Neutrosophic Crisp Generalized αg-Continuous Functions

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Abstract

This paper provided new concepts of neutrosophic crisp continuous functions named neutrosophic crisp αg-continuous, neutrosophic crisp $g\alpha$-continuous, neutrosophic crisp $g\alpha g$-continuous, neutrosophic crisp $g\alpha g^{*}$-continuous and neutrosophic crisp $g\alpha g^{**}$-continuous functions and their relations.
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1. Introduction

The concept of topological space from the point of view of neutrosophic crisp was presented by Salama et al. [1], and we were denoted simply neutrosophic crisp topological space by Neu_{CTS}. Furthermore, the ideas of α-topological spaces were extended by Salam et al. [2], employing the latter concept and calling them neutrosophic crisp α-topological spaces. Moreover, the explanations of semi-α-closed sets in Neu_{CTS} were established by Al-Hamido et al. [3]. Additionally, the weakly classes of open and closed mappings corresponding to the above view were invented by Al-Obaidi et al. [4,5]. On the other hand, the note of generalized homeomorphism with respect to the neutrosophic concept was founded by PAGE et al. [6]. Subsequently, the generalized alpha generalized continuous function in the neutrosophic proposition, some classes of weakly continuous functions in the Neu_{CTS}, and open and closed sets of separation axioms in Neu_{CTS} were deliberate by Imran et al. [7-10]. The article endeavours to consider new categories of continuous functions in Neu_{CTS} and we name neutrosophic crisp αg-continuous, neutrosophic crisp $g\alpha$-continuous, neutrosophic crisp $g\alpha g$-continuous, neutrosophic crisp $g\alpha g^{*}$-continuous, and neutrosophic crisp $g\alpha g^{**}$-continuous functions and their interaction.

2. Preliminaries

During this article, the Neu_{CTSs} are written as the following pairs $(\mathcal{X},\tau)$, $(\mathcal{Y},\zeta)$ and $(\mathcal{Z},\eta)$ or written simply as $\mathcal{X},\mathcal{Y}$ and $\mathcal{Z}$, correspondingly. To symbolize the neutrosophic crisp closure, interior, and complement of any subset $\mathcal{G}$ of $\mathcal{X}$, we have these symbols $Neu_{C}cl(\mathcal{G})$, $Neu_{C}int(\mathcal{G})$ and $\underline{\mathcal{G}} = \mathcal{X}_{Neu} - \mathcal{G}$, correspondingly.

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Definition 2.1: [1]
An object $\mathcal{G} = \langle \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3 \rangle$ is called a neutrosophic crisp set and is denoted by Neu_CS where $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ are mutually exclusive subsets of non-empty understudy space $\mathcal{X}$.

Definition 2.2: [1]
A neutrosophic crisp topology τ (simply denoted by Neu_{CT}) on a non-empty $\mathcal{X}$ is a collection of Neu_CSs in $\mathcal{X}$ calming the forthcoming conditions:
(i) $\phi_{Neu}, \mathcal{X}_{Neu} \in \tau$,
(ii) $\mathcal{G}_1 \cap \mathcal{G}_2 \in \tau$ whenever $\mathcal{G}_1, \mathcal{G}_2 \in \tau$,
(iii) $\cup \mathcal{G}_k \in \tau$ for any collection $\{\mathcal{G}_k | k \in \Delta\} \subseteq \tau$.
The Neu_{CTS} is given as the pair $(\mathcal{X}, \tau)$ where its element $\mathcal{G}$ is titled a neutrosophic crisp open set and denoted by Neu_COS and its complement $\underline{\mathcal{G}}$ is titled as neutrosophic crisp closed set and denoted by Neu_CCS.

Definition 2.3: [2]
A neutrosophic crisp subset $\mathcal{G}$ of a Neu_{CTS} $(\mathcal{X}, \tau)$ is stated to be a neutrosophic crisp α-open set (in brief $Neu_{C\alpha}$OS) if $\mathcal{G} \subseteq Neu_C int(Neu_C cl(Neu_C int(\mathcal{G})))$ and a neutrosophic crisp α-closed set (in short $Neu_{C\alpha}$CS) if $Neu_C cl(Neu_C int(Neu_C cl(\mathcal{G}))) \subseteq \mathcal{G}$. The neutrosophic crisp α-closure of $\mathcal{G}$ of a Neu_{CTS} $(\mathcal{X}, \tau)$ is the overlapping of the whole $Neu_{C\alpha}$CSs that include $\mathcal{G}$ and it is symbolized by $Neu_{C\alpha} cl(\mathcal{G})$.

