Modified Approach for Optimization of Real-Life Transportation Problem in Neutrosophic Environment: Suggested Modifications

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Abstract

Singh et al. [1] presented a modified approach for solving transportation problems in neutrosophic environments. They stated that their modified approach for solving transportation problem had addressed the mathematical problems found in Thamaraiselvi and Santhi [2] approaches. But they also stopped to handle the existing problems in Thamaraiselvi and Santhi's approach. After a deep study of Singh et al.'s method, it is observed that Singh et al. have considered several mathematical incorrect assumptions in their proposed method, and hence it is scientifically incorrect to use Singh et al.'s method and Thamaraiselvi and Santhi's method in their present forms. This research aims to make the researchers aware of the mathematical incorrect assumptions, considered by Singh et al. in their proposed method as well as to suggest the required modifications in Singh et al.'s method and Thamaraiselvi and Santhi's method.

Keywords: Transportation problem; neutrosophic set; trapezoidal neutrosophic number.

1. Introduction

Various approaches have been presented in the literature for finding the optimal solution to transportation problems in which all parameters are real numbers [3] [4]. According to real-life situations, these real numbers are not always valid. Since experts may provide their opinion about these parameters in terms of linguistic variables like high, very high, low, and so on, fuzzy numbers and their extensions are presented to deal with such situations.

Thamaraiselvi and Santhi [2] were the first in using neutrosophic numbers for optimizing real-life transportation problems. They presented two types of neutrosophic transportation problems. In the first type, the cost for transporting the unit quantity of the product is presented as a trapezoidal neutrosophic number, whereas availability and demands are represented as real numbers. In the second type of neutrosophic transportation problem, the cost for transporting a unit quantity of the product, product's availability, and demand of the product is represented as trapezoidal neutrosophic numbers. However, after a deep study of these existing

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methods for solving neutrosophic transportation problems, it is observed that a mathematical incorrect assumption has been used in these existing methods.

For modifying existing errors in Thamaraiselvi and Santhi methods, Singh et al. [1] presented a modified approach for solving real-life transportation problems in a neutrosophic environment. However, they also failed in their suggested approach for solving transportation problems, since there exist several incorrect mathematical assumptions in their proposed method. Hence, it is scientifically incorrect to use Singh et al.'s method and also Thamaraiselvi and Santhi's method for solving transportation problems.

Therefore, there is a need to modify these existing methods. In this research, these existing methods are modified and the correct results of existing transportation problems are obtained by the modified methods.

2. Incorrect Mathematical Assumptions Considered in Singh et al.'s Approaches

In this part, the unclear mathematical assumptions of Singh et al.'s methods are pointed out. Thamaraiselvi and Santhi [2] assumed that if we have two trapezoidal neutrosophic numbers $\tilde{a}_1, \tilde{a}_2$ then $S(\tilde{a}+\tilde{b}) = S(\tilde{a}) + S(\tilde{b})$. However, Singh et al. [1] in section 3 indicated that the methods proposed by Thamaraiselvi and Santhi [2] are not valid due to one reason which is $S(\tilde{a}+\tilde{b}) \neq S(\tilde{a}) + S(\tilde{b})$, and constructed their research on this concept. Also, in section 4 Singh et al. [1] illustrated that there is a need for finding the exact relation between $S(\sum_{i=1}^{n} \tilde{a}_i)$ and $\sum_{i=1}^{n} S(\tilde{a}_i)$. They claimed that the exact relation among $S(\sum_{i=1}^{n} \tilde{a}_i)$ and $\sum_{i=1}^{n} S(\tilde{a}_i)$ is obtained as follows: $S(\sum_{i=1}^{n} \tilde{a}_i) =$

$$S((\textstyle\sum_{i=1}^{n} a_{i1}, \sum_{i=1}^{n} a_{i2}, \sum_{i=1}^{n} a_{i3}, \sum_{i=1}^{n} a_{i4}); \underset{1\le i\le n}{min}(\mu_{\tilde{a}}), \underset{1\le i\le n}{max}(\gamma_{\tilde{a}}), \underset{1\le i\le n}{max}(\lambda_{\tilde{a}})) =$$

$$\frac{1}{16}\left(\textstyle\sum_{i=1}^{n} a_{i1} + \sum_{i=1}^{n} a_{i2} + \sum_{i=1}^{n} a_{i3} + \sum_{i=1}^{n} a_{i4}\right)[\underset{1\le i\le n}{min}(\mu_{\tilde{a}}) + (1 - \underset{1\le i\le n}{max}(\gamma_{\tilde{a}})) + (1 - \underset{1\le i\le n}{max}(\lambda_{\tilde{a}}))]. \tag{1}$$

They also used the score function proposed by Thamaraiselvi and Santhi [1] although they objected to it in section 3:

$$S(\tilde{a}_i) = \frac{1}{16}(a_{i1} + a_{i2} + a_{i3} + a_{i4}).[\mu_{\tilde{a}} + (1 - \gamma_{\tilde{a}}) + (1 - \lambda_{\tilde{a}})] \tag{2}$$

