Comparison of Some Entropy Measures for Non-Central Fisher Probability Distribution

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Abstract

In this paper, many entropy measures of noncentral Fisher distribution were driven including Shannon, Renyi, Sharma, Havrda, Arimoto and Tsallis. A comparison between these entropies was made according to distribution's shift parameter, distribution's degrees of freedom, shape parameter and truncation parameter. The entropy that had less relative loss was said to be better than the other. There were significant differences according to all studied parameters except the shift parameter and we found that the best entropy of the mentioned entropies for noncentral Fisher distribution was Renyi entropy.

Keywords: Goodness of Fit. Entropy; Noncentral Distributions; Relative Loss; Truncated Distribution; Shift Parameter.

1. Introduction

Entropy was first introduced in the 19th century by Clausius discussing the heat cycle which eventually led to an application in the second law of thermodynamics [1]. Then the idea of entropy of random variables was developed by Shannon (1948), first in information theory and now it is used in various disciplines. For example, in physic entropy measures the number of ways in which a thermodynamic system is arranged [2-5], in information theory entropy measures the average amount of information in each message [6-8], in statistics entropy measures uncertainty and dispersion and in probability entropy is used to quantify the probability distribution of a random variable and its uncertainty [9-12].

Shannon defined a formal measure of entropy in the following form [13]:

$$H_S(X) = -\int_x f(x)\log_2 f(x)\,dx \qquad (1)$$

Alfred Réyni generalized Shannon's entropy adding shift parameter α which makes the measure sensitive to the shape of probability distributions [14]:

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$$H_{\alpha}(X) = \frac{1}{1-\alpha}\log_2 \int_x f(x)^{\alpha}dx;\ \alpha > 0, \alpha \neq 1 \qquad (2)$$

Idea of After Alfred Réyni's extension depending on shift parameters transferred to many researchers to make their own generalization of entropy measure like Havrda and Charvat [15], Arimoto [16], Sharma and Mittal [17]and Tsallis [18]as following respectively:

$$HC^{\alpha}(X) = \frac{1}{2^{1-\alpha}-1}\left[\int_0^{\infty} \{f(x)\}^{\alpha}dx - 1\right];\ \alpha > 0, \alpha \neq 1 \qquad (3)$$

$$A_{\alpha}(X) = \frac{\alpha}{1-\alpha}\left[\left(\int_0^{\infty} (f(x)^{\alpha}dx\right)^{\frac{1}{\alpha}} - 1\right]; \alpha > 0, \alpha \neq 1 \qquad (4)$$

$$HSM_{\alpha}(X) = \frac{1}{2^{1-\alpha}-1}\left[\exp\left[(\alpha-1)\int_0^{\infty} f(x)\ln f(x)dx\right] - 1\right]; \alpha > 0, \alpha \neq 1 \quad (5)$$

$$T_{\alpha}(X) = \frac{1}{\alpha-1}\left[1 - \int_x f(x)^{\alpha}dx\right]; \alpha > 0, \alpha \neq 1 \qquad (6)$$

In probability theory, many researches were made to determine the suitable entropy measure according to various probability distributions and these studies had proved that each probability distribution has its own suitable entropy measure and sometimes these researches didn't reach a significant result [19-22].

2. Preliminaries

2.1 Noncentral Fisher Distribution [23,24]:

Let $\chi^2_{n_1}(\delta)$ be a noncentral chi-square random variable with degrees of freedom n_1 and noncentrality parameter δ and let $\chi^2_{n_2}$ be a chi-square random variable independent of $\chi^2_{n_1}(\delta)$ with degrees of freedom n_2 then the following ratio:

$$X = \frac{\frac{\chi^2_{n1}(\delta)}{n_1}}{\frac{\chi^2_{n2}}{n_2}} \qquad (7)$$

Follows noncentral Fisher distribution and denoted by $F_{n1,n2}(\delta)$ and has a cumulative distribution function, probability density function, expected value, variance and noncentral moments given respectively as follows:

$$F(x \mid n_1, n_2, \delta) = \sum_{k=0}^{\infty} \frac{exp\left(-\frac{\delta}{2}\right)\left(\frac{\delta}{2}\right)^k}{k!} P\left(F_{n_1+2k,n_2} \le \frac{n_1 x}{n_1+2k}\right), \qquad (8)$$
$$n_1 > 0, n_2 > 0, \delta > 0$$

$$f(x) = \frac{\Gamma\left(\frac{2i+n_1+n_2}{2}\right)\left(\frac{n_1}{n_2}\right)^{\frac{(2i+n_1)}{2}} x^{\frac{(2i+n_1-2)}{2}} e^{-\frac{\delta}{2}}\left(\frac{\delta}{2}\right)^i}{\Gamma\left(\frac{n_2}{2}\right)\Gamma\left(\frac{2i+n_1}{2}\right)i!\left(1+\frac{n_1}{n_2}x\right)^{\frac{(2i+n_1+n_2)}{2}}} \qquad (9)$$

$$E(X) = \frac{n_2(n_1+\delta)}{n_1(n_2-2)},\ n_2 > 2 \qquad (10)$$

$$Var(X) = \frac{2{n_2}^2[(n_1+\delta)^2 + (n_1+2\delta)(n_2-2)]}{{n_1}^2(n_2-2)^2(n_2-4)},\ n_2 > 4 \qquad (11)$$

$$E(X^k) = \frac{\Gamma[(n_2-2k)/2]\Gamma[(n_1+2k)/2]{n_2}^k}{\Gamma(n_2/2){n_1}^k}\sum_{j=0}^{k}\binom{k}{j}\frac{(\delta/2)^j}{\Gamma[(n_1+2j)/2]},\ n_2 > 2k \qquad (12)$$

2.2 Truncated distribution

Let X be a positive random variable with probability density function $f(x)$and cumulative distribution function $F(x)$, then the density of the random variable Y cut from X on the interval $[0, t)$is defined by: [21]

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$$f(y;t) = \frac{f(y)}{F(t)} \quad (13)$$

2.3 Relative Loss

Relative loss is a useful measure to compare different entropy measures and choose the most efficient entropy for a given data [21], the lower the relative loss, the better the entropy measure is. Relative loss when a random variable Y is used instead of a random variable X is defined as follows [19]:

$$s_H(t) = \frac{H(X)-H(Y)}{H(X)} \quad (14)$$

3. Results and Discussion

The entropies corresponding to the noncentral Fisher distribution and its relative loss were calculated for different values of $n_1, n_2, \delta, T, \alpha$ which were selected based on literature studies [19-22], [25,26], and following tables show the results:

Table 1: Shannon's Relative Loss.