Definition 2.4: [12]
Let $\mathcal{G}$ be a neutrosophic crisp subset, and let $\mathcal{M}$ be a Neu_COS in a Neu_{CTS} $(\mathcal{X}, \tau)$ where $\mathcal{G} \subseteq \mathcal{M}$ then $\mathcal{G}$ is called a neutrosophic crisp generalized closed set (in a word, Neu_{Cg}CS) if $Neu_C cl(\mathcal{G}) \subseteq \mathcal{M}$ and the complement of a Neu_{Cg}CS is a Neu_{Cg}OS in $(\mathcal{X}, \tau)$.

Definition 2.5: [12]
A neutrosophic crisp subset $\mathcal{G}$ of Neu_{CTS} $(\mathcal{X}, \tau)$ is said to be:
(i) a neutrosophic crisp αg-closed set (in a word, $Neu_{C\alpha g}$CS) if $Neu_{C\alpha} cl(\mathcal{G}) \subseteq \mathcal{M}$ whenever $\mathcal{G} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a Neu_COS in a $(\mathcal{X}, \tau)$. The complement of a $Neu_{C\alpha g}$CS is a $Neu_{C\alpha g}$OS in $(\mathcal{X}, \tau)$.
(ii) a neutrosophic crisp $g\alpha$-closed set (in a word, $Neu_{Cg\alpha}$CS) if $Neu_{C\alpha} cl(\mathcal{G}) \subseteq \mathcal{M}$ whenever $\mathcal{G} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a $Neu_{C\alpha}$OS in a $(\mathcal{X}, \tau)$. The complement of a $Neu_{Cg\alpha}$CS is a $Neu_{Cg\alpha}$OS in $(\mathcal{X}, \tau)$.

Definition 2.6: [12]
A neutrosophic crisp subset $\mathcal{G}$ of a Neu_{CTS} $(\mathcal{X}, \tau)$ is said to be a neutrosophic crisp $g\alpha g$-closed set (briefly $Neu_{Cg\alpha g}$CS) if $Neu_C cl(\mathcal{G}) \subseteq \mathcal{M}$ whenever $\mathcal{G} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a $Neu_{C\alpha g}$OS in $(\mathcal{X}, \tau)$. The family of all $Neu_{Cg\alpha g}$CSs of a Neu_{CTS} $(\mathcal{X}, \tau)$ is denoted by $Neu_{Cg\alpha g}$C$(\mathcal{X})$. The complement of a $Neu_{Cg\alpha g}$CS is a $Neu_{Cg\alpha g}$OS in $(\mathcal{X}, \tau)$. The family of all $Neu_{Cg\alpha g}$OSs of a Neu_{CTS} $(\mathcal{X}, \tau)$ is denoted by $Neu_{Cg\alpha g}$O$(\mathcal{X})$.

Proposition 2.7: [2,12]
In a Neu_{CTS} $(\mathcal{X}, \tau)$, then the following statements hold and the opposite of each statements are not valid:
(i) Each Neu_COS (resp. Neu_CCS) is a $Neu_{C\alpha}$OS (resp. $Neu_{C\alpha}$CS).
(ii) Each Neu_COS (resp. Neu_CCS) is a Neu_{Cg}OS (resp. Neu_{Cg}CS).
(iii) Each Neu_{Cg}OS (resp. Neu_{Cg}CS) is a $Neu_{C\alpha g}$OS (resp. $Neu_{C\alpha g}$CS).
(iv) Each $Neu_{C\alpha}$OS (resp. $Neu_{C\alpha}$CS) is a $Neu_{Cg\alpha}$OS (resp. $Neu_{Cg\alpha}$CS).
(v) Each $Neu_{Cg\alpha}$OS (resp. $Neu_{Cg\alpha}$CS) is a $Neu_{C\alpha g}$OS (resp. $Neu_{C\alpha g}$CS).
(vi) Each Neu_COS (resp. Neu_CCS) is a $Neu_{Cg\alpha g}$OS (resp. $Neu_{Cg\alpha g}$CS).
(vii) Each $Neu_{Cg\alpha g}$OS (resp. $Neu_{Cg\alpha g}$CS) is a Neu_{Cg}OS (resp. Neu_{Cg}CS).
(viii) Each $Neu_{Cg\alpha g}$OS (resp. $Neu_{Cg\alpha g}$CS) is a $Neu_{Cg\alpha}$OS (resp. $Neu_{Cg\alpha}$CS).

