, FH_ySos). A FH_ySs $(\tilde{\theta_1}, \zeta)$ is called a fuzzy hypersoft closed set (briefly, FH_yScs) if its complement $(\tilde{\theta_1}, \zeta)^c$ is FH_ySos.

Definition 2.12. [4] Let (Θ, Υ, τ) be a FH_ySts over Θ and $(\tilde{\theta_1}, \zeta)$ be a FH_ySs in Θ. Then,

(i) the fuzzy hypersoft interior (briefly, FH_ySint) of $(\tilde{\theta_1}, \zeta)$ is defined as $FH_ySint(\tilde{\theta_1}, \zeta) = \cup\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ where $(\tilde{\theta_2}, \zeta)$ is $FH_ySos\}$.

(ii) the fuzzy hypersoft closure (briefly, FH_yScl) of $(\tilde{\theta_1}, \zeta)$ is defined as $FH_yScl(\tilde{\theta_1}, \zeta) = \cap\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \supseteq (\tilde{\theta_1}, \zeta)$ where $(\tilde{\theta_2}, \zeta)$ is $FH_yScs\}$.

Definition 2.13. [5] Let (Θ, Υ, τ) be a FH_ySts over Θ and $(\tilde{\theta_1}, \zeta)$ be a FH_ySs in Θ. Then, $(\tilde{\theta_1}, \zeta)$ is called the fuzzy hypersoft semiopen set (briefly, FH_ySSos) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl(int(\tilde{\theta_1}, \zeta))$.

A FH_ySs $(\tilde{\theta_1}, \zeta)$ is called a fuzzy hypersoft semiclosed set (briefly, FH_ySScs) if its complement $(\tilde{\theta_1}, \zeta)^c$ is a FH_ySSos.

Definition 2.14. [10] Consider two neutrosophic hypersoft sets $(\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ over Θ. The cotangent similarity measure for these two sets based on the cotangent function is given by

$$S_{Ct}((\tilde{\theta_1}, \zeta), (\tilde{\theta_2}, \zeta)) = \frac{1}{n}\sum_{i=1}^{n} cot[\tfrac{\pi}{4} + \tfrac{\pi}{4}(|\mu^i_{\theta_1} - \mu^i_{\theta_2}| \vee |\sigma^i_{\theta_1} - \sigma^i_{\theta_2}| \vee |\nu^i_{\theta_1} - \nu^i_{\theta_2}|)]$$

where $\vee$ denotes the maximum operator.

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

3 More on Open Maps in Fuzzy Hypersoft Topological Spaces

In this section, various forms of open sets, closed sets, respective interior and closure operators, fuzzy hypersoft open maps, fuzzy hypersoft semi open maps, fuzzy hypersoft pre open maps, fuzzy hypersoft δ open maps, fuzzy hypersoft δ semi open maps, fuzzy hypersoft δ pre open maps, fuzzy hypersoft e open maps, fuzzy hypersoft δ α open maps and fuzzy hypersoft e^* open maps are introduced and their related properties are discussed.

Definition 3.1. Let (Θ, Υ, τ) be a fuzzy hypersoft topological space (briefly, FH_ySts) over Θ. An fuzzy hypersoft set (briefly, FH_ySs) $(\tilde{\theta_1}, \zeta)$ is said to be a fuzzy hypersoft regular open set (briefly, FH_ySros) if $(\tilde{\theta_1}, \zeta) = FH_ySint(FH_yScl(\tilde{\theta_1}, \zeta))$. The complement of FH_ySros is called a fuzzy hypersoft regular closed set (briefly, FH_ySrcs) in Θ.

Definition 3.2. Let (Θ, Υ, τ) be a FH_ySts over Θ and $(\tilde{\theta_1}, \zeta)$ be a FH_ySs on Θ. Then the fuzzy hypersoft (briefly, FH_yS)

(i) δ-interior (briefly, FH_ySint) of $(\tilde{\theta_1}, \zeta)$ is defined by $FH_yS\delta int(\tilde{\theta_1}, \zeta) = \bigcup\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a FH_ySros in $\Theta\}$

(ii) δ-closure (briefly, FH_yScl) of $(\tilde{\theta_1}, \zeta)$ is defined by $FH_yS\delta cl(\tilde{\theta_1}, \zeta) = \bigcap\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \supseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a FH_ySrcs in $\Theta\}$

Definition 3.3. Let (Θ, Υ, τ) be a FH_ySts over Θ. An FH_ySs $(\tilde{\theta_1}, \zeta)$ is said to be a fuzzy hypersoft

(i) semi-regular if $(\tilde{\theta_1}, \zeta)$ is both FH_ySSos and FH_ySScs.

(ii) pre open set (briefly, $FH_yS\mathcal{P}os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_ySint(FH_yScl(\tilde{\theta_1}, \zeta)$

(iii) δ-open set (briefly, $FH_yS\delta os$) if $(\tilde{\theta_1}, \zeta) = FH_yS\delta int(\tilde{\theta_1}, \zeta)$

(iv) δ-pre open set (briefly, $FH_yS\delta\mathcal{P}os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_ySint(FH_yS\delta cl(\tilde{\theta_1}, \zeta))$

(v) δ-semi open set (briefly, $FH_yS\delta\mathcal{S}os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl(FH_yS\delta int(\tilde{\theta_1}, \zeta))$

(vi) e-open set (briefly, FH_ySeos) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl\big(FH_yS\delta int(\tilde{\theta_1}, \zeta)\big) \cup FH_ySint\big(FH_yS\delta cl(\tilde{\theta_1}, \zeta)\big)$.

(vii) δ α-open set (briefly, $FH_yS\delta\alpha os$) if $(\tilde{\theta_1}, \zeta) \subseteq FH_ySint\big(FH_yScl(FH_yS\delta int(\tilde{\theta_1}, \zeta))\big)$.

(viii) e^*-open set (briefly, FH_ySe^*os) if $(\tilde{\theta_1}, \zeta) \subseteq FH_yScl\big(FH_ySint(FH_yS\delta cl(\tilde{\theta_1}, \zeta))\big)$.

The complement of $FH_yS\delta os$ (resp. $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, FH_ySeos, $FH_yS\delta\alpha os$ & FH_ySe^*os) is called a $FH_yS\delta$ (resp. FH_yS pre, $FH_yS\delta$ pre, $FH_yS\delta$ semi, FH_ySe, $FH_yS\delta\alpha$ & FH_ySe^*) closed set (briefly, $FH_yS\delta cs$ (resp. $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$, FH_ySeos, $FH_yS\delta\alpha cs$ & FH_ySe^*cs)) in Θ.

The family of all $FH_yS\delta os$ (resp. $FH_yS\delta cs$, FH_ySros, FH_ySrcs, $FH_yS\mathcal{P}os$, $FH_yS\mathcal{P}cs$ $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}os$, $FH_yS\delta\mathcal{S}cs$, FH_ySeos, FH_ySecs, $FH_yS\delta\alpha os$, $FH_yS\delta\alpha cs$, FH_ySe^*os & FH_ySe^*cs) of Θ is denoted by $FH_yS\delta OS(\Theta)$ (resp. $FH_yS\delta CS(\Theta)$, $FH_ySrOS(\Theta)$, $FH_ySrOS(\Theta)$, $FH_yS\mathcal{P}OS(\Theta)$, $FH_yS\mathcal{P}CS(\Theta)$, $FH_yS\delta\mathcal{P}OS(\Theta)$, $FH_yS\delta\mathcal{P}CS(\Theta)$, $FH_yS\delta\mathcal{S}OS(\Theta)$, $FH_yS\delta\mathcal{S}CS(\Theta)$, $FH_ySeOS(\Theta)$, $FH_ySeCS(\Theta)$, $FH_yS\delta\alpha OS(\Theta)$, $FH_yS\delta\alpha CS(\Theta)$, $FH_ySe^*OS(\Theta)$ & $FH_ySe^*CS(\Theta)$).

Definition 3.4. Let (Θ, Υ, τ) be a FH_ySts over Θ and $(\tilde{\theta_1}, \zeta)$ be a FH_ySs on Θ. Then the fuzzy hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $FH_yS\delta\mathcal{P}int$ (resp. $FH_yS\delta\mathcal{S}int$)) of $(\tilde{\theta_1}, \zeta)$ is defined by FH_yS $\delta\mathcal{P}int(\tilde{\theta_1}, \zeta) = \bigcup\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a $FH_yS\delta\mathcal{P}os$ (resp. $FH_yS\delta\mathcal{S}os$) in $\Theta\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $FH_yS\delta\mathcal{P}cl$ (resp. $FH_yS\delta\mathcal{S}cl$)) of $(\tilde{\theta_1}, \zeta)$ is defined by FH_yS $\delta\mathcal{P}cl(\tilde{\theta_1}, \zeta) = \bigcap\{(\tilde{\theta_2}, \zeta) : (\tilde{\theta_2}, \zeta) \supseteq (\tilde{\theta_1}, \zeta)$ and $(\tilde{\theta_2}, \zeta)$ is a $FH_yS\delta\mathcal{P}cs$ (resp. $FH_yS\delta\mathcal{S}cs$) in $\Theta\}$

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

(iii) e interior (briefly, $FH_ySeint(\tilde{\theta_1}, \zeta)$ is defined by $FH_ySeint(\tilde{\theta_1}, \zeta) = \bigcup\{(\tilde{L}, \zeta) : (\tilde{L}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ & $(\tilde{L}, \zeta)$ is a FH_ySeos in $\Theta\}$.

(iv) e closure (briefly, $FH_ySecl(\tilde{\theta_1}, \zeta)$ is defined by $FH_ySecl(\tilde{\theta_1}, \zeta) = \bigcap\{(\tilde{L}, \zeta) : (\tilde{\theta_1}, \zeta) \subseteq (\tilde{L}, \zeta)$ & $(\tilde{\theta_1}, \zeta)$ is a FH_ySecs in $\Theta\}$.

Definition 3.5. Consider any two FH_ySts (Θ, L, τ) and (Ω, M, σ). A map $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is called as FH_yS

(i) continuous (briefly, FH_ySCts) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_ySos in (Θ, L, τ).

(ii) semi-continuous (briefly, $FH_yS\mathcal{S}Cts$) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_yS $\mathcal{S}os$ in (Θ, L, τ).

(iii) pre-continuous (briefly, $FH_yS\mathcal{P}Cts$) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_yS $\mathcal{P}os$ in (Θ, L, τ).

(iv) δ continuous (briefly, $FH_yS\delta Cts$) if the inverse image of each FH_ySos in (Ω, M, σ) is a $FH_yS\delta os$ in (Θ, L, τ).

(v) δ semi continuous (briefly, $FH_yS\delta\mathcal{S}Cts$) if the inverse image of each FH_ySos in (Ω, M, σ) is a $FH_yS\delta\mathcal{S}os$ in (Θ, L, τ).