T	**0.25**			**0.5**			**0.75**			**1**		
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	4.619	3.787	2.785	3.114	2.757	2.344	2.225	2.123	2.061	1.629	1.675	1.852
$n_1 = n_2$	13.55	5.434	2.985	9.46	4.333	2.74	6.568	3.569	2.567	4.301	2.95	1.59
$n_1 > n_2$	3.2	2.29	1.92	2.38	1.99	1.65	1.87	1.81	1.55	1.48	1.67	1.45
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		
$n_1 < n_2$	1.209	1.339	1.686	0.907	1.08	1.548	0.688	0.876	1.43	0.526	0.715	2.27
$n_1 = n_2$	2.6	2.42	1.39	1.39	1.94	1.2	0.688	1.52	1.04	0.307	1.15	8.88
$n_1 > n_2$	1.17	1.56	1.39	0.913	1.47	1.32	0.702	1.38	1.27	0.526	1.3	1.22

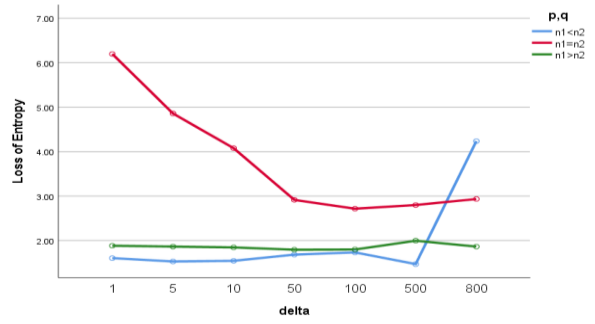

Figure 1: Shannon's relative loss for mutable values of delta.

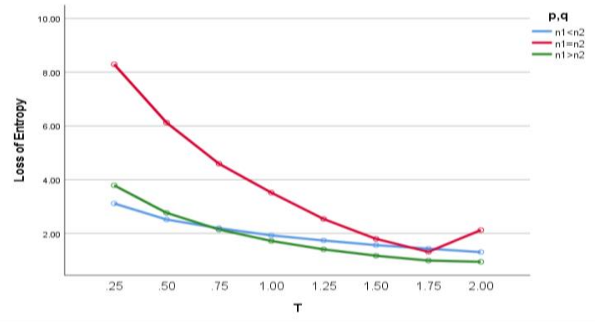

figure 2: Shannon's relative loss for mutable values of T.

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We notice from Table (1) and Figures (1-2) that the relative becomes bigger when $n_1 = n_2$ and it decreases when T, δ increase.

Table 2: Renyi's Relative Loss.

						Alpha=0.5						
T		**0.25**			**0.5**			**0.75**			**1**	
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$\boldsymbol{n_1 < n_2}$	3.135	2.834	2.395	2.224	2.126	2.026	1.697	1.701	1.792	1.344	1.405	1.62
$\boldsymbol{n_1 = n_2}$	6.898	4.088	2.652	4.87	3.258	2.431	3.49	2.694	2.274	2.44	2.245	2.147
$\boldsymbol{n_1 > n_2}$	2.602	2.027	1.828	1.969	1.754	1.725	1.577	1.59	1.665	1.286	1.469	1.622
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T		**1.25**			**1.5**			**1.75**			**2**	
$\boldsymbol{n_1 < n_2}$	1.09	1.184	1.484	0.901	1.011	1.372	0.755	0.874	1.277	0.641	0.762	1.195
$\boldsymbol{n_1 = n_2}$	1.639	1.865	2.037	1.055	1.537	1.941	0.655	1.251	1.854	0.397	1.003	1.774
$\boldsymbol{n_1 > n_2}$	1.054	1.371	1.587	0.863	1.287	1.558	0.702	1.212	1.532	0.568	1.144	1.509
						Alpha=1.5						
T	**0.25**			**0.5**			**0.75**			**1**		
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$\boldsymbol{n_1 < n_2}$	5.76	4.406	2.988	3.811	3.177	2.513	2.641	2.41	2.208	1.854	1.862	1.981
$\boldsymbol{n_1 = n_2}$	23.817	6.388	3.168	16.588	5.101	2.911	11.369	4.201	2.728	7.199	3.461	2.579
$\boldsymbol{n_1 > n_2}$	3.583	2.438	2.06	2.663	2.126	1.95	2.073	1.936	1.886	1.622	1.793	1.839
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		
$\boldsymbol{n_1 < n_2}$	1.308	2.232	1.8	0.927	1.132	1.649	0.662	0.886	1.52	0.477	0.695	1.406
$\boldsymbol{n_1 = n_2}$	4.032	2.815	2.451	1.955	2.236	2.337	0.83	1.719	2.235	0.32	1.268	2.141
$\boldsymbol{n_1 > n_2}$	1.255	1.676	1.802	0.952	1.573	1.77	0.704	1.482	1.742	0.505	1.397	**1.717**

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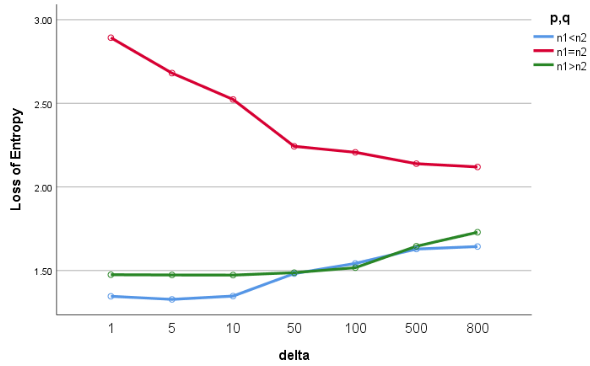

figure 3: Renyi's relative loss for mutable values of delta & alpha=0.5.

