



Neutrosophic Pre-compactness

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Abstract

The purpose of this article is to study some covering properties in neutrosophic topological spaces via neutrosophic pre-open sets. We define neutrosophic pre-open cover, neutrosophic pre-compactness, neutrosophic countably pre-compactness and neutrosophic pre-Lindelöfness and study various properties connecting them. We study some properties involving neutrosophic continuous and neutrosophic pre-continuous functions. We also define neutrosophic pre-base, neutrosophic pre-subbase, neutrosophic pre*-open function, neutrosophic pre-irresolute function and study some properties. In addition to that, we define and study neutrosophic local pre-compactness.

Keywords: Neutrosophic pre-compact space; Neutrosophic countably pre-compact space; Neutrosophic pre-Lindelöf space; Neutrosophic Np-base; Neutrosophic Np-subbase; Neutrosophic pre-irresolute function; Neutrosophic local pre-compact space.

1 Introduction

The notion of neutrosophic set was coined by Florentin Smarandache.^{13–15} Since then, Smarandache and other researchers^{6,18,20} have studied and further developed the theory of neutrosophic sets. Neutrosophic set is an extended form of intuitionistic fuzzy set developed by K.Atanassov¹ in 1986. The concept of neutrosophic sets has found various applications in different fields, particularly in situations where uncertainty, vagueness, and indeterminacy are present.

In 2012, Salama & Alblawi¹⁶ developed the concept of neutrosophic topological space, which is a generalization of the intuitionistic fuzzy topological space that was originally proposed by D.Coker⁵ in 1997. In 2016, Karatas and Kuru⁹ redefined the single-valued neutrosophic set operations and introduced neutrosophic topology. The authors then investigated some important properties of neutrosophic topological spaces. Since then, many researchers^{2,4,10,11,17,19} have further developed various aspects of neutrosophic topology. The idea of fuzzy pre-compact space was introduced by Jaber⁸ in 2020. Rao & Rao,¹² in 2017, developed the concepts of neutrosophic pre-open and pre-closed sets and thereafter, Arokiarani et.al.³ developed the idea of neutrosophic pre-open, pre-closed, pre-continuous functions. Recently, Dey & Ray⁷ studied separation properties using neutrosophic pre-open sets.

In this write-up, we first define neutrosophic pre-open cover using neutrosophic pre-open sets. After that, we define neutrosophic pre-compact space, neutrosophic countably pre-compact space and neutrosophic pre-Lindelöf space via neutrosophic pre-open covers and study various covering properties involving them. We also define neutrosophic pre-base, neutrosophic pre-subbase, neutrosophic pre*-open function, neutrosophic pre-irresolute function and investigate some covering properties. In the long run, we define and study neutrosophic local pre-compactness.

2 Preliminaries

2.1 Definition:¹³

Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x) \rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^{-0}, 1^+[$, i.e., $\mathcal{T}_A : X \rightarrow]^{-0}, 1^+[$, $\mathcal{I}_A : X \rightarrow]^{-0}, 1^+[$, $\mathcal{F}_A : X \rightarrow]^{-0}, 1^+[$ and $-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

2.2 Definition:²⁰

Let X be the universe of discourse. A single valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x) \rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

2.3 Definition:⁹

Let $A, B \in \mathcal{N}(X)$. Then

- (i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.
- (ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.
- (iii) (Intersection): The intersection of A and B , denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x) \rangle : x \in X\}$.
- (iv) (Union): The union of A and B , denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x) \rangle : x \in X\}$.
- (v) (Complement): The complement of the NS A , denoted by A^c , is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x) \rangle : x \in X\}$.
- (vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.
- (vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.4 Definition:¹¹

Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X . An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x) \rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x) \rangle : x \in X\}$ will be denoted by $x_{\alpha, \beta, \gamma}$. For the NP $x_{\alpha, \beta, \gamma}$, x will be called its support. The complement of the NP $x_{\alpha, \beta, \gamma}$ will be denoted by $x_{\alpha, \beta, \gamma}^c$ or $(x_{\alpha, \beta, \gamma})^c$.

2.5 Definition:¹⁷

Let X and Y be two non-empty sets and $f : X \rightarrow Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then

(1) Image of A under f is defined by

$$f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y) \rangle : y \in Y\}, \text{ where}$$

$$f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

(2) Pre-image of B under f is defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x) \rangle : x \in X\}$$

2.6 Theorem:¹⁷

Let $f : X \rightarrow Y$ be a function. Also let $A, A_i \in \mathcal{N}(X), i \in I$ and $B, B_j \in \mathcal{N}(Y), j \in J$. Then the following hold.

- (i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2), B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.
- (ii) $A \subseteq f^{-1}(f(A))$ and if f is injective then $A = f^{-1}(f(A))$.
- (iii) $f^{-1}(f(B)) \subseteq B$ and if f is surjective then $f^{-1}(f(B)) = B$.
- (iv) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ and $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.
- (v) $f(\cup A_i) = \cup f(A_i), f(\cap A_i) \subseteq \cap f(A_i)$ and if f is injective then $f(\cap A_i) = \cap f(A_i)$.
- (vi) $f^{-1}(\tilde{\emptyset}_Y) = \tilde{\emptyset}_X, f^{-1}(\tilde{Y}) = \tilde{X}$.
- (vii) $f(\tilde{\emptyset}_X) = \tilde{\emptyset}_Y, f(\tilde{X}) = \tilde{Y}$ if f is surjective.

2.7 Definition:⁹

Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

- (i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ .
- (ii) An arbitrary union of neutrosophic sets in τ is in τ .
- (iii) The intersection of any two neutrosophic sets in τ is in τ .

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X . The members of τ are called neutrosophic open sets in X . If for a neutrosophic set $A, A^c \in \tau$ then A is said to be a neutrosophic closed set in X .

2.8 Definition:⁹

Let (X, τ) be a NTS and $A \in \mathcal{N}(X)$. Then the neutrosophic

- (i) interior of A , denoted by $\text{int}(A)$, is defined as $\text{int}(A) = \cup\{G : G \in \tau \text{ and } G \subseteq A\}$.
- (ii) closure of A , denoted by $\text{cl}(A)$, is defined as $\text{cl}(A) = \cap\{G : G \text{ is a neutrosophic closed set and } G \supseteq A\}$.

2.9 Definition:¹²

Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. Then

- (i) A is called a neutrosophic pre-open set (NPO, for short) in X iff $A \subseteq \text{int}(\text{cl}(A))$.
- (ii) A is called a neutrosophic pre-closed (NPC, for short) set in X iff $\text{cl}(\text{int}(A)) \subseteq A$.

If G is an NPO (resp. NPC) set in X then we may also say that G is a τ -NPO (resp. τ -NPC) set.

2.10 Theorem:¹²

Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. Then

- (i) A is an NPC set in X if and only if A^c is an NPO set in X .
- (ii) Every neutrosophic open set in an NTS is an NPO set.
- (iii) Every neutrosophic closed set in an NTS is an NPC set.

2.11 Definition:¹⁹

Let f be a function from an NTS (X, τ) to the NTS (Y, σ) . Then

- (i) f is called a neutrosophic open function if $f(G) \in \sigma$ for all $G \in \tau$
- (ii) f is called a neutrosophic continuous function if $f^{-1}(G) \in \tau$ for all $G \in \sigma$.

