On various Inverse of Neutrosophic Fuzzy Matrices

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Abstract

In this article, we discuss various Inverses Minimum norm g-inverse, Least square g-inverse, Moore Penrose inverse, Group Inverse, Generalized Symmetric Neutrosophic Fuzzy Matrices. Also we describes secondary k-column symmetric Neutrosophic fuzzy matrices are produced. It is discussed how s-k-column symmetric, s-column symmetric, k-column symmetric, and column symmetric Neutrosophic fuzzy matrices relate to one another. For an Neutrosophic fuzzy matrices to be an s-k-column symmetric Neutrosophic fuzzy matrices, necessary and sufficient requirements are identified.

Keywords: Neutrosophic fuzzy matrices; Minimum norm g-inverse; Least square g-inverse; Moore Penrose inverse; Group Inverse; Column symmetric Neutrosophic fuzzy matrices.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [11] in 1965. The traditional fuzzy sets are characterized by the membership value or the grade of membership value. Some- times it may be very difficult to assign the membership value for fuzzy sets. Intuitionistic fuzzy sets introduced by Atanassov [10] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership (or simply membership) and falsity-membership(or nonmembership)values. It does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache [14] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.

We discuss the relation between group inverse and Moore-Penrose inverse of a range symmetric fuzzy matrix. We exhibit group inverse as a generalization of Drazin inverse analogous to that of complex matrix [1] .Under certain conditions we obtain the common solution for matrix equations of the type AX = C, XB = D. Finally, we discuss the existence of group inverse for product of two square fuzzy matrices having group inverses. The concept of this chapter is a development of the theory of generalized inverses of fuzzy matrices analogous to that for comlex metrix in [1]

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based on the work of Kim and Roush[2] ,Cho[3,4], Meenakshi and sriram [5,6], Inbam [7] and Inbam[8]. Anandhkumar , Kamalakannan, Chithra, Broumi Said have studied Pseudo Similarity of Neutrosophic Fuzzy matrices. Many generalization for range symmetric (or) EP complex matrix we refer for range symmetric matrix [9] for kernel symmetric matrix. In this article, we discuss various Inverse of Neutrosophic fuzzy matrices . It is well known that generalized inverses exists for a complex matrix. However this is fail for Neutrosophic fuzzy matrices that is , for $A \in (NF)_{mn}$ under the max-min Neutrosophic fuzzy operations the matrix equation AXA = A need not have a solution X. If A has a generalized inverse (g-inverse) ,then A is said to be regular. The concept of generalized inverse presents a very interesting area of research in matrix theory, in the same way a regular matrix as one of which g-inverse exists, lays the foundation for research in Neutrosophic Fuzzy matrix theory. We discuss Various g-inverse associated with a regular matrix and obtain characterization of set of all inverses. Finally we give the example of column symmetric Neutrosophic fuzzy matrix, column symmetric Neutrosophic fuzzy matrix analogues to that of an EP matrix in the complex field .First we present equivalent characterizations of a column symmetric Neutrosophic fuzzy matrix and then derive equivalent conditions for a Neutrosophic fuzzy matrix to be column symmetric Neutrosophic fuzzy matrices.

1.1 Notations: For NFM of A ∈ $(NF)_n$,

A^T : Transpose of A,
R(A) : Row space of A,
C(A) : Column space of A,
A^+ : Moore-Penrose inverse of A ,
$(NF)_n$: Square Neutrosophic Fuzzy Matrix.
$A^\#$:Group inverse of A.

2. DEFINITIONS AND THEOREMS

2.1.Minimum norm g-inverse, Least square g-inverse and Moore Penrose inverse

In this section we shall see the existence and construction of minimum norm g-inverse and least square g-inverses of a Neutrosophic fuzzy matrix. We conclude with a formula for the moore Penrose inverses.

