Pseudo Similarity of Neutrosophic Fuzzy matrices

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Abstract

In this paper, first we shall define Pseudo Similarity for Neutrosophic Fuzzy Matrices and prove that Pseudo Similarity relation on pair of Neutrosophic Fuzzy Matrices. Also, we derive some relation between Pseudo Similarity and Idempotent matrices. Finally, we give in varies inverse of Neutrosophic Fuzzy Matrices.

Keywords: Neutrosophic Fuzzy Matrices (NFMs); G – inverse; Idempotent NFMs; Pseudo Similarity NFMs.

1. Introduction

It is well known that generalized inverses exist for a complex matrix. However, this is the failure for fuzzy matrices, that is for $P \in F_{mn}$ under the max-min fuzzy operations the matrix equation PXP = P need not have a solution X. If P has a generalized inverse (g-inverse) then P is said to be regular. The concept of generalized inverse presents a very interesting area of research in matrix theory in the same way a regular matrix as one of which g-inverse exists, lays the foundation for research in fuzzy matrix theory.

Matrices play a prime role in several areas of science and engineering to represent any binary operation. Such as unpredictability are generally reality handled with the help of the topics like probability, fuzzy set, etc., Zadeh in 1965 [1] introduced concepts of fuzzy sets. Thomson [2] has developed fuzzy matrices and the convergence of the powers of fuzzy matrices. The theory of intuitionistic fuzzy sets as a generalization of fuzzy sets was initiated by Atanassov [3]. Pradhan and Pal [4] have discussed Some Results on Generalized Inverse of Intuitionistic Fuzzy Matrices. Pradhan and Pal [5] have Convergence of max arithmetic mean – min arithmetic mean powers of intuitionistic fuzzy matrices. Also, they found the interval–valued fuzzy matrices as a generalization of the fuzzy matrices was introduced and developed by Shyamal and Pal [6] established max–min operation in fuzzy algebra. Bhowmik, Pal [7] introduced some results of Intuitionistic fuzzy matrices. Meenakshi and Inbam [8] described about Minus Partial Order in fuzzy matrices. Theory of Moore – Penrose Inverse of Intuitionistic Fuzzy Matrices was investigated by Sriram and Murugadas [9]. The extension of intuitionistic fuzzy sets is neutrosophic sets which was introduced by Smarandache [10]. Uma et.al.[11] extend the results on the generalized Inverse of Fuzzy Neutrosophic Soft Matrices .

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In this section, we define pseudo-similarity of NFMs and exhibit the pseudo-similarity relation preserve idempotency and regularity of the NFMs P and Q.

2. Preliminaries and Theorems

Definition:2.1 The NFMs $P \in F_m$ and $Q \in F_n$ are said to be pseudo-similar, denoted by P≈Q if there exists an idempotent NFMs $L \in F_{m\times n}$ and $M \in F_{n\times m}$ such that P = LQM and Q = MPL.

The NFMs $P = \begin{bmatrix} (0.4,0,0) & (0.4,0,0) \\ (0,1,1) & (0,1,1) \end{bmatrix}$ and $Q = \begin{bmatrix} (0.4,0,0) & (0,1,1) \\ (0.4,0,0) & (0,1,1) \end{bmatrix}$ are pseudo-similar with respect to the idempotent NFM L= $\begin{bmatrix} (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{bmatrix}$.

Definition:2.2 The NFMs $P \in F_m$ and $Q \in F_n$ are said to be semi-similar, denoted by P≈Q if there exists NFM $L \in F_{m\times n}$ and $M \in F_{n\times m}$ such that P = LQM and Q = MPL.

Definition :2.3 If P∈ F_{mn} and K ∈ F_{nm} are NFMs which satisfies the conditions PKP = P and $(PK)^T$ = PK, then K is called least square generalized inverse of P (or) P{1,3} inverse of P.

Definition : 2.4 If P∈ F_{mn} and K ∈ F_{nm} are NFMs which satisfies the conditions PKP= P and $(KP)^T$ = KP, then K is called minimum norm generalized inverse of P (or) P{1,4} inverse of P.

Definition2.5 Sum and product of a Neutrosophic Fuzzy Set
Let X be a non - empty set. A neutrosophic fuzzy sets P and Q is of the form P = { x,$\mu_{P(x)}$, ,σ_P(x),ν_P(x): x ∈ X} and Q = { x,$\mu_{Q(x)}$, ,σ_Q(x),ν_Q(x) : x ∈ X} then, the sum, and product of two Neutrosophic fuzzy sets is defined by,

P+Q = $\{x, (\mu_P(x) \vee \mu_Q(x), \sigma_P(x) \vee \sigma_Q(x), \nu_P(x) \wedge \nu_Q(x)\}$

PQ = $\{x, (\mu_P(x) \wedge \mu_Q(x), 1-\sigma_P(x) \wedge 1-\sigma_Q(x), \nu_P(x) \vee \nu_Q(x)\}$

Teorem:2.1 Let P and Q be two NFMs of order $m \times n$. Then the below conditions are equivalent

(i) $P \approx$ Q

(ii) If P and Q be two NFMs. Such that P = LQM, Q = MPL and $LM \in F_m$ is idempotent.