2.12 Definition:³

Let f be a function from an NTS (X, τ) to the NTS (Y, σ) . Then f is called a neutrosophic

- (i) pre-open function if $f(G)$ is an NPO set in Y for every neutrosophic open set G in X .
- (ii) pre-continuous function if $f^{-1}(G)$ is an NPO set in X for every $G \in \sigma$.

2.13 Proposition:⁷

Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X, τ) . Then

- (i) $G|_Y$ is a $\tau|_Y$ -NPO set in Y for every τ -NPO set G in X .
- (ii) $G|_Y$ is a $\tau|_Y$ -NPC set in Y for every τ -NPC set G in X .

3 Neutrosophic pre-compactness**3.1 Definition:**

Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. A collection $C = \{G_\lambda : \lambda \in \Delta\}$ of NPO sets of X is called a neutrosophic pre-open cover (NPOC, in short) of A iff $A \subseteq \bigcup_{\lambda \in \Delta} G_\lambda$. In particular, C is said to be an NPOC of X iff $\tilde{X} = \bigcup_{\lambda \in \Delta} G_\lambda$.

Let C be an NPOC of the NS A and $C' \subseteq C$. Then C' is called a neutrosophic pre-open subcover (NPOSC, in short) of C if C' is also an NPOC of A .

An NPOC C of an NS A is said to be countable (resp. finite) if C consists of a countable (resp. finite) number of NPO sets.

3.2 Definition:

An NS A in an NTS (X, τ) is said to be a neutrosophic pre-compact set iff every NPOC of A has a finite NPOSC. In particular, the space X is said to be a neutrosophic pre-compact space iff every NPOC of X has a finite NPOSC.

An NTS (X, τ) is said to be a neutrosophic countably pre-compact space iff every countable NPOC of X has a finite NPOSC.

An NTS (X, τ) is said to be a neutrosophic pre-Lindelöf space iff every NPOC of X has a countable NPOSC.

3.3 Proposition:

Every neutrosophic pre-compact space is neutrosophic countably pre-compact.

Proof: Obvious.

3.4 Proposition:

In an NTS, every neutrosophic pre-compact set is neutrosophic compact.

Proof: Let A be a neutrosophic pre-compact set of an NTS (X, τ) . Let $\mathcal{C} = \{G_i : i \in \Delta\}$ be an NOC of A . Since every neutrosophic open set is an NPO set [by 2.10], so G_i is an NPO set for each $i \in \Delta$. Therefore $\mathcal{C}$ is an NPOC of A . Since A is pre-compact, so there exists a finite subcollection $\{G_i^1, G_i^2, \dots, G_i^m\}$, say, of $\mathcal{C}$ such that $A \subseteq G_i^1 \cup G_i^2 \cup \dots \cup G_i^m$. Thus the NOC $\mathcal{C}$ of A has a finite NOSC $\{G_i^1, G_i^2, \dots, G_i^m\}$. Hence A is a neutrosophic compact set.

3.5 Example :

Converse of 3.4 is not true. We establish it by the following example.

Let $X = \{a, b\}$, $A = \{\langle a, 1, 0, 0 \rangle, \langle b, 0, 1, 1 \rangle\}$, $\tau = \{\tilde{X}, \tilde{\emptyset}, A\}$ and for each $n \in \mathbb{N} = \{1, 2, 3, \dots\}$, we define $H_n = \{\langle a, \frac{n}{n+1}, \frac{1}{n+2}, \frac{1}{n+3} \rangle, \langle b, 0, 1, 1 \rangle\}$. Obviously (X, τ) is an NTS and H_n is an NPO set in X . Clearly A is a neutrosophic compact set. Now $\cup_{n \in \mathbb{N}} H_n = A$. So, $\{H_n : n \in \mathbb{N}\}$ is an NPOC of A . It is clear that the NPOC $\{H_n : n \in \mathbb{N}\}$ of A has no finite NPOSC. Therefore A is not a neutrosophic pre-compact set in X .

3.6 Proposition:

Every neutrosophic pre-compact space is a neutrosophic compact space.

Proof: Obvious from 3.4.

3.7 Remark:

Converse of 3.6 is not true. We establish it by the following example.

Let us consider the NTS $(\mathbb{N}, \tau)$, where $\tau = \{\tilde{\emptyset}, \tilde{\mathbb{N}}\}$, $\mathbb{N} = \{1, 2, 3, \dots\}$. Clearly $(\mathbb{N}, \tau)$ is a neutrosophic compact space. We show that $(\mathbb{N}, \tau)$ is not a neutrosophic pre-compact space.

For $n \in \mathbb{N}$, we define $G_n = \{\langle x, \mathcal{T}_{G_n}(x), \mathcal{I}_{G_n}(x), \mathcal{F}_{G_n}(x) : x \in X \rangle\}$, where $\mathcal{T}_{G_n}(x) = 1, \mathcal{I}_{G_n}(x) = 0, \mathcal{F}_{G_n}(x) = 0$ if $x = n$ and $\mathcal{T}_{G_n}(x) = 0, \mathcal{I}_{G_n}(x) = 1, \mathcal{F}_{G_n}(x) = 1$ if $x \neq n$. Clearly, for each $n \in \mathbb{N}$, G_n is an NPO set in $(\mathbb{N}, \tau)$. Obviously the collection $\mathcal{C} = \{G_n : n \in \mathbb{N}\}$ is an NPOC of $\mathbb{N}$ but it has no finite NPOSC. Therefore $(\mathbb{N}, \tau)$ is not a neutrosophic pre-compact space.

Thus $(\mathbb{N}, \tau)$ is a neutrosophic compact space but not a neutrosophic pre-compact space.

3.8 Proposition:

In an NTS, union of two neutrosophic pre-compact sets is neutrosophic pre-compact.

Proof: Let A and B be two neutrosophic pre-compact sets of an NTS (X, τ) . Let $\mathcal{C} = \{G_i : i \in \Delta\}$ be an NPOC of $A \cup B$. Then $A \cup B \subseteq \cup_{i \in \Delta} G_i$. Since $A \subseteq A \cup B$, so $\mathcal{C}$ is an NPOC of A . Since A is neutrosophic pre-compact, so there exists a finite subcollection $\{G_i^1, G_i^2, \dots, G_i^m\}$ of $\mathcal{C}$ such that $A \subseteq G_i^1 \cup G_i^2 \cup \dots \cup G_i^m$. Similarly, since B is neutrosophic pre-compact, so there exists a finite subcollection $\{H_i^1, H_i^2, \dots, H_i^n\}$ of $\mathcal{C}$ such that $B \subseteq H_i^1 \cup H_i^2 \cup \dots \cup H_i^n$. Therefore $A \cup B \subseteq G_i^1 \cup G_i^2 \cup \dots \cup G_i^m \cup H_i^1 \cup H_i^2 \cup \dots \cup H_i^n$. Thus there exists a finite subcollection $\{G_i^1, G_i^2, \dots, G_i^m, H_i^1, H_i^2, \dots, H_i^n\}$ of $\mathcal{C}$ such that $A \cup B \subseteq G_i^1 \cup G_i^2 \cup \dots \cup G_i^m \cup H_i^1 \cup H_i^2 \cup \dots \cup H_i^n$. Therefore $A \cup B$ is neutrosophic pre-compact. Hence proved.

3.9 Proposition:

In an NTS, finite union of neutrosophic pre-compact sets is neutrosophic pre-compact.

Proof: Immediate from 3.8.

3.10 Proposition:

In an NTS, union of a neutrosophic pre-compact set and a neutrosophic compact set is a neutrosophic compact set.