Definition 2.1 Sum and product of a Neutrosophic Fuzzy Set

Let X be a non - empty set. A neutrosophic fuzzy sets A and B is of the form A = { x,$\mu_{P(x)}$, ,$\sigma_P(x)$,$\nu_P(x)$: x ∈ X} and B = { x,$\mu_{Q(x)}$, ,$\sigma_Q(x)$,$\nu_Q(x)$: x ∈ X} then, the sum, and product of two Neutrosophic fuzzy sets is defined by,

A+B = $\left\{x,(\mu_P(x) \vee \mu_Q(x), \sigma_P(x) \vee \sigma_Q(x), \nu_P(x) \wedge \nu_Q(x)\right\}$

AB = $\left\{x,(\mu_P(x) \wedge \mu_Q(x), 1-\sigma_P(x) \wedge 1-\sigma_Q(x), \nu_P(x) \vee \nu_Q(x)\right\}$

Definition 2.2 For $A \in (NF)_{mn}$ and $L \in (NF)_{nm}$ is said to be {1,3} inverse of A if ALA = A and $(AL)^T$ = AL

Example2.1 Let us consider NFMs $A = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (1,0,0)) & (1,0,0) \end{bmatrix}$, $L = \begin{bmatrix} (0,0,1) & (1,0,0) \\ (1,0,0)) & (1,0,0) \end{bmatrix}$

$$LA = \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (1,1,0) \end{bmatrix}$$

$$ALA = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (1,0,0)) & (1,0,0) \end{bmatrix} \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (1,1,0) \end{bmatrix} = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (1,0,0)) & (1,0,0) \end{bmatrix}$$

ALA = A

$$AL = \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (1,1,0) \end{bmatrix}$$

$(AL)^T$ = AL

Definition:2.3 For $A \in (NF)_{mn}$ and $L \in (NF)_{nm}$ is said to be {1,4} inverse of A if ALA=A and $(LA)^T$ = LA

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Example: 2.2 Let us consider, $A=\begin{bmatrix}(1,0,0) & (1,0,0)\\(1,0,0) & (1,0,0)\end{bmatrix}, L=\begin{bmatrix}(0,0,1) & (0,0,1)\\(1,0,0) & (1,0,0)\end{bmatrix}$

$$ALA=\begin{bmatrix}(1,0,0) & (1,0,0)\\(1,0,0) & (1,0,0)\end{bmatrix}\begin{bmatrix}(0,0,1) & (0,0,1)\\(1,0,0) & (1,0,0)\end{bmatrix}\begin{bmatrix}(1,0,0) & (1,0,0)\\(1,0,0) & (1,0,0)\end{bmatrix}=\begin{bmatrix}(1,0,0) & (1,0,0)\\(1,0,0) & (1,0,0)\end{bmatrix}$$

ALA = A

$(LA)^T$ = LA

Remark:2.1 From the above definition , it is clear that $L \in A\{1,3\}$ if and only if $L^T \in A^T\{1,4\}$

Definition:2.4 For $A \in (NF)_{mn}$ and $L \in (NF)_{nm}$ is both {1,3} inverse and {1,4} inverse ,then L is said to be Moore-Penrose inverse of A, and is denoted by A^+

Theorem:2.1 For $A \in (NF)_{mn}$, A has a least square inverse if and only if A^TA is a regular matrix and R(A^T A) = R(A)

Proof: Let A has a least square inverse X (say)

Then AXA = A and $(AX)^T$ = AX

A^T(AXA) = A^TA

(A^TAX)A = A^TA

$R(A^TA) \subseteq R(A)$

Also, $(AX)^T A = AXA$

$\Rightarrow X^T A^T A = A$

$\Rightarrow X^T\left(A^T A\right) = A$

$R(A) \subseteq R(A^T A)$

$R(A) = R(A^T A)$

$R(A) = R(XA)$

Hence, $R(A^T A) = R(A)R(XA)$

Since, $R(A^T A) \supseteq R(XA)$

$YA^T A = XA$ for some positive integer Y

$A^T A(YA^T A) = A^T A(XA)$

$\left(A^T A\right)Y\left(A^T A\right) = A^T\left(AXA\right) = A^T A$

Thus A^TA is a regular fuzzy matrix.

Conversely, let A^TA be a regular fuzzy matrix and R(A) = R(A^TA) iff A is regular fuzzy matrix.