Definition 2.8: [12]
The intersection of all $Neu_{Cg\alpha g}$CSs in a Neu_{CTS} $(\mathcal{X}, \tau)$ containing $\mathcal{G}$ is called neutrosophic crisp $g\alpha g$-closure of $\mathcal{G}$ and is denoted by $Neu_{Cg\alpha g} cl(\mathcal{G})$, $Neu_{Cg\alpha g} cl(\mathcal{G}) = \cap\{\mathcal{H}: \mathcal{G} \subseteq \mathcal{H}, \mathcal{H}$ is a $Neu_{Cg\alpha g}$CS$\}$.

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Definition 2.9: [12]
A Neu_{CTS} $(\mathcal{X},\tau)$ is said to be a neutrosophic crisp $T_{\frac{1}{2}}$-space (briefly $Neu_C T_{\frac{1}{2}}$-space) if every Neu_{Cg}CS in it is a Neu_CCS.

Definition 2.10: [12]
A Neu_{CTS} $(\mathcal{X},\tau)$ is said to be a neutrosophic crisp $T_{g\alpha g}$-space (briefly $Neu_C T_{g\alpha g}$-space) if every $Neu_{Cg\alpha g}$CS in it is a Neu_CCS.

Proposition 2.11: [12]
Each $Neu_C T_{\frac{1}{2}}$-space is a $Neu_C T_{g\alpha g}$-space, but the opposite is not valid in general.

Definition 2.12: [1]
A function $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ is said to be neutrosophic crisp continuous (in short Neu_C-continuous) if $\hbar^{-1}(\mathcal{G})$ is a Neu_CCS (Neu_COS) in $(\mathcal{X},\tau)$ for every Neu_CCS (Neu_COS) $\mathcal{G}$ in $(\mathcal{Y},\zeta)$.

Definition 2.13: [11]
A function $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ is said to be neutrosophic crisp g-continuous (in short Neu_{Cg}-continuous) if $\hbar^{-1}(\mathcal{G})$ is a Neu_{Cg}CS (Neu_{Cg}OS) in $(\mathcal{X},\tau)$ for every Neu_CCS (Neu_COS) $\mathcal{G}$ in $(\mathcal{Y},\zeta)$.

Definition 2.14: [2]
A function $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ is said to be neutrosophic crisp α-continuous (in short $Neu_{C\alpha}$-continuous) if $\hbar^{-1}(\mathcal{G})$ is a $Neu_{C\alpha}$CS ($Neu_{C\alpha}$OS) in $(\mathcal{X},\tau)$ for every Neu_CCS (Neu_COS) $\mathcal{G}$ in $(\mathcal{Y},\zeta)$.

Remark 2.15: [2,11]
(i) Each Neu_C-continuous function is a Neu_{Cg}-continuous, but the opposite is not valid in general.
(ii) Each Neu_C-continuous function is a $Neu_{C\alpha}$-continuous, but the opposite is not valid in general.

3. Neutrosophic Crisp Generalized αg-Continuous Functions

Definition 3.1:
A function $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ is said to be neutrosophic crisp αg-continuous (in short $Neu_{C\alpha g}$-continuous) if $\hbar^{-1}(\mathcal{G})$ is a $Neu_{C\alpha g}$CS ($Neu_{C\alpha g}$OS) in $(\mathcal{X},\tau)$ for every Neu_CCS (Neu_COS) $\mathcal{G}$ in $(\mathcal{Y},\zeta)$.

Definition 3.2:
A function $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ is said to be neutrosophic crisp $g\alpha$-continuous (in short $Neu_{Cg\alpha}$-continuous) if $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha}$CS ($Neu_{Cg\alpha}$OS) in $(\mathcal{X},\tau)$ for every Neu_CCS (Neu_COS) $\mathcal{G}$ in $(\mathcal{Y},\zeta)$.

Definition 3.3:
A function $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ is said to be neutrosophic crisp $g\alpha g$-continuous (in short $Neu_{Cg\alpha g}$-continuous) if $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS ($Neu_{Cg\alpha g}$OS) in $(\mathcal{X},\tau)$ for every Neu_CCS (Neu_COS) $\mathcal{G}$ in $(\mathcal{Y},\zeta)$.