Proof: Obvious.

3.11 Definition:

Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X, τ) . Then the set of all NPO sets $G|_Y$ in Y for which G is an NPO set in X will be denoted by $NPO(Y)$, i.e., $NPO(Y) = \{G|_Y \subseteq Y : G|_Y \text{ is an NPO set in } Y \text{ and } G \subseteq X \text{ is an NPO set in } X\}$.

3.12 Proposition:

Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. Then A is neutrosophic pre-compact in X iff every cover of A by the NPO sets in $NPO(Y)$ has a finite subcover.

Proof: Necessary part: Let $\mathcal{C} = \{G_i|_Y : i \in \Delta\}$ be a cover of A , where $G_i|_Y \in NPO(Y)$ for each $i \in \Delta$. Then $A \subseteq \bigcup_{i \in \Delta} (G_i|_Y) \Rightarrow A \subseteq (\bigcup_{i \in \Delta} G_i)|_Y \Rightarrow A \subseteq \bigcup_{i \in \Delta} G_i$. Clearly G_i is an NPO set in X [by 3.11] for each $i \in \Delta$ and so, $\mathcal{C}^* = \{G_i : i \in \Delta\}$ is an NPOC of A in X . Since A is pre-compact in X , so there exists a finite subcollection $\{G_{i_k} : k = 1, 2, 3, \dots, n\}$ of $\mathcal{C}^*$ such that $A \subseteq \bigcup_{k=1}^n G_{i_k} \Rightarrow A \subseteq (\bigcup_{k=1}^n G_{i_k})|_Y \Rightarrow A \subseteq \bigcup_{k=1}^n (G_{i_k}|_Y)$. Thus the cover $\mathcal{C}$ of A has a finite subcover $\{G_{i_k}|_Y : k = 1, 2, 3, \dots, n\}$.

Sufficient part: Let $\mathcal{B} = \{G_i : i \in \Delta\}$ be an NPOC of A in X , where G_i is an NPO set in X for each $i \in \Delta$. Then $A \subseteq \bigcup_{i \in \Delta} G_i \Rightarrow A \subseteq (\bigcup_{i \in \Delta} G_i)|_Y \Rightarrow A \subseteq \bigcup_{i \in \Delta} (G_i|_Y)$. Since $G_i|_Y \in NPO(Y)$ for each $i \in \Delta$ [by 2.13], so $\mathcal{B}^* = \{G_i|_Y : i \in \Delta\}$ is a cover of A by the NPO sets in $NPO(Y)$. Therefore, by hypothesis, there exists a finite subcollection $\{G_{i_k}|_Y : k = 1, 2, 3, \dots, n\}$ of $\mathcal{B}^*$ such that $A \subseteq \bigcup_{k=1}^n (G_{i_k}|_Y) \Rightarrow A \subseteq (\bigcup_{k=1}^n G_{i_k})|_Y \Rightarrow A \subseteq \bigcup_{k=1}^n G_{i_k}$. Thus the NPOC $\mathcal{B}$ of A has a finite NPOSC $\{G_{i_k} : k = 1, 2, 3, \dots, n\}$. Therefore, A is neutrosophic pre-compact in X .

3.13 Proposition:

Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. Then A is neutrosophic pre-Lindelöf (resp. neutrosophic countably pre-compact) in X iff every cover (resp. countable cover) of A by the NPO sets in $NPO(Y)$ has a countable (resp. finite) subcover.

Proof: Obvious from 3.12.

3.14 Proposition:

Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. If A is neutrosophic pre-compact in X then A is neutrosophic compact in Y .

Proof: Let $\mathcal{C} = \{G_i|_Y : i \in \Delta\}$ be an NOC of A in Y , where $G_i|_Y \in \tau|_Y$ for each $i \in \Delta$. Then $A \subseteq \bigcup_{i \in \Delta} (G_i|_Y) \Rightarrow A \subseteq \bigcup_{i \in \Delta} G_i$. Obviously $G_i \in \tau$ and so, G_i is an NPO set in X for each $i \in \Delta$. Therefore, $\mathcal{C}^* = \{G_i : i \in \Delta\}$ is an NPOC of A in X . Since A is pre-compact in X , so there exists a finite subcollection $\{G_{i_k} : k = 1, 2, 3, \dots, n\}$ of $\mathcal{C}^*$ such that $A \subseteq \bigcup_{k=1}^n G_{i_k} \Rightarrow A \subseteq (\bigcup_{k=1}^n G_{i_k})|_Y \Rightarrow A \subseteq \bigcup_{k=1}^n (G_{i_k}|_Y)$. Thus the NOC $\mathcal{C}$ of A has a finite NOSC $\{G_{i_k}|_Y : k = 1, 2, 3, \dots, n\}$. Therefore A is neutrosophic compact in Y .

3.15 Proposition:

Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X, τ) and $A \subseteq Y$. If A is pre-compact in Y then A is pre-compact in X .

Proof: Obvious.

3.16 Proposition:

If G is an NPC subset of a neutrosophic pre-compact space (X, τ) such that $G \cap G^c = \tilde{\emptyset}$ then G is neutrosophic pre-compact.

Proof: Let $\mathcal{C} = \{H_i : i \in \Delta\}$ be an NPOC of G . Then $G \subseteq \cup_{i \in \Delta} H_i$. Since G^c is an NPO set and since $G \cap G^c = \tilde{\emptyset}$, i.e., $G \cup G^c = \tilde{X}$, so $\mathcal{D} = \{H_i : i \in \Delta\} \cup \{G^c\}$ is an NPOC of X . As X is neutrosophic pre-compact, so there exists a finite subcollection $\mathcal{D}' = \{H_{i_1}, H_{i_2}, \dots, H_{i_n}\} \cup \{G^c\}$ of $\mathcal{D}$ such that $X \subseteq H_{i_1} \cup H_{i_2} \cup \dots \cup H_{i_n} \cup G^c$. Therefore $G \subseteq H_{i_1} \cup H_{i_2} \cup \dots \cup H_{i_n} \cup G^c$. But $G \cap G^c = \tilde{\emptyset}$, so $G \subseteq H_{i_1} \cup H_{i_2} \cup \dots \cup H_{i_n}$. Thus the NPOC $\mathcal{C}$ of G has a finite NPOSC $\{H_{i_1}, H_{i_2}, \dots, H_{i_n}\}$. Hence G is neutrosophic pre-compact set.

3.17 Proposition:

If G is an NPC subset of a neutrosophic pre-compact space (X, τ) such that $G \cap G^c = \tilde{\emptyset}$ then G is neutrosophic compact.

Proof: Immediate from 3.16.

3.18 Proposition:

If G is a neutrosophic closed subset of a neutrosophic pre-compact space (X, τ) such that $G \cap G^c = \tilde{\emptyset}$ then G is neutrosophic pre-compact.

Proof: Immediate from 3.16.

3.19 Proposition:

If G is a neutrosophic closed subset of a neutrosophic pre-compact space (X, τ) such that $G \cap G^c = \tilde{\emptyset}$ then G is neutrosophic compact.

Proof: Immediate from 3.18.