Since A = XA^TA and take $Y=\left(A^TA\right)^- A^T$

$$\begin{aligned}AY &= XA^TA\left(A^TA\right)^- A^T\\ &= XA^TA\left(A^TA\right)^- A^TAX^T\\ &= X\left(A^TA\right)\left(A^TA\right)^-\left(A^TA\right)X^T\\ &= X(A^TAX^T) \quad = XA^T\end{aligned}$$

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$\left(AY\right)^{T} = (XA^{T})^{T}$

$= AX^{T}$

$= XA^{T}AA^{T}$

$= XA^{T}$

$= AY$

Thus A has a least square inverse.

Theorem: 2.2 For $A \in \left(NF\right)_{mn}$, A has a minimum norm g-inverse if and only if AA^T is a regular matrix and $C(AA^T) = C(A)$

Proof: A has a {1,4} inverse

$\Leftrightarrow$ A^T has a {1,3} inverse (Remark 2.1)

$\Leftrightarrow$ AA^T is regular and $R(AA^T) = R(A^T)$ (By Theorem 2.1)

$\Leftrightarrow$ AA^T is regular and $C(AA^T) = C(A^T)$

Theorem:2.3 Let $A \in \left(NF\right)_{mn}$ be the regular Neutrosophic fuzzy matrix , with A^TA is a regular Neutrosophic fuzzy matrix and $C(A^T) = C(A^TA)$, then $Y = \left(A^{T}A\right)^{-} A^{T} \in A\{1,2,3\}$

Proof: Since $A \in \left(NF\right)_{mn}$ be a regular Neutrosophic fuzzy matrix, A^TA is a regular Neutrosophic fuzzy matrix and $C(A^T) = C(A^TA)$,

Therefore, $C(A^{T}) \subseteq C(A^{T}A) \Rightarrow A^{T}AX = A^{T}$ for some $X \in (NF)_{nm}$

Taking transpose on both sides we get,

$$A = X^{T}A^{T}A$$

$$AYA = \left(X^{T}A^{T}A\right)\left(\left(A^{T}A\right)^{-} A^{T}\right)A$$

$$= X^{T}\left(A^{T}A\right)\left(A^{T}A\right)^{-}\left(A^{T}A\right)$$

$$= X^{T}A^{T}A = A$$

$$YAY = Y\left(X^{T}A^{T}A\right)\left(A^{T}A\right)^{-} A^{T}$$

$$= YX^{T}\left(A^{T}A\right)\left(A^{T}A\right)^{-}\left(A^{T}AX\right)$$

$$= YX^{T}\left(A^{T}A\right)\left(A^{T}A\right)^{-}\left(A^{T}A\right)X$$

$$= YX^{T}A^{T}AX = YAX$$

$$= \left[\left(A^{T}A\right)^{-} A^{T}\right]AX$$

$$= \left(A^{T}A\right)^{-}\left(A^{T}AX\right)$$

$$= \left(A^{T}A\right)^{-} A^{T} = Y$$

$$AY = \left(X^{T}A^{T}A\right)\left(A^{T}A\right)^{-} A^{T}$$

$$= X^{T}\left(A^{T}A\right)\left(A^{T}A\right)^{-}\left(A^{T}AX\right)$$

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$$= X^T \left(A^T A\right)\left(A^T A\right)^{-}\left(A^T A\right)X$$

$$= X^T A^T AX \quad = AX$$

$$(AY)^T = (AX)^T$$

$$= X^T A^T$$

$$= X^T A^T AX$$

$$= AX = AY$$

Thus $Y \in A\{1,2,3\}$.

Theorem:2.4 Let $A \in (NF)_{mn}$,be a regular Neutrosophic matrix with AA^T is a regular Neutrosophic fuzzy matrix and $R(AA^T) = R(A^T)$ then $Z = A^T\left(AA^T\right)^{-} \in A\{1,2,4\}$.

Proof: Similar to the proof of theorem 2.3 .