From (1) and (2) they concluded that, $S(\sum_{i=1}^{n} \tilde{a}_i) =$

$$(\underset{1\le i\le n}{min}(\mu_{\tilde{a}_i}) + \underset{1\le i\le n}{max}(1 - \gamma_{\tilde{a}_i}) + \underset{1\le i\le n}{max}(1 - \lambda_{\tilde{a}_i}))* \sum_{i=1}^{n}\left(\frac{S(\tilde{a}_i)}{\mu_{\tilde{a}_i} + (1 - \gamma_{\tilde{a}_i}) + (1 - \lambda_{\widetilde{\tilde{a}_i}})}\right) \tag{3}$$

For showing that the previous assumptions and modifications are incorrect let us define a well-known fact,

If $a_1 = a_2 = a_3 = a_4$ then the trapezoidal number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); 1, 0, 0\rangle$ will be transformed into a real number $a = \langle (a, a, a, a); 1, 0, 0\rangle$ and hence in this case $S(a) = a$.

Let us consider the following example for proving existing error in this suggestion of Singh et al. [1],

If we have two trapezoidal neutrosophic numbers $\tilde{a}_1, \tilde{a}_2$ since $\tilde{a}_1 = \langle (1,1,1,1); 1, 0, 0\rangle$, $\tilde{a}_2 = \langle (2,2,2,2); 1, 0, 0\rangle$, and applying Eq. (1) then,

$S(\sum_{i=1}^{n} \tilde{a}_i) = S(\langle (1,1,1,1); 1, 0, 0\rangle + \langle (2,2,2,2); 1, 0, 0\rangle) = S(3,3,3,3,1,0,0) = 3$,according to addition operation and known fact of trapezoidal neutrosophic numbers. Since in Eq. (1) of Singh et al. [1], $S(\sum_{i=1}^{n} \tilde{a}_i) =$

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$= \frac{1}{16}\left(\sum_{i=1}^{n} a_{i1} + \sum_{i=1}^{n} a_{i2} + \sum_{i=1}^{n} a_{i3} + \sum_{i=1}^{n} a_{i4}\right)[\min_{1\le i\le n}(\mu_{\tilde{a}}) + (1 - \max_{1\le i\le n}(\gamma_{\tilde{a}})) + (1 - \max_{1\le i\le n}(\lambda_{\tilde{a}}))] = \frac{1}{16}(3 + 3 + 3 + 3)[1+1+1] = 2.25 \approx 2.$

It appears that $3 \neq 2$,and hence $S(\sum_{i=1}^{n} \tilde{a}_i) \neq \frac{1}{16}\left(\sum_{i=1}^{n} a_{i1} + \sum_{i=1}^{n} a_{i2} + \sum_{i=1}^{n} a_{i3} + \sum_{i=1}^{n} a_{i4}\right)[\min_{1\le i\le n}(\mu_{\tilde{a}}) + (1 - \max_{1\le i\le n}(\gamma_{\tilde{a}})) + (1 - \max_{1\le i\le n}(\lambda_{\tilde{a}}))]$.

Also, they used the score function proposed by Thamaraiselvi and Santhi [2] although it is incorrect as appears in the following example:

If we have one trapezoidal neutrosophic number $\tilde{a}_1$, since $\tilde{a}_1 = \langle(2, 2,2,2\,); 1\,,0, 0\rangle$ and by applying Eq. (2) then,

$S(\tilde{a}_1) = \frac{1}{16}(2 + 2 + 2 + 2).\,[\,1 + (1 - 0) + (1 - 0)] = 1.5$, but according to known fact $S((2, 2,2,2\,); 1\,,0, 0)$ must equal 2. Then $1.5 \neq 2$.

As Singh et al. [1] concluded Eq. (3) from two mathematical incorrect assumptions, then Eq. (3) will also be incorrect as follows:

If we have two trapezoidal neutrosophic numbers $\tilde{a}_1$, $\tilde{a}_2$, since $\tilde{a}_1 = \langle(4, 4,4,4\,); 1\,,0, 0\rangle$ and $\tilde{a}_2 = \langle(0, 0,0,0\,); 1\,,0, 0\rangle$ then,

The value of $S(\sum_{i=1}^{n} \tilde{a}_i) = S(\langle(4, 4,4,4\,); 1\,,0, 0\rangle + \langle(0, 0,0,0\,); 1\,,0, 0\rangle)$, and according to knowing the fact, this value must equal 4.