(vi) δ pre continuous (briefly, $FH_yS\delta\mathcal{P}Cts$) if the inverse image of each FH_ySos in (Ω, M, σ) is a $FH_yS\delta\mathcal{P}os$ in (Θ, L, τ).

(vii) e continuous (briefly, FH_ySeCts) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_ySeos in (Θ, L, τ).

(viii) $\delta\alpha$ continuous (briefly, $FH_yS\delta\alpha Cts$) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_yS $\delta\alpha os$ in (Θ, L, τ).

(ix) e^* continuous (briefly, FH_ySe^*Cts) if the inverse image of each FH_ySos in (Ω, M, σ) is a FH_yS e^*os in (Θ, L, τ).

(x) e-irresolute (resp. irresolute, δ irresolute, $\mathcal{P}$ irresolute, $\delta\mathcal{P}$ irresolute, $\delta\mathcal{S}$ irresolute, $\delta\alpha$ irresolute, e^* irresolute) (briefly, FH_ySeIrr (resp. FH_ySIrr , $FH_yS\delta Irr$, $FH_yS\mathcal{P}Irr$, $FH_yS\delta\mathcal{P}Irr$, $FH_yS\delta\mathcal{S}$ Irr, $FH_yS\delta\alpha Irr$, FH_ySe^*Irr)) if the inverse image of every FH_ySeos (resp. FH_ySos, $FH_yS\delta os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\alpha os$ & FH_ySe^*os) in (Ω, M, σ) is a FH_ySeos (resp. FH_ySos, $FH_yS\delta os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\alpha os$ & FH_ySe^*os) in (Θ, L, τ).

Definition 3.6. Consider any two FH_ySts (Θ, L, τ) and (Ω, M, σ). A map $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is called as FH_yS

(i) open (briefly, FH_ySO) if the image of each FH_ySos in (Θ, L, τ) is a FH_ySos in (Ω, M, σ).

(ii) semi-open (briefly, $FH_yS\mathcal{S}O$) if the image of each FH_ySos in (Θ, L, τ) is a $FH_yS\mathcal{S}os$ in (Ω, M, σ).

(iii) pre-open (briefly, $FH_yS\mathcal{P}O$) if the image of each FH_ySos in (Θ, L, τ) is a $FH_yS\mathcal{P}os$ in (Ω, M, σ).

(iv) δ open (briefly, $FH_yS\delta O$) if the image of each FH_ySos in (Θ, L, τ) is a $FH_yS\delta os$ in (Ω, M, σ).

(v) δ semi open (briefly, $FH_yS\delta\mathcal{S}O$) if the image of each FH_ySos in (Θ, L, τ) is a $FH_yS\delta\mathcal{S}os$ in (Ω, M, σ).

(vi) δ pre open (briefly, $FH_yS\delta\mathcal{P}O$) if the image of each FH_ySos in (Θ, L, τ) is a $FH_yS\delta\mathcal{P}os$ in (Ω, M, σ).

(vii) e open (briefly, FH_ySeO) if the image of each FH_ySos in (Θ, L, τ) is a FH_ySeos in (Ω, M, σ).

(viii) $\delta\alpha$ open (briefly, $FH_yS\delta\alpha O$) if the image of each FH_ySos in (Θ, L, τ) is a $FH_yS\delta\alpha os$ in (Ω, M, σ).

(ix) e^* open (briefly, FH_ySe^*O) if the image of each FH_ySos in (Θ, L, τ) is a FH_ySe^*os in (Ω, M, σ).

https://doi.org/10.54216/IJNS.210203
Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Proposition 3.7. The statements are correct, but the converse need not be true.

(i) Every $FH_ySδO$ is a FH_ySO.

(ii) Every FH_ySO is a $FH_yS\mathcal{S}O$.

(iii) Every FH_ySO is a $FH_yS\mathcal{P}O$.

(iv) Every $FH_ySδO$ is a $FH_ySδ\mathcal{S}O$.

(v) Every $FH_ySδO$ is a $FH_ySδ\mathcal{P}O$.

(vi) Every $FH_ySδO$ is a FH_ySeO.

(vii) Every $FH_ySδ\mathcal{S}O$ is a FH_ySeO.

(viii) Every $FH_ySδ\mathcal{P}O$ is a FH_ySeO.

(ix) Every $FH_ySδ\mathcal{S}O$ is a FH_ySe^*O.

(x) Every $FH_ySδαO$ is a FH_ySe^*O.

(xi) Every $FH_ySδαO$ is a $FH_ySδ\mathcal{P}O$.

(xii) Every FH_ySe^*O is a FH_ySeO.

Proof. Consider a mapping $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$.

(i) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδO$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδos$ in Ω. Since all $FH_ySδos$ are FH_ySos, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySos in Ω. Hence $\mathfrak{h}$ is a FH_ySO.

(ii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is FH_ySO, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySos in Ω. Since all FH_ySos are $FH_yS\mathcal{S}os$,[5] $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\mathcal{S}os$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\mathcal{S}O$.

(iii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is FH_ySO, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySos in Ω. Since all FH_ySos are $FH_yS\mathcal{P}os$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\mathcal{P}os$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\mathcal{P}O$.

(iv) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδO$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδos$ in Ω. Since all $FH_ySδos$ are $FH_ySδ\mathcal{S}os$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδ\mathcal{S}os$ in Ω. Hence $\mathfrak{h}$ is a $FH_ySδ\mathcal{S}O$.

(v) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδO$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδos$ in Ω. Since all $FH_ySδos$ are $FH_ySδ\mathcal{P}os$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδ\mathcal{P}os$ in Ω. Hence $\mathfrak{h}$ is a $FH_ySδ\mathcal{P}O$.

(vi) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδO$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδos$ in Ω. Since all $FH_ySδos$ are FH_ySeos, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySeos in Ω. Hence $\mathfrak{h}$ is a FH_ySeO.

(vii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδ\mathcal{S}O$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδ\mathcal{S}os$ in Ω. Since all $FH_ySδ\mathcal{S}os$ are FH_ySeos, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySeos in Ω. Hence $\mathfrak{h}$ is a FH_ySeO.

(viii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδ\mathcal{P}O$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδ\mathcal{P}os$ in Ω. Since all $FH_ySδ\mathcal{P}os$ are FH_ySeos, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySeos in Ω. Hence $\mathfrak{h}$ is a FH_ySeO.

(ix) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδ\mathcal{S}O$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδ\mathcal{S}os$ in Ω. Since all $FH_ySδ\mathcal{S}os$ are FH_ySe^*os, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySe^*os in Ω. Hence $\mathfrak{h}$ is a FH_ySe^*O.

(x) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδαO$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδαos$ in Ω. Since all FH_yS $δαos$ are FH_ySe^*os, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySe^*os in Ω. Hence $\mathfrak{h}$ is a FH_ySe^*O.

(xi) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is $FH_ySδαO$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδαos$ in Ω. Since all FH_yS $δαos$ are $FH_ySδ\mathcal{P}os$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_ySδ\mathcal{P}os$ in Ω. Hence $\mathfrak{h}$ is a $FH_ySδ\mathcal{P}O$.

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

(xii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Θ. Since $\mathfrak{h}$ is FH_ySe^*O, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySe^*os in Ω. Since all FH_ySe^*os are FH_ySeos, $\mathfrak{h}(\tilde{\theta_2}, \zeta)$ is FH_ySeos in Ω. Hence $\mathfrak{h}$ is a FH_ySeO.

□

Proposition 3.8. (i) $FH_yS\delta\alpha O$ and $FH_yS\delta SO$ are independent to each other.

(ii) FH_ySe^*O and $FH_yS\delta\mathcal{P}O$ are independent to each other.

Remark 3.9. The diagram shows FH_ySO maps in FH_ySts.

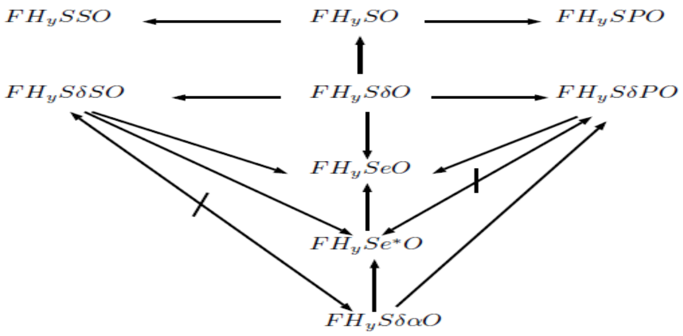

Figure 1. FH_yS open maps in FH_ySts

Example 3.10. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$, $(\tilde{\theta_4}, \zeta_3)$, $(\tilde{\theta_5}, \zeta_1)$, $(\tilde{\theta_6}, \zeta_2)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_5}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_6}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.3}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.6}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$$(\tilde{\psi_1},\zeta_1)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\varphi_1}{0.5},\frac{\varphi_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\varphi_1}{0.5},\frac{\varphi_2}{0.5}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\Omega,\Upsilon)},\tilde{1}_{(\Omega,\Upsilon)},(\tilde{\psi_1},\zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h}=(\omega,\nu):(\Omega,M)\to(\Theta,L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_1)=\chi_2,\omega(\varphi_2)=\chi_1,$$
$$\nu(c_2,d_1)=(a_1,b_1),\nu(c_1,d_2)=(a_2,b_1),\nu(c_2,d_2)=(a_1,b_2)$$

$$\mathfrak{h}(\tilde{\psi_1},\zeta_1)=(\tilde{\theta_5},\zeta_1)$$

$(\tilde{\psi_1},\zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1},\zeta_1)=(\tilde{\theta_5},\zeta_1)$ is FH_ySSos in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySSO but not FH_ySO because $\mathfrak{h}(\tilde{\psi_1},\zeta_1)=(\tilde{\theta_5},\zeta_1)$ is not FH_ySos in Θ.

Example 3.11. Let $\Theta=\{\chi_1,\chi_2\}$ and $\Omega=\{\varphi_1,\varphi_2\}$ be the FH_yS initial universes and the attributes be $L=\Upsilon_1\times\Upsilon_2$ and $M=\Upsilon_1'\times\Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1=\{a_1,a_2,a_3\},\Upsilon_2=\{b_1,b_2\}$$
$$\Upsilon_1'=\{c_1,c_2,c_3\},\Upsilon_2'=\{d_1,d_2\}.$$

Let $(\Theta,L),(\Omega,M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1},\zeta_1)$, $(\tilde{\theta_2},\zeta_2)$, $(\tilde{\theta_3},\zeta_3)$, $(\tilde{\theta_4},\zeta_3)$ & $(\tilde{\theta_5},\zeta_1)$ over the universe Θ be

$$(\tilde{\theta_1},\zeta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.7},\frac{\chi_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_2},\zeta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\chi_1}{0.5},\frac{\chi_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_3},\zeta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.7},\frac{\chi_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\chi_1}{0.5},\frac{\chi_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_4},\zeta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.7},\frac{\chi_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\chi_1}{0.5},\frac{\chi_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_5},\zeta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0.4}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.5},\frac{\chi_2}{0.5}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\Theta,\Upsilon)},\tilde{1}_{(\Theta,\Upsilon)},(\tilde{\theta_1},\zeta_1),(\tilde{\theta_2},\zeta_2),(\tilde{\theta_3},\zeta_3),(\tilde{\theta_4},\zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1},\zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1},\zeta_1)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\varphi_1}{0.4},\frac{\varphi_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\varphi_1}{0.5},\frac{\varphi_2}{0.5}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\Omega,\Upsilon)},\tilde{1}_{(\Omega,\Upsilon)},(\tilde{\psi_1},\zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h}=(\omega,\nu):(\Omega,M)\to(\Theta,L)$ be a FH_yS mapping as follows:

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$ is $FH_yS\mathcal{P}os$ in Θ.
$\therefore$ $\mathfrak{h}$ is $FH_yS\mathcal{P}O$. But $\mathfrak{h}$ is not FH_ySO because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$ is not FH_ySos in Θ.