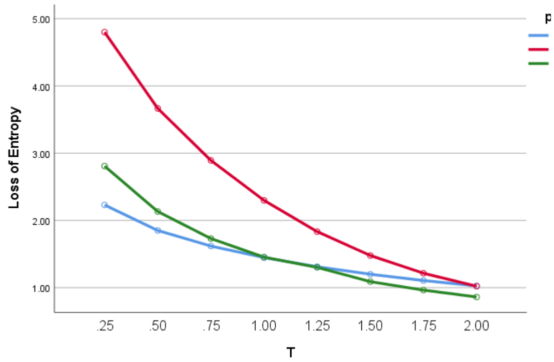

figure 4: Renyi's relative loss for mutable values of T & alpha=0.5.

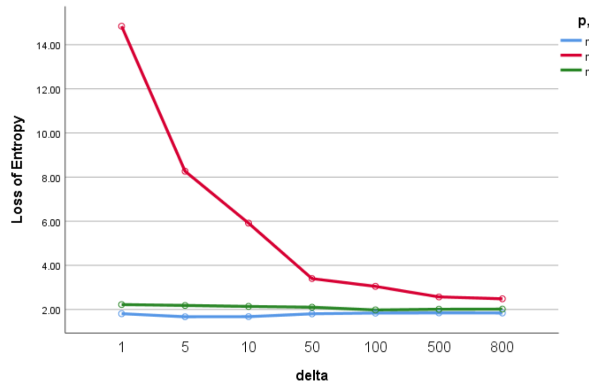

Figure 5: Renyi's relative loss for mutable values of delta & alpha=1.5

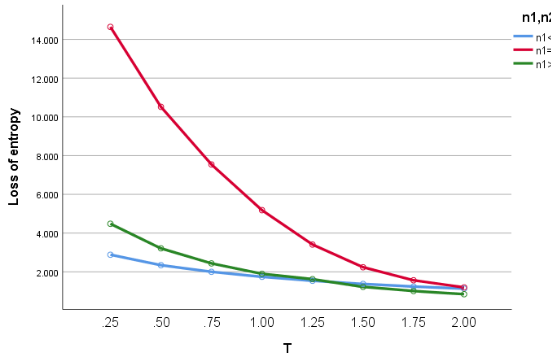

Figure 6: Renyi's relative loss for mutable values of T & alpha=1.5.

When $\alpha = 0.5$ we notice according to table (2) and figures (3-4) that relative becomes bigger when $n_1 = n_2$ and it decreases when T increase but increases when δ increases.

When $\alpha = 1.5$ we notice according to table (2) and figures (5-6) that relative becomes bigger when $n_1 = n_2$ and it decreases when T, δ increase.

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Table 3: Havrda and Charvat Relative Loss.

						Alpha=0.5						
T		**0.25**			**0.5**			**0.75**			**1**	
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	1.825	1.611	1.251	1.602	1.474	1.225	1.4	1.344	1.199	1.22	1.222	1.173
$n_1 = n_2$	3.718	2.175	1.341	3.184	2.016	1.325	2.632	1.861	1.312	2.065	1.7	1.298
$n_1 > n_2$	1.754	1.343	1.14	1.538	1.285	1.132	1.357	1.242	1.127	1.192	1.204	1.123
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T		**1.25**			**1.5**			**1.75**			**2**	
$n_1 < n_2$	1.062	1.11	1.148	0.927	1.007	1.122	0.811	0.914	1.097	0.713	0.83	1.073
$n_1 = n_2$	1.52	1.532	1.285	1.048	1.358	1.271	0.683	1.179	1.257	0.427	1.002	1.243
$n_1 > n_2$	1.039	1.169	1.119	0.895	1.136	1.116	0.762	1.105	1.113	0.64	1.074	1.11
						Alpha=1.5						
T	**0.25**			**0.5**			**0.75**			**1**		
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	14.726	12.721	14.926	6.48	6.095	8.286	3.569	3.636	5.624	2.161	2.382	4.188
$n_1 = n_2$	63.721	19.509	15.795	31.693	11.656	11.782	17.274	7.901	9.546	9.189	5.578	8.026
$n_1 > n_2$	6.847	5.608	7.409	3.995	4.039	6.094	2.68	3.283	5.429	1.878	2.795	4.99
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		
$n_1 < n_2$	1.381	3.174	3.291	0.915	1.174	2.677	0.624	0.858	2.232	0.435	0.639	1.895
$n_1 = n_2$	4.533	3.979	6.903	2.028	2.821	6.033	0.825	1.964	5.336	0.312	1.332	4.763
$n_1 > n_2$	1.332	2.439	4.663	0.941	2.16	4.402	0.657	1.93	4.184	0.451	1.736	3.996

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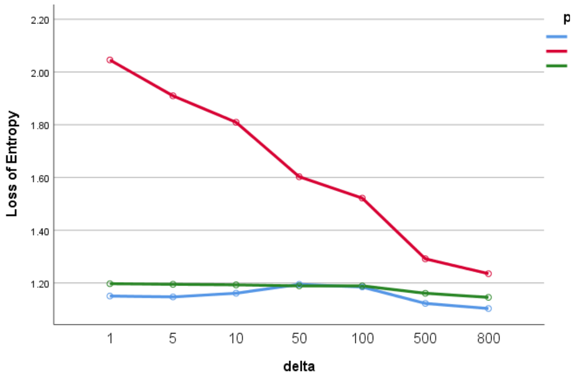

Figure 7: Havrda and Charvat relative loss for mutable values of delta & alpha=0.5.

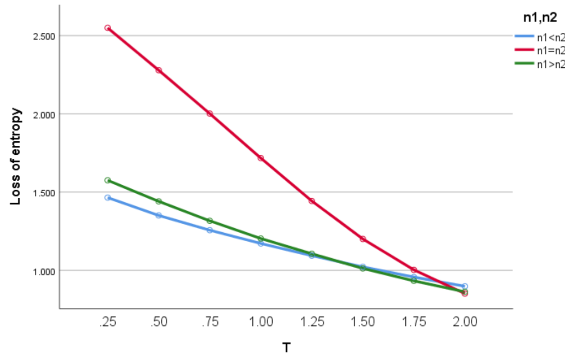

Figure 8: Havrda and Charvat relative loss for mutable values of T & alpha=0.5.