Theorem: 2.5 Let $A \in (IF)_{mn}$, be a regular Neutrosophic fuzzy matrix then $A^+ = A^{(1,4)} A A^{(1,3)}$

Proof: Let $X = A^{(1,4)}\ A\ A^{(1,3)}$ one can easily verify that $X \in A\{1,2\}$

$$AX = A\left(A^{(1,4)} AA^{(1,3)}\right) = \left(AA^{(1,4)}A\right)A^{(1,3)} = AA^{(1,3)}$$

$$(AX)^T = \left(AA^{(1,3)}\right)^T = AA^{(1,3)} = AX$$

$$XA = \left(AA^{(1,3)}A\right) = A^{(1,4)}\left(AA^{(1,3)}A\right) = A^{(1,4)}A$$

$$(XA)^T = \left(A^{(1,4)}A\right)^T = A^{(1,4)}A = XA$$

Thus, $X \in A\{1,2,3,4\}$

Hence $X = A^{(1,4)}\ AA^{(1,3)} = A^+$

3.Group Inverse

Definition:3.1 For $A \in (NF)_n$ the group inverse A, denoted as $A^\#$ is commutative semi inverse of A, that is $AA^\#A = A$, $A^\#A\ A^\# = A^\#$ and $A\ A^\# = A^\#A$. For complex matrix its well-known that $A^\#$ exists if and only if $\text{rank}(A^2) = \text{rank}(A)$.However, it need not be true for Neutrosophic Fuzzy matrices. This is illustrated in the following example.

Example 3.1 $A = \begin{bmatrix} (0,0,1) & (1,0,0) \\ (1,0,0) & (1,0,0) \end{bmatrix}, A^2 = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) \end{bmatrix}$

Here, $\rho_r(A) = 2 = \rho_c(A)$ and $\rho_r(A^2) = 2 = \rho_c(A^2)$. Direct computations show that there is no X such that AXA = A. Thus A is not regular.

Theorem: 3.1 For $A \in (NF)_n$ and $X \in \{1,2\}$ such that $R(X) \subseteq R(A^T)$ and $R(X) \subseteq R(A^T)$ then $X = A^T$ is the unique Moore-Penrose inverse of A.

Proof: $X \in A\{1\} \Rightarrow X^T \in A^T\{1\}$. Hence A^T is regular and X^T is a group inverse of A^T

$$R(X) \subseteq R\left(A^T\right) \Rightarrow X = X(A^T)^{-}A^T \text{ for all } (A^T)^{-} \in A^T\{1\}$$

$$\Rightarrow X = XX^T A^T \quad (\text{Since } (X^T) \in A^T\{1\}$$

$$\Rightarrow X^T = AXX^T$$

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Now, $AX = \left(AXX^T A^T\right)$

$= \left(AXX^T\right) A^T$

$= X^T A^T$

$= \left(AX\right)^T$

Similarly, $C(X) \subseteq C(A^T)$

$\Rightarrow X = A^T \left(A^T\right)^- X$ for all $(A^T)^- \in A^T\{1\}$

$\Rightarrow X = A^T X^T X$ (Since $(X^T) \in A^T\{1\}$

$\Rightarrow X^T = X^T XA$

Hence, $XA = \left(A^T X^T X\right) A$

$= A^T \left(X^T XA\right)$

$= A^T X^T$

$= (XA)^T$

Thus, $X \in A\{1,2\}$, $AX = (AX)^T$ and $XA = (XA)^T \Rightarrow X = A^+$

A^+ is unique and $X = A^T$.

Theorem: 3.2 For $A \in \left(NF\right)_n$ .If $A^\#$ and A^+ exists, then the following hold.