By considering the right-hand side of Eq.(3) i.e. $(\min_{1\le i\le n}(\mu_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \gamma_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \lambda_{\tilde{a}_i}))* \sum_{i=1}^{n}\left(\frac{S(\tilde{a}_i)}{\mu_{\tilde{a}_i} + (1 - \gamma_{\tilde{a}_i}) + (1 - \lambda_{\widetilde{a_i}})}\right)$, then the value will equal

$$(1 + (1 - 0) + (1 - 0)) * \left[\frac{\frac{1}{16}(4+4+4+4).[\,1+(1-0)+(1-0)]}{(1+(1-0)+(1-0))} + 0\right] = 3.$$

Since $4 \neq 3$ then,

$$S(\sum_{i=1}^{n} \tilde{a}_i) \neq (\min_{1\le i\le n}(\mu_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \gamma_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \lambda_{\tilde{a}_i}))* \sum_{i=1}^{n}\left(\frac{S(\tilde{a}_i)}{\mu_{\tilde{a}_i} + (1 - \gamma_{\tilde{a}_i}) + (1 - \lambda_{\widetilde{a_i}})}\right)$$

Let us consider another example,

If we have two trapezoidal neutrosophic numbers $\tilde{a}_1$, $\tilde{a}_2$ since $\tilde{a}_1 = \langle(2, 2,2,2\,); 1\,,0, 0\rangle$, $\tilde{a}_2 = \langle(5, 5,5,5\,); 1\,,0, 0\rangle$ then the value of $S(\sum_{i=1}^{n} \tilde{a}_i) = S(\langle(2, 2,2,2\,); 1\,,0, 0\rangle + \langle(5, 5,5,5\,); 1\,,0, 0\rangle) = S(\langle(7, 7,7,7\,); 1\,,0, 0\rangle)$, and according to the known fact, this value must equal 7.

By considering the right-hand side i.e. $(\min_{1\le i\le n}(\mu_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \gamma_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \lambda_{\tilde{a}_i}))* \sum_{i=1}^{n}\left(\frac{S(\tilde{a}_i)}{\mu_{\tilde{a}_i} + (1 - \gamma_{\tilde{a}_i}) + (1 - \lambda_{\widetilde{a_i}})}\right)$, then the value will equal

$$(1 + (1 - 0) + (1 - 0)) * \left(\frac{\frac{1}{16}(2+2+2+2).[\,1+(1-0)+(1-0)]}{(1+(1-0)+(1-0))} + \frac{\frac{1}{16}(5+5+5+5).[\,1+(1-0)+(1-0)]}{(1+(1-0)+(1-0))}\right) = 5.25.$$

Since $7 \neq 5.25$ then, $S(\sum_{i=1}^{n} \tilde{a}_i) \neq$

$$(\min_{1\le i\le n}(\mu_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \gamma_{\tilde{a}_i}) + \max_{1\le i\le n}(1 - \lambda_{\tilde{a}_i}))* \sum_{i=1}^{n}\left(\frac{S(\tilde{a}_i)}{\mu_{\tilde{a}_i} + (1 - \gamma_{\tilde{a}_i}) + (1 - \lambda_{\widetilde{a_i}})}\right).$$

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Also, let us solve the same example which was solved by Singh et al. [1] for proving that Thamaraiselvi and Santhi's methods [2] are not valid.

Let $\tilde{a}_1=\langle(2,4,6,8\ );0.1\ ,0.2,0.3\rangle$, $\tilde{a}_2=\langle(3,6,9,12\ );0.4\ ,0.5,0.6\rangle$ be two trapezoidal neutrosophic numbers and then $\tilde{a}_1+\tilde{a}_2=$ $\langle(2,4,6,8\ );0.1\ ,0.2,0.3\rangle+\langle(3,6,9,12\ );0.4\ ,0.5,0.6\rangle=\langle(5,10,15,20\ );0.1\ ,0.5,0.6\rangle$. By using Eq. (1) then

$$S(\tilde{a}_1+\tilde{a}_2)=\frac{1}{16}\,(5+10+15+20)*[0.1+0.5+0.4]\ =3.125.$$

Since $S(\sum_{i=1}^{n}\tilde{a}_i)=$

$\left(\min\limits_{1\le i\le n}(\mu_{\tilde{a}_i})+\max\limits_{1\le i\le n}(1-\gamma_{\tilde{a}_i})+\max\limits_{1\le i\le n}(1-\lambda_{\tilde{a}_i})\right)*\sum_{i=1}^{n}(\frac{S(\tilde{a}_i)}{\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\widetilde{\tilde{a}_i}})})$ according to Singh et al. [1] modifications then,

$$3.125\neq(0.1+0.8+0.7)*\frac{\frac{1}{16}(2+4+6+8)*[0.1+0.8+0.7]}{[0.1+0.8+0.7]}+\frac{\frac{1}{16}(3+6+9+12)*[0.4+0.5+0.4]}{0.4+0.5+0.4}$$

$$3.125\neq 5.$$

Hence, the suggested modifications proposed also by Singh et al. [2] for solving transportation problems are not valid.