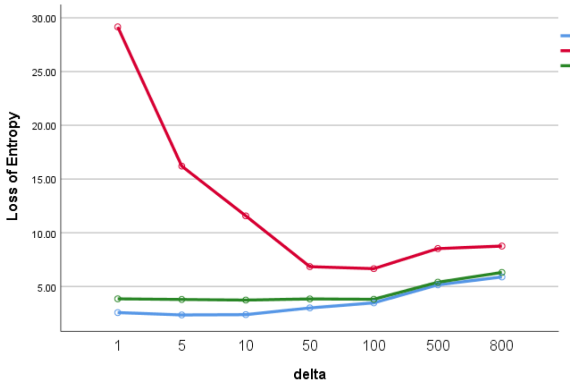

Figure 9: Havrda and Charvat relative loss for mutable values of delta & alpha=1.5.

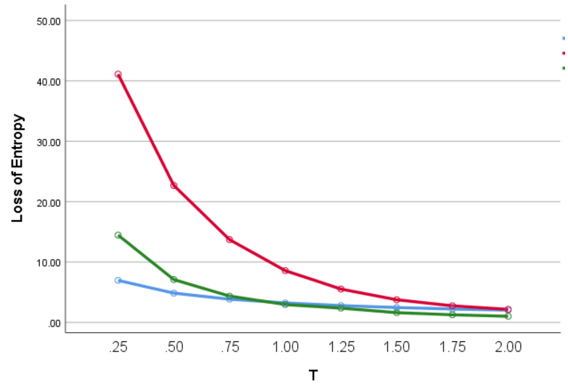

Figure 10: Havrda and Charvat relative loss for mutable values of T & alpha=1.5.

When $\alpha = 0.5$ we notice according to table (3) and figures (7-8) that relative becomes bigger when $n_1 = n_2$ and it decreases when T, δ increase.

When $\alpha = 1.5$ we notice according to table (3) and figures (9-10) that relative becomes bigger when $n_1 = n_2$ and it decreases when T increase but increases when δ increases.

Table 4: Arimoto's Relative Loss.

Alpha=0.5												
T	**0.25**			**0.5**			**0.75**			**1**		
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$\boldsymbol{n_1 < n_2}$	1.355	1.228	1.051	1.301	1.202	1.05	1.225	1.164	1.047	1.136	1.116	1.044
$\boldsymbol{n_1 = n_2}$	2.463	1.527	1.084	2.321	1.497	1.083	2.099	1.455	1.081	1.792	1.398	1.08
$\boldsymbol{n_1 > n_2}$	1.37	1.115	1.022	1.298	1.104	1.022	1.216	1.093	1.021	1.126	1.082	1.021
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		

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$n_1 < n_2$	1.042	1.062	1.04	0.948	1.004	1.035	0.859	0.945	1.029	0.777	0.885	1.023
$n_1 = n_2$	1.422	1.326	1.079	1.042	1.235	1.077	0.71	1.126	1.075	0.457	1.002	1.073
$n_1 > n_2$	1.027	1.071	1.021	0.923	1.06	1.02	0.815	1.048	1.02	0.707	1.035	1.02
Alpha=1.5												
T	**0.25**			**0.5**			**0.75**			**1**		
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	10.44	8.578	8.031	5.356	4.804	5.247	3.209	3.135	3.939	2.048	2.182	3.153
$n_1 = n_2$	44.638	12.917	8.516	25.189	8.622	6.895	14.927	6.289	5.914	8.449	4.702	5.206
$n_1 > n_2$	5.411	4.1	4.507	3.454	3.178	3.918	2.446	2.698	3.605	1.783	2.371	3.391
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		
$n_1 < n_2$	1.355	2.794	2.621	0.919	1.159	2.231	0.636	0.868	1.933	0.449	0.658	1.695
$n_1 = n_2$	4.356	3.519	4.655	2.003	2.599	4.209	0.827	1.875	3.837	0.314	1.309	3.52
$n_1 > n_2$	1.304	2.123	3.228	0.945	1.922	3.095	0.673	1.751	2.982	0.469	1.603	2.884

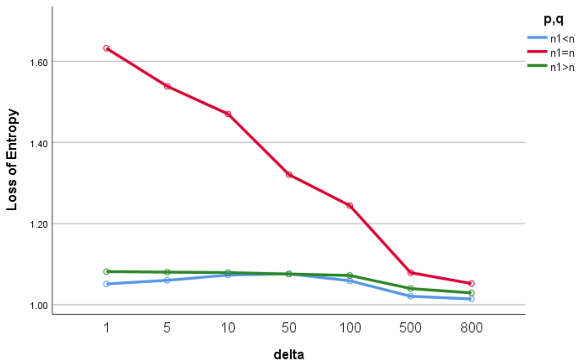

Figure 11: Arimoto relative loss for mutable values of delta & alpha=0.5.

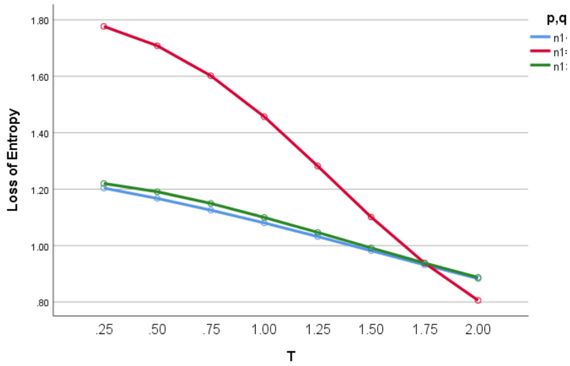

Figure 12: Arimoto relative loss for mutable values of T & alpha=0.5.