3.20 Proposition:

Let (X, τ) be an NTS. An NS $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x) \rangle : x \in X\}$ in X is neutrosophic pre-compact iff for every collection $\mathcal{C} = \{G_\lambda : \lambda \in \Delta\}$ of NPO sets of X satisfying $\mathcal{T}_A(x) \leq \bigvee_{\lambda \in \Delta} \mathcal{T}_{G_\lambda}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{I}_{G_\lambda}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{F}_{G_\lambda}(x))$, there exists a finite subcollection $\{G_{\lambda_k} : k = 1, 2, 3, \dots, n\}$ such that $\mathcal{T}_A(x) \leq \bigvee_{k=1}^n \mathcal{T}_{G_{\lambda_k}}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{I}_{G_{\lambda_k}}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{F}_{G_{\lambda_k}}(x))$.

Proof: Necessary Part : Let $C = \{G_\lambda : \lambda \in \Delta\}$ be any collection of NPO sets of X satisfying $\mathcal{T}_A(x) \leq \bigvee_{\lambda \in \Delta} \mathcal{T}_{G_\lambda}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{I}_{G_\lambda}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{F}_{G_\lambda}(x))$. Now $1 - \mathcal{I}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{I}_{G_\lambda}(x)) \Rightarrow 1 - \mathcal{I}_A(x) \leq 1 - \mathcal{I}_{G_\beta}(x)$ for some $\beta \in \Delta \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_\beta}(x) \Rightarrow \mathcal{I}_A(x) \geq \bigwedge_{\lambda \in \Delta} \mathcal{I}_{G_\lambda}(x)$. Similarly $1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{F}_{G_\lambda}(x)) \Rightarrow \mathcal{F}_A(x) \geq \bigwedge_{\lambda \in \Delta} \mathcal{F}_{G_\lambda}(x)$. Therefore $A \subseteq \bigcup_{\lambda \in \Delta} G_\lambda$, i.e., C is an NPOC of A . Since A is neutrosophic pre-compact, so C has a finite NPOSC $\{G_{\lambda_k} : k = 1, 2, 3, \dots, n\}$, say. Therefore $A \subseteq \bigcup_{k=1}^n G_{\lambda_k}$. Then $\mathcal{T}_A(x) \leq \bigvee_{k=1}^n \mathcal{T}_{G_{\lambda_k}}(x)$, $\mathcal{I}_A(x) \geq \bigwedge_{k=1}^n \mathcal{I}_{G_{\lambda_k}}(x)$ and $\mathcal{F}_A(x) \geq \bigwedge_{k=1}^n \mathcal{F}_{G_{\lambda_k}}(x)$. Now $\mathcal{I}_A(x) \geq \bigwedge_{k=1}^n \mathcal{I}_{G_{\lambda_k}}(x) \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_{\lambda_m}}(x)$ for some m , $1 \leq m \leq n \Rightarrow 1 - \mathcal{I}_A(x) \leq 1 - \mathcal{I}_{G_{\lambda_m}}(x)$ for some m , $1 \leq m \leq n \Rightarrow 1 - \mathcal{I}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{I}_{G_{\lambda_k}}(x))$. Similarly $\mathcal{F}_A(x) \geq \bigwedge_{k=1}^n \mathcal{F}_{G_{\lambda_k}}(x) \Rightarrow 1 - \mathcal{F}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{F}_{G_{\lambda_k}}(x))$. Thus $\mathcal{T}_A(x) \leq \bigvee_{k=1}^n \mathcal{T}_{G_{\lambda_k}}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{I}_{G_{\lambda_k}}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{F}_{G_{\lambda_k}}(x))$.

Sufficient Part : Let $C = \{G_\lambda : \lambda \in \Delta\}$ be an NPOC of A . Then $A \subseteq \bigcup_{\lambda \in \Delta} G_\lambda$, i.e., $\mathcal{T}_A(x) \leq \bigvee_{\lambda \in \Delta} \mathcal{T}_{G_\lambda}(x)$, $\mathcal{I}_A(x) \geq \bigwedge_{\lambda \in \Delta} \mathcal{I}_{G_\lambda}(x)$ and $\mathcal{F}_A(x) \geq \bigwedge_{\lambda \in \Delta} \mathcal{F}_{G_\lambda}(x)$. Now $\mathcal{I}_A(x) \geq \bigwedge_{\lambda \in \Delta} \mathcal{I}_{G_\lambda}(x) \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_\alpha}(x)$ for some $\alpha \in \Delta \Rightarrow 1 - \mathcal{I}_A(x) \leq 1 - \mathcal{I}_{G_\alpha}(x) \Rightarrow 1 - \mathcal{I}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{I}_{G_\lambda}(x))$. Similarly $\mathcal{F}_A(x) \geq \bigwedge_{\lambda \in \Delta} \mathcal{F}_{G_\lambda}(x) \Rightarrow 1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{F}_{G_\lambda}(x))$. Thus the collection C satisfies the condition $\mathcal{T}_A(x) \leq \bigvee_{\lambda \in \Delta} \mathcal{T}_{G_\lambda}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{I}_{G_\lambda}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda \in \Delta} (1 - \mathcal{F}_{G_\lambda}(x))$. By the hypothesis, there exists a finite subcollection $\{G_{\lambda_k} : k = 1, 2, 3, \dots, n\}$ such that $\mathcal{T}_A(x) \leq \bigvee_{k=1}^n \mathcal{T}_{G_{\lambda_k}}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{I}_{G_{\lambda_k}}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{F}_{G_{\lambda_k}}(x))$. Now $1 - \mathcal{I}_A(x) \leq \bigvee_{k=1}^n (1 - \mathcal{I}_{G_{\lambda_k}}(x)) \Rightarrow 1 - \mathcal{I}_A(x) \leq 1 - \mathcal{I}_{G_{\lambda_m}}(x)$ for some m , $1 \leq m \leq n \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_{\lambda_m}}(x) \Rightarrow \mathcal{I}_A(x) \geq \bigwedge_{k=1}^n \mathcal{I}_{G_{\lambda_k}}(x)$. Similarly we shall have $\mathcal{F}_A(x) \geq \bigwedge_{k=1}^n \mathcal{F}_{G_{\lambda_k}}(x)$. Therefore $A \subseteq \bigcup_{k=1}^n G_{\lambda_k}$, i.e., the NPOC C of A has a finite NPOSC $\{G_{\lambda_k} : k = 1, 2, 3, \dots, n\}$. Therefore, A is neutrosophic pre-compact set.

Hence proved.

3.21 Proposition:

Let (X, τ) be an NTS. Then X is neutrosophic compact iff for every collection $C = \{G_\lambda : \lambda \in \Delta\}$ of NPO sets of X satisfying $\bigvee_{\lambda \in \Delta} \mathcal{T}_{G_\lambda}(x) = 1$, $\bigvee_{\lambda \in \Delta} (1 - \mathcal{I}_{G_\lambda}(x)) = 1$ and $\bigvee_{\lambda \in \Delta} (1 - \mathcal{F}_{G_\lambda}(x)) = 1$, there exists a finite subcollection $\{G_{\lambda_k} : k = 1, 2, 3, \dots, n\}$ such that $\bigvee_{k=1}^n \mathcal{T}_{G_{\lambda_k}}(x) = 1$, $\bigvee_{k=1}^n (1 - \mathcal{I}_{G_{\lambda_k}}(x)) = 1$ and $\bigvee_{k=1}^n (1 - \mathcal{F}_{G_{\lambda_k}}(x)) = 1$.

Proof: Immediate from 3.20.