Proposition 3.4:
(i) Each Neu_{Cg}-continuous function is a $Neu_{C\alpha g}$-continuous.
(ii) Each $Neu_{C\alpha}$-continuous function is a $Neu_{Cg\alpha}$-continuous.
(iii) Each $Neu_{Cg\alpha}$-continuous function is a $Neu_{C\alpha g}$-continuous.
Proof:
(i) Let $\hbar: (\mathcal{X},\tau) \longrightarrow (\mathcal{Y},\zeta)$ be a Neu_{Cg}-continuous function and let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y},\zeta)$, since $\hbar$ is a Neu_{Cg}-continuous then $\hbar^{-1}(\mathcal{G})$ is a Neu_{Cg}CS in $(\mathcal{X},\tau)$, which implies $\hbar^{-1}(\mathcal{G})$ is a $Neu_{C\alpha g}$CS in $(\mathcal{X},\tau)$. Hence $\hbar$ is a $Neu_{C\alpha g}$-continuous. The proof is obvious for others. ▪
The opposite of Proposition 3.4 need not be true as shown in the following examples.

Example 3.5:

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Suppose $\mathcal{X} = \{x_1, x_2, x_3\}$ and $\mathcal{Y} = \{y_1, y_2, y_3\}$. Then $\tau = \{\phi_{Neu}, \langle\{x_3\}, \phi, \phi\rangle, \langle\{x_1, x_2\}, \phi, \phi\rangle, \mathcal{X}_{Neu}\}$ and $\zeta = \{\phi_{Neu}, \langle\{y_2\}, \phi, \phi\rangle, \langle\{y_1, y_3\}, \phi, \phi\rangle, \mathcal{Y}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{X}$ and $\mathcal{Y}$, respectively. Define the function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ via $\hbar(\langle\{x_1\}, \phi, \phi\rangle) = \langle\{y_2\}, \phi, \phi\rangle$, $\hbar(\langle\{x_2\}, \phi, \phi\rangle) = \langle\{y_3\}, \phi, \phi\rangle, \hbar(\langle\{x_3\}, \phi, \phi\rangle) = \langle\{y_1\}, \phi, \phi\rangle$. Then $\hbar$ is a $Neu_{C\alpha g}$-continuous function but not Neu_{Cg}-continuous.

Example 3.6:
In the Example (3.5), then $\hbar$ is a $Neu_{Cg\alpha}$-continuous function but not $Neu_{C\alpha}$-continuous.

Example 3.7:
Suppose $\mathcal{X} = \{x_1, x_2, x_3\}$ and $\mathcal{Y} = \{y_1, y_2, y_3\}$. Then $\tau = \{\phi_{Neu}, \langle\{x_1\}, \phi, \phi\rangle, \mathcal{X}_{Neu}\}$ and $\zeta = \{\phi_{Neu}, \langle\{y_2\}, \phi, \phi\rangle, \mathcal{Y}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{X}$ and $\mathcal{Y}$, respectively. Define the function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ via $\hbar(\langle\{x_1\}, \phi, \phi\rangle) = \langle\{y_1\}, \phi, \phi\rangle$, $\hbar(\langle\{x_2\}, \phi, \phi\rangle) = \langle\{y_3\}, \phi, \phi\rangle, \hbar(\langle\{x_3\}, \phi, \phi\rangle) = \langle\{y_2\}, \phi, \phi\rangle$. Then $\hbar$ is a $Neu_{C\alpha g}$-continuous function but not $Neu_{Cg\alpha}$-continuous.

Theorem 3.8:
Let $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a function:
(i) If $(\mathcal{X}, \tau)$ is a $Neu_C T_{\frac{1}{2}}$-space then $\hbar$ is a Neu_{Cg}-continuous iff it is a $Neu_{Cg\alpha g}$-continuous.
(ii) If $(\mathcal{X}, \tau)$ is a $Neu_C T_{g\alpha g}$-space then $\hbar$ is a Neu_C-continuous iff it is a $Neu_{Cg\alpha g}$-continuous.
Proof:
(i) Let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$. Since $\hbar$ is a Neu_{Cg}-continuous, $\hbar^{-1}(\mathcal{G})$ is a Neu_{Cg}CS in $(\mathcal{X}, \tau)$. By $(\mathcal{X}, \tau)$ is a $Neu_C T_{\frac{1}{2}}$-space, which implies $\hbar^{-1}(\mathcal{G})$ is a Neu_CCS. By Proposition (2.7), $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. Hence, $\hbar$ is a $Neu_{Cg\alpha g}$-continuous.
Conversely, suppose that $\hbar$ is a $Neu_{Cg\alpha g}$-continuous. Let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$. Then $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. By Proposition (2.7), $\hbar^{-1}(\mathcal{G})$ is a Neu_{Cg}CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a Neu_{Cg}-continuous.
(ii) Let $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a Neu_C-continuous function and let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$, since $\hbar$ is a Neu_C-continuous then $\hbar^{-1}(\mathcal{G})$ is a Neu_CCS in $(\mathcal{X}, \tau)$, which implies $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a $Neu_{Cg\alpha g}$-continuous.
Conversely, suppose that $\hbar$ is a $Neu_{Cg\alpha g}$-continuous. Let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$. Then $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. By $(\mathcal{X}, \tau)$ is a $Neu_C T_{g\alpha g}$-space, which implies $\hbar^{-1}(\mathcal{G})$ is a Neu_CCS in $(\mathcal{X}, \tau)$. Hence, $\hbar$ is a Neu_C-continuous. ▪

