An Introduction To The Symbolic 3-Plithogenic Modules

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Abstract

The objective of this paper is to define and study for the first time the concept of symbolic 3-plithogenic module based on symbolic 3-plithogenic sets and classical modules.Also, many related substructures will be defined and handled such as AH-functions, AH-submodules, and symbolic 3-plithogenic homomorphisms.

Keywords: 3-plithogenic symbolic set; 3-plithogenic module; 3-plithogenic homomorphism

1. Introduction

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17 ,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31], where the concepts such as symbolic AH-ideals, and AH-homomorphisms were presented and discussed. In [35-39] many algebraic structures about symbolic 2-plithogenic structures were studied such as number theory, algebraic equations, and symbolic 3-plithogenic rings.

In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:
$3-SP_R=\{a_0+a_1P_1+a_2P_2+a_3P_3;\ a_i\in R, {P_j}^2=P_j, P_i\times P_j=P_{max(i,j)}\}$.
Smarandache has defined algebraic operations on $3-SP_R$ as follows:
Addition:
$[a_0+a_1P_1+a_2P_2+a_3P_3]+[b_0+b_1P_1+b_2P_2+b_3P_3]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2+(a_3+b_3)P_3$.
Multiplication:
$[a_0+a_1P_1+a_2P_2+a_3P_3].[b_0+b_1P_1+b_2P_2+b_3P_3]=a_0b_0+P_1[a_0b_1+a_1b_0+a_1b_1]+P_2[a_0b_2+a_1b_2+a_2b_2+a_2b_0+a_2b_1]+P_3[a_0b_3+a_1b_3+a_2b_3+a_3b_3+a_3b_0+a_3b_1+a_3b_2]$.
It is clear that $(3-SP_R)$ is a ring.

Main Discussion

Definition.
Let M be a module over the ring R, let $3-SP_R$ be the corresponding symbolic 3-plithogenic ring.
$3-SP_R=\{x+yP_1+zP_2+tP_3;\ x,y,z,t\in R, {P_i}^2=P_i, P_1P_2=P_2P_1=P_2\}$.
We define the symbolic 3-plithogenic module as follows:
$3-SP_M=M+MP_1+MP_2+MP_3=\{a+bP_1+cP_2+dP_3;\ a,b,c,d\in M\}$.
Operations on $3-SP_M$ can be defined as follows:
Addition: $(+): 3-SP_M\to 3-SP_M$, such that:

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$[x_0+x_1P_1+x_2P_2+x_3P_3]+[y_0+y_1P_1+y_2P_2+y_3P_3]=(x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2+(x_3+y_3)P_3$.
Multiplication: $(.):3-SP_R\times 3-SP_M\to 3-SP_M$, such that:
$[a+bP_1+cP_2+dP_3].[x_0+x_1P_1+x_2P_2+x_3P_3]=ax_0+(ax_1+bx_0+bx_1)P_1+(ax_2+bx_2+cx_0+cx_1+cx_2)P_2+(ax_3+bx_3+cx_3+dx_0+dx_1+dx_2+dx_3)P_3$.
where $x_i,y_i\in V,a,b,c,d\in R$

Theorem.
Let $(3-SP_M,+,.)$ Is a module over the ring $3-SP_R$.

Proof.
Let$X=x_0+x_1P_1+x_2P_2+x_3P_3, Y=y_0+y_1P_1+y_2P_2+y_3P_3\in 3-SP_V$, $A=a_0+a_1P_1+a_2P_2+a_3P_3, B=b_0+b_1P_1+b_2P_2+b_3P_3\in 3-SP_R$ we have:
1.$X=X,(X+Y)+Z=X+(Y+Z),X+(-X)=-X+X=0,X+0=0+X=X$
Also
$A(X+Y)=(a_0+a_1P_1+a_2P_2+a_3P_3)[(x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2+(x_3+y_3)P_3]=A.X+A.Y$
$(A+B)X=A.X+B.X$
$(A.B).X=A(B.X)$

Example.
Let $V=Z^3$ be a module over the ring of integers $Z=R$.
The corresponding symbolic 3-plithogenic module over $3-SP_Z$ is:
$3-SP_{Z^3}=\{(x_0,y_0,z_0)+(x_1,y_1,z_1)P_1+(x_2,y_2,z_2)P_2+(x_3,y_3,z_3)P_3; x_i,y_i,z_i\in Z\}$

Definition.
Let $3-SP_M$ be a symbolic 3-plithogenic module over $3-SP_M$, let V_0,V_1,V_2,V_3 be the three submodules of M, we define the AH-submodule as follows:
$W=V_0+V_1P_1+V_2P_2+V_3P_3=\{x+yP_1+zP_2+tP_3;\ x\in V_0,y\in V_1,z\in V_2,t\in V_3\}$.
If $V_0=V_1=V_2=V_3$, then W is called an AHS-submodule.