(iii) If P and Q be two NFMs. Such that P = LQM, Q = MPL and $ML \in F_n$ is idempotent.

Proof: (i) implies (ii) and (i) implies (iii) are trivial.
Since L = LQL implies $LM \in F_m$ are both idempotent matrices.
(ii) implies (i) P = LQM = (LM)P(LM) = (LML)Q(MLM).
Similarly Q = MPL = (MLM)P(LML)
Put $L' = LML$ and $M' = MLM$
Then $P = L'QM'$. and $M' = MLM$
Further using LM is idempotent, we get
$L'M' = (LML)(MLM) = LM$
and $(L'M')(L'M') = (LM)(LM) = L'M'$
Thus $L'M'$ is idempotent.
Set $L'' = L'M'L'$ and $M'' = M'L'M'$.
Then $P = L'QM' = L'M'PL'M' = (L'M'L')(M'L'M')$.
Therefore $P = L''QM''$.
Similarly Q = $M'PL' = (M'L'M')\,\mathrm{P}(L'M'L') = \mathrm{M}''\mathrm{P}L''$.
By using $L'M'$ is idempotent. We have $L''M''L'' = (L'M'L')(M'L'M')(L'M'L') = L'M'L' = L''$.

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Therefore, $P \approx$ Q
Therefore (i) holds
(iv) Implies (i) Can be proved in the same manner.

Example:2.1 Consider the NFMs

$P = \begin{bmatrix} (0.4,0,0) & (0.4,0,0) \\ (0,1,1) & (0,1,1) \end{bmatrix}$ and Q = $\begin{bmatrix} (0.4,0,0) & (0,1,1) \\ (0.4,0,0) & (0,1,1) \end{bmatrix}$.

Then, $L = \begin{bmatrix} (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{bmatrix}$ and $M = \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix}$

$$LQM = \begin{bmatrix} (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{bmatrix} \begin{bmatrix} (0.4,0,0) & (0,1,1) \\ (0.4,0,0) & (0,1,1) \end{bmatrix} \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix}$$
$$= \begin{bmatrix} (0.4,0,0) & (0.4,0,0) \\ (0,1,1) & (0,1,1) \end{bmatrix}$$

$LQM = P.$
and
$$MPL = \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix} \begin{bmatrix} (0.4,0,0) & (0.4,0,0) \\ (0,1,1) & (0,1,1) \end{bmatrix} \begin{bmatrix} (1,0,0) & (0,0,1) \\ (0,0,1) & (0,0,1) \end{bmatrix}$$
$$= \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix}$$
$MPL = Q$ holds.
Thus P and Q are Pseudo – Similar, hence $P \approx$ Q.
Again we satisfy the conditions,
$$LM = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix} \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix} = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix}$$
Now, $(LM)^2 = \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix} \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix}$
$$= \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix} = LM.$$
$$ML = \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix} \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix}$$
$$= \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix}$$
Thus, $(ML)^2 = \begin{bmatrix} (1,1,0) & (1,1,0) \\ (1,1,0) & (0,1,1) \end{bmatrix} \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix}$
$$= \begin{bmatrix} (1,0,0) & (1,0,0) \\ (0,0,1) & (0,0,1) \end{bmatrix} = ML.$$

Theorem 2.2 Let $P \in F_m$ and $Q \in F_n$ be two NFMs such that **P**≈Q. Then P is idempotent if and only if Q is idempotent.

Proof: Since P≈Q then from the definition there exists idempotent NFMs $L \in F_{m\times n}$ and $M \in F_{n\times m}$ such that P = LQM , Q = MP and LML=L. That is LMP = LMLQM = LQM = P
Suppose P is idempotent .Then P^2 = P
Now Q^2 = (MPL)(MPL) = MP(LMP)L = MP^2L = MPL = Q
Therefore, Q is idempotent.

Theorem 2.3 If $P \in F_n$ be a symmetric and idempotent NFM then P itself a {1,3} inverse .

Proof : Since P is idempotent ,P^T = P and P is idempotent P^2 = P
Now KP = P if K = I_n
Then PKP = PP = P^2 = P
i.e) $P \in P\{1\}$
Now $(PL)^T = L^TP^T = L^TP = P^TP$ (Taking L = P, as P itself a g –inverse)
= PP = PL
Therefore, $P \in P\{1,3\}$.

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Theorem 2.4 If $P \in F_n$ be a symmetric and idempotent NFM then P itself a {1,4} inverse .

Proof : Since P is idempotent ,$P^T = P$ and P is idempotent $P^2 = P$

Now KP = P if $K = I_n$

Then $PKP = PP = P^2 = P$

i.e) $P \in P\{1\}$

Now $(LP)^T = P^T L^T = PL^T = P P^T$ (Taking L = P, as P itself a g –inverse)

= PP = LP

Therefore, $P \in P\{1,4\}$.