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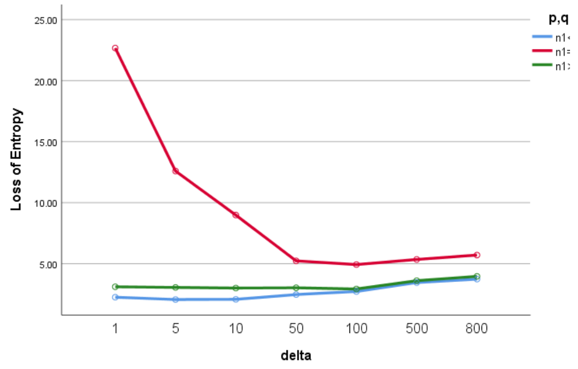

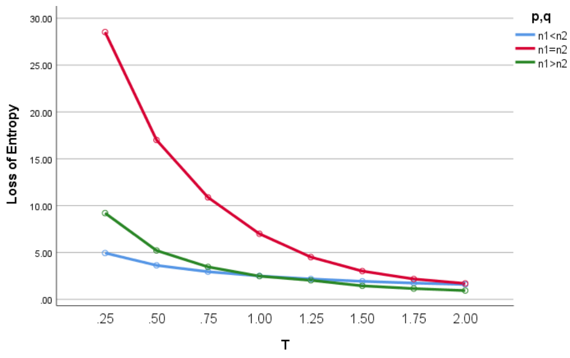

Figure 13: Arimoto relative loss for mutable values of delta & alpha=1.5.

Figure 14: Arimoto relative loss for mutable values of T & alpha=1.5.

When $\alpha = 0.5$ we notice according to table (4) and figures (11-12) that relative becomes bigger when $n_1 = n_2$ and it decreases when T, δ increase.

When $\alpha = 1.5$ we notice according to table (4) and figures (13-14) that relative becomes bigger when $n_1 = n_2$ and it decreases when T increase but increases when δ increases.

Table 5: Sharma and Mittal Relative Loss.

Alpha=0.5												
T	**0.25**			**0.5**			**0.75**			**1**		
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	2.07	1.668	1.181	1.869	1.573	1.172	1.63	1.455	1.162	1.381	1.324	1.149
$n_1 = n_2$	5.393	2.185	1.224	4.86	2.097	1.22	4.144	1.992	1.216	3.239	1.8665	0.699
$n_1 > n_2$	1.776	1.271	0.786	1.611	1.2431	0.888	1.4512	1.2202	0.996	1.282	1.1981	1.111
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	1.25			1.5			1.75			2		
$n_1 < n_2$	1.143	1.186	1.135	0.931	1.049	1.12	0.75	0.917	1.103	0.601	0.796	1.085
$n_1 = n_2$	2.255	1.717	1.197	1.345	1.5361	1.548	0.7081	1.3347	1.666	0.327	1.1095	1.605
$n_1 > n_2$	1.113	1.177	1.221	0.938	1.156	1.322	0.7714	1.1344	1.4	0.612	1.1119	1.457
Alpha=1.5												
T		**0.25**			**0.5**			**0.75**			**1**	
$\boldsymbol{\delta}$	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	18.42	18.25	31.66	6.705	7.106	13.948	3.418	3.817	8.211	2.019	2.364	5.525

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$n_1 = n_2$	59.87	30.89	8.24	8.263	7.8765	7.647	11.864	8.8172	6.779	6.01	5.7268	5.878
$n_1 > n_2$	8.263	7.876	7.647	4.215	4.992	5.792	2.669	3.8161	4.955	1.794	3.0867	4.347
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	1.25			1.5			1.75			2		
$n_1 < n_2$	1.297	3.09	4.017	0.88	1.123	3.073	0.621	0.825	2.436	0.451	0.623	1.982
$n_1 = n_2$	3.042	3.885	4.932	1.447	2.6351	3.952	0.668	1.8046	3.092	0.287	1.2129	2.346
$n_1 > n_2$	1.256	2.601	3.901	0.882	2.2255	3.514	0.6266	1.9394	3.194	0.439	1.6993	2.901

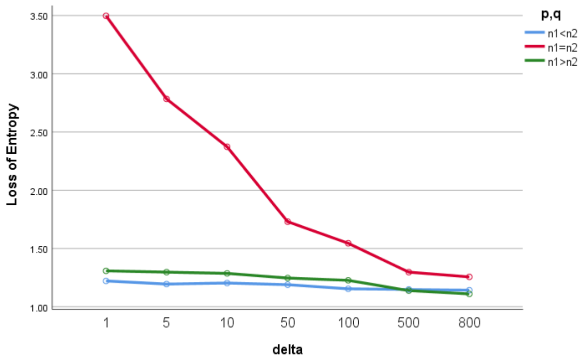

Figure 15: Sharma and Mittal relative loss for mutable values of delta & alpha=0.5.

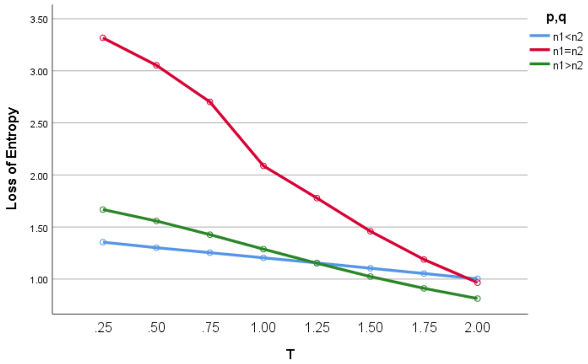

Figure 16: Sharma and Mittal relative loss for mutable values of T & alpha=0.5.

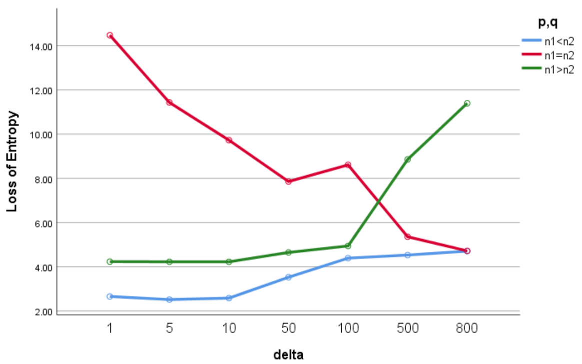

Figure 17: Sharma and Mittal relative loss for mutable values of delta & alpha=1.5.

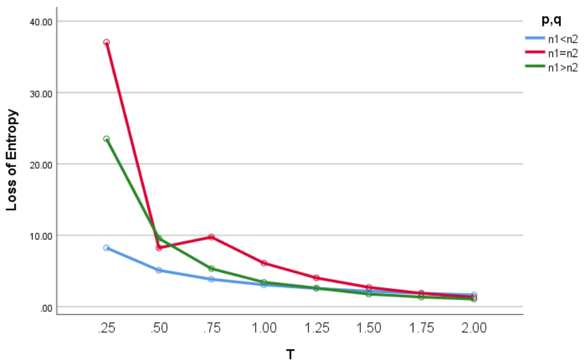

Figure 18: Sharma and Mittal relative loss for mutable values of T & alpha=1.5.