3. Suggested Modifications and Correct Relations

In this section the suggested modifications and correct relations are presented. Singh et al. [1] claimed that the correct way for modifying the incorrect mathematical assumption of Thamaraiselvi and Santhi's method [2] is to find the exact relation among $S(\sum_{i=1}^{n}\tilde{a}_i)$ and $\sum_{i=1}^{n}S(\tilde{a}_i)$. However, they also failed in finding the exact relation among $S(\sum_{i=1}^{n}\tilde{a}_i)$ and $\sum_{i=1}^{n}S(\tilde{a}_i)$.

Therefore, in this part the exact relation among $S(\sum_{i=1}^{n}\tilde{a}_i)$ and $\sum_{i=1}^{n}S(\tilde{a}_i)$ is obtained.

Let $\tilde{a}_i=\langle(a_{i1},a_{i2},a_{i3},a_{i4});\mu_{\tilde{a}_i},\gamma_{\tilde{a}_i},\lambda_{\tilde{a}_i}\rangle$ be trapezoidal neutrosophic number then,

$$S(\tilde{a}_i)=\frac{1}{12}(a_{i1}+\ a_{i2}+\ a_{i3}+a_{i4})*\ [\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})] \tag{4}$$

Since $\sum_{i=1}^{n}\tilde{a}_i\ =\langle(\sum_{i=1}^{n}a_{i1}\,,\sum_{i=1}^{n}a_{i2}\,,\sum_{i=1}^{n}a_{i3}\,,\sum_{i=1}^{n}a_{i4});\ \min\limits_{1\le i\le n}(\mu_{\tilde{a}_i})+\max\limits_{1\le i\le n}(\gamma_{\tilde{a}_i})+\max\limits_{1\le i\le n}(\lambda_{\tilde{a}_i})\rangle$ then,

$$S(\sum_{i=1}^{n}\tilde{a}_i)=S\,\langle(\sum_{i=1}^{n}a_{i1}\,,\sum_{i=1}^{n}a_{i2}\,,\sum_{i=1}^{n}a_{i3}\,,\sum_{i=1}^{n}a_{i4});\ \min\limits_{1\le i\le n}(\mu_{\tilde{a}_i}),\max\limits_{1\le i\le n}(\gamma_{\tilde{a}_i}),\max\limits_{1\le i\le n}(\lambda_{\tilde{a}_i})\rangle\ =\ \frac{1}{12}\,(\ \sum_{i=1}^{n}a_{i1}+\sum_{i=1}^{n}a_{i2}+\sum_{i=1}^{n}a_{i3}+\sum_{i=1}^{n}a_{i4})*\ [\min\limits_{1\le i\le n}(\mu_{\tilde{a}_i})+\left(1-\max\limits_{1\le i\le n}(\gamma_{\tilde{a}_i})\right)+(1-\max\limits_{1\le i\le n}(\lambda_{\tilde{a}_i}))]. \tag{5}$$

From (4) and (5), $S(\sum_{i=1}^{n}\tilde{a}_i)=$

$$(\min\limits_{1\le i\le n}(\mu_{\tilde{a}_i})+\left(1-\max\limits_{1\le i\le n}(\gamma_{\tilde{a}_i})\right)+\left(1-\max\limits_{1\le i\le n}(\lambda_{\tilde{a}_i})\right)*\ \sum_{i=1}^{n}\frac{\frac{1}{12}(a_{i1}+a_{i2}+a_{i3}+a_{i4})*[\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}{\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})} \tag{6}$$

For verifying suggested modifications let us consider the following examples:

Let $\tilde{a}_1=\langle(2,2,2,2\ );1\ ,0,0\rangle$, $\tilde{a}_2=\langle(5,5,5,5\ );1\ ,0,0\rangle$ be two trapezoidal neutrosophic numbers and then,

$\tilde{a}_1+\tilde{a}_2=\langle(2,2,2,2\ );1\ ,0,0\rangle+\langle(5,5,5,5\ );1\ ,0,0\rangle\ =\langle(7,7,7,7\ );1\ ,0,0\rangle.$

Furthermore,

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$S(\tilde{a}_1+\tilde{a}_2)=S\langle(7,7,7,7);1,0,0\rangle=\frac{1}{12}(7+7+7+7)*[1+1+1]=7$, which also corresponds to the previously defined fact "If $a_1=a_2=a_3=a_4$ then the trapezoidal number $\tilde{a}=\langle(a_1,a_2,a_3,a_4);1,0,0\rangle$ will be transformed into a real number $a=\langle(a,a,a,a);1,0,0\rangle$ and hence in this case $S(a)=a$".