Example 3.12. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$, $(\tilde{\theta_4}, \zeta_3)$ & $(\tilde{\theta_5}, \zeta_1)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_2}, \zeta_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_3}, \zeta_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_4}, \zeta_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_5}, \zeta_1) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.4}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{matrix} \right\}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \left\{ \begin{matrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.3}, \frac{\varphi_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.7}, \frac{\varphi_2}{0.5}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.5}\} \rangle \end{matrix} \right\}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is FH_ySos in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySO. But $\mathfrak{h}$ is not $FH_yS\delta O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is not $FH_yS\delta os$ in Θ.

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Example 3.13. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$ & $(\tilde{\theta_4}, \zeta_3)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.6}, \frac{\varphi_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.7}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is $FH_yS\delta Sos$ in Θ.
$\therefore$ $\mathfrak{h}$ is $FH_yS\delta SO$. But $\mathfrak{h}$ is not $FH_yS\delta O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is not $FH_yS\delta os$ in Θ.

Example 3.14. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$ & $(\tilde{\theta_4}, \zeta_3)$ over the universe Θ be

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$$(\tilde{\theta_1}, \zeta_1) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.8}, \frac{\chi_2}{0.6} \} \rangle, \\ \langle (a_2, b_1), \{ \frac{\chi_1}{0.7}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_2}, \zeta_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.2}, \frac{\chi_2}{0.3} \} \rangle, \\ \langle (a_1, b_2), \{ \frac{\chi_1}{0.5}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_3}, \zeta_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.2}, \frac{\chi_2}{0.3} \} \rangle, \\ \langle (a_2, b_1), \{ \frac{\chi_1}{0.7}, \frac{\chi_2}{0.5} \} \rangle, \\ \langle (a_1, b_2), \{ \frac{\chi_1}{0.5}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_4}, \zeta_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.8}, \frac{\chi_2}{0.6} \} \rangle, \\ \langle (a_2, b_1), \{ \frac{\chi_1}{0.7}, \frac{\chi_2}{0.5} \} \rangle, \\ \langle (a_1, b_2), \{ \frac{\chi_1}{0.5}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \left\{ \begin{matrix} \langle (c_2, d_1), \{ \frac{\varphi_1}{0.3}, \frac{\varphi_2}{0.2} \} \rangle, \\ \langle (c_1, d_2), \{ \frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.7} \} \rangle, \\ \langle (c_2, d_2), \{ \frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$
$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is $FH_yS\delta\mathcal{P}os$ in Θ.
$\therefore$ $\mathfrak{h}$ is $FH_yS\delta\mathcal{P}O$. But $\mathfrak{h}$ is not $FH_yS\delta O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is not $FH_yS\delta os$ in Θ.

Example 3.15. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon'_1 \times \Upsilon'_2$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon'_1 = \{c_1, c_2, c_3\}, \Upsilon'_2 = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$ & $(\tilde{\theta_4}, \zeta_3)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.8}, \frac{\chi_2}{0.6} \} \rangle, \\ \langle (a_2, b_1), \{ \frac{\chi_1}{0.7}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_2}, \zeta_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.2}, \frac{\chi_2}{0.3} \} \rangle, \\ \langle (a_1, b_2), \{ \frac{\chi_1}{0.5}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_3}, \zeta_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.2}, \frac{\chi_2}{0.3} \} \rangle, \\ \langle (a_2, b_1), \{ \frac{\chi_1}{0.7}, \frac{\chi_2}{0.5} \} \rangle, \\ \langle (a_1, b_2), \{ \frac{\chi_1}{0.5}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

$$(\tilde{\theta_4}, \zeta_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{ \frac{\chi_1}{0.8}, \frac{\chi_2}{0.6} \} \rangle, \\ \langle (a_2, b_1), \{ \frac{\chi_1}{0.7}, \frac{\chi_2}{0.5} \} \rangle, \\ \langle (a_1, b_2), \{ \frac{\chi_1}{0.5}, \frac{\chi_2}{0.5} \} \rangle \end{matrix} \right\}$$

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.6}, \frac{\varphi_2}{0.8}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.7}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is FH_ySeos in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySeO. But $\mathfrak{h}$ is not $FH_yS\delta O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is not $FH_yS\delta os$ in Θ.

Example 3.16. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$ & $(\tilde{\theta_4}, \zeta_3)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.3}, \frac{\varphi_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

https://doi.org/10.54216/IJNS.210203
Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is FH_ySeos in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySeO. But $\mathfrak{h}$ is not $FH_yS\delta SO$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is not $FH_yS\delta Sos$ in Θ.

Example 3.17. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$, $(\tilde{\theta_4}, \zeta_3)$, $(\tilde{\theta_5}, \zeta_1)$, $(\tilde{\theta_6}, \zeta_2)$ & $(\tilde{\theta_7}, \zeta_1)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_5}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_6}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.3}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_7}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.3}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.4}, \frac{\varphi_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.3}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$$

https://doi.org/10.54216/IJNS.210203
Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$ is FH_ySeos in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySeO. But $\mathfrak{h}$ is not $FH_yS\delta\mathcal{P}O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$ is not $FH_yS\delta\mathcal{P}os$ in Θ.

Example 3.18. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$ & $(\tilde{\theta_4}, \zeta_3)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ωbe defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.3}, \frac{\varphi_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is FH_ySe^*os in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySe^*O. But $\mathfrak{h}$ is not $FH_yS\delta\mathcal{S}O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is not $FH_yS\delta\mathcal{S}os$ in Θ.

Example 3.19. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2, a_3\}, \Upsilon_2 = \{b_1, b_2\}$$
$$\Upsilon_1' = \{c_1, c_2, c_3\}, \Upsilon_2' = \{d_1, d_2\}.$$

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta_1)$, $(\tilde{\theta_2}, \zeta_2)$, $(\tilde{\theta_3}, \zeta_3)$, $(\tilde{\theta_4}, \zeta_3)$ & $(\tilde{\theta_5}, \zeta_1)$ over the universe Θ be

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_5}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.4}, \frac{\varphi_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ is FH_ySts.

Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2 = \chi_1), \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$ is FH_ySe^*os in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySe^*O but not $FH_yS\delta\alpha O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$ is not $FH_yS\delta\alpha os$ in Θ.
Also, $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_5}, \zeta_1)$ is $FH_yS\delta\mathcal{P}os$ in Θ.
$\therefore$ $\mathfrak{h}$ is $FH_yS\delta\mathcal{P}O$ but not $FH_yS\delta\alpha O$ as $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)$ is not $FH_yS\delta\alpha os$ in Θ.

Example 3.20. Let $\Theta = \{\chi_1, \chi_2\}$ and $\Omega = \{\varphi_1, \varphi_2\}$ be the FH_yS initial universes and the attributes be $L = \Upsilon_1 \times \Upsilon_2$ and $M = \Upsilon_1' \times \Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1 = \{a_1, a_2\}, \Upsilon_2 = \{b_1\}$$
$$\Upsilon_1' = \{c_1, c_2\}, \Upsilon_2' = \{d_1\}.$$

Let $(\Theta, L), (\Omega, M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1}, \zeta)$, $(\tilde{\theta_2}, \zeta)$, $(\tilde{\theta_3}, \zeta)$, $(\tilde{\theta_4}, \zeta)$, $(\tilde{\theta_5}, \zeta)$, $(\tilde{\theta_6}, \zeta)$, $(\tilde{\theta_7}, \zeta)$ & $(\tilde{\theta_8}, \zeta)$ over the universe Θ be

https://doi.org/10.54216/IJNS.210203
Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$$(\tilde{\theta_1},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_2},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0},\frac{\chi_2}{0}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_3},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_4},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_5},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_6},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_7},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0}\}\rangle\end{Bmatrix}$$

$$(\tilde{\theta_8},\zeta)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\chi_1}{0.8},\frac{\chi_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\chi_1}{0.2},\frac{\chi_2}{0.5}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\Theta,\Upsilon)},\tilde{1}_{(\Theta,\Upsilon)},(\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta),(\tilde{\theta_3},\zeta),(\tilde{\theta_4},\zeta),(\tilde{\theta_5},\zeta),(\tilde{\theta_6},\zeta),(\tilde{\theta_7},\zeta),(\tilde{\theta_8},\zeta)\}$ is FH_ySts.

Let the FH_ySs $(\tilde{\psi_1},\zeta)$ over the universe Ω be defined as

$$(\tilde{\psi_1},\zeta)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\varphi_1}{0},\frac{\varphi_2}{0.8}\}\rangle,\\ \langle(c_1,d_1),\{\frac{\varphi_1}{0},\frac{\varphi_2}{0}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\Omega,\Upsilon)},\tilde{1}_{(\Omega,\Upsilon)},(\tilde{\psi_1},\zeta)\}$ is FH_ySts.

Let $\mathfrak{h}=(\omega,\nu):(\Omega,M)\to(\Theta,L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2)=\chi_1,\omega(\varphi_1)=\chi_2,$$
$$\nu(c_2,d_1)=(a_1,b_1),\nu(c_1,d_1)=(a_2,b_1)$$
$$\mathfrak{h}(\tilde{\psi_1},\zeta)=(\tilde{\theta_1},\zeta)$$

$(\tilde{\psi_1},\zeta)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1},\zeta)=(\tilde{\theta_1},\zeta)$ is FH_ySe^*os in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySe^*O. But $\mathfrak{h}$ is not FH_ySeO because $\mathfrak{h}(\tilde{\psi_1},\zeta)=(\tilde{\theta_1},\zeta)$ is not FH_ySeos in Θ.