When $\alpha = 0.5$ we notice according to table (5) and figures (15-16) that relative becomes bigger when $n_1 = n_2$ and it decreases when T, δ increase.

When $\alpha = 1.5$ we notice according to table (5) and figures (17-18) that relative becomes bigger when $n_1 = n_2$ and it decreases when T increase but increases when δ increases.

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Table 6: Tsallis's Relative Loss.

Alpha=0.5												
T	**0.25**			**0.5**			**0.75**			**1**		
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	1.825	1.611	1.251	1.602	1.474	1.225	1.4	1.344	1.199	1.22	1.222	1.173
$n_1 = n_2$	3.718	2.175	1.341	3.184	2.016	1.325	2.632	1.861	1.312	2.065	1.7	1.298
$n_1 > n_2$	1.754	1.343	1.14	1.538	1.285	1.132	1.357	1.242	1.127	1.192	1.204	1.123
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		
$n_1 < n_2$	1.062	1.11	1.148	0.927	1.007	1.122	0.811	0.914	1.097	0.713	0.83	1.073
$n_1 = n_2$	1.52	1.532	1.285	1.048	1.358	1.271	0.683	1.179	1.257	0.427	1.002	1.243
$n_1 > n_2$	1.039	1.169	1.119	0.895	1.136	1.116	0.762	1.105	1.113	0.64	1.074	1.11
Alpha=1.5												
T	**0.25**			**0.5**			**0.75**			**1**		
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
$n_1 < n_2$	14.726	12.72	14.926	6.48	6.095	8.286	3.569	3.636	5.624	2.161	2.382	4.188
$n_1 = n_2$	63.721	19.51	15.795	31.693	11.66	11.782	17.274	7.901	9.546	9.189	5.578	8.026
$n_1 > n_2$	6.847	5.608	7.409	3.995	4.039	6.094	2.68	3.283	5.429	1.878	2.795	4.99
δ	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**	**5**	**50**	**500**
T	**1.25**			**1.5**			**1.75**			**2**		
$n_1 < n_2$	1.381	3.174	3.291	0.915	1.174	2.677	0.624	0.858	2.232	0.435	0.639	1.895
$n_1 = n_2$	4.533	3.979	6.903	2.028	2.821	6.033	0.825	1.964	5.336	0.312	1.332	4.763
$n_1 > n_2$	1.332	2.439	4.663	0.941	2.16	4.402	0.657	1.93	4.184	0.451	1.736	3.996

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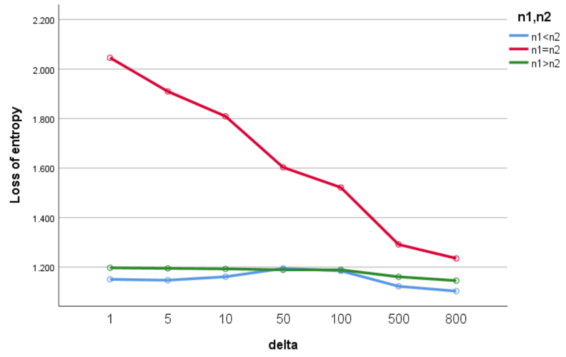

Figure 19: Tsallis's relative loss for mutable values of delta & alpha=0.5.

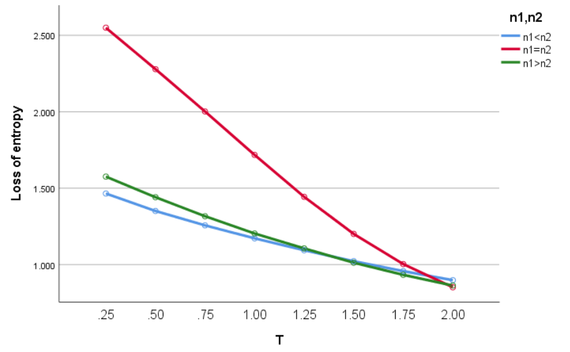

Figure 20: Tsallis's relative loss for mutable values of T & alpha=0.5.

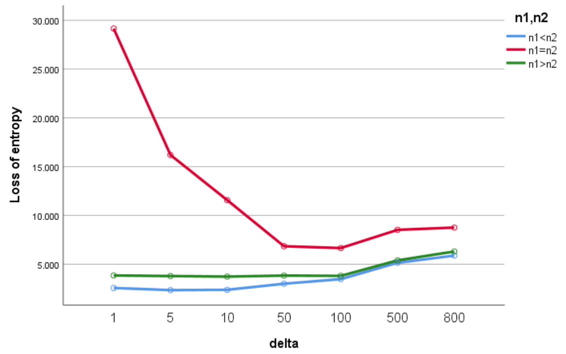

Figure 21: tsallis's relative loss for mutable values of delta & alpha=1.5.

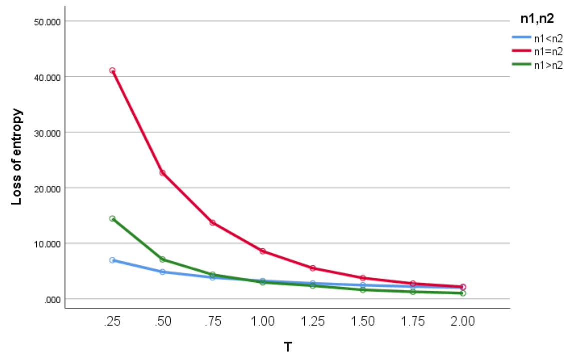

Figure 22: tsallis's relative loss for mutable values of T & alpha=1.5.

When $\alpha = 0.5$ we notice according to table (6) and figures (19-20) that relative becomes bigger when $n_1 = n_2$ and it decreases when T increase but increases when δ increases.