Since $S(\sum_{i=1}^{n}\tilde{a}_i)=\min_{1\le i\le n}(\mu_{\tilde{a}_i})+\left(1-\max_{1\le i\le n}(\gamma_{\tilde{a}_i})\right)+\left(1-\max_{1\le i\le n}(\lambda_{\tilde{a}_i})\right)*\sum_{i=1}^{n}\frac{\frac{1}{12}(a_{i1}+a_{i2}+a_{i3}+a_{i4})*[\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}{\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}$ then,

$$S(\sum_{i=1}^{n}\tilde{a}_i)=3*(\left\{\frac{\frac{1}{12}(2+2+2+2)*[1+1+1]}{[1+1+1]}\right\}+\left\{\frac{\frac{1}{12}(5+5+5+5)*[1+1+1]}{[1+1+1]}\right\})=7.$$

It is obvious that, $S(\sum_{i=1}^{n}\tilde{a}_i)=\min_{1\le i\le n}(\mu_{\tilde{a}_i})+\left(1-\max_{1\le i\le n}(\gamma_{\tilde{a}_i})\right)+\left(1-\max_{1\le i\le n}(\lambda_{\tilde{a}_i})\right)*\sum_{i=1}^{n}\frac{\frac{1}{12}(a_{i1}+a_{i2}+a_{i3}+a_{i4})*[\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}{\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}$,

Let us consider the same example presented in [1] and apply our suggested function,

Let $\tilde{a}_1=\langle(2,4,6,8);0.1,0.2,0.3\rangle$, $\tilde{a}_2=\langle(3,6,9,12);0.4,0.5,0.6\rangle$ be two trapezoidal neutrosophic numbers and then,

$\tilde{a}_1+\tilde{a}_2=\langle(2,4,6,8);0.1,0.2,0.3\rangle+\langle(3,6,9,12);0.4,0.5,0.6\rangle=\langle(5,10,15,20);0.1,0.5,0.6\rangle$.

Furthermore,

$$S(\langle(5,10,15,20);0.1,0.5,0.6\rangle)=\frac{1}{12}(5+10+15+20)*[0.1+0.5+0.4]=4.166.$$

Also, $\min_{1\le i\le n}(\mu_{\tilde{a}_i})+\left(1-\max_{1\le i\le n}(\gamma_{\tilde{a}_i})\right)+\left(1-\max_{1\le i\le n}(\lambda_{\tilde{a}_i})\right)*\sum_{i=1}^{n}\frac{\frac{1}{12}(a_{i1}+a_{i2}+a_{i3}+a_{i4})*[\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}{\mu_{\tilde{a}_i}+\left(1-\gamma_{\tilde{a}_i}\right)+(1-\lambda_{\tilde{a}_i})}=$

$$1*\left\{\frac{\frac{1}{12}(2+4+6+8)*[0.1+0.8+0.7]}{[0.1+0.8+0.7]}\right\}+\left\{\frac{\frac{1}{12}(3+6+9+12)*[0.4+0.5+0.4]}{[0.4+0.5+0.4]}\right\}=4.166.$$

4. Modified Approach for Solving Transportation Problem

It is obvious from Section 2 that several incorrect mathematical assumptions have been considered in Singh et al.'s approach [1]. Consequently, it is scientifically incorrect to use Singh et al.'s approach in its present form.

In this part, the required modifications of Singh et al.'s approach [1] are suggested.

4.1. Type I of Neutrosophic Transportation Problem

In the first type of transportation problem introduced in the neutrosophic environment, the decision-maker is indeterminate about transportation cost from source to destination. In this type, there is no uncertainty about the demand and supply of the product. The mathematical formulation of this type is as follows:

Minimize $\tilde{z}=\sum_{i=0}^{m}\sum_{j=0}^{n}x_{ij}\,\tilde{c}_{ij}$

Subject to

$$\sum_{j=0}^{n}x_{ij}=a_i\ ,\qquad i=1,2,\dots,m$$

$$\sum_{i=0}^{m}x_{ij}=b_j\ ,\qquad j=1,2,\dots,n, \tag{7}$$

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$x_{ij} \geq 0.$

Where x_{ij} is the number of units which transported from i^{th}source to j^{th} destination.

$\tilde{c}_{ij}$ is the cost of one unit quantity of product which transported from i^{th}source to j^{th} destination.

x_i is the total availability of product at the source.

x_j is the total demand for the product at the destination.

4.1.1. Modified Approach for Finding Neutrosophic Solution for Type I

The steps for finding the solution for type I are as follows:

Step 1. Use the obtained relation in section 3 for transforming the neutrosophic transportation problem into its equivalent crisp form.

Step 2. Represent crisp form of transportation problem into tabular form.

Step 3. Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell cost in the same row or column.

Step 4. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.

Step 5. Repeat the previous steps until all demands are satisfied and suppliers are fully exhausted.

Step 6. Once an initial basic feasible solution has been determined, the next step is to solve the model for the optimal solution.

Step 7. Finally, determine the total minimum neutrosophic transportation cost through compensation in $\sum_{i=0}^{m}\sum_{j=0}^{n} x_{ij}\, \tilde{c}_{ij}$.