Example 3.21. Let $\Theta=\{\chi_1,\chi_2\}$ and $\Omega=\{\varphi_1,\varphi_2\}$ be the FH_yS initial universes and the attributes be $L=\Upsilon_1\times\Upsilon_2$ and $M=\Upsilon_1'\times\Upsilon_2'$ respectively. The attributes are given as:

$$\Upsilon_1=\{a_1,a_2,a_3\},\Upsilon_2=\{b_1,b_2\}$$
$$\Upsilon_1'=\{c_1,c_2,c_3\},\Upsilon_2'=\{d_1,d_2\}.$$

Let $(\Theta,L),(\Omega,M)$ be the classes of FH_yS sets. Let the FH_ySs's $(\tilde{\theta_1},\zeta_1)$, $(\tilde{\theta_2},\zeta_2)$, $(\tilde{\theta_3},\zeta_3)$, $(\tilde{\theta_4},\zeta_3)$, $(\tilde{\theta_5},\zeta_1)$, $(\tilde{\theta_6},\zeta_2)$ & $(\tilde{\theta_7},\zeta_1)$ over the universe Θ be

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$$(\tilde{\theta_1}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_2}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_4}, \zeta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_5}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_6}, \zeta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.3}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\chi_1}{0.5}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_7}, \zeta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\chi_1}{0.3}, \frac{\chi_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\Theta,\Upsilon)}, \tilde{1}_{(\Theta,\Upsilon)}, (\tilde{\theta_1}, \zeta_1), (\tilde{\theta_2}, \zeta_2), (\tilde{\theta_3}, \zeta_3), (\tilde{\theta_4}, \zeta_3)\}$ is FH_ySts.

Let the FH_ySs's $(\tilde{\psi_1}, \zeta_1)$ & $(\tilde{\psi_2}, \zeta_1)$ over the universe Ω be defined as

$$(\tilde{\psi_1}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.3}, \frac{\varphi_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{\psi_2}, \zeta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\varphi_1}{0.4}, \frac{\varphi_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\varphi_1}{0.5}, \frac{\varphi_2}{0.3}\}\rangle \end{Bmatrix}$$

$\sigma_1 = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_1}, \zeta_1)\}$ and $\sigma_2 = \{\tilde{0}_{(\Omega,\Upsilon)}, \tilde{1}_{(\Omega,\Upsilon)}, (\tilde{\psi_2}, \zeta_1)\}$ are FH_ySts's. Let $\mathfrak{h} = (\omega, \nu) : (\Omega, M) \to (\Theta, L)$ be a FH_yS mapping as follows:

$$\omega(\varphi_2) = \chi_1, \omega(\varphi_1) = \chi_2,$$
$$\nu(c_2, d_1) = (a_1, b_1), \nu(c_1, d_2) = (a_2, b_1), \nu(c_2, d_2) = (a_1, b_2)$$

$$\mathfrak{h}_1(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$$
$$\mathfrak{h}_2(\tilde{\psi_2}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$$

$(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}_1(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is $FH_yS\delta\alpha os$ in Θ.
$\therefore$ $\mathfrak{h}_1$ is $FH_yS\delta\alpha O$ but not $FH_yS\delta SO$ because $\mathfrak{h}_1(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_3}, \zeta_3)$ is not $FH_yS\delta Sos$ in Θ.
Also, $\mathfrak{h}_2(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$ is $FH_yS\delta Sos$ in Θ.
$\therefore$ $\mathfrak{h}_2$ is $FH_yS\delta SO$ but not $FH_yS\delta\alpha O$ because $\mathfrak{h}_2(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$ is not $FH_yS\delta\alpha os$ in Θ.
Hence, $FH_yS\delta\alpha O$ and $FH_yS\delta SO$ are independent to each other.

Example 3.22. In the Example 3.17, $(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$ is FH_ySe^*os in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySe^*O but not $FH_yS\delta\mathcal{P}O$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_7}, \zeta_1)$ is not $FH_yS\delta\mathcal{P}os$ in Θ.
Also, in the Example 3.20, $(\tilde{\psi_1}, \zeta_1)$ is FH_ySos in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is $FH_yS\delta\mathcal{P}os$ in Θ.
$\therefore$ $\mathfrak{h}$ is $FH_yS\delta\mathcal{P}O$ but not FH_ySe^*O because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1) = (\tilde{\theta_1}, \zeta_1)$ is not FH_ySe^*os in Θ.
Hence, FH_ySe^*O and $FH_yS\delta\mathcal{P}O$ are independent to each other.

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Theorem 3.23. A map $\mathfrak{h} : (\Omega, M, \sigma) \to (\Theta, L, \tau)$ is FH_ySeO (resp. FH_ySO, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\mathcal{S}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$, FH_ySe^*O) iff the image of each FH_yScs in Ω is FH_ySecs (resp. FH_yScs, $FH_yS\delta cs$, $FH_yS\mathcal{P}cs$, $FH_yS\mathcal{S}cs$, $FH_yS\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$, $FH_yS\delta\alpha cs$, FH_ySe^*cs) in Θ.

Proof. Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Ω. This implies that $(\tilde{\theta_1}, \zeta)^c$ is FH_ySos in Ω. Since $\mathfrak{h}$ is FH_ySeO, $\mathfrak{h}((\tilde{\theta_1}, \zeta)^c)$ is FH_ySeos in Θ. Since $\mathfrak{h}((\tilde{\theta_1}, \zeta)^c) = (\mathfrak{h}(\tilde{\theta_1}, \zeta))^c$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_ySecs in Θ.

Conversely, let $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in Ω. Then $(\tilde{\theta_1}, \zeta)^c$ is a FH_yScs in Ω. By hypothesis, $\mathfrak{h}((\tilde{\theta_1}, \zeta)^c)$ is FH_ySecs in Θ. Since, $\mathfrak{h}((\tilde{\theta_1}, \zeta)^c) = (\mathfrak{h}(\tilde{\theta_1}, \zeta))^c$, $(\mathfrak{h}(\tilde{\theta_1}, \zeta))^c$ is FH_ySecs in Θ. Therefore, $(\mathfrak{h}(\tilde{\theta_1}, \zeta))$ is a FH_ySeos in Θ. Hence, $\mathfrak{h}$ is FH_ySeO.

The other cases are similar. □

Theorem 3.24. A mapping $\mathfrak{h} : (\Omega, M, \sigma) \to (\Theta, L, \tau)$ is FH_ySeO iff for every FH_ySs $(\tilde{\theta_1}, \zeta)$ of (Ω, M, σ), $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) \subseteq FH_ySeint(\mathfrak{h}(\tilde{\theta_1}, \zeta))$.

Proof. Necessity: Let $\mathfrak{h}$ be a FH_ySeO mapping and $(\tilde{\theta_1}, \zeta)$ be a FH_ySos in (Ω, M, σ).

Now, $FH_ySint(\tilde{\theta_1}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ implies $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) \subseteq \mathfrak{h}(\tilde{\theta_1}, \zeta)$. Since $\mathfrak{h}$ is a FH_ySeO mapping, $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta))$ is FH_ySeos in (Θ, L, τ) such that $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) \subseteq \mathfrak{h}(\tilde{\theta_1}, \zeta)$. Therefore $\mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) \subseteq FH_ySeint(\mathfrak{h}(\tilde{\theta_1}, \zeta))$.

Sufficiency: Assume $(\tilde{\theta_1}, \zeta)$ is a FH_ySos of (Ω, M, σ).

Then $\mathfrak{h}(\tilde{\theta_1}, \zeta) = \mathfrak{h}(FH_ySint(\tilde{\theta_1}, \zeta)) \subseteq FH_ySeint(\mathfrak{h}(\tilde{\theta_1}, \zeta))$. But $FH_ySeint(\mathfrak{h}(\tilde{\theta_1}, \zeta)) \subseteq \mathfrak{h}(\tilde{\theta_1}, \zeta)$. So $\mathfrak{h}(\tilde{\theta_1}, \zeta) = FH_ySeint(\tilde{\theta_1}, \zeta)$ which implies $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is a FH_ySeos of (Θ, L, τ) and hence $\mathfrak{h}$ is a FH_ySeO. □

Theorem 3.25. If $\mathfrak{h} : (\Omega, M, \sigma) \to (\Theta, L, \tau)$ is a FH_ySeO mapping, then $FH_ySint(\mathfrak{h}^{-1}(\tilde{\theta_1}, \zeta)) \subseteq \mathfrak{h}^{-1}(FH_ySeint(\tilde{\theta_1}, \zeta))$ for every FH_ySs $(\tilde{\theta_1}, \zeta)$ of (Θ, L, τ).

Proof. Let $(\tilde{\theta_1}, \zeta)$ be a FH_ySs of (Θ, L, τ). Then $FH_ySint(\mathfrak{h}^{-1}(\tilde{\theta_1}, \zeta))$ is a FH_ySos in (Ω, M, σ). Since $\mathfrak{h}$ is FH_ySeO, $\mathfrak{h}(FH_ySint(\mathfrak{h}^{-1}(\tilde{\theta_1}, \zeta)))$ is FH_ySeO in (Θ, L, τ) and hence we have $\mathfrak{h}(FH_ySint(\mathfrak{h}^{-1}(\tilde{\theta_1}, \zeta))) \subseteq FH_ySeint(\mathfrak{h}(\mathfrak{h}^{-1}(\tilde{\theta_1}, \zeta))) \subseteq FH_ySeint(\tilde{\theta_1}, \zeta)$. Thus $FH_ySint(\mathfrak{h}^{-1}(\tilde{\theta_1}, \zeta)) \subseteq \mathfrak{h}^{-1}(FH_ySeint(\tilde{\theta_1}, \zeta))$. □

Theorem 3.26. A mapping $\mathfrak{h} : (\Omega, M, \sigma) \to (\Theta, L, \tau)$ is FH_ySeO iff for each FH_ySs $(\tilde{\theta_2}, \zeta)$ of (Θ, L, τ) and for each FH_yScs $(\tilde{\theta_1}, \zeta)$ of (Ω, M, σ) containing $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)$, there is a FH_ySecs $(\tilde{\psi}, \zeta)$ of (Θ, L, τ) such that $(\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$ and $\mathfrak{h}^{-1}(\tilde{\psi}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$.

Proof. Necessity: Assume $\mathfrak{h}$ is a FH_ySeO mapping. Let $(\tilde{\theta_2}, \zeta)$ be the FH_yScs of (Θ, L, τ) and $(\tilde{\theta_1}, \zeta)$ is a FH_yScs of (Ω, M, σ) such that $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$. Then $(\tilde{\psi}, \zeta) = (\mathfrak{h}(\tilde{\theta_1}, \zeta)^c)^c$ is FH_ySecs of (Θ, L, τ) such that $\mathfrak{h}^{-1}(\tilde{\psi}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$.