Example.
Consider $3-SP_{R^3}$, we have $V_0=\{(a,0,0);\ a\in R\},V_1=\{(0,b,0);\ b\in R\},V_2=\{(0,0,c);\ c\in R\}$ are three submodules of $M=R^3$.
$W=V_0+V_1P_1+V_2P_2+V_2P_3=\{(a,0,0)+(0,b,0)P_1+(0,0,c)P_2+(0,0,d)P_2;\ a,b,c,d\in R\}$ is an AH-submodule of $3-SP_{R^3}$.
$T=V_1+V_1P_1+V_1P_2+V_1P_3=\{(0,a,0)+(0,b,0)P_1+(0,c,0)P_2+(0,d,0)P_2;\ a,b,c,d\in R\}$ is an AHS-submodule.

Theorem.
Let $3-SP_M$ be a symbolic 3-plithogenic module over $3-SP_R$, let W be an AHS-submodule of $3-SP_M$, then W is a submodule of $3-SP_M$.

Proof.
The proof is similar to the case of 2-plithogenic modules.

Definition.
Let V,W be two modules over the ring R. Let $3-SP_V$, $3-SP_W$ be the corresponding symbolic 3-plithogenic modules over $3-SP_R$.
Let $L_0,L_1,L_2,L_3:V\to W$ be three homomorphisms, we define the AH-homomorphism as follows:
$L:3-SP_V\to 3-SP_W, L=L_0+L_1P_1+L_2P_2+L_3P_3; L(x+yP_1+zP_2+dP_3)=L_0(x)+L_1(y)P_1+L_2(z)P_2+L_3(d)P_3$.
If $L_0=L_1=L_2=L_3$, then L is called AHS-homomorphism.

Definition.
Let $L=L_0+L_1P_1+L_2P_2+L_3P_3:3-SP_V\to 3-SP_W$ be an AH-homomorphism, we define:

1. $AH-ker(L)=ker(L_0)+ker(L_1)P_1+ker(L_2)P_2+ker(L_3)P_3=\{x+yP_1+zP_2+dP_3\}; x\in ker(L_0),y\in ker(L_1),z\in ker(L_2),d\in ker(L_3)$.
2. $AH-Im(L)=Im(L_0)+Im(L_1)P_1+Im(L_2)P_2+Im(L_3)P_3=\{a+bP_1+cP_2+dP_3\}; a\in Im(L_0),b\in Im(L_1),c\in Im(L_2),d\in Im(L_3)$

If L is AHS-homomorphism, then we get $AHS-kernel, AHS-Image$.

Theorem.
Let $L=L_0+L_1P_1+L_2P_2+L_3P_3:3-SP_V\to 3-SP_W$ be an AH-homomorphism, then:
$AH-ker(L)$ is AH-submodule of $3-SP_V$.
$AH-Im(L)$ is AH-submodule of $3-SP_W$.

Example.
Take $V=Z^3$, $W=Z^3$, $L_0,L_1,L_2:V\to W$ such that:
$L_0(x,y,z)=(x,y),L_1(x,y,z)=(2x,z),L_2(x,y,z)=(x-y,y-z)$

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The corresponding AH-homomorphism is:

$L = L_0 + L_1P_1 + L_2P_2 + L_2P_3 : 3 - SP_{Z^3} \to 3 - SP_{Z^2}$:

$$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2 + +(x_3, y_3, z_3)P_3]$$
$$= L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2 + L_2(x_3, y_3, z_3)P_3$$
$$= (x_0, y_0) + (2x_1, z_1)P_1 + (x_2 - y_2, y_2 - z_2)P_2 + (x_3 - y_3, y_3 - z_3)P_3$$

$$\begin{cases} ker(L_0) = \{(0,0,z_0);\ z_0 \in R\} \\ ker(L_1) = \{(0, y_1, 0);\ y_1 \in R\} \\ ker(L_2) = \{(x_2, x_2, x_2);\ x_2 \in R\} \\ AH - ker(L) = \{(0,0,z_0) + (0, y_1, 0)P_1 + (x_2, x_2, x_2)P_2 + (x_3, x_3, x_3)P_3; z_0, y_1, x_2, x_3 \in Z\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0) = R^2 \\ Im(L_1) = R^2 \\ Im(L_2) = R^2 \\ AH - Im(L) = R^2 + R^2P_1 + R^2P_2 + R^2P_3 = 3 - SP_W \end{cases}$$

3. Conclusion

In this paper we have defined the concept of symbolic 3-plithogenic modules over a symbolic 3-plithogenic ring, where we have presented some of their elementary properties such as basis, linear transformations, and AH-submodules. On the other hand, we have suggested many examples to clarify the validity of our work.

References

[1] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[2] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[3] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.

[4] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

[5] Celik, M., and Hatip, A., " On The Refined AH-Isometry And Its Applications In Refined Neutrosophic Surfaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[6] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[7] M. Ibrahim. A. Agboola, B.Badmus and S. Akinleye. On refined Neutrosophic Vector Spaces . International Journal of Neutrosophic Science, Vol. 7, 2020, pp. 97-109.