4.2. Type II of Neutrosophic Transportation Problem

In the second type of transportation problem, the decision-maker will be indeterminate about supply, demand units, and transportation costs.

The mathematical formulation of this model is as follows:

Minimize $\tilde{z} = \sum_{i=0}^{m}\sum_{j=0}^{n} \tilde{x}_{ij}\, \tilde{c}_{ij}$

Subject to

$$\sum_{j=0}^{n} \tilde{x}_{ij} = \tilde{a}_i \ , \qquad i = 1,2,\dots,m$$
$$\sum_{i=0}^{m} \tilde{x}_{ij} = \tilde{b}_j \quad j = 1,2,\dots,n, \tag{8}$$
$$\tilde{x}_{ij} \geq 0.$$

4.2.1. Modified Approach for Finding Neutrosophic Solution for Type II

For solving this type of transportation problem does the following:

Step 1. Use the obtained relation in section 3 for transforming the neutrosophic cost of transportation problem into its equivalent crisp value.

Step 2. Keep neutrosophic values of demand and supply units as the same. For modifying neutrosophic values of demand and supply units in each iteration, subtraction of single-valued trapezoidal neutrosophic numbers is applied.

Step 3. Calculate penalty by finding the absolute difference between the minimum and next to a minimum of transportation costs for each row and column.

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Step 4. Correspond to the largest penalty make the maximum allotment in the cell with the minimum cost of transportation.

Step 5. Repeat the previous steps until all demands are satisfied and suppliers are fully exhausted.

Step 6. Once an initial basic feasible solution has been determined, the next step is to solve the model for the optimal solution.

Step 7. Finally, determine total minimum neutrosophic transportation cost through compensation in $\sum_{i=0}^{m}\sum_{j=0}^{n} x_{ij}\,\tilde{c}_{ij}$.

5. Correct Solutions of Numerical Examples

Since Singh et al. [1] failed in finding the correct modifications of Thamaraiselvi and Santhi's method [2], then the solutions obtained by Singh et al. [1] are also not exact. In this part, the correct solutions to these examples are obtained by the suggested modified method.

5.1. Correct Solution of Neutrosophic Transportation Problem of Type I

The input data of the neutrosophic transportation problem of Type I presented in Table 1.

Table 1: Input data for the neutrosophic transportation problem.

	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	$\boldsymbol{D_3}$	$\boldsymbol{D_4}$
$\boldsymbol{Q_1}$	$\mathbf{(3,5,6,8);0.6,0.5,0.4}$	$\mathbf{(5,8,10,14);0.3,0.6,0.6}$	$\mathbf{(12,15,19,22);0.6,0.4,0.5}$	$\mathbf{(14,17,21,28);0.8,0.}$
$\boldsymbol{Q_2}$	$\mathbf{(0,1,3,6);0.7,0.5,0.3}$	$\mathbf{(5,7,9,11);0.9,0.7.0.5}$	$\mathbf{(15,17,19,22);0.4,0.8,0.4}$	$\mathbf{(9,11,14,16);0.5,0.4}$
$\boldsymbol{Q_3}$	$\mathbf{(4,8,11,15);0.6,0.3,0.2}$	$\mathbf{(1,3,4,6);0.6,0.3,0.5}$	$\mathbf{(5,7,8,10);0.5,0.4,0.7}$	$\mathbf{(5,9,14,19);0.3,0.7,}$
Demand	**17**	**23**	**28**	**12**

5.1.1. Correct Solution of Neutrosophic Initial Basic Feasible Solution for Type I

After applying the modified approach, the crisp form of transportation problem will be as in Table 2.

Table 2: Crisp form of transportation problem.

	D_1	D_2	D_3	D_4	Supply
Q_1	1	2	4	5	26
Q_2	1	2	5	3	24
Q_3	2	1	2	3	30
Demand	17	23	28	12	

The penalty for each row and column presented in Table 3.

Table 3: Tabular representation with penalties.

	D_1	D_2	D_3	D_4	Supply	Penalty
Q_1	1	2	4	5	26	1

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Q_2	1	2	5	3	24	1
Q_3	2	1	2	3	30	1
Demand	17	23	28	12		
Penalty	0	1	2*	0		

The first allotment with penalties appears in Table 4.

Table 4: The first allotment with penalties.

	D_1	D_2	D_3	D_4	Supply	Penalty
Q_1	1	2	–	5	26	1
Q_2	1	2	–	3	24	1
Q_3	2	1	28	3	2	1
Demand	17	23	0	12		
Penalty	0	1	–	0		

After many iterations, we get the complete allocation of transportation units as in Table 5.

Table 5: The complete allocation of the problem.

	D_1	D_2	D_3	D_4	Supply
Q_1	17	9	–	–	–
Q_2	–	12	–	12	–
Q_3	–	2	28	–	–
Demand	–	–	–	–	

The initial basic feasible solution is,

$x_{11} = 17$,

$x_{12} = 9$,

$x_{22} = 12$,

$x_{24} = 12$,

$x_{32} = 2$,

$x_{33} = 28$.