Sufficiency: Assume $(\tilde{\theta_1}, \zeta)$ is a FH_ySos of (Ω, M, σ). Then $\mathfrak{h}^{-1}((\mathfrak{h}(\tilde{\theta_1}, \zeta))^c) \subseteq (\tilde{\theta_1}, \zeta)^c$ and $(\tilde{\theta_1}, \zeta)^c$ is FH_yScs in (Ω, M, σ). By hypothesis, there is a FH_ySecs $(\tilde{\theta_2}, \zeta)$ of (Θ, L, τ) such that $(\mathfrak{h}(\tilde{\theta_1}, \zeta))^c \subseteq (\tilde{\theta_2}, \zeta)$ and $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)^c$. Therefore $(\tilde{\theta_1}, \zeta) \subseteq (\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))^c$. Hence $(\tilde{\theta_2}, \zeta)^c \subseteq \mathfrak{h}(\tilde{\theta_1}, \zeta) \subseteq \mathfrak{h}((\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))^c) \subseteq (\tilde{\theta_2}, \zeta)^c$ which implies $\mathfrak{h}(\tilde{\theta_1}, \zeta) = (\tilde{\theta_2}, \zeta)^c$. Since $(\tilde{\theta_2}, \zeta)^c$ is FH_ySeos of (Θ, L, τ), $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySeO in (Θ, L, τ) and thus $\mathfrak{h}$ is FH_ySeO mapping. □

Theorem 3.27. A mapping $\mathfrak{h} : (\Omega, M, \sigma) \to (\Theta, L, \tau)$ is FH_ySeO iff $\mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2}, \zeta)) \subseteq FH_yScl(\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta))$ for every FH_ySs $(\tilde{\theta_2}, \zeta)$ of (Θ, L, τ).

Proof. Necessity: Assume $\mathfrak{h}$ is a FH_ySeO mapping. For any FH_ySs $(\tilde{\theta_2},\zeta)$ of (Θ,L,τ), $\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta) \subseteq FH_yScl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta))$. Hence, by Theorem 3.26, there exists a FH_ySecs $(\tilde{\theta_1},\zeta)$ in (Θ,L,τ) such that $(\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_1},\zeta)$ & $\mathfrak{h}^{-1}(\tilde{\theta_1},\zeta) \subseteq FH_yScl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta))$. Therefore we obtain that $\mathfrak{h}^{-1}(FH_ySecl(\tilde{\theta_2},\zeta)) \subseteq \mathfrak{h}^{-1}(\tilde{\theta_1},\zeta) \subseteq FH_yScl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta))$.

Sufficiency: Assume $(\tilde{\theta_2},\zeta)$ is a FH_ySs of (Θ,L,τ) and $(\tilde{\theta_1},\zeta)$ is a FH_yScs of (Ω,M,σ) containing $\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta)$. Put $(\tilde{\psi},\zeta) = FH_yScl(\tilde{\theta_2},\zeta)$, then $(\tilde{\theta_2},\zeta) \subseteq (\tilde{\psi},\zeta)$ and $(\tilde{\psi},\zeta)$ is FH_ySec and $\mathfrak{h}^{-1}(\tilde{\psi},\zeta) \subseteq FH_yScl(\mathfrak{h}^{-1}(\tilde{\theta_2},\zeta)) \subseteq (\tilde{\theta_1},\zeta)$. Then by Theorem 3.26, $\mathfrak{h}$ is FH_ySeO mapping. □

Theorem 3.28. If $\mathfrak{h} : (\Omega,M,\sigma) \to (\Theta,L,\tau)$ and $g : (\Theta,L,\tau) \to (P,\rho,Q)$ be two FH_yS mappings and $\mathfrak{g} \circ \mathfrak{h} : (\Omega,M,\sigma) \to (P,\rho,Q)$ is FH_ySeO. If $g : (\Theta,L,\tau) \to (P,\rho,Q)$ is FH_ySeIrr, then $\mathfrak{h} : (\Omega,M,\sigma) \to (\Theta,L,\tau)$ is FH_ySeO mapping.

Proof. Let $(\tilde{\theta_1},\zeta)$ be a FH_ySos in (Ω,M,σ). Then $(\mathfrak{g}\circ\mathfrak{h})(\tilde{\theta_1},\zeta)$ is FH_ySeos of (P,ρ,Q) because $\mathfrak{g}\circ\mathfrak{h}$ is FH_ySeO mapping. Since $\mathfrak{g}$ is FH_ySeIrr and $(\mathfrak{g}\circ\mathfrak{h})(\tilde{\theta_1},\zeta)$ is FH_ySeos of (P,ρ,Q), $\mathfrak{g}^{-1}(\mathfrak{g}\circ\mathfrak{h}(\tilde{\theta_1},\zeta)) = \mathfrak{h}(\tilde{\theta_1},\zeta)$ is FH_ySeos in (Θ,L,τ). Hence $\mathfrak{h}$ is FH_ySeO mapping. □

Theorem 3.29. Let $\mathfrak{h} : (\Omega,M,\sigma) \to (\Theta,L,\tau)$ be a FH_ySO map and $\mathfrak{g} : (\Theta,L,\tau) \to (P,N,\rho)$ be a FH_ySeO (resp. FH_ySO, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\mathcal{S}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$, FH_yS e^*O) map, then $\mathfrak{g}\circ\mathfrak{h} : (\Omega,M,\sigma) \to (P,N,\rho)$ is a FH_ySeO (resp. FH_ySO, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\mathcal{S}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, $FH_yS\delta\alpha O$, FH_ySe^*O).

Proof. Let $(\tilde{\theta_1},\zeta)$ be a FH_ySos in Ω. Then $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is a FH_ySos in Θ, by hypothesis. Since $\mathfrak{g}$ is a FH_ySeO map, $\mathfrak{g}(\mathfrak{h}(\tilde{\theta_1},\zeta))$ is a FH_ySeos in P. Hence $\mathfrak{g}\circ\mathfrak{h}$ is a FH_ySeO map.

The other cases are similar. □

Remark 3.30. The Theorems 3.24, 3.25, 3.26 and 3.27 are also true for FH_ySos, $FH_yS\mathcal{P}os$, $FH_yS\mathcal{S}os$, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\delta\alpha os$ & FH_ySe^*os of their respective closure and interior operators.

4 More on Closed Maps in Fuzzy Hypersoft Topological Spaces

Definition 4.1. A mapping $\mathfrak{h} : (\Theta,L,\tau) \to (\Omega,M,\sigma)$ is FH_yS *e-closed* (resp. *closed, δ closed, δ-semi closed, δ-pre closed* & *e^*-closed*) (briefly, FH_ySeC (resp.FH_ySC, $FH_yS\delta C$, $FH_yS\delta\mathcal{S}C$, $FH_yS\delta\mathcal{P}C$ & FH_ySe^*C)) if the image of every FH_yS closed set of (Θ,L,τ) is FH_ySec (resp. FH_ySc, $FH_yS\delta c$, $FH_yS\delta\mathcal{S}c$, $FH_yS\delta\mathcal{P}c$ & FH_ySe^*c) set in (Ω,M,σ).

Theorem 4.2. The statements are correct, but the converse need not be true.

(i) Every $FH_yS\delta C$ is a FH_ySC.

(ii) Every FH_ySC is a $FH_yS\mathcal{S}C$.

(iii) Every FH_ySC is a $FH_yS\mathcal{P}C$.

(iv) Every $FH_yS\delta C$ is a $FH_yS\delta\mathcal{S}C$.

(v) Every $FH_yS\delta C$ is a $FH_yS\delta\mathcal{P}C$.

(vi) Every $FH_yS\delta C$ is a FH_ySeC.

(vii) Every $FH_yS\delta\mathcal{S}C$ is a FH_ySeC.

(viii) Every $FH_yS\delta\mathcal{P}C$ is a FH_ySeC.

(ix) Every $FH_yS\delta\mathcal{S}C$ is a FH_ySe^*C.

(x) Every $FH_yS\delta\alpha C$ is a FH_ySe^*C.

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

(xi) Every $FH_yS\delta\alpha C$ is a $FH_yS\delta\mathcal{P}C$.

(xii) Every FH_ySe^*C is a FH_ySeC.

Proof. Consider a mapping $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$.

(i) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta cs$ in Ω. Since all $FH_yS\delta cs$ are FH_yScs, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_yScs in Ω. Hence $\mathfrak{h}$ is a FH_ySC.

(ii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is FH_ySC, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_yScs in Ω. Since all FH_yScs are $FH_yS\mathcal{S}cs$,[5] $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\mathcal{S}cs$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\mathcal{S}C$.

(iii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is FH_ySC, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_yScs in Ω. Since all FH_yScs are $FH_yS\mathcal{P}cs$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\mathcal{P}cs$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\mathcal{P}C$.

(iv) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta cs$ in Ω. Since all $FH_yS\delta cs$ are $FH_yS\delta\mathcal{S}cs$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\mathcal{S}cs$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\delta\mathcal{S}C$.

(v) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta cs$ in Ω. Since all $FH_yS\delta cs$ are $FH_yS\delta\mathcal{P}cs$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\mathcal{P}cs$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\delta\mathcal{P}C$.

(vi) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta cs$ in Ω. Since all $FH_yS\delta cs$ are FH_ySecs, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySecs in Ω. Hence $\mathfrak{h}$ is a FH_ySeC.

(vii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta\mathcal{S}C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\mathcal{S}cs$ in Ω. Since all $FH_yS\delta\mathcal{S}cs$ are FH_ySecs, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySecs in Ω. Hence $\mathfrak{h}$ is a FH_ySeC.

(viii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta\mathcal{P}C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\mathcal{P}cs$ in Ω. Since all $FH_yS\delta\mathcal{P}cs$ are FH_ySecs, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySecs in Ω. Hence $\mathfrak{h}$ is a FH_ySeC.

(ix) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta\mathcal{S}C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\mathcal{S}cs$ in Ω. Since all $FH_yS\delta\mathcal{S}cs$ are FH_ySe^*cs, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySe^*cs in Ω. Hence $\mathfrak{h}$ is a FH_ySe^*C.

(x) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta\alpha C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\alpha cs$ in Ω. Since all $FH_yS\delta\alpha cs$ are FH_ySe^*cs, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySe^*cs in Ω. Hence $\mathfrak{h}$ is a FH_ySe^*C.

(xi) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is $FH_yS\delta\alpha C$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\alpha cs$ in Ω. Since all $FH_yS\delta\alpha cs$ are $FH_yS\delta\mathcal{P}cs$, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is $FH_yS\delta\mathcal{P}cs$ in Ω. Hence $\mathfrak{h}$ is a $FH_yS\delta\mathcal{P}C$.

(xii) Let $(\tilde{\theta_1}, \zeta)$ be a FH_yScs in Θ. Since $\mathfrak{h}$ is FH_ySe^*C, $\mathfrak{h}(\tilde{\theta_1}, \zeta)$ is FH_ySe^*cs in Ω. Since all FH_ySe^*cs are FH_ySecs, $\mathfrak{h}(\tilde{\theta_2}, \zeta)$ is FH_ySecs in Ω. Hence $\mathfrak{h}$ is a FH_ySeC.

□

Proposition 4.3. (i) $FH_yS\delta\alpha C$ and $FH_yS\delta\mathcal{S}C$ are independent to each other.

(ii) FH_ySe^*C and $FH_yS\delta\mathcal{P}C$ are independent to each other.

Remark 4.4. The diagram shows FH_ySC maps in FH_ySts.