Proposition 3.9:
(i) Each $Neu_{Cg\alpha g}$-continuous function is a $Neu_{C\alpha g}$-continuous.
(ii) Each $Neu_{Cg\alpha g}$-continuous function is a $Neu_{Cg\alpha}$-continuous.
Proof:
(i) Let $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a $Neu_{Cg\alpha g}$-continuous function and let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$, since $\hbar$ is a $Neu_{Cg\alpha g}$-continuous then $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$, which implies $\hbar^{-1}(\mathcal{G})$ is a $Neu_{C\alpha g}$CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a $Neu_{C\alpha g}$-continuous.
(ii) Let $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a $Neu_{Cg\alpha g}$-continuous function and let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$, since $\hbar$ is a $Neu_{Cg\alpha g}$-continuous then $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$, which implies $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha}$CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a $Neu_{Cg\alpha}$-continuous. ▪
The opposite of Proposition 3.9 is not true in general, as illustrated by the example below:

Example 3.10:
Suppose $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ and $\mathcal{Y} = \{y_1, y_2, y_3, y_4\}$. Then $\tau = \{\phi_{Neu}, \langle\{x_1\}, \phi, \phi\rangle, \langle\{x_2, x_4\}, \phi, \phi\rangle, \langle\{x_1, x_2, x_4\}, \phi, \phi\rangle, \mathcal{X}_{Neu}\}$ and $\zeta = \{\phi_{Neu}, \langle\{y_2\}, \phi, \phi\rangle, \langle\{y_1, y_3\}, \phi, \phi\rangle, \langle\{y_1, y_2, y_3\}, \phi, \phi\rangle, \mathcal{Y}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{X}$ and $\mathcal{Y}$, respectively. Define the function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ where $\hbar(\langle\{x_1\}, \phi, \phi\rangle) = \langle\{y_3\}, \phi, \phi\rangle$, $\hbar(\langle\{x_2\}, \phi, \phi\rangle) = \langle\{y_1\}, \phi, \phi\rangle, \hbar(\langle\{x_3\}, \phi, \phi\rangle) = \langle\{y_4\}, \phi, \phi\rangle, \hbar(\langle\{x_4\}, \phi, \phi\rangle) = \langle\{y_2\}, \phi, \phi\rangle$. Then $\hbar$ is a $Neu_{Cg\alpha}$-continuous and $Neu_{C\alpha g}$-continuous but not $Neu_{Cg\alpha g}$-continuous.