3.22 Definition:

Let (X, τ) be an NTS. A collection $\{G_\lambda : \lambda \in \Delta\}$ of neutrosophic sets of X is said to have the finite intersection property (FIP, in short) iff every finite subcollection $\{G_{\lambda_k} : k = 1, 2, \dots, n\}$ of $\{G_\lambda : \lambda \in \Delta\}$ satisfies the condition $\bigcap_{k=1}^n G_{\lambda_k} \neq \emptyset$, where Δ is an index set.

3.23 Proposition:

An NTS (X, τ) is neutrosophic pre-compact iff every collection of NPC sets with the FIP has a non-empty intersection.

Proof: Necessary part: Let $\mathcal{A} = \{N_i : i \in \Delta\}$ be an arbitrary collection of NPC sets with the FIP. We show that $\bigcap_{i \in \Delta} N_i \neq \emptyset$. On the contrary, suppose that $\bigcap_{i \in \Delta} N_i = \emptyset$. Then $(\bigcap_{i \in \Delta} N_i)^c = (\emptyset)^c \Rightarrow \bigcup_{i \in \Delta} N_i^c = \tilde{X}$. Therefore $\mathcal{B} = \{N_i^c : N_i \in \mathcal{A}\}$ is an NPOC of X and so, $\mathcal{B}$ has a finite NPOSC $\{N_{i_1}^c, N_{i_2}^c, \dots, N_{i_k}^c\}$, say. Then $\bigcup_{j=1}^k N_{i_j}^c = \tilde{X} \Rightarrow \bigcap_{j=1}^k N_{i_j} = \emptyset$, which is a contradiction as $\mathcal{A}$ has FIP. Therefore $\bigcap_{i \in \Delta} N_i \neq \emptyset$.

Sufficient part: On the contrary, suppose that X is not neutrosophic pre-compact. Then there must exist an NPOC of X which will have no finite NPOSC. Let $\mathcal{C} = \{G_i : i \in \Delta\}$ be an NPOC of X which has no

finite subcover. Then for every finite subcollection $\{G_{i_1}, G_{i_2}, \dots, G_{i_k}\}$ of $\mathcal{C}$, we have $\bigcup_{j=1}^k G_{i_j} \neq \tilde{X} \Rightarrow \bigcap_{j=1}^k G_{i_j}^c \neq \tilde{\emptyset}$. Therefore $\{G_i^c : G_i \in \mathcal{C}\}$ is a collection of NPC sets having the FIP. By the assumption, $\bigcap_{i \in \Delta} G_i^c \neq \tilde{\emptyset} \Rightarrow \bigcup_{i \in \Delta} G_i \neq \tilde{X}$. This implies that $\mathcal{C}$ is not an NPOC of X , which is a contradiction. Therefore, NPOC $\mathcal{C}$ of X must have a finite NPOSC. Therefore X is neutrosophic pre-compact.

Hence proved.

3.24 Proposition:

Let f be a neutrosophic pre-continuous function from an NTS (X, τ) to the NTS (Y, σ) . If A is neutrosophic pre-compact set in X then $f(A)$ is neutrosophic compact set in Y .

Proof: Let $\mathcal{B} = \{G_\lambda : \lambda \in \Delta\}$ be an NOC of $f(A)$. Then $f(A) \subseteq \bigcup_{\lambda \in \Delta} G_\lambda \Rightarrow f^{-1}(f(A)) \subseteq f^{-1}(\bigcup_{\lambda \in \Delta} G_\lambda) \Rightarrow f^{-1}(f(A)) \subseteq \bigcup_{\lambda \in \Delta} f^{-1}(G_\lambda) \Rightarrow A \subseteq \bigcup_{\lambda \in \Delta} f^{-1}(G_\lambda)$. Since G_λ is σ -open NS in Y , so $f^{-1}(G_\lambda)$ is τ -NPO set in X as f is pre-continuous. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NPOC of A . Since A is neutrosophic pre-compact, so C has a finite NPOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \dots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $A \subseteq \bigcup_{i=1}^n f^{-1}(G_{\lambda_i}) \Rightarrow f(A) \subseteq f(\bigcup_{i=1}^n f^{-1}(G_{\lambda_i})) \Rightarrow f(A) \subseteq \bigcup_{i=1}^n f(f^{-1}(G_{\lambda_i})) \Rightarrow f(A) \subseteq \bigcup_{i=1}^n G_{\lambda_i}$. Thus the NOC $\mathcal{B}$ of $f(A)$ has a finite NOSC. Therefore $f(A)$ is neutrosophic compact. Hence proved.

3.25 Proposition:

Let f be a neutrosophic continuous function from an NTS (X, τ) to the NTS (Y, σ) . If A is neutrosophic pre-compact in X then $f(A)$ is neutrosophic compact in Y .

Proof: Obvious from 3.24.

3.26 Proposition:

Let f be a neutrosophic pre-continuous function from an NTS (X, τ) onto the NTS (Y, σ) . If (X, τ) is neutrosophic pre-compact then (Y, σ) is neutrosophic compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $\mathcal{B} = \{G_\lambda : \lambda \in \Delta\}$ be an NOC of Y . Then $\bigcup_{\lambda \in \Delta} G_\lambda = \tilde{Y} \Rightarrow f^{-1}(\bigcup_{\lambda \in \Delta} G_\lambda) = f^{-1}(\tilde{Y}) \Rightarrow \bigcup_{\lambda \in \Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is σ -open NS in Y , so $f^{-1}(G_\lambda)$ is τ -NPO set in X as f is pre-continuous. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NPOC of X . Since X is pre-compact, so C has a finite NPOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \dots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $\bigcup_{i=1}^n f^{-1}(G_{\lambda_i}) = \tilde{X} \Rightarrow f(\bigcup_{i=1}^n f^{-1}(G_{\lambda_i})) = f(\tilde{X}) \Rightarrow \bigcup_{i=1}^n f(f^{-1}(G_{\lambda_i})) = \tilde{Y} \Rightarrow \bigcup_{i=1}^n G_{\lambda_i} = \tilde{Y}$. Thus the NOC $\mathcal{B}$ of Y has a finite NOSC. Therefore Y is neutrosophic compact. Hence proved.

3.27 Proposition:

Let f be a neutrosophic continuous function from an NTS (X, τ) onto the NTS (Y, σ) . If (X, τ) is neutrosophic pre-compact then (Y, σ) is neutrosophic compact.

Proof: Obvious from 3.26.

3.28 Proposition:

Let f be a neutrosophic pre-continuous function from an NTS (X, τ) onto the NTS (Y, σ) . If X is neutrosophic countably pre-compact then Y is neutrosophic countably compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $\mathcal{A} = \{G_\lambda : \lambda \in \Delta\}$ be a countable NOC of Y . Then $\cup_{\lambda \in \Delta} G_\lambda = \tilde{Y} \Rightarrow f^{-1}(\cup_{\lambda \in \Delta} G_\lambda) = f^{-1}(\tilde{Y}) \Rightarrow \cup_{\lambda \in \Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is σ -open NS in Y , so $f^{-1}(G_\lambda)$ is τ -NPO set in X as f is pre-continuous. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NPOC of X . Obviously C is countable as A is countable. Again since X is neutrosophic countably pre-compact, so C has a finite NPOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \dots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $\cup_{i=1}^n f^{-1}(G_{\lambda_i}) = \tilde{X} \Rightarrow f(\cup_{i=1}^n f^{-1}(G_{\lambda_i})) = f(\tilde{X}) \Rightarrow \cup_{i=1}^n f(f^{-1}(G_{\lambda_i})) = \tilde{Y} \Rightarrow \cup_{i=1}^n G_{\lambda_i} = \tilde{Y}$. Thus the countable NOC $\mathcal{A}$ of Y has a finite NPOSC. Hence Y is neutrosophic countably compact.