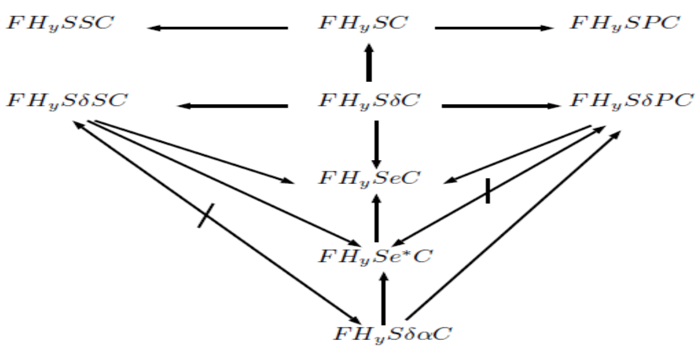

Figure 2. FH_yS closed maps in FH_ySts

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Example 4.5. In Example 3.10, $\mathfrak{h}$ is FH_ySSC mapping but not FH_ySC mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_5, \zeta_1)^c$ is FH_ySScs but not FH_yScs in Θ.

Example 4.6. In Example 3.11, $\mathfrak{h}$ is $FH_yS\mathcal{P}C$ mapping but not FH_ySC mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_5, \zeta_1)^c$ is $FH_yS\mathcal{P}cs$ but not FH_yScs in Θ.

Example 4.7. In Example 3.12, $\mathfrak{h}$ is FH_ySC mapping but not $FH_yS\delta C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_3, \zeta_3)^c$ is FH_yScs but not $FH_yS\delta cs$ in Θ.

Example 4.8. In Example 3.13, $\mathfrak{h}$ is $FH_yS\delta\mathcal{S}C$ mapping but not $FH_yS\delta C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_1, \zeta_1)^c$ is $FH_yS\delta\mathcal{S}cs$ but not $FH_yS\delta cs$ in Θ.

Example 4.9. In Example 3.14, $\mathfrak{h}$ is $FH_yS\delta\mathcal{P}C$ mapping but not $FH_yS\delta C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_3, \zeta_3)^c$ is $FH_yS\delta\mathcal{P}cs$ but not $FH_yS\delta cs$ in Θ.

Example 4.10. In Example 3.15, $\mathfrak{h}$ is FH_ySeC mapping but not $FH_yS\delta C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_1, \zeta_1)^c$ is FH_ySecs but not $FH_yS\delta cs$ in Θ.

Example 4.11. In Example 3.16, $\mathfrak{h}$ is FH_ySeC mapping but not $FH_yS\delta\mathcal{S}C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_3, \zeta_3)^c$ is FH_ySecs but not $FH_yS\delta\mathcal{S}cs$ in Θ.

Example 4.12. In Example 3.17, $\mathfrak{h}$ is FH_ySeC mapping but not $FH_yS\delta\mathcal{P}C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_7, \zeta_1)^c$ is FH_ySecs but not $FH_yS\delta\mathcal{P}cs$ in Θ.

Example 4.13. In Example 3.18, $\mathfrak{h}$ is FH_ySe^*C mapping but not $FH_yS\delta\mathcal{S}C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_3, \zeta_3)^c$ is FH_ySe^*cs but not $FH_yS\delta\mathcal{S}cs$ in Θ.

Example 4.14. In Example 3.19, $\mathfrak{h}$ is FH_ySe^*C (resp. $FH_yS\delta\mathcal{P}C$) mapping but not $FH_yS\delta\alpha C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_5, \zeta_1)^c$ is FH_ySe^*cs (resp. $FH_yS\delta\mathcal{P}cs$) but not $FH_yS\delta\alpha cs$ in Θ.

Example 4.15. In Example 3.20, $\mathfrak{h}$ is FH_ySe^*C mapping but not FH_ySeC mapping because $(\tilde{\psi_1}, \zeta)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta)^c = (\tilde{\varphi}_1, \zeta)^c$ is FH_ySe^*cs but not FH_ySecs in Θ.

Example 4.16. In Example 3.21, $\mathfrak{h}_1$ is $FH_yS\delta\alpha C$ mapping but not $FH_yS\delta\mathcal{S}C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}_1(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_3, \zeta_3)^c$ is $FH_yS\delta\alpha cs$ but not $FH_yS\delta\mathcal{S}cs$ in Θ. Also, $\mathfrak{h}_2$ is $FH_yS\delta\mathcal{S}C$ mapping but not $FH_yS\delta\alpha C$ mapping because $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}_2(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\varphi}_7, \zeta_1)^c$ is $FH_yS\delta\mathcal{S}cs$ but not $FH_yS\delta\alpha cs$ in Θ.

Example 4.17. Consider the Example 3.22. In the Example 3.17, $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\theta_7}, \zeta_1)^c$ is FH_ySe^*cs in Θ.
$\therefore$ $\mathfrak{h}$ is FH_ySe^*C but not $FH_yS\delta\mathcal{P}C$ because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\theta_7}, \zeta_1)^c$ is not $FH_yS\delta\mathcal{P}cs$ in Θ.
Also, in the Example 3.20, $(\tilde{\psi_1}, \zeta_1)^c$ is FH_yScs in Ω and $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\theta_1}, \zeta_1)^c$ is $FH_yS\delta\mathcal{P}cs$ in Θ.
$\therefore$ $\mathfrak{h}$ is $FH_yS\delta\mathcal{P}C$ but not FH_ySe^*C because $\mathfrak{h}(\tilde{\psi_1}, \zeta_1)^c = (\tilde{\theta_1}, \zeta_1)^c$ is not FH_ySe^*cs in Θ.
Hence, FH_ySe^*C and $FH_yS\delta\mathcal{P}C$ are independent to each other.

Theorem 4.18. A mapping $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is FH_ySeC (resp. FH_ySC, $FH_yS\delta C$, $FH_yS\mathcal{P}C$, FH_ySSC, $FH_yS\delta\mathcal{P}C$, $FH_yS\delta\mathcal{S}C$, $FH_yS\delta\alpha C$, FH_ySe^*C) map iff for each FH_ySs $(\tilde{\theta_2}, \zeta)$ of (Ω, M, σ) and for each FH_ySos $(\tilde{\theta_1}, \zeta)$ of (Θ, L, τ) containing $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta)$, there is a FH_ySeos (resp. FH_ySos, $FH_yS\delta os$, $FH_yS\mathcal{P}os$, FH_ySSos, $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$, $FH_yS\delta\alpha os$, FH_ySe^*os) $(\tilde{\psi}, \zeta)$ of (Ω, M, σ) such that $(\tilde{\theta_2}, \zeta) \subseteq (\tilde{\psi}, \zeta)$ and $\mathfrak{h}^{-1}(\tilde{\psi}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$.

Proof. Necessity: Assume $\mathfrak{h}$ is a FH_ySeC mapping. Let $(\tilde{\theta_2}, \zeta)$ be the FH_yScs of (Ω, M, σ) and $(\tilde{\theta_1}, \zeta)$ is a FH_ySos of (Θ, L, τ) such that $\mathfrak{h}^{-1}(\tilde{\theta_2}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$. Then $(\tilde{\psi}, \zeta) = \Omega - \mathfrak{h}^{-1}((\tilde{\theta_1}, \zeta)^c)$ is FH_ySeos of (Ω, M, σ) such that $\mathfrak{h}^{-1}(\tilde{\psi}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)$.

Sufficiency: Assume $(\tilde{\theta_1}, \zeta)$ is a FH_yScs of (Θ, L, τ). Then $(\mathfrak{h}(\tilde{\theta_1}, \zeta))^c$ is a FH_ySs of (Ω, M, σ) and $(\tilde{\theta_1}, \zeta)^c$ is FH_ySos in (Θ, L, τ) such that $\mathfrak{h}^{-1}((\mathfrak{h}(\tilde{\theta_1}, \zeta))^c) \subseteq (\tilde{\theta_1}, \zeta)^c$. By hypothesis, there is a FH_ySeos $(\tilde{\psi}, \zeta)$ of (Ω, M, σ) such that $(\mathfrak{h}(\tilde{\theta_1}, \zeta))^c \subseteq (\tilde{\psi}, \zeta)$ and $\mathfrak{h}^{-1}(\tilde{\psi}, \zeta) \subseteq (\tilde{\theta_1}, \zeta)^c$. Therefore $(\tilde{\theta_1}, \zeta) \subseteq (\mathfrak{h}^{-1}(\tilde{\psi}, \zeta))^c$. Hence

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

$(\tilde{\psi},\zeta)^c \subseteq \mathfrak{h}(\tilde{\psi},\zeta) \subseteq \mathfrak{h}((\mathfrak{h}^{-1}(\tilde{\psi},\zeta))^c) \subseteq (\tilde{\psi},\zeta)^c$ which implies $\mathfrak{h}(\tilde{\theta_1},\zeta) = (\tilde{\psi},\zeta)^c$. Since $(\tilde{\psi},\zeta)^c$ is FH_ySecs of (Ω, M,σ), $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is FH_ySec in (Ω, M,σ) and thus $\mathfrak{h}$ is FH_ySeC mapping.

The other cases are similar. □

Theorem 4.19. If $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is FH_ySC and $\mathfrak{g} : (\Omega, M, \sigma) \to (P, \rho, Q)$ is FH_ySeC (resp. FH_ySC, $FH_yS\delta C$, $FH_yS\mathcal{P}C$, $FH_yS\mathcal{S}C$, $FH_yS\delta\mathcal{P}C$, $FH_yS\delta\mathcal{S}C$, $FH_yS\delta\alpha C$, FH_ySe^*C) map, then $\mathfrak{g}\circ\mathfrak{h} : (\Theta, L, \tau) \to (P, \rho, Q)$ is FH_ySeC (resp. FH_ySC, $FH_yS\delta C$, $FH_yS\mathcal{P}C$, $FH_yS\mathcal{S}C$, $FH_yS\delta\mathcal{P}C$, $FH_yS\delta\mathcal{S}C$, $FH_yS\delta\alpha C$, FH_ySe^*C).

Proof. Let $(\tilde{\theta_1},\zeta)$ be a FH_yScs in (Θ, L, τ). Then $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is FH_yScs of (Ω, M, σ) because $\mathfrak{h}$ is FH_ySC mapping. Now $(\mathfrak{g}\circ\mathfrak{h})(\tilde{\theta_1},\zeta) = \mathfrak{g}(\mathfrak{h}(\tilde{\theta_1},\zeta))$ is FH_ySecs in (P, ρ, Q) because $\mathfrak{g}$ is FH_ySeC mapping. Thus $\mathfrak{g}\circ\mathfrak{h}$ is FH_ySeC mapping.

The other cases are similar. □

Theorem 4.20. If $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ is FH_ySeC map, then $FH_ySecl(\mathfrak{h}(\tilde{\theta_1},\zeta)) \subseteq \mathfrak{h}(FH_yScl(\tilde{\theta_1},\zeta))$.