Hence the minimum total neutrosophic cost is,

Minimize $\tilde{z} = \sum_{i=0}^{3} \sum_{j=0}^{4} x_{ij}\, \tilde{c}_{ij}$

$= \langle 17 * (3,5,6,8); 0.6,0.5,0.4 \rangle + \langle 9 * (5,8,10,14); 0.3,0.6,0.6 \rangle +$

$\langle 12 * (5,7,9,11); 0.9,0.7.0.5 \rangle + \langle 12 * (9,11,14,16); 0.5,0.4,0.7 \rangle +$

$\langle 2 * (1,3,4,6); 0.6,0.3,0.5 \rangle + \langle 28 * (5,7,8,10); 0.5,0.4,0.7 \rangle$

$= \langle (406,575,700,878); 0.3,0.7,0.7 \rangle$.

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5.1.2. Correct Solution of Neutrosophic optimal Solution for Type I

After applying the modified approach, the crisp form of transportation problem will be as in Table 2. The steps for calculating optimal solution presented with detail in [2,6] and started from step 2, since in step 1 we utilized the modified relation presented in section 3 for converting neutrosophic cost value into crisp value. After choosing the smallest value in each row and subtract it from the corresponding row entries, and doing the approach for each column, we obtain Table 6. Since Table 6 doesn't satisfy the optimal solution condition, then we have to draw the minimum number of horizontal and vertical lines that cover all zeros and revise the reduced table via finding the smallest element from uncovered entries, subtract it from all uncovered entries and add it to the entries at the intersection of any two lines. The modified table with zero point presented in Table 7.

Table 6: Table with zero point.

	D_1	D_2	D_3	D_4	Supply
Q_1	0	1	2	2	26
Q_2	0	1	3	0	24
Q_3	1	0	0	0	30
Demand	17	23	28	12	

Table 7: Modified table with zero point.

	D_1	D_2	D_3	D_4	Supply
Q_1	0	0	1	2	26
Q_2	0	0	2	0	24
Q_3	2	0	0	1	30
Demand	17	23	28	12	

The schedule with complete allocation presented in Table 8.

Table 8: Table with complete allocation.

	D_1	D_2	D_3	D_4	Supply
Q_1	17	9	–	–	–
Q_2	–	12	–	12	–
Q_3	–	2	28	–	–
Demand	–	–	–	–	

The optimal solution is:

$x_{11} = 17,$

$x_{12} = 9,$

$x_{22} = 12,$

$x_{24} = 12,$

$x_{32} = 2,$

$x_{33} = 28.$

Using the optimal solution, the minimum total neutrosophic is,

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$$\text{Minimize } \tilde{z} = \sum_{i=0}^{3}\sum_{j=0}^{4} x_{ij}\,\tilde{c}_{ij} =$$

$$= \langle 17 * (3,5,6,8); 0.6,0.5,0.4\rangle + \langle 9 * (5,8,10,14); 0.3,0.6,0.6\rangle +$$

$$\langle 12 * (5,7,9,11); 0.9,0.7.0.5\rangle + \langle 12 * (9,11,14,16); 0.5,0.4,0.7\rangle +$$

$$\langle 2 * (1,3,4,6); 0.6,0.3,0.5\rangle + \langle 28 * (5,7,8,10); 0.5,0.4,0.7\rangle = \langle (406,575,700,878); 0.3,0.7,0.7\rangle.$$

5.2. Correct Solution of Neutrosophic Transportation Problem of Type II

The neutrosophic transportation problem of type II presented in Table 9 as follows:

Table 9: Input data for type II of neutrosophic transportation problem.

	D_1	D_2	D_3	D_4	Supply
Q_1	(3,5,6,8); 0.6,0.5,0.	(5,8,10,14); 0.3,0.6,	(12,15,19,22); 0.6,	(14,17,21,28); 0.8,	(22, 26, 28, 32); 0.6,0.5,0.5
Q_2	(0,1,3,6); 0.7,0.5,0.	(5,7,9,11); 0.9,0.7.0	(15,17,19,22); 0.4,	(9,11,14,16); 0.5,0.	(17, 22, 27, 31); 0.6,0.4,0.5
Q_3	(4,8,11,15); 0.6,0.3,	(1,3,4,6); 0.6,0.3,0.	(5,7,8,10); 0.5,0.4,0	(5,9,14,19); 0.3,0.7,	(21, 28, 32, 37); 0.8,0.5,0.4
Demand	(13,16,18,22); 0.6,	(17,21,24,28); 0.6,	(24,29,32,35); 0.9,	(6,10,13,15); 0.7,0.	

The neutrosophic transportation problem with crisp cost is presented in Table 10.

Table 10: Neutrosophic transportation problem with crisp cost.