Example 2.2 Consider the NFMs

$$P = \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.4) \\ (0.5,0.5,0.4) & (0.5,0.5,0.1) \end{bmatrix}$$

$$P^2 = \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.4) \\ (0.5,0.5,0.4) & (0.5,0.5,0.1) \end{bmatrix} \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.4) \\ (0.5,0.5,0.4) & (0.5,0.5,0.1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.4) \\ (0.5,0.5,0.4) & (0.5,0.5,0.1) \end{bmatrix}$$

$P^2 = P$ and $P^T = P$

$$K = I_n, K = \begin{bmatrix} (1,1,0) & (0,0,1) \\ (0,0,1) & (1,1,0) \end{bmatrix}$$

$$KP = \begin{bmatrix} (1,1,0) & (0,0,1) \\ (0,0,1) & (1,1,0) \end{bmatrix} \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.4) \\ (0.5,0.5,0.4) & (0.5,0.5,0.1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.4) \\ (0.5,0.5,0.4) & (0.5,0.5,0.1) \end{bmatrix}$$

This shows that P is symmetric and idempotent satisfy the condition PLP = P for L = P, itself. Again, $(PP)^{:T} = (P^2)^T$ = P = PP.

Therefore, $P \in P\{1,3\}$ and $P\{1,4\}$

Theorem:2.5 Let $P, Q \in F_{m\times n}$ be two NFMs. If P is regular then

(i) $R(Q) \subseteq R(P)$ iff $Q = QP^{-}P$ for each $P^{-} \in P\{1\}$

(ii) $C(Q) \subseteq C(P)$ iff $Q = PP^{-}Q$ for each $P^{-} \in P\{1\}$

Proof: (i) $R(Q) \subseteq R(P)$ then every row of Q is linear combination of the row of P.

Hence $Q_{i^*} = \sum l_{ij} P_{j^*}$, *where* $l_{ij} \in < F >$.

That is , $Q = LP$ (for some $L \in F_m$)

$or, Q = LPP^{-}P$ (since $P = PP^{-}P$)

$or, Q = QP^{-}P$

Conversely, if $Q = QP^{-}P$, then $Q = LPP^{-}P$ (for some $L \in F_m$)

or, Q = LP (since $P = PP^{-}P$)

This implies that $R(Q) \subseteq R(P)$

(ii) $C(Q) \subseteq C(P)$

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That is , $Q = PM$ (for some $M \in F_n$)

$or, Q = PP^{-}PM$ (since $P = PP^{-}P$)

$or, Q = P^{-}PQ$

Conversely, if $Q = P^{-}PQ$, then $Q = PP^{-}PM$ (for some $M \in F_n$)

or, Q = PM (since $P = PP^{-}P$)

This implies that $C(Q) \subseteq C(P)$

Example2.3 Let us consider NFMs

$$P = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.1) \end{bmatrix}$$

$$Q = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.2) \end{bmatrix}$$

One of the g-inverse of P is $P^{-} = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.1) \end{bmatrix}$ for which $Q = QP^{-}P$ holds.

Also Q = LA for $L = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.2) \end{bmatrix}$ holds.

So, $R(Q) \subseteq R(P)$

Similarly, The result is true for column space also.

Theorem:2.6 Let $P \in F_{m \times n}$ be a regular NFMs and L be a g -inverse of P then PL and LP are idempotent.

Proof: Since (PL)(PL) = (PLP)L

= PL (as PLP = P)

Also, (LP)(LP) = (LPL)P

= LP (as LPL = L)

Example2.4 Let us consider NFMs

$$P = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.1) \end{bmatrix}$$

$$L = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.7, 0.5, 0.4) & (0.4, 0.5, 0.1) \end{bmatrix}$$

$$PL = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.7, 0.5, 0.4) & (0.4, 0.5, 0.1) \end{bmatrix}$$

$$(PL)^2 = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.1) \end{bmatrix} \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.1) \end{bmatrix}$$

$$(PL)^2 = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.4) & (0.5, 0.5, 0.1) \end{bmatrix} = PL$$

$$LP = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \end{bmatrix}$$

$$(LP)^2 = \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \end{bmatrix} \begin{bmatrix} (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \\ (0.5, 0.5, 0.2) & (0.5, 0.5, 0.2) \end{bmatrix}$$

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$$(LP)^2 = \begin{bmatrix} (0.5,0.5,0.2) & (0.5,0.5,0.2) \\ (0.5,0.5,0.2) & (0.5,0.5,0.2) \end{bmatrix} = LP$$

3. Conclusion

In this paper, we discussed the properties of Pserudo – Inverse, Semi – Similar of Idempotent Neutrosophic Fuzzy Matrices. The concept of group – inverse of Neutrosophic Fuzzy Matrices is traduced by using the definition of g – inverse in Idempotent Neutrosophic Fuzzy Matrices. Future we shall planned to find the related properties of g – inverse Neutrosophic Fuzzy Matrices.

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