Proof. Obvious. □

Theorem 4.21. Let $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ and $\mathfrak{g} : (\Omega, M, \sigma) \to (P, \rho, Q)$ are FH_ySeC mappings. If every FH_ySecs of (Ω, M, σ) is FH_yScs, then $\mathfrak{g}\circ\mathfrak{h} : (\Theta, L, \tau) \to (P, \rho, Q)$ is FH_ySeC.

Proof. Let $(\tilde{\theta_1},\zeta)$ be a FH_yScs in (Θ, L, τ). Then $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is FH_ySecs of (Ω, M, σ) because $\mathfrak{h}$ is FH_ySeC mapping. By hypothesis, $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is FH_yScs of (Ω, M, σ). Now $\mathfrak{g}(\mathfrak{h}(\tilde{\theta_1},\zeta)) = (\mathfrak{g}\circ\mathfrak{h})(\tilde{\theta_1},\zeta)$ is FH_ySecs in (P, ρ, Q) because $\mathfrak{g}$ is FH_ySeC mapping. Thus $\mathfrak{g}\circ\mathfrak{h}$ is FH_ySeC mapping. □

Theorem 4.22. Let $\mathfrak{h} : (\Theta, L, \tau) \to (\Omega, M, \sigma)$ be a bijective mapping. Then the following statements are equivalent:

(i) $\mathfrak{h}$ is a FH_ySeO (resp. FH_ySO, $FH_yS\delta O$, $FH_yS\mathcal{P}O$, $FH_yS\mathcal{S}O$, $FH_yS\delta\mathcal{P}O$, $FH_yS\delta\mathcal{S}O$, FH_yS $\delta\alpha O$, FH_ySe^*O) mapping.

(ii) $\mathfrak{h}$ is a FH_ySeC (resp. FH_ySC, $FH_yS\delta C$, $FH_yS\mathcal{P}C$, $FH_yS\mathcal{S}C$, $FH_yS\delta\mathcal{P}C$, $FH_yS\delta\mathcal{S}C$, FH_yS $\delta\alpha C$, FH_ySe^*O) mapping.

(iii) $\mathfrak{h}^{-1}$ is FH_ySeCts (resp. FH_ySCts, $FH_yS\delta Cts$, $FH_yS\mathcal{P}Cts$, $FH_yS\mathcal{S}Cts$, $FH_yS\delta\mathcal{P}Cts$, FH_yS $\delta\mathcal{S}Cts$, $FH_yS\delta\alpha Cts$, FH_ySe^*Cts) mapping.

Proof. (i) ⇒ (ii): Let us assume that $\mathfrak{h}$ is a FH_ySeO mapping. By definition, $(\tilde{\theta_1},\zeta)$ is a FH_ySos in (Θ, L, τ), then $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is a FH_ySeos in (Ω, M, σ). Here, $(\tilde{\theta_1},\zeta)$ is FH_yScs in (Θ, L, τ). Then $Y-(\tilde{\theta_1},\zeta)$ is a FH_ySos in (Θ, L, τ). By assumption, $\mathfrak{h}(Y-(\tilde{\theta_1},\zeta))$ is a FH_ySeos in (Ω, M, σ). Hence, $Z-\mathfrak{h}(Y-(\tilde{\theta_1},\zeta))$ is a FH_ySecs in (Ω, M, σ). Therefore, $\mathfrak{h}$ is a FH_ySeC mapping.

(ii) ⇒ (iii): Let $(\tilde{\theta_1},\zeta)$ be a FH_yScs in (Θ, L, τ). By (ii), $\mathfrak{h}(\tilde{\theta_1},\zeta)$ is a FH_ySecs in (Ω, M, σ). Hence, $\mathfrak{h}(\tilde{\theta_1},\zeta) = (\mathfrak{h}^{-1})^{-1}(\tilde{\theta_1},\zeta)$. So $\mathfrak{h}^{-1}$ is a FH_ySecs in (Ω, M, σ). Hence, $\mathfrak{h}^{-1}$ is FH_ySeCts.

(iii) ⇒ (i): Let $(\tilde{\theta_1},\zeta)$ be a FH_ySos in (Θ, L, τ). By (iii), $(\mathfrak{h}^{-1})^{-1}(\tilde{\theta_1},\zeta) = \mathfrak{h}(\tilde{\theta_1},\zeta)$ is a FH_ySeO mapping.

The other cases are similar. □

Remark 4.23. The Theorems 4.18, 4.20 and 4.21 are also true for FH_yScs, $FH_yS\mathcal{P}cs$, $FH_yS\mathcal{S}cs$, FH_yS $\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$, $FH_yS\delta\alpha cs$ & FH_ySe^*cs of their respective closure operators.

https://doi.org/10.54216/IJNS.210203
Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

5 Cotangent Similarity Measure for Fuzzy Hypersoft Sets

In this section, we use cotangent functions to construct a new similarity measure for $FH_ySs's$.

Definition 5.1. Consider two FH_ySs's $(\tilde{\theta_1},\zeta)$ and $(\tilde{\theta_2},\zeta)$ over Θ. The cotangent similarity measure for these two sets based on the cotangent function is given by

$$S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = \frac{1}{n}\sum_{i=1}^{n} cot[\frac{\pi}{4}+\frac{\pi}{4}(|\mu^i_{\tilde{\theta_1}}-\mu^i_{\tilde{\theta_2}}|)]$$

Proposition 5.2. The cotangent similarity measure $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta))$, satisfies the following properties:

(i) $0 \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) \leq 1$.

(ii) $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = S_{Ct}((\tilde{\theta_2},\zeta),(\tilde{\theta_1},\zeta))$.

(iii) $(\tilde{\theta_1},\zeta) = (\tilde{\theta_2},\zeta)$ iff $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = 1$.

(iv) If $(\tilde{\theta_3},\zeta)$ is a FH_ySs in Θ and $(\tilde{\theta_1},\zeta) \subseteq (\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_3},\zeta)$, then $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta))$ and $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_2},\zeta),(\tilde{\theta_3},\zeta))$.

Proof. (i) Since the value of cotangent function and the membership value of FH_ySs's are in the interval $[0,1]$, the similarity measure based on the cotangent functions which is arithmetic mean of these cotangent functions, are also in $[0,1]$. Therefore, $0 \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) \leq 1$.

(ii) Proof is obvious.

(iii) For any two FH_ySs's $(\tilde{\theta_1},\zeta)$ and $(\tilde{\theta_2},\zeta)$ in Θ, if $(\tilde{\theta_1},\zeta) = (\tilde{\theta_2},\zeta)$, then $\mu^i_{(\tilde{\theta_1},\zeta)} = \mu^i_{(\tilde{\theta_2},\zeta)}$, for $i = 1,2,...,n$. Thus, we obtain $|\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_2},\zeta)}| = 0$.

And so the cotangent similarity measure $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = 1$. Conversely, let $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = 1$. Since $cot\frac{\pi}{4} = 1$, this implies that

$|\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_2},\zeta)}| = 0$.

Therefore, we obtain $\mu^i_{(\tilde{\theta_1},\zeta)} = \mu^i_{(\tilde{\theta_2},\zeta)}$, for $i = 1,2,3,...,n$. Hence, $(\tilde{\theta_1},\zeta) = (\tilde{\theta_2},\zeta)$.

(iv) If $(\tilde{\theta_1},\zeta) \subseteq (\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_3},\zeta)$, then $\mu^i_{(\tilde{\theta_1},\zeta)} \leq \mu^i_{(\tilde{\theta_2},\zeta)} \leq \mu^i_{(\tilde{\theta_3},\zeta)}$, for $i = 1,2,3,...,n$.

Thus, we have

$|\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_2},\zeta)}| \leq |\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_3},\zeta)}|$

$|\mu^i_{(\tilde{\theta_2},\zeta)} - \mu^i_{(\tilde{\theta_3},\zeta)}| \leq |\mu^i_{(\tilde{\theta_1},\zeta)} - \mu^i_{(\tilde{\theta_3},\zeta)}|$

Hence, $(\tilde{\theta_1},\zeta) \subseteq (\tilde{\theta_2},\zeta) \subseteq (\tilde{\theta_3},\zeta)$. Then, $S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta))$ and

$S_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_3},\zeta)) \leq S_{Ct}((\tilde{\theta_2},\zeta),(\tilde{\theta_3},\zeta))$.

As the cotangent function is decreasing with the interval $[0,\frac{\pi}{4}]$, the proof is completed. □

Similarly, the weighted version of cotangent similarity measure is given as

$$WS_{Ct}((\tilde{\theta_1},\zeta),(\tilde{\theta_2},\zeta)) = \frac{1}{n}\sum_{i=1}^{n} W_i cot[\frac{\pi}{4}+\frac{\pi}{4}(|\mu^i_{\tilde{\theta_1}}-\mu^i_{\tilde{\theta_2}}|)]$$

where $0 \leq W_1, W_2, W_3, ..., W_n \leq 1$ with $\sum_{i=1}^{n} W_i = 1$.

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

6 Algorithm

In this section, the algorithm based on the proposed similarity measure is given.

As per the medical history, the various symptoms of Covid-19 are Fever, Headache, Body pain, Dry Cough, Difficulty in breathing and Chest pain. We categorize these symptoms as the distinct set of severe symptoms, most common symptom and less common symptoms.

Severe symptoms = Difficulty in breathing, Chest pain

Most common symptoms = Fever, Dry cough

Less common symptoms = Headache, Body pain

We can formulate the symptoms of the Covid-19 patients collected from the hospital records as FH_ySs's by considering the membership values as 'Covid-19' and 'No Covid-19'. Now, consider the patients visiting hospital with Covid-19 symptoms. Let us formulate those patients' symptoms as the FH_ySs's using the defined category of the symptoms. Using the proposed cotangent similarity measure, the examination can be done by comparing the symptoms of the Covid-19 patients and the patients visiting hospital with the symptoms related to Covid-19. Thus, a decision can be made whether the patients have the possibility of suffering from Covid-19 or not.

We next give the implementation steps of the proposed algorithm based on cotangent similarity measure for FH_ySs's in which the flow chart of the proposed alogorithm is shown in the figure.

Step 1: Formulate the symptoms of Covid-19 patients as a FH_ySs by considering the degree of association between the Covid-19 patients and the Covid-19 symptoms.

Step 2: Formulate the symptoms of the two patients visited the hospital as FH_ySs's by considering the association between the patients and the Covid-19 symptoms.

Step 3: Find the similarity between the symptoms of the Covid-19 patients and the 1st patient visited hospital using the proposed cotangent similarity measure.

Step 4: Find the similarity between the symptoms of the Covid-19 patients and the 2nd patient visited hospital using the proposed cotangent similarity measure.

Step 5: Compare both the similarity measures. The more the similarity, there is a larger possibility for the patient to be suffering from Covid-19.