(i) $A^\# = A^\# A^T A = AA^T A^\#$

(ii) $(A^\#)^+$ exists and $(A^\#)^+ = A^T A^3 A^T$

(iii) $(A^2A^T)^+$ exists and $(A^2A^T)^+ = A^\# AA^T$

(iv) $(A^T A^2)^+$ exists and $(A^T A^2)^+ = A^T A A^\#$

Proof: (i) Since $R(A^\#) = R(A^\# A)$ and $C(A^\#) = C(AA^\#)$

$R(A^\#) = R(A^\# A) \subseteq R(A) = R(A^+ A)$

$\Rightarrow R(A^\#) \subseteq R(A^+ A)$

$\Rightarrow A^\# = A^\# \left(A^+ A\right)^- A^+ A$ for all $\left(A^+ A\right)^- \in A^+ A\{1\}$

$\Rightarrow A^\# = A^\# \left(A^+ A\right) A^+ A$ ($A^+ A$ itself is a $\{1\}$ inverse of $A^+ A$)

$\Rightarrow A^\# = A^\# A^T A$

Similarly, we have $A^\# = AA^T A^\#$

(ii) Let $X = A^T A^3 A^T$

Claim: $X = (A^\#)^+$

$A^\# XA^\#$

$= A^\# \left(A^T AAAA^T\right) A^\#$

$= \left(A^\# A^T A\right) A \left(AA^T A^\#\right)$

$= A^\# AA^\#$ $(By\ (i))$

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Similarly, we can prove that $XA^{\#}X = X$

Also, $A^{\#}X$

$= A^{\#}\left(A^{T}AAAA^{T}\right)$

$= \left(A^{\#}A^{T}A\right)\left(AAA^{T}\right)$

$= A^{\#}AAA^{T}$

$= AA^{T}AA^{T}$

$= AA^{T}$

$A^{\#}X = AA^{T} \Rightarrow \left(A^{\#}X\right)$ is symmetric.

Similarly we can prove that $XA^{\#} = A^{T}A$ implies $(XA^{\#})$ is symmetric.

Thus, $X \in A^{\#}\{1,2,3,4\}$.

Hence $(A^{\#})^{+}$ exists and $(A^{\#})^{+} = X = A^{T}A^{3}A^{T}$

(iii) Let $Y=A^{\#}AA^{T}$ we show that Y is the moore-Penrose inverse of $A^{2}A^{T}$ using the existence of A^{+} and $A^{\#}$.

Now, $(A^{2}A^{T})Y(A^{2}A^{T})$

$= (A^{2}A^{T})A^{\#}AA^{T}(A^{2}A^{T})$

$= (A^{2}A^{T})AA^{T}$

$= A^{2}\left(A^{T}AA^{T}\right)$

$= A^{2}\left(A^{+}AA^{+}\right)$

$= A^{2}A^{+}$

$= A^{2}A^{T}$

Thus $Y \in \left(A^{2}A^{T}\right)\{1\}$

$Y(A^{2}A^{T})Y = YAA^{T}$

$= \left(A^{\#}AA^{T}\right)AA^{T}$

$= A^{\#}A\left(A^{+}AA^{+}\right)$

$= A^{\#}AA^{+}$

$= A^{\#}AA^{T}$

$= Y$

Thus $Y \in \left(A^{2}A^{T}\right)\{2\}$

$\left(A^{2}A^{T}\right)Y = \left(A^{2}A^{T}\right)\left(A^{\#}AA^{T}\right)$

$= A\left(AA^{T}A^{\#}AA^{T}\right)$

$= A\left(AA^{T}A^{\#}\right)AA^{T}$

$= \left(AA^{\#}A\right)A^{T} \qquad (By(i))$

$= AA^{T}$

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AA^T is symmetric $\Rightarrow Y \in \left(A^2A^T\right)\{3\}$.

$Y\left(A^2A^T\right) = \left(A^{\#}AA^T\right)A^2A^T$

$= A\left(A^{\#}A^TA\right)AA^T$

$= \left(AA^{\#}A\right)A^T = AA^T$

AA^T is symmetric $\Rightarrow Y \in \left(A^2A^T\right)\{4\}$.

Hence, $Y \in \left(A^2A^T\right)\{1,2,3,4\}$

Thus $(A^2A^T)^+$ exists and $(A^2A^T)^+$ =Y= $A^{\#}AA^T$. Similarly it can be prove that $(A^T A^2)^+$ exists and $(A^T A^2)^+$ $=A^TAA^{\#}$.