3.29 Proposition:

Let f be a neutrosophic continuous function from an NTS (X, τ) onto the NTS (Y, σ) . If X is neutrosophic countably pre-compact then Y is neutrosophic countably compact.

Proof: Immediate from 3.28.

3.30 Proposition:

Let f be a neutrosophic pre-continuous function from an NTS (X, τ) onto the NTS (Y, σ) . If X is neutrosophic pre-Lindelöf then Y is neutrosophic Lindelöf.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $\mathcal{A} = \{A_i : i \in \Delta\}$ be an NOC of Y . Then $\tilde{Y} = \cup_{i \in \Delta} A_i \Rightarrow f^{-1}(\tilde{Y}) = f^{-1}(\cup_{i \in \Delta} A_i) \Rightarrow \tilde{X} = \cup_{i \in \Delta} f^{-1}(A_i) \Rightarrow \{f^{-1}(A_i) : i \in \Delta\}$ is an NPOC of X . Since X is neutrosophic pre-Lindelöf, so $\{f^{-1}(A_i) : i \in \Delta\}$ has a countable NPOSC $\mathcal{B} = \{f^{-1}(A_{i_k}) : k = 1, 2, 3, \dots\}$, say. Therefore $\tilde{X} = f^{-1}(A_{i_1}) \cup f^{-1}(A_{i_2}) \cup f^{-1}(A_{i_3}) \cup \dots$. This gives $f(\tilde{X}) = f[f^{-1}(A_{i_1}) \cup f^{-1}(A_{i_2}) \cup f^{-1}(A_{i_3}) \cup \dots] \Rightarrow \tilde{Y} = f(f^{-1}(A_{i_1})) \cup f(f^{-1}(A_{i_2})) \cup f(f^{-1}(A_{i_3})) \cup \dots \Rightarrow \tilde{Y} = A_{i_1} \cup A_{i_2} \cup A_{i_3} \cup \dots \Rightarrow \{A_{i_k} : k = 1, 2, 3, \dots\}$ an NOC of Y . Since $\mathcal{B}$ is countable, so $\{A_{i_k} : k = 1, 2, 3, \dots\}$ is also countable. Therefore the NOC $\mathcal{A}$ of Y has a countable NOSC $\{A_{i_k} : k = 1, 2, 3, \dots\}$ and so, Y is neutrosophic Lindelöf. Hence proved.

3.31 Proposition:

Let f be a neutrosophic continuous function from an NTS (X, τ) onto the NTS (Y, σ) . If X is neutrosophic pre-Lindelöf then Y is neutrosophic Lindelöf.

Proof: Immediate from 3.30.

3.32 Proposition:

Let f be a neutrosophic pre-open function from an NTS (X, τ) to the NTS (Y, σ) . If $A \subseteq Y$ is neutrosophic pre-compact in Y then $f^{-1}(A)$ is neutrosophic compact in X .

Proof: Let $\mathcal{B} = \{G_\lambda : \lambda \in \Delta\}$ be an NOC of $f^{-1}(A)$. Then $f^{-1}(A) \subseteq \cup_{\lambda \in \Delta} G_\lambda \Rightarrow A \subseteq f(\cup_{\lambda \in \Delta} G_\lambda) \Rightarrow A \subseteq \cup_{\lambda \in \Delta} f(G_\lambda)$. Since G_λ is τ -open set, so $f(G_\lambda)$ is σ -NPO set for each $\lambda \in \Delta$ as f is a pre-open function. Therefore, $\mathcal{C} = \{f(G_\lambda) : \lambda \in \Delta\}$ is an NPOC of A . Since A is neutrosophic pre-compact, so $\mathcal{C}$ has a finite NPOSC $\{f(G_{\lambda_1}), f(G_{\lambda_2}), f(G_{\lambda_3}), \dots, f(G_{\lambda_n})\}$, say. Therefore $A \subseteq \cup_{i=1}^n f(G_{\lambda_i}) \Rightarrow A \subseteq f(\cup_{i=1}^n G_{\lambda_i}) \Rightarrow f^{-1}(A) \subseteq \cup_{i=1}^n G_{\lambda_i}$. Thus the NOC $\mathcal{B}$ of $f^{-1}(A)$ has a finite NOSC $\{G_{\lambda_1}, G_{\lambda_2}, G_{\lambda_3}, \dots, G_{\lambda_n}\}$. Therefore $f^{-1}(A)$ is neutrosophic compact in X . Hence proved.

3.33 Proposition:

Let f be a neutrosophic pre-open function from an NTS (X, τ) onto the NTS (Y, σ) . If (Y, σ) is neutrosophic pre-compact (resp. neutrosophic countably pre-compact, neutrosophic pre-Lindelöf) then (X, τ) is neutrosophic compact (resp. neutrosophic countably compact, neutrosophic Lindelöf).

Proof: Immediate from 3.32 as f is onto.

3.34 Proposition:

Let f be a neutrosophic open function from an NTS (X, τ) onto the NTS (Y, σ) . If (Y, σ) is neutrosophic pre-compact (resp. neutrosophic countably pre-compact, neutrosophic pre-Lindelöf) then (X, τ) is neutrosophic compact (resp. neutrosophic countably compact, neutrosophic Lindelöf).

Proof: Obvious from 3.33.

3.35 Definition:

Let f be a function from an NTS (X, τ) to the NTS (Y, σ) . Then f is called a neutrosophic pre*-open function if $f(G)$ is an NPO set in Y for every NPO set G in X .

3.36 Proposition:

Let f be a neutrosophic pre*-open function from an NTS (X, τ) to the NTS (Y, σ) and $A \in \mathcal{N}(Y)$. If A is neutrosophic pre-compact in Y then $f^{-1}(A)$ is neutrosophic pre-compact in X .

Proof: Let $\mathcal{B} = \{G_\lambda : \lambda \in \Delta\}$ be an NPOC of $f^{-1}(A)$. Then $f^{-1}(A) \subseteq \bigcup_{\lambda \in \Delta} G_\lambda \Rightarrow A \subseteq f(\bigcup_{\lambda \in \Delta} G_\lambda) \Rightarrow A \subseteq \bigcup_{\lambda \in \Delta} f(G_\lambda)$. Since G_λ is τ -NPO set, so $f(G_\lambda)$ is σ -NPO set for each $\lambda \in \Delta$ as f is a pre*-open function. Therefore, $\mathcal{C} = \{f(G_\lambda) : G_\lambda \in \mathcal{B}\}$ is an NPOC of A . Since A is neutrosophic pre-compact, so $\mathcal{C}$ has a finite NPOSC $\{f(G_{\lambda_1}), f(G_{\lambda_2}), f(G_{\lambda_3}), \dots, f(G_{\lambda_n})\}$, say. Therefore $A \subseteq \bigcup_{i=1}^n f(G_{\lambda_i}) \Rightarrow A \subseteq f(\bigcup_{i=1}^n G_{\lambda_i}) \Rightarrow f^{-1}(A) \subseteq \bigcup_{i=1}^n G_{\lambda_i}$. Thus the NPOC $\mathcal{B}$ of $f^{-1}(A)$ has a finite NPOSC $\{G_{\lambda_1}, G_{\lambda_2}, G_{\lambda_3}, \dots, G_{\lambda_n}\}$. Therefore $f^{-1}(A)$ is neutrosophic pre-compact in X . Hence proved.