[8] Von Shtawzen, O., " On A Novel Group Derived From A Generalization Of Integer Exponents and Open Problems", Galoitica journal Of Mathematical Structures and Applications, Vol 1, 2022.

[9] Hatip, A., "An Introduction To Weak Fuzzy Complex Numbers ", Galoitica Journal Of Mathematical Structures and Applications, Vol.3, 2023.

[10] Merkepci, H., and Ahmad, K., " On The Conditions Of Imperfect Neutrosophic Duplets and Imperfect Neutrosophic Triplets", Galoitica Journal Of Mathematical Structures And Applications, Vol.2, 2022.

[11] Von Shtawzen, O., " Conjectures For Invertible Diophantine Equations Of 3-Cyclic and 4-Cyclic Refined Integers", Journal Of Neutrosophic And Fuzzy Systems, Vol.3, 2022.

[12] Abobala, M., and Zeina, M.B., " A Study Of Neutrosophic Real Analysis By Using One Dimensional Geometric AH-Isometry", Galoitica Journal Of Mathematical Structures And Applications, Vol.3, 2023.

[13] F. Smarandache, Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers,

Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers. In *Symbolic NeutrosophicTheory*, Chapter 7, pages 186-193, Europa Nova, Brussels, Belgium, 2015.

[14] F. Smarandache, Plithogeny, Plithogenic Set, Logic, Probability, and Statistics, 141 pages, Pons Editions,Brussels, Belgium, 2017. arXiv.org (Cornell University), Computer Science - Artificial Intelligence, 03Bxx:

[15] Florentin Smarandache, Physical Plithogenic Set, 71st Annual Gaseous Electronics Conference, Session LW1, Oregon Convention Center Room, Portland, Oregon, USA, November 5–9, 2018.

[16] Florentin Smarandache: Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited, *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166.

Doi: https://doi.org/10.54216/GJMSA.060102
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[17] Florentin Smarandache, Plithogenic Algebraic Structures. Chapter in “Nidus idearum Scilogs, V: joining the dots” (third version), Pons Publishing Brussels, pp. 123-125, 2019.

[18] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[19] Khaldi, A., " A Study On Split-Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[20] Ahmad, K., " On Some Split-Complex Diophantine Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[21]. Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", Neoma Journal Of Mathematics and Computer Science, 2023.

[22] A. Alrida Basher, Katy D. Ahmad, Rosina Ali, An Introduction to the Symbolic Turiyam Groups and AH-Substructures,,*Journal of Neutrosophic and Fuzzy Systems*, Vol. 03, No. 02, pp. 43-52, 2022.

[23] Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.

[24] T.Chalapathi and L. Madhavi,. "Neutrosophic Boolean Rings", Neutrosophic Sets and Systems, Vol. 33,pp. 57-66, 2020.

[25] G. Shahzadi, M. Akram and A. B. Saeid, "An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis," *Neutrosophic Sets and Systems,* vol. 18, pp. 80-88, 2017.

[26] Sarkis, M., " On The Solutions Of Fermat's Diophantine Equation In 3-refined Neutrosophic Ring of Integers", Neoma Journal of Mathematics and Computer Science, 2023.

[27] J. Anuradha and V. S, "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka," *Neutrosophic Sets and Systems,* vol. 31, pp. 179-199, 2020.

[28] Celik, M., and Olgun, N., " An Introduction To Neutrosophic Real Banach And Hillbert Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[29] Celik, M., and Olgun, N., " On The Classification Of Neutrosophic Complex Inner Product Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[30] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[31] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[32] Abobala, M., and Hatip, A., "An Algebraic Approach To Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[33] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[34] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011

[35] Taffach, N., " An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol 54, 2023.

[36] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules

Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

[37] Khaldi, A., Ben Othman, K., Von Shtawzen, O., Ali, R., and Mosa, S., " On Some Algorithms for Solving Different Types of Symbolic 2-Plithogenic Algebraic Equations", Neutrosophic Sets and Systems, Vol 54, 2023.

[38] Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.

Doi: https://doi.org/10.54216/GJMSA.060102
Received: December 15, 2022 Revised: April 01, 2023 Accepted: May 01, 2023

[39] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and TheirAlgebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.

[40] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.

[41] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[42] Multu, F., Tercan, A., and Yasar, R., Eventually semisimple weak FI-extending modules, Mathematica BOHEMICA, Vol 148, 2023

[43] Yasar, R., C11-modules via left exact modules preradicals, Turkish journal of mathematics, vol 45, 2021.

[44] Yasar, R., On weak Projection invariant semisimple modules, Fundamental journal of mathematics and applications, 2021.

[45] Yasar, R., and Tercan, A., Extending property on EC-fully submodules, Sakarya university Journal of Science, 2018.

Doi: https://doi.org/10.54216/GJMSA.060102
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