	D_1	D_2	D_3	D_4	Supply
Q_1	1	2	4	5	(22, 26, 28, 32); 0.6,0.5,0.5
Q_2	1	2	5	3	(17, 22, 27, 31); 0.6,0.4,0.5
Q_3	2	1	2	3	(21, 28, 32, 37); 0.8,0.5,0.4
Demand	(13,16,18,22); 0.	(17,21,24,28); 0.	(24,29,32,35); 0.	(6,10,13,15); 0.7,	

By applying the modified approach to the neutrosophic transportation problem of type II, the table with a complete allocation is presented in Table 11.

Table 11: Table with complete allocation for type II of neutrosophic transportation problem.

	D_1	D_2	D_3	D_4	Supply
Q_1	(13,16,18,22);	(9,10,10,10); 0.	−	−	−
Q_2	−	(11,12,14,16);	−	(6,10,13,15); 0.	−

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Q_3	$-$	$(-3,-1,0,2); 0.$	$(24,29,32,35);$	$-$	$-$
Demand	$-$	$-$	$-$	$-$	

From Table 11 the optimal solution will be as follows:

$\tilde{x}_{11} = (13,16,18,22); 0.6,0.5,0.5,$

$\tilde{x}_{12} = (9,10,10,10); 0.6,0.5,0.5,$

$\tilde{x}_{22} = (11,12,14,16); 0.6,0.4,0.5,$

$\tilde{x}_{24} = (6,10,13,15); 0.7,0.3,0.4,$

$\tilde{x}_{32} = (-3,-1,0,2); 0.8,0.5,0.4,$

$\tilde{x}_{33} = (24,29,32,35); 0.9,0.5,0.3.$

Using the optimal solution, the minimum total neutrosophic cost is as follows:

Minimize $\tilde{z} = \sum_{i=0}^{3} \sum_{j=0}^{4} \tilde{x}_{ij}\, \tilde{c}_{ij} =$

$$\langle (13,16,18,22); 0.6,0.5,0.5 \rangle * \langle (3,5,6,8); 0.6,0.5,0.4 \rangle$$
$$+ \ (\langle (9,10,10,10); 0.6,0.5,0.5 \rangle * \langle (5,8,10,14); 0.3,0.6,0.6 \rangle)$$
$$+(\langle (11,12,14,16); 0.6,0.4,0.5 \rangle * \langle (5,7,9,11); 0.9,0.7.0.5 \rangle)$$
$$+ \ (\langle (6,10,13,15);\ 0.7,0.3,0.4 \rangle * \langle (9,11,14,16); 0.5,0.4,0.7 \rangle)$$
$$+ \ (\langle (-3,-1,0,2); 0.8,0.5,0.4 \rangle * \langle (1,3,4,6); 0.6,0.3,0.5 \rangle)$$
$$+ \ (\langle (24,29,32,35); 0.9,0.5,0.3 \rangle * \langle (5,7,8,10); 0.5,0.4,0.7 \rangle)$$
$$= \ (310,554,772,1094); 0.3,0.7,0.7$$

By looking at the optimal solution obtained by Singh et al. [1] as in Table 12, we note a great error that contradicts the allocation process of the transportation problem. As we know that all supply units must be fully exhausted, and all demands must be satisfied. However, if we look at Q_3D_4 and Q_2D_4 which must take total quantity equal $(6,10,13,15); 0.7,0.3,0.4$, we note that the summation of $Q_2\ D_4$ and $Q_3\ D_4$ will equal$(-24,2,21,42); 0.5,0.5,0.6$. The same problem appears also in demand units of D_1 .

This means that Singh et al. [1] failed not only in finding the correct relation among $S(\sum_{i=1}^{n} \tilde{a}_i)$ and $\sum_{i=1}^{n} S(\tilde{a}_i)$, but also in knowing the basic concepts of the transportation problem.

Table 12: Table with complete allocation for type II of neutrosophic transportation problem according to Singh et al.[1].

	D_1	D_2	D_3	D_4	Supply
Q_1	$(-6,2,7,11); 0.7,0$	$(17,21,24,28); 0.$			$-$
Q_2	$(2,9,16,27); 0.5,0.$			$(-10,6,18,29); 0.$	$-$
Q_3			$(24,29,32,35); 0.$	$(-14,-4,3,13); 0$	$-$
Demand	$-$	$-$	$-$	$-$	

6. Conclusions and Future Directions

Since Singh et al. [1] pointed out that Thamaraiselvi and Santhi [2] used incorrect mathematical assumptions in their approaches, they suggested a modified approach for solving the neutrosophic transportation problem.

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However, they failed also in handling incorrect assumptions of Thamaraiselvi and Santhi's method. Therefore, the required modifications are suggested in this research. Also, the correct results of the two types of neutrosophic transportation problems are presented.

In the future we tend to use cut sets and indeterminacy for presenting a new parametric index for solving transportation problems and various types of neutrosophic optimization problems [5].

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