Theorem:**3.3** For $A, B \in (\mathrm{N}F)_n^{\#}, C, D \in (IF)_n$ with $R(C) \subseteq R(B), C(D) \subseteq C(A),$ then AX = C, XB = D have a common solution if and only if AD = CB $C(C) \subseteq C(A)\, and\, R(D) \subseteq R(B)$.

Proof: Suppose the common solution exists, then

CB = (AX)B = A(XB) = AD

$C = AX \Rightarrow C(C) \subseteq C(A)$

$D = XB \Rightarrow R(D) \subseteq R(B)$

Conversely, by taking X = $A^{\#}C+DB^{\#}+A^{\#}ADB^{\#}$

We have AX = $A(A^{\#}C+DB^{\#}+A^{\#}ADB^{\#})$

= $AA^{\#}C+ ADB^{\#}+ AA^{\#}ADB^{\#}$

= $AA^{\#}C+ ADB^{\#}+ ADB^{\#}$

= $AA^{\#}C+ ADB^{\#}$

= $AA^{\#}C+ CBB^{\#}$ (By AD = CB)

= $AA^{\#}C+ CBB^{\#}$

$= AA^{\#}C + CB^{\#}B$ $(By\ \ BB^{\#} = B^{\#}B)$

$= C + C$ (By $C(C) \subseteq C(A)$ and $R(B) \subseteq C(B)$)

= C

4. Column symmetric Neutrosophic Fuzzy Matrices

Definition: 4.1 Let A be a NFM , if $C[A] = C[A^T]$ then A is called as Column symmetric.

Example: 4.1 Let us consider $A = \begin{bmatrix} (0.6,0.5,0.4) & (0.1,0.2,0.4) & (0.4,0.2,0.5) \\ (0.1,0.2,0.4) & (0,0,1) & (0.3,0.6,0.4) \\ (0.4,0.2,0.5) & (0.3,0.6,0.4) & (0.6,0.2,0.4) \end{bmatrix}$,

The following matrices are not column symmetric

$$P = \begin{bmatrix} (0.5,0.5,0) & (0.4,0.6,1) & (0.4,0.6,1) \\ (0.4,0.6,1) & (0.5,0.5,0) & (0.6,0.5,0) \\ (0.4,0.6,1) & (0.5,0.5,0) & (0.5,0.5,0) \end{bmatrix},\ P^T = \begin{bmatrix} (0.5,0.5,0) & (0.4,0.6,1) & (0.4,0.6,1) \\ (0.4,0.6,1) & (0.5,0.5,0) & (0.5,0.5,0) \\ (0.4,0.6,1) & (0.6,0.5,0) & (0.5,0.5,0) \end{bmatrix},$$

$\left((0.5,0.5,0)\quad (0.4,0.6,1)\quad (0.4,0.6,1)\right)^T \in C(\mathrm{P})$, $\left((0.5,0.5,0)\quad (0.4,0.6,1)\quad (0.4,0.6,1)\right)^T \in C(\mathrm{P}^T)$

$\left((0.4,0.6,1)\quad (0.5,0.5,0)\quad (0.5,0.5,0)\right)^T \in C(\mathrm{P})$, $\left((0.4,0.6,1)\quad (0.5,0.5,0)\quad (0.5,0.5,0)\right)^T \in C(\mathrm{P}^T)$

$\left((0.4,0.6,1)\quad (0.6,0.5,0)\quad (0.5,0.5,0)\right)^T \in C(\mathrm{P})$, $\left((0.4,0.6,1)\quad (0.5,0.5,0)\quad (0.6,0.5,0)\right)^T \notin C(\mathrm{P}^T)$

$C(\mathrm{P}) \notin C(\mathrm{P}^T)$

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Definition 4.2: A NFM A belongs to F_n is s-symmetric NFM iff $A = VA^TV$.

Definition 4.3: A NFM A belongs to F_n is s-column(Cs) symmetric NFM iff $C(A) = C(VA^TV)$.