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Theorem 3.11:
A function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is $Neu_{Cg\alpha g}$-continuous iff $\hbar(Neu_{Cg\alpha g}cl(\mathcal{G})) \subseteq Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$, for every $\mathcal{G} \subseteq \mathcal{X}$.
Proof:
Let $\hbar$ be $Neu_{Cg\alpha g}$-continuous and $\mathcal{G} \subseteq \mathcal{X}$. Then $\hbar(\mathcal{G}) \subseteq \mathcal{Y}$. Since $\hbar$ is a $Neu_{Cg\alpha g}$-continuous and $Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$ is Neu_CCS in $(\mathcal{Y}, \zeta)$, $\hbar^{-1}(Neu_{Cg\alpha g}cl(\hbar(\mathcal{G})))$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. Since $\hbar(\mathcal{G}) \subseteq Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$, $t^{-1}(\hbar(\mathcal{G})) \subseteq \hbar^{-1}(Neu_{Cg\alpha g}cl(\hbar(\mathcal{G})))$, then $Neu_{Cg\alpha g}cl(\mathcal{G}) \subseteq Neu_{Cg\alpha g}cl(\hbar^{-1}(Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))) = \hbar^{-1}(Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$. Thus $Neu_{Cg\alpha g}cl(\mathcal{G}) \subseteq \hbar^{-1}(Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$. Therefore $\hbar(Neu_{Cg\alpha g}cl(\mathcal{G})) \subseteq Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$, for every $\mathcal{G} \subseteq \mathcal{X}$.
Conversely, let $\hbar(Neu_{Cg\alpha g}cl(\mathcal{G})) \subseteq Neu_{Cg\alpha g}cl(\hbar(\mathcal{G}))$, for every $\mathcal{G} \subseteq \mathcal{X}$. If $\mathcal{H}$ is a Neu_CCS in $(\mathcal{Y}, \zeta)$, since $\hbar^{-1}(\mathcal{H}) \subseteq \mathcal{X}$, $\hbar(Neu_{Cg\alpha g}cl(\hbar^{-1}(\mathcal{H}))) \subseteq Neu_{Cg\alpha g}cl(\hbar(\hbar^{-1}(\mathcal{H}))) = Neu_{Cg\alpha g}cl(\mathcal{H}) = \mathcal{H}$. That is $\hbar(Neu_{Cg\alpha g}cl(\hbar^{-1}(\mathcal{H}))) \subseteq \mathcal{H}$, hence $Neu_{Cg\alpha g}cl(\hbar^{-1}(\mathcal{H})) \subseteq \hbar^{-1}(\mathcal{H})$ but $\hbar^{-1}(\mathcal{H}) \subseteq Neu_{Cg\alpha g}cl(\hbar^{-1}(\mathcal{H}))$. This mean $Neu_{Cg\alpha g}cl(\hbar^{-1}(\mathcal{H})) = \hbar^{-1}(\mathcal{H})$. Therefore $\hbar^{-1}(\mathcal{H})$ is $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a $Neu_{Cg\alpha g}$-continuous. ▪

Definition 3.12:
A function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be neutrosophic crisp $g\alpha g^*$-continuous (in short $Neu_{Cg\alpha g^*}$-continuous) if $\hbar^{-1}(\mathcal{G})$ is a Neu_CCS (Neu_COS) in $(\mathcal{X}, \tau)$ for every $Neu_{Cg\alpha g}$CS ($Neu_{Cg\alpha g}$OS) $\mathcal{G}$ in $(\mathcal{Y}, \zeta)$.

Definition 3.13:
A function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ is said to be neutrosophic crisp $g\alpha g^{**}$-continuous (in short $Neu_{Cg\alpha g^{**}}$-continuous) if $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS ($Neu_{Cg\alpha g}$OS) in $(\mathcal{X}, \tau)$ for every $Neu_{Cg\alpha g}$CS ($Neu_{Cg\alpha g}$OS) $\mathcal{G}$ in $(\mathcal{Y}, \zeta)$.

Proposition 3.14:
(i) Each $Neu_{Cg\alpha g^*}$-continuous function is a $Neu_{Cg\alpha g^{**}}$-continuous.
(ii) Each $Neu_{Cg\alpha g}$-continuous function is a $Neu_{Cg\alpha g^{**}}$-continuous.
Proof:
(i) Let $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a $Neu_{Cg\alpha g^*}$-continuous function and let $\mathcal{G}$ be a $Neu_{Cg\alpha g}$CS in $(\mathcal{Y}, \zeta)$. Since $\hbar$ is a $Neu_{Cg\alpha g^*}$-continuous, then $\hbar^{-1}(\mathcal{G})$ is a Neu_CCS in $(\mathcal{X}, \tau)$, which implies $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a $Neu_{Cg\alpha g^{**}}$-continuous.
(ii) Let $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ be a $Neu_{Cg\alpha g}$-continuous function and let $\mathcal{G}$ be a Neu_CCS in $(\mathcal{Y}, \zeta)$, which implies $\mathcal{G}$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{Y}, \zeta)$. Since $\hbar$ is a $Neu_{Cg\alpha g}$-continuous, then $\hbar^{-1}(\mathcal{G})$ is a $Neu_{Cg\alpha g}$CS in $(\mathcal{X}, \tau)$. Hence $\hbar$ is a $Neu_{Cg\alpha g^{**}}$-continuous. ▪
However, the opposite of Proposition 3.14 is untrue as shown by the following examples.