3.37 Proposition:

Let f be a neutrosophic pre*-open function from an NTS (X, τ) onto the NTS (Y, σ) . If (Y, σ) is neutrosophic pre-compact (resp. neutrosophic countably pre-compact, neutrosophic pre-Lindelöf) then (X, τ) is also neutrosophic pre-compact (resp. neutrosophic countably pre-compact, neutrosophic pre-Lindelöf).

Proof: Immediate from 3.36 as f is onto.

3.38 Definition:

Let f be a neutrosophic function from an NTS (X, τ) to the NTS (Y, σ) . Then f is called a neutrosophic pre-irresolute function if $f^{-1}(G)$ is an NPO set in X for every NPO set G in Y .

3.39 Proposition:

Let f be a neutrosophic pre-irresolute function from an NTS (X, τ) to the NTS (Y, σ) . If A is neutrosophic pre-compact in X then $f(A)$ is also neutrosophic pre-compact in Y .

Proof: Let $\mathcal{B} = \{G_\lambda : \lambda \in \Delta\}$ be an NPOC of $f(A)$ in Y . Then $f(A) \subseteq \bigcup_{\lambda \in \Delta} G_\lambda \Rightarrow A \subseteq f^{-1}(\bigcup_{\lambda \in \Delta} G_\lambda) \Rightarrow A \subseteq \bigcup_{\lambda \in \Delta} f^{-1}(G_\lambda)$. Since G_λ is σ -NPO set in Y , so $f^{-1}(G_\lambda)$ is a τ -NPO set in X as f is a neutrosophic pre-irresolute function. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NPOC of A in X . Since A is neutrosophic pre-compact in X , so C has a finite NPOSC $\{f^{-1}(G_{\lambda_1}), f^{-1}(G_{\lambda_2}), \dots, f^{-1}(G_{\lambda_n})\}$, say. Therefore $A \subseteq \bigcup_{i=1}^n f^{-1}(G_{\lambda_i}) \Rightarrow f(A) \subseteq f(\bigcup_{i=1}^n f^{-1}(G_{\lambda_i})) \Rightarrow f(A) \subseteq \bigcup_{i=1}^n f(f^{-1}(G_{\lambda_i})) \Rightarrow f(A) \subseteq \bigcup_{i=1}^n G_{\lambda_i}$. Thus the NPOC $\mathcal{B}$ of $f(A)$ has a finite NPOSC $\{G_{\lambda_1}, G_{\lambda_2}, \dots, G_{\lambda_n}\}$. Therefore $f(A)$ is neutrosophic pre-compact. Hence proved.

3.40 Proposition:

Let f be a neutrosophic pre-irresolute function from an NTS (X, τ) onto the NTS (Y, σ) . If (X, τ) is neutrosophic pre-compact (resp. neutrosophic countably pre-compact, neutrosophic pre-Lindelöf) then (Y, σ) is also neutrosophic pre-compact (resp. neutrosophic countably pre-compact, neutrosophic pre-Lindelöf).

Proof: Obvious from 3.39.

3.41 Definition :

Let (X, τ) be an NTS and $NPO(X)$ be the collection of all NPO sets in X . A subcollection $\mathcal{B}$ of $NPO(X)$ is called a neutrosophic pre-base (Np-base, for short) for X iff for each $A \in NPO(X)$, there exists a subcollection $\{A_i : i \in \Delta\}$ of $\mathcal{B}$ such that $A = \bigcup \{A_i : i \in \Delta\}$, where Δ is an index set.

A subcollection $\mathcal{B}_*$ of $NPO(X)$ is called a neutrosophic pre-subbase (Np-subbase, for short) for X iff the finite intersection of members of $\mathcal{B}_*$ forms a neutrosophic pre-base for X .

3.42 Definition:

An NTS (X, τ) is said to be neutrosophic pre- C_{II} space iff X has a countable neutrosophic pre-base, i.e., an NTS (X, τ) is said to be pre- C_{II} space iff there exists a countable subcollection $\mathcal{B}$ of $NPO(X)$ such that every member of $NPO(X)$ can be expressed as the union of some members of $\mathcal{B}$.

3.43 Proposition:

Let $\mathcal{B}$ be an Np-base for an NTS (X, τ) . Then X is neutrosophic pre-compact iff every NPOC of X by the members of $\mathcal{B}$ has a finite NPOSC.

Proof: Necessary Part : Obvious.

Sufficient Part : Let $\mathcal{B} = \{B_\alpha : \alpha \in \Delta\}$ be the Np-base. Also let $\mathcal{C} = \{G_\lambda : \lambda \in \Delta\}$ be an NPOC of X . Then each member G_λ of $\mathcal{C}$ is the union of some members of $\mathcal{B}$ and the totality of such members of $\mathcal{B}$ is evidently an NPOC of X . By the hypothesis, this collection of members of $\mathcal{B}$ has a finite NPOSC $\mathcal{D} = \{B_{\alpha_j} : j = 1, 2, 3, \dots, n\}$, say. Clearly for each B_{α_j} in $\mathcal{D}$, there is a G_{λ_j} in $\mathcal{C}$ such that $B_{\alpha_j} \subseteq G_{\lambda_j}$. Therefore the finite subcollection $\{G_{\lambda_j} : j = 1, 2, 3, \dots, n\}$ of $\mathcal{C}$ is an NPOC of X , i.e., the NPOC $\mathcal{C}$ of X has a finite NPOSC $\{G_{\lambda_j} : j = 1, 2, 3, \dots, n\}$. Therefore X is neutrosophic pre-compact.

3.44 Proposition:

Let (X, τ) be a neutrosophic countably pre-compact space. If X is pre- C_{II} then X neutrosophic pre-compact.

Proof: Let $\mathcal{D} = \{A_i : i \in \Delta\}$ be any NPOC of X . Since X is pre- C_{II} , so there exists a countable Np-base $\mathcal{B} = \{B_n : n = 1, 2, 3, \dots\}$ for X . Then each $A_i \in \mathcal{D}$ can be expressed as the union of some members of $\mathcal{B}$. Let $A_i = \bigcup_{k=1}^{i_0} B_{n_k}$, where $B_{n_k} \in \mathcal{B}$ and i_0 may be infinity. Clearly $\mathcal{B}_0 = \{B_{n_k}\}$ is an NPOC of X . Also $\mathcal{B}_0$ is countable as $\mathcal{B}_0 \subseteq \mathcal{B}$. Therefore, $\mathcal{B}_0$ is a countable NPOC of X . Since X is countably pre-compact, so $\mathcal{B}_0$ has a finite NPOSC $\mathcal{B}'$, say. Since by construction, each member of $\mathcal{B}'$ is contained in one member A_i of $\mathcal{D}$, so these A_i 's form a finite NPOC of X . Thus the NPOC $\mathcal{D}$ of X has a finite NPOSC. Therefore X is neutrosophic pre-compact. Hence Proved.

3.45 Remark:

In view of 3.3 and 3.44, it is evident that if an NTS (X, τ) is pre- C_{II} then neutrosophic pre-compactness and neutrosophic countably pre-compactness are equivalent.

3.46 Proposition:

If an NTS (X, τ) is pre- C_{II} then it is neutrosophic pre-Lindelöf.