When $\alpha = 1.5$ we notice according to table (6) and figures (21-22) that relative becomes bigger when $n_1 = n_2$ and it decreases when T, δ increase

4. Statistical Analysis

Analysis of Variance (ANOVA) was applied to the numerical results given in Tables (1-6) to check out whether there are statistically significant differences in the relative loss according to the different values of $n_1, n_2, \delta, T, \alpha$ f. Results of ANOVA test is given table (7):

Table 7: ANOVA table of differences

	DF	Sum Sq	Mean Sq	F	P-Value
Entropy	5	900	180	5.175	0.000101***
n_1	2	4582	2291	65.897	< 2e-16***
δ	6	1227	204	5.882	0.00000428***
T	7	11942	1706	49.066	< 2e-16***
α	1	6190	6190	178.035	< 2e-16***
Residuals	1994	69328	35		
Signify. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	' 0.1 ' ' 1

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We conclude from table (7) that there are highly significant differences in relative loss according to all parameters. Applying Tukey HSD to perform a pairwise comparison between different groups yields to results in table (8) below:

Table (8): Comparison between relative loss of different entropy measures

Compared entropies	Diff	p adj	Best entropy
Havrda and Charvat-Arimoto	0.95	0.293	No sig. differences
Renyi-Arimoto	-0.606	0.767	No sig. differences
Shannon-Arimoto	-0.579	0.8	No sig. differences
Sharmaa and mittal-Arimoto	0.713	0.62	No sig. differences
Tsallis-Arimoto	0.95	0.293	No sig. differences
Renyi-Havrda and Charvat	-1.557	0.008	Renyi
Shannon-Havrda and Charvat	-1.529	0.01	Shannon
Sharmaa and mittal-Havrda and Charvat	-0.237	0.995	No sig. differences
Tsallis-Havrda and Charvat	0	1	No sig. differences
Shannon-Renyi	0.027	1	No sig. differences
Sharmaa and mittal-Renyi	1.32	0.044	Renyi
Tsallis-Renyi	1.557	0.008	Renyi
Sharmaa and mittal-Shannon	1.292	0.052	Shannon
Tsallis-Shannon	1.529	0.01	Shannon
Tsallis-Sharmaa and mittal	0.237	0.995	No sig. differences

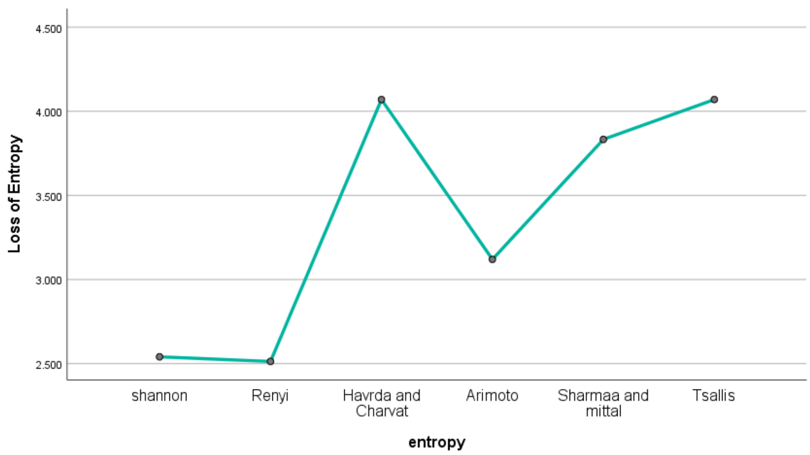

Figure 23: Relative loss of all measures.

We notice that generally and without taking into consideration any of the parameters $n_1, n_2, \delta, T, \alpha$, performance of entropy measures goes in this ascending order: Renyi, Shannon, Arimoto, Sharma and Mittal, Havrda and Charvat then Tsallis.

Table 9: TukeyHSD for n1&n2

Compared degrees of freedom	diff	p adj	Best degree of freedom
50-10	3.388	0	10
100-10	0.421	0.398	No sig. differences
100-50	-2.967	0	100

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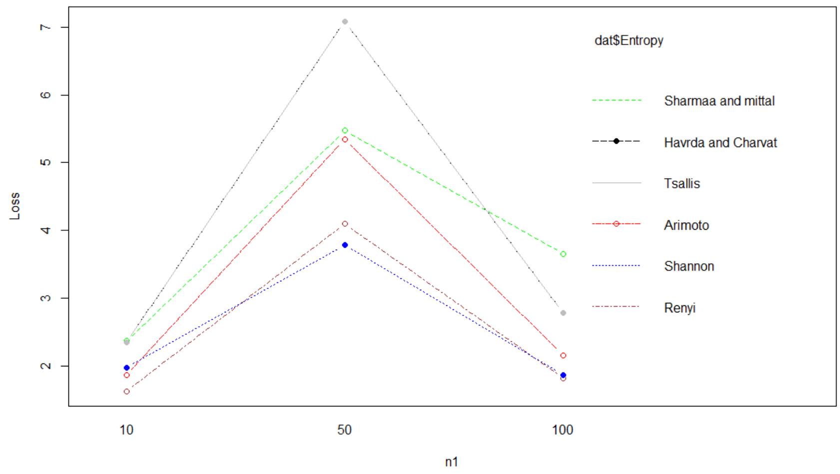

Figure 24: Change in relative loss according to n_1

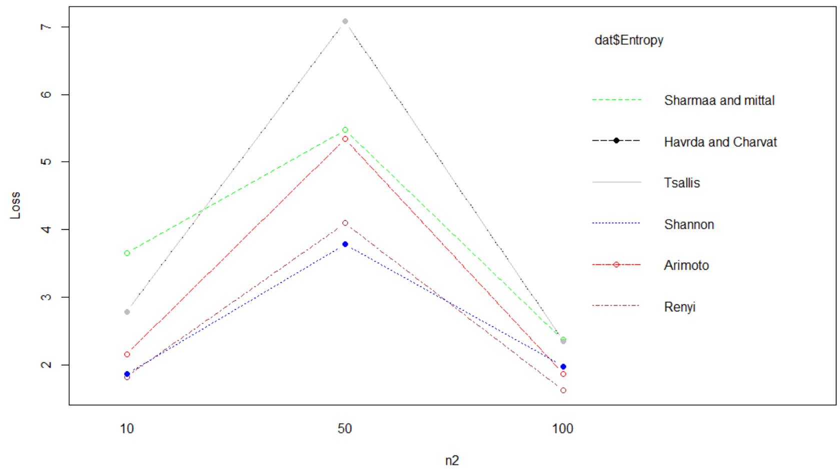

Figure 25: Change of relative loss according to n_2

We conclude from table (9) and figure (1) that performance of entropy is sorted ascending as: Renyi, Shannon, Arimoto, Tsallis, Havrda and Charvat then Sharma and Mittal.