Example 3.15:
Suppose $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ and $\mathcal{Y} = \{y_1, y_2, y_3, y_4\}$. Then $\tau = \{\phi_{Neu}, \langle\{x_4\}, \phi, \phi\rangle, \langle\{x_1, x_3\}, \phi, \phi\rangle, \langle\{x_1, x_3, x_4\}, \phi, \phi\rangle, \mathcal{X}_{Neu}\}$ and $\zeta = \{\phi_{Neu}, \langle\{y_1\}, \phi, \phi\rangle, \langle\{y_2, y_3\}, \phi, \phi\rangle, \langle\{y_1, y_2, y_3\}, \phi, \phi\rangle, \mathcal{Y}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{X}$ and $\mathcal{Y}$, respectively. Define the function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ as $\hbar(\langle\{x_1\}, \phi, \phi\rangle) = \langle\{y_1\}, \phi, \phi\rangle$, $\hbar(\langle\{x_2\}, \phi, \phi\rangle) = \langle\{y_4\}, \phi, \phi\rangle$, $\hbar(\langle\{x_3\}, \phi, \phi\rangle) = \langle\{y_2\}, \phi, \phi\rangle$, $\hbar(\langle\{x_4\}, \phi, \phi\rangle) = \langle\{y_3\}, \phi, \phi\rangle$. Then $\hbar$ is a $Neu_{Cg\alpha g^{**}}$-continuous but not $Neu_{Cg\alpha g^*}$-continuous.

Example 3.16:
Suppose $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ and $\mathcal{Y} = \{y_1, y_2, y_3, y_4\}$. Then $\tau = \{\phi_{Neu}, \langle\{x_3\}, \phi, \phi\rangle, \langle\{x_1, x_4\}, \phi, \phi\rangle, \langle\{x_1, x_3, x_4\}, \phi, \phi\rangle, \mathcal{X}_{Neu}\}$ and $\zeta = \{\phi_{Neu}, \langle\{y_4\}, \phi, \phi\rangle, \langle\{y_1, y_3\}, \phi, \phi\rangle, \langle\{y_1, y_3, y_4\}, \phi, \phi\rangle, \mathcal{Y}_{Neu}\}$ are Neu_{CTSs} on $\mathcal{X}$ and $\mathcal{Y}$, respectively. Define the function $\hbar: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ as $\hbar(\langle\{x_1\}, \phi, \phi\rangle) = \langle\{y_1\}, \phi, \phi\rangle$, $\hbar(\langle\{x_2\}, \phi, \phi\rangle) = \langle\{y_2\}, \phi, \phi\rangle$, $\hbar(\langle\{x_3\}, \phi, \phi\rangle) = \langle\{y_3\}, \phi, \phi\rangle$, $\hbar(\langle\{x_4\}, \phi, \phi\rangle) = \langle\{y_4\}, \phi, \phi\rangle$. Then $\hbar$ is a $Neu_{Cg\alpha g^{**}}$-continuous but not $Neu_{Cg\alpha g}$-continuous.

Theorem 3.17:
Let $\hbar_1: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Y}, \zeta)$ and $\hbar_2: (\mathcal{Y}, \zeta) \longrightarrow (\mathcal{Z}, \eta)$ be two functions, then:
(i) If $\hbar_1$ and $\hbar_2$ are $Neu_{Cg\alpha g^*}$-continuous, then $\hbar_2 \circ \hbar_1: (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is a $Neu_{Cg\alpha g^*}$-continuous function.

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(ii) If $\hbar_1$ and $\hbar_2$ are $Neu_{Cg\alpha g^{**}}$-continuous, then $\hbar_2 \circ \hbar_1 : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is a $Neu_{Cg\alpha g^{**}}$-continuous function.
(iii) If $\hbar_1$ is a $Neu_{Cg\alpha g^{**}}$-continuous and $\hbar_2$ is a $Neu_{Cg\alpha g^{*}}$-continuous, then $\hbar_2 \circ \hbar_1 : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is a $Neu_{Cg\alpha g^{**}}$-continuous function.
(iv) If $\hbar_1$ is a Neu_C-continuous and $\hbar_2$ is a $Neu_{Cg\alpha g}$-continuous ($Neu_{Cg\alpha g^{*}}$-continuous, $Neu_{Cg\alpha g^{**}}$-continuous), then $\hbar_2 \circ \hbar_1 : (\mathcal{X}, \tau) \longrightarrow (\mathcal{Z}, \eta)$ is a $Neu_{Cg\alpha g}$-continuous ($Neu_{Cg\alpha g^{*}}$-continuous, $Neu_{Cg\alpha g^{**}}$-continuous) function.