Definition 4.4: A NFM A belongs to F_n is s-k-column symmetric(Cs) NFM iff $C(A) = C(KVA^TVK)$.
Definition:4.5 Let the function is defined $\kappa(x)=(x_{k[1]}, x_{k[2]}, x_{k[3]},\ldots, x_{k[n]}) \in F_{n\times 1}$ for $x = (x_1, x_2,\ldots,x_n) \in F_{[1\times n]}$, where K is involuntary, the following are satisfied the associated permutation matrix where V is a permutation matrix, $VV^T = V^TV = I_n$ then $V^T = V$ and $C(A) = C(VA), C(A) = C(KA)$

Remark 4.1: We observe that s-k-symmetric NFM is s-k-column symmetric NFM since $A = KVA^TVK$ if A is s-k-symmetric NFM. Thus, $C(A) = C(KVA^TVK)$,indicating that A is an NFM with s-k-column symmetry.

Theorem 4.1: For NFM A belongs to F_n , the following statements are equivalent : (i) $C(A) = C(A^T)$.

(ii) $A^T = AH = KA$ for some IFM H, K and $\rho(A) = r$.

Lemma 4.1: For NFM A belongs to F_n and a permutation matrix P, C(A)= C(B) iff $C(PAP^T) = C(PBP^T)$

Theorem 4.2 . For N F M A belongs to F_n the following are equivalent

(i) $C(A) = C(KVA^TVK)$
(ii) $C(KVA) = C((KVA)^T)$
(iii) $C(AKV) = C((AKV)^T)$
(iv) $C(VA) = C(K(VA)^TK)$
(v) $C(AK) = C(V(AK)^TV)$
(vi) $C(A^T) = C(KV(A)VK)$
(vii) $C(A) = C(A^TVK)$
(viii) $C(A^T) = C(AKV)$
(ix) $A = VKA^TVKH_1$ for $H_1 \in F_n$
(x) $A = H_1KVA^TVK$ for $H_1 \in F_n$
(xi) $A^T = KVAVKH$ for $H \in F_n$
(xii) $A^T = HKVAKV$ for $H \in F_n$

Proof: (i) *iff* (ii) iff (iv) *A* is s- κ- Cs

$\Leftrightarrow C(A) = C(KVA^TVK)$

$\Leftrightarrow C(KVA) = C((KVA)^T$ [By Definition:4.5]

$\Leftrightarrow$ *KVA* is Cs

$\Leftrightarrow$ *VA* is κ- Cs [By Theorem 3.5 in [9]]

Therefore, (i) iff (ii) iff (iv) hold.
(i) iff (iii) iff (v)

A is s- κ - Cs $\Leftrightarrow C(A) = C(KVA^TVK)$ [By Definition 4.3]

$\Leftrightarrow C(KVA) = C((KVA)^T)$ [By Definition:4.5]

$\Leftrightarrow C(VK(KVA)(VK)^T = C((VK)A^TVK(VK)^T)$ [By Lemma 2.2]

$\Leftrightarrow C(AKV) = C((AKV)^T)$

$\Leftrightarrow$ AKV is Cs

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$\Leftrightarrow$ AK is s-Cs

Therefore, (i) iff (iii) iff (v) hold.

(ii) $\Leftrightarrow$ (vii)

KVA is Cs $\Leftrightarrow$ $C(KVA) = C((KVA)^T)$

$\Leftrightarrow$ $C(A) = C((KVA)^T)$ [By Definition:4.5]

$\Leftrightarrow$ $C(A) = C(A^T VK)$

Therefore, (ii) iff (vii) hold.

(iii) iff (viii):

AVK is Cs $\Leftrightarrow$ $C(AVK) = C((AVK)^T)$

$\Leftrightarrow$ $C(AVK) = C(A^T)$ [By Definition:4.5]

Therefore, (iii) iff (viii) hold.

(i) iff (vi)

A is s- κ- Cs $\Leftrightarrow$ $C(A) = C(KVA^T VK)$

$\Leftrightarrow$ $C(KVA) = C((KVA)^T)$ [By Definition:4.5]

$\Leftrightarrow$ $(KVA)^T$ is Cs

$\Leftrightarrow$ $A^T VK$ is Cs

$\Leftrightarrow$ A^T is s- κ - Cs

Therefore, (i) *iff* (vi) hold.