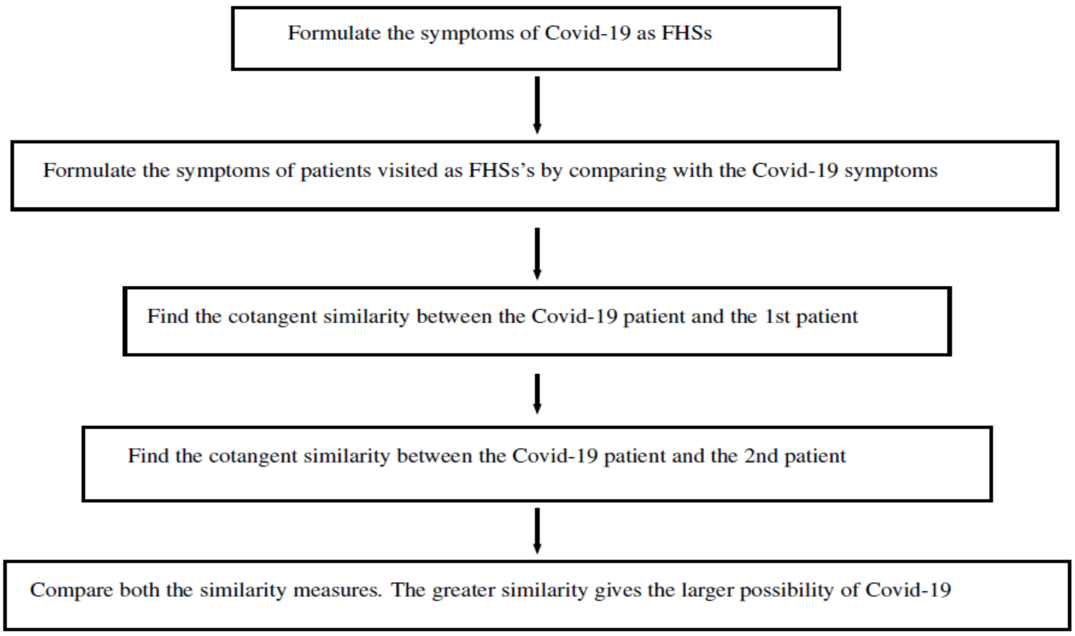

Figure 3. Flowchart of the proposed algorithm

7 Application in Covid-19 Diagnosis using Cotangent Similarity Measure

Example 7.1. Consider 2 patients visiting hospital with the following symptoms: Fever, Dry cough, Head ache, Body pain, Difficulty in breathing and Chest pain. The symptoms of Covid-19 patients can be categorized as
Severe symptoms = Difficulty in breathing, Chest pain
Most common symptoms = Fever, Dry cough
Less common symptoms = Headache, Body pain
Using the fuzzy hypersoft model problem, the examination can be done whether the patients have the possibility of suffering from Covid-19 or not. Let Θ be the universal set $\Theta = \{\chi_1, \chi_2\}$ = {Covid-19, No Covid-19}. The attributes are given as:

$$\Upsilon_1 = \{a_1 = \text{Difficulty in breathing}, a_2 = \text{Chest pain}\}$$
$$\Upsilon_2 = \{b_1 = \text{Fever}, b_2 = \text{Dry cough}\}$$
$$\Upsilon_3 = \{c_1 = \text{Headache}, c_2 = \text{Body pain}\}$$

We define the fuzzy hypersoft sets which give the degree of association between the Covid-19 patients and the Covid-19 symptoms and between the 2 patients visited and their symptoms.

The FH_ySs $(\tilde{\theta_1}, \zeta)$ describes the evaluation of the Covid-19 patients and their symptoms as per the hospital records.

$$(\tilde{\theta_1}, \zeta) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{\chi_1}{1.0}, \frac{\chi_2}{0.2}\}\rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.1}\}\rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.2}\}\rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.2}\}\rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.1}\}\rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.1}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.1}\}\rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\chi_1}{0.9}, \frac{\chi_2}{0.1}\}\rangle \end{Bmatrix}$$

The FH_ySs's $(\tilde{\theta_2}, \zeta)$ and $(\tilde{\theta_3}, \zeta)$ describe the evaluation of the 2 patients visited and their symptoms respectively.

$$(\tilde{\theta_2}, \zeta) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\}\rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\}\rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\chi_1}{0.0}, \frac{\chi_2}{0.9}\}\rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\}\rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\chi_1}{0.2}, \frac{\chi_2}{0.9}\}\rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.8}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\}\rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\chi_1}{0.1}, \frac{\chi_2}{0.9}\}\rangle \end{Bmatrix}$$

$$(\tilde{\theta_3}, \zeta) = \begin{Bmatrix} \langle (a_1, b_1, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_1, b_1, c_2), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.2}\}\rangle, \\ \langle (a_1, b_2, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_1, b_2, c_2), \{\frac{\chi_1}{0.6}, \frac{\chi_2}{0.4}\}\rangle, \\ \langle (a_2, b_1, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.2}\}\rangle, \\ \langle (a_2, b_2, c_1), \{\frac{\chi_1}{0.8}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.3}\}\rangle, \\ \langle (a_2, b_1, c_2), \{\frac{\chi_1}{0.7}, \frac{\chi_2}{0.2}\}\rangle \end{Bmatrix}$$

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

Using the proposed cotangent similarity measure, we get
$S_C((\tilde{\theta_1}, \zeta), (\tilde{\theta_2}, \zeta)) = 0.1943$
$S_C((\tilde{\theta_1}, \zeta), (\tilde{\theta_3}, \zeta)) = 0.7369.$
As the similarity between the Covid-19 patient and the 2nd patient is lesser than 1st patient, there is larger possibility for the 2nd patient suffering from Covid-19.

8 Conclusions

In this paper, FH_ySO, FH_ySSO, $FH_yS\delta O$, $FH_yS\delta SO$, $FH_yS\delta\mathcal{P}O$, FH_ySeO, $FH_yS\delta\,\alpha O$ and FH_ySe^*O maps and their respective closed maps are introduced in $FH_ySts.$ Also, their properties and relations between them are analyzed with the examples. Further, a cotangent similarity measure for FH_ySs's is introduced and an application in diagnosing Covid-19 using cotangent similarity measure is discussed with an example. In future, these findings can be extended to various forms of fuzzy hypersoft homeomorphic functions and separation axioms.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] A. Acikgoz and F. Esenbel, *Neutrosophic soft δ-topology and neutrosophic soft compactness*, AIP Conference Proceedings 2183, **030002**, (2019).

[2] M. Abbas, G. Murtaza and F. Smarandache, *Basic operations on hypersoft sets and hypersoft point*, Neutrosophic Sets and Systems, **35**, (2020), 407-421.

[3] M. Ahsan, M. Saeed and A.U. Rahman, *A Theoretical and Analytical Approach for Fundamental Framework of Composite Mappings on Fuzzy Hypersoft Classes*, Neutrosophic Sets and Systems, **45**, (2021), 268-285.

[4] D. Ajay and J. Joseline Charisma, *Neutrosophic hypersoft topological spaces*, Neutrosophic Sets and Systems, **40**, (2021), 178-194.

[5] D. Ajay, J. Joseline Charisma, N. Boonsatit, P. Hammachukiattikul and G. Rajchakit *Neutrosophic semiopen hypersoft sets with an application to MAGDM under the COVID-19 scenario*, Hindawi Journal of Mathematics, **2021**, (2021), 1-16.

[6] C. G. Aras and S. Bayramov, *Neutrosophic Soft Continuity in Neutrosophic Soft Topological Spaces*, Filomat, **34:10**, (2020), 3495 - 3506.

[7] V. Chandrasekar, D. Sobana and A. Vadivel, *On Fuzzy e-open Sets, Fuzzy e-continuity and Fuzzy e-compactness in Intuitionistic Fuzzy Topological Spaces*, Sahand Communications in Mathematical Analysis (SCMA), **12** (1) (2018), 131-153.

[8] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182-190.

[9] E. Ekici, *On e-open sets, $\mathcal{DP}^\star$-sets and $\mathcal{DP}\epsilon^\star$-sets and decomposition of continuity*, The Arabian Journal for Science and Engineering, **33** (2A) (2008), 269-282.

[10] M. N. Jafar, M. Saeed, M. Saqlain and M. Yang, *Trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection*, IEEE Access, **9**, (2021), 129178-129187.

[11] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., **37**, (1999), 19-31.

[12] P. Revathi, K. Chitirakala and A. Vadivel, *Soft e-separation axioms in neutrosophic soft topological spaces*, Journal of Physics: Conference Series, **2070**, (2021), 012028.

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023

[13] P. Revathi, K. Chitirakala and A. Vadivel, *Neutrosophic Soft e-Open Maps, Neutrosophic Soft e-Closed Maps and Neutrosophic Soft e-Homeomorphisms in Neutrosophic Soft Topological Spaces*, Springer Proceedings in Mathematics and Statistics, **384**, (2022), 47-58.

[14] S. Saha, *Fuzzy δ-continuous mappings*, Journal of Mathematical Analysis and Applications, **126** (1987), 130-142.

[15] M. Saqlain, N. Jafar, S. Moin, M. Saeed and S. Broumi, *Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Set* , Neutrosophic Sets and Systems , **32** , (2020) , 317-329.

[16] V. Seenivasan and K. Kamala, *Fuzzy e-continuity and fuzzy e-open sets*, Annals of Fuzzy Mathematics and Informatics, **8** (2014), 141-148.

[17] M. Shabir and M. Naz, *On soft topological spaces*, Comput. Math. Appl., **61**, (2011), 1786-1799.

[18] F. Smarandache, *Extension of soft set to hypersoft set, and then to plithogenic hypersoft set* , Neutrosophic Sets and Systems, **22**, (2018), 168-170.

[19] A. Vadivel and C. John Sundar, *Neutrosophic δ-Open Maps and Neutrosophic δ-Closed Maps*, International Journal of Neutrosophic Science, **13** (2) (2021), 66-74.

[20] A. Vadivel and C. John Sundar, *$N_{nc}\delta$-Open Sets*, South East Asian Journal of Mathematics and Mathematical Sciences, **18** (3) (2022), 207-216.

[21] A. Vadivel, M. Seenivasan and C. John Sundar, *An introduction to δ-open sets in a neutrosophic topological spaces*, Journal of Physics: Conference series, **1724** (2021), 012011.

[22] A. Vadivel, P. Thangaraja and C. John Sundar, *Neutrosophic e-Continuous Maps and Neutrosophic e-Irresolute Maps*, Turkish Journal of Computer and Mathematics Education, **12** (1S) (2021), 369-375.

[23] A. Vadivel, P. Thangaraja and C. John Sundar, *Neutrosophic e-open maps, neutrosophic e-closed maps and neutrosophic e homeomorphisms in neutrosophic topological spaces*, AIP Conference Proceedings, 2364 (2021), 020016.

[24] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (3), (1965), 338–353.

https://doi.org/10.54216/IJNS.210203

Received: January 19, 2023 Revised: April 23, 2023 Accepted: May 05, 2023