Proof: Let $\mathcal{A} = \{A_i : i \in \Delta\}$ be an NPOC of X . Since X is pre- C_{II} , so there exists a countable Np-base $\mathcal{B} = \{B_n : n = 1, 2, 3, \dots\}$ for X . Then each $A_i \in \mathcal{A}$ can be expressed as the union of some members of $\mathcal{B}$. Let $A_i = \bigcup_{k=1}^{i_0} B_{n_k}$, where $B_{n_k} \in \mathcal{B}$ and i_0 may be infinity. Let $\mathcal{B}_0 = \{B_{n_k}\}$. Then $\mathcal{B}_0$ is an NPOC of X . Also $\mathcal{B}_0$ is countable as $\mathcal{B}_0 \subseteq \mathcal{B}$. Therefore, $\mathcal{B}_0$ is a countable NPOC of X . By construction, each member of $\mathcal{B}_0$ is contained in one A_i of $\mathcal{A}$. So, these A_i 's form a countable NPOC of X . Thus the NPOC $\mathcal{A}$ of X has a countable NPOSC. Therefore X is neutrosophic pre-Lindelöf.

3.47 Proposition:

Let β be an Np-subbase of an NTS (X, τ) . Then X is neutrosophic pre-compact iff for every collection of NPC sets chosen from β^c having the FIP, there is a non-empty intersection.

Proof: Necessary part : Immediate from 3.23.

Sufficient Part : On the contrary, let us suppose that X is not pre-compact. Then by 3.23, there exists a collection $\mathcal{C} = \{G_i : i \in I\}$ of NPC of X having the FIP such that $\bigcap_{i \in \Delta} G_i = \tilde{\emptyset}$. The collection of all such collections $\mathcal{C}$ can be arranged in an order by using the classical inclusion ($\subseteq$) and the collection will certainly have an upper bound. Therefore by Zorn's lemma, there will be a maximal collection of all the collections $\mathcal{C}$. Let $\mathcal{P} = \{P_j : j \in J\}$ be the maximal collection. This collection $\mathcal{P}$ has the following properties :

(i) $\tilde{\emptyset} \notin \mathcal{P}$ (ii) $P \in \mathcal{P}, P \subseteq Q \Rightarrow Q \in \mathcal{P}$ (iii) $P, Q \in \mathcal{P} \Rightarrow P \cap Q \in \mathcal{P}$ (iv) $\bigcap (\mathcal{P} \cap \beta^c) = \tilde{\emptyset}$. Clearly the property (iv) delivers a contradiction to the hypothesis. Therefore X is pre-compact.

Hence proved.

4 Neutrosophic local pre-compactness**4.1 Definition:**

An NTS (X, τ) is said to be a neutrosophic locally pre-compact space iff for every NP $x_{\alpha, \beta, \gamma}$ in X , there exists a τ -NPO set G such that $x_{\alpha, \beta, \gamma} \in G$ and G is neutrosophic pre-compact in X .

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4.2 Proposition:

Every neutrosophic pre-compact space is a neutrosophic locally pre-compact space.

Proof: Let (X, τ) be a neutrosophic pre-compact space and let $x_{\alpha, \beta, \gamma}$ be an NP in X . Since X is neutrosophic pre-compact and since $\tilde{X}$ is an NPO set such that $x_{\alpha, \beta, \gamma} \in \tilde{X}$, so, (X, τ) is a neutrosophic locally pre-compact space.

4.3 Proposition:

Let f be a neutrosophic pre^{*}-open and pre-irresolute function from an NTS space (X, τ) to the NTS (Y, σ) . If (Y, σ) neutrosophic locally pre-compact then (X, τ) is also a neutrosophic locally pre-compact space.

Proof: Let $x_{\alpha, \beta, \gamma}$ be any NP in X . Then there exists an NP $y_{p, q, r}$ in Y such that $f(x_{\alpha, \beta, \gamma}) = y_{p, q, r}$. Since $y_{p, q, r} \in Y$ and Y neutrosophic locally pre-compact, so there exists a σ -NPO set G such that $y_{p, q, r} \in G$ and G is neutrosophic pre-compact in Y . Now $y_{p, q, r} \in G \Rightarrow f(x_{\alpha, \beta, \gamma}) \in G \Rightarrow x_{\alpha, \beta, \gamma} \in f^{-1}(G)$. Since f is neutrosophic pre^{*}-open and G is neutrosophic pre-compact in Y , so by 3.36, $f^{-1}(G)$ is neutrosophic pre-compact in X . Again since f is a neutrosophic pre-irresolute function, so $f^{-1}(G)$ is a τ -NPO set. Thus for any any NP $x_{\alpha, \beta, \gamma}$ in X , there exists a τ -NPO set $f^{-1}(G)$ such that $x_{\alpha, \beta, \gamma} \in f^{-1}(G)$ and $f^{-1}(G)$ is neutrosophic pre-compact in X . Therefore (X, τ) is neutrosophic locally pre-compact space.

4.4 Proposition:

Let f be a neutrosophic pre^{*}-open and pre-irresolute function from an NTS (X, τ) onto the NTS (Y, σ) . If X is neutrosophic locally pre-compact then Y is neutrosophic locally pre-compact.

Proof: Let $y_{p, q, r}$ be any NP in Y . Since f is onto, so there is an NP $x_{\alpha, \beta, \gamma}$ in X such that $f(x_{\alpha, \beta, \gamma}) = y_{p, q, r}$. Since $x_{\alpha, \beta, \gamma} \in X$ and X neutrosophic locally pre-compact, so there exists a τ -NPO set G such that $x_{\alpha, \beta, \gamma} \in G$ and G is neutrosophic pre-compact in X . Now $x_{\alpha, \beta, \gamma} \in G \Rightarrow f(x_{\alpha, \beta, \gamma}) \in f(G) \Rightarrow y_{p, q, r} \in f(G)$. Since f is neutrosophic pre-irresolute and G is neutrosophic pre-compact in X , so by 3.39, $f(G)$ is neutrosophic pre-compact in Y . Again since f is a neutrosophic pre^{*}-open function, so $f(G)$ is a σ -NPO set. Thus for any any NP $y_{p, q, r}$ in Y , there exists a σ -NPO set $f(G)$ such that $y_{p, q, r} \in f(G)$ and $f(G)$ is neutrosophic pre-compact in Y . Therefore (Y, σ) is neutrosophic locally pre-compact space.

5 Conclusions

In this article, we have defined neutrosophic pre-open cover with the help of neutrosophic pre-open sets and then we have defined neutrosophic pre-compact space, neutrosophic countably pre-compact space, neutrosophic pre-Lindelöf space and investigated various covering properties. We have proved that every neutrosophic pre-compact space is a neutrosophic compact space but the converse is not true. We have shown that if a neutrosophic topological space is neutrosophic pre- C_{II} then neutrosophic pre-compactness and neutrosophic countably pre-compactness are equivalent. In 3.40, we have established that neutrosophic pre-compactness (resp. neutrosophic countably pre-compactness, neutrosophic pre-Lindelöfness) is preserved under a neutrosophic pre-irresolute function. In 3.47, we have also stated and proved “Alexander subbase lemma” in case of a neutrosophic pre-compact space with the help of neutrosophic pre-subbase. In the long run, we have defined neutrosophic locally pre-compact space and put forward a few propositions with proofs. Hope that the findings in this article will assist the research fraternity to move forward for the development of different aspects of neutrosophic topology.

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