(i) *iff* (xi) *iff* (x)

A is s- κ- Cs $\Leftrightarrow$ $C(A) = C(KVA^T VK)$

$\Leftrightarrow$ $C(A^T) = C(KVAVK)$

$\Leftrightarrow$ $A^T = KVAVKH$ [By Theorem 2.1]

$\Leftrightarrow$ $A = H_1 KV A^T VK$ for $H_1 \in F_n$

Therefore, (i) iff (xi) *iff* (x) hold.

(ii) iff (xii) iff (ix)

KV A is Cs $\Leftrightarrow$ *VA* is κ- Cs

$\Leftrightarrow$ $C(VA) = C(K(VA)^T K)$

$\Leftrightarrow$ $C(A) = C(A^T VK)$ [By Definition:4.5]

$\Leftrightarrow$ $C(A^T) = C(KVA)$

$\Leftrightarrow$ $A^T = HKVA$ for $H \in F_n$ [By Theorem 4.1]

$\Leftrightarrow$ $A^T = HKVAKV$

$\Leftrightarrow$ $A = VKA^T VKH_1$ for $H_1 \in F_n$

Therefore, (ii) iff (xii) iff (ix) hold.

Corollary 4.1: For NFM A belongs to F_n the following are equivalent:

(i) $C(A) = C(VA^T V)$

(ii) $C(VA) = C(VA)^T$

(iii) $C(AV) = C(AV)^T$

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(iv) A is s-column symmetric

(v) $C(A^T) = C(VAV)$

(vi) $C(A) = C(A^TV)$

(vii) $C(A^T) = C(AV)$

(viii) $C(KVA) = C((VA)^T)$

(ix) $A = VA^TVH_1$ for $H_1 \in F_n$

(x) $A = H_1VA^TV$ for $H_1 \in F_n$

(xi) $A^T = VAVH$ for $H \in F_n$

(xii) $A^T = HVAV$ for $H \in F$

Theorem 4.3: *For IFM A belongs to* F_n. *Then any two of the following conditions imply theother one:*

(i) $C(A) = C(KA^TK)$

(ii) $C(A) = C(VKA^TKV)$

(iii) $C(A^T) = C((VKA)^T)$

Proof: (i) and (ii) iff (iii)

A is s- κ –Cs $\Rightarrow C(A) = C(A^TVK)$ [By Theorem 4.2]

$\Rightarrow C(KAK) = C(KA^TK)$ [By Lemma 4.1]

Hence (i) and (ii) $\Rightarrow C(A^T) = C((VAK)^T)$

Therefore, (iii) hold.

(i) and (iii) iff (ii)

A is κ- Cs $\Rightarrow C(A) = C(KA^TK)$

$\Rightarrow C(KAK) = C(A^T)$ [By Lemma 4.1]

Hence (i) and (iii) $\Rightarrow C(KAK)= C((VAK)^T)$

$\Rightarrow C(A) = C(A^T VK)$

$\Rightarrow C(A) = C((KVA)^T)$

$\Rightarrow$ A is s- κ Cs

[By Theorem 4.2]

Therefore, (ii) hold.

(ii) and (iii) $\Rightarrow$ (i)

A is s- κ – Cs $\Rightarrow C(A)= C(A^TVK)$

$\Rightarrow C(KAK)= C(KA^TV)$ [By Definition:4.5]

Hence (ii) and (iii) $\Rightarrow C(KAK) = C(A^T)$

$\Rightarrow C(A) = C(KA^TK)$ [By Lemma 4.1]

$\Rightarrow$ A is κ – Cs

Therefore, (1) hold. Hence the Theorem

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5. Conclusion

we discuss various Inverse Minimum norm g-inverse, Least square g-inverse, Moore Penrose inverse, Group Inverse, of Intuitionistic Fuzzy Matrices as well as examples. The concept of generalized inverse presents a very interesting area of research in matrix theory, in the same way a regular matrix as one of which g-inverse exists, lays the foundation for research in Intuitionistic Fuzzy matrix theory. We discuss Various g-inverse associated with a regular matrix and obtain characterization of set of all inverses.

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