Proof:

(i) Let $\mathcal{P} \subseteq \mathcal{Z}$ be a $Neu_{Cg\alpha g}$CS, since $\hbar_2$ is a $Neu_{Cg\alpha g^{*}}$-continuous then ${\hbar_2}^{-1}(\mathcal{P})$ is a Neu_CCS in $\mathcal{Y}$. Since every Neu_CCS is a $Neu_{Cg\alpha g}$CS, therefore ${\hbar_2}^{-1}(\mathcal{P})$ is a $Neu_{Cg\alpha g}$CS in $\mathcal{Y}$. Since $\hbar_1$ is $Neu_{Cg\alpha g^{*}}$-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{P}))$ is a Neu_CCS in $\mathcal{X}$. Thus $(\hbar_2 \circ \hbar_1)^{-1}(\mathcal{P})$ is a Neu_CCS in $\mathcal{X}$. Hence $\hbar_2 \circ \hbar_1$ is a $Neu_{Cg\alpha g^{*}}$-continuous.

(ii) Let $\mathcal{P} \subseteq \mathcal{Z}$ be a $Neu_{Cg\alpha g}$CS, since $\hbar_2$ is a $Neu_{Cg\alpha g^{**}}$-continuous then ${\hbar_2}^{-1}(\mathcal{P})$ is a $Neu_{Cg\alpha g}$CS in $\mathcal{Y}$. Since $\hbar_1$ is $Neu_{Cg\alpha g^{**}}$-continuous, ${\hbar_1}^{-1}({\hbar_2}^{-1}(\mathcal{P}))$ is a $Neu_{Cg\alpha g}$CS in $\mathcal{X}$. Thus $(\hbar_2 \circ \hbar_1)^{-1}(\mathcal{P})$ is a $Neu_{Cg\alpha g}$CS in $\mathcal{X}$. Hence $\hbar_2 \circ \hbar_1$ is a $Neu_{Cg\alpha g^{**}}$-continuous. The proof is obvious for others. ▪

Remark 3.18:

The subsequent illustration indicates the relative among the various kinds of Neu_C-continuous functions:

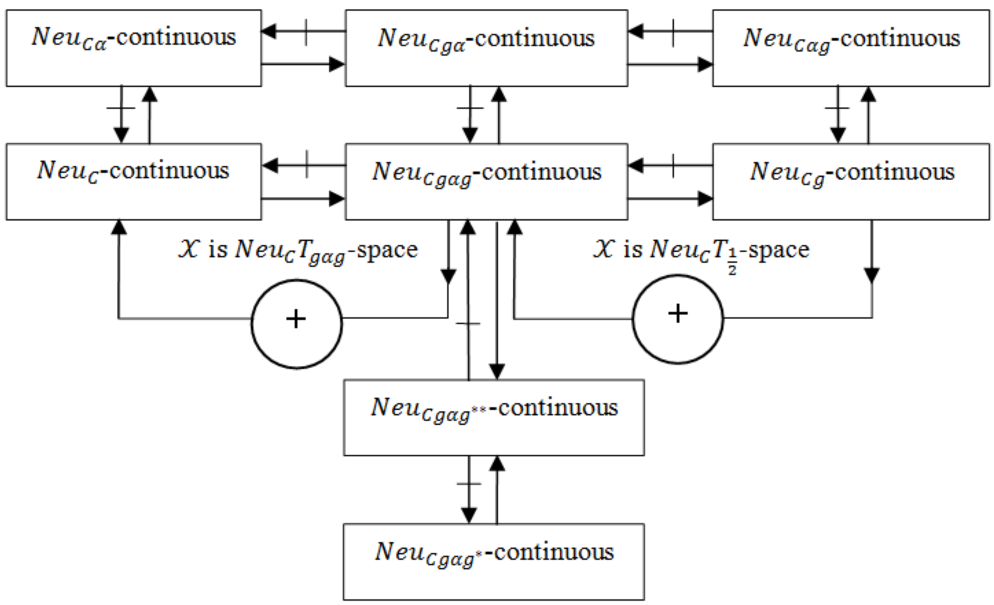

Fig.1

4. Conclusion

To summarize, $Neu_{Cg\alpha g}$-continuous functions were introduced as new mathematical tools that can be realized. We also presented some new notions that are necessary for this study. As a result, we investigated some of these functions characterizations and explained how they relate to other types of functions for instance $Neu_{C\alpha g}$-continuous, $Neu_{Cg\alpha}$-continuous, $Neu_{Cg\alpha g^{*}}$-continuous and $Neu_{Cg\alpha g^{**}}$-continuous functions. We also showed that the opposite is not true in general except under particular conditions by providing a series of examples that illustrate that. In the future, we anticipate that many additional studies will be able to be conducted in the using these concepts from Neu_{CTS}.

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