We conclude from table (9) and figure (2) that performance of entropy is sorted ascending as: Renyi, Arimoto, Shannon, Tsallis, Havrda and Charvat then Sharma and Mittal.

Table 10: TukeyHSD for delta

Compared deltas	diff	p adj	Preferred delta
5-1	-1.415	0.061	No sig. differences
10-1	-1.953	0.001	10
50-1	-2.446	0	50
100-1	-2.415	0	500
500-1	-2.099	0	800
800-1	-1.763	0.006	No sig. differences
10-5	-0.537	0.93	No sig. differences
50-5	-1.03	0.355	No sig. differences

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100-5	-1	0.393	No sig. differences
500-5	-0.683	0.807	No sig. differences
800-5	-0.348	0.992	No sig. differences
50-10	-0.493	0.954	No sig. differences
100-10	-0.462	0.966	No sig. differences
500-10	-0.146	1	No sig. differences
800-10	0.19	1	No sig. differences
100-50	0.03	1	No sig. differences
500-50	0.347	0.992	No sig. differences
800-50	0.682	0.808	No sig. differences
500-100	0.317	0.995	No sig. differences
800-100	0.652	0.839	No sig. differences
800-500	0.335	0.994	No sig. differences

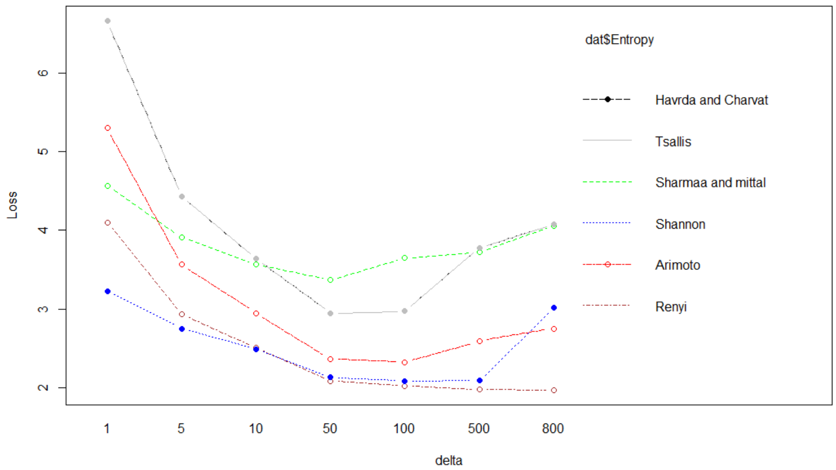

Figure 26: Relative loss according to δ

So, according to various values of delta, the performance of entropy measures is sorted ascending as follows: Renyi, Arimoto, Shannon, Sharma and Mittal, Tsallis, then Havrda and Charvat.

Table 11: TukeyHSD for T

Compared T	diff	p adj	The best T
0.5-0.25	-3.774	0	0.5
0.75-0.25	-5.166	0	0.75
1-0.25	-6.158	0	1
1.25-0.25	-6.782	0	1.25
1.5-0.25	-7.25	0	1.5
1.75-0.25	-7.543	0	2
2-0.25	-7.679	0	2.75
0.75-0.5	-1.392	0.139	No sig. differences
1-0.5	-2.383	0	1
1.25-0.5	-3.008	0	1.25
1.5-0.5	-3.476	0	1.5

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1.75-0.5	-3.769	0	1.75
2-0.5	-3.905	0	2
1-0.75	-0.992	0.56	No sig. differences
1.25-0.75	-1.616	0.044	1.25
1.5-0.75	-2.084	0.002	1.5
1.75-0.75	-2.377	0	1.75
2-0.75	-2.513	0	2
1.25-1	-0.624	0.935	No sig. differences
1.5-1	-1.093	0.428	No sig. differences
1.75-1	-1.386	0.143	No sig. differences
2-1	-1.521	0.074	No sig. differences
1.5-1.25	-0.468	0.987	No sig. differences
1.75-1.25	-0.761	0.834	No sig. differences
2-1.25	-0.897	0.682	No sig. differences
1.75-1.5	-0.293	0.999	No sig. differences
2-1.5	-0.429	0.992	No sig. differences
2-1.75	-0.136	1	No sig. differences

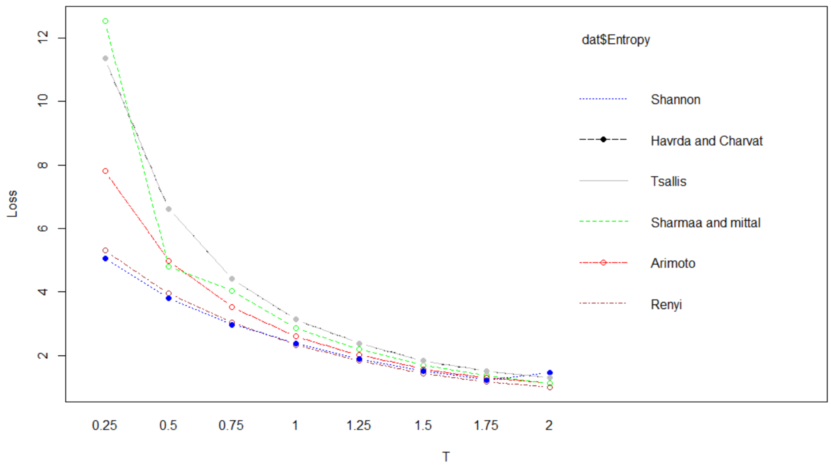

Figure 27: Relative loss according to T

So, performance of entropy measures according to T is sorted in ascending order as follows: Renyi, Arimoto, Sharma and Mittal, Tsallis, Havrda and Charvat then Shannon.

Table 12: TukeyHSD for Alpha

Alpha	diff	p adj	The best Alpha
1.5-0.5	3.504	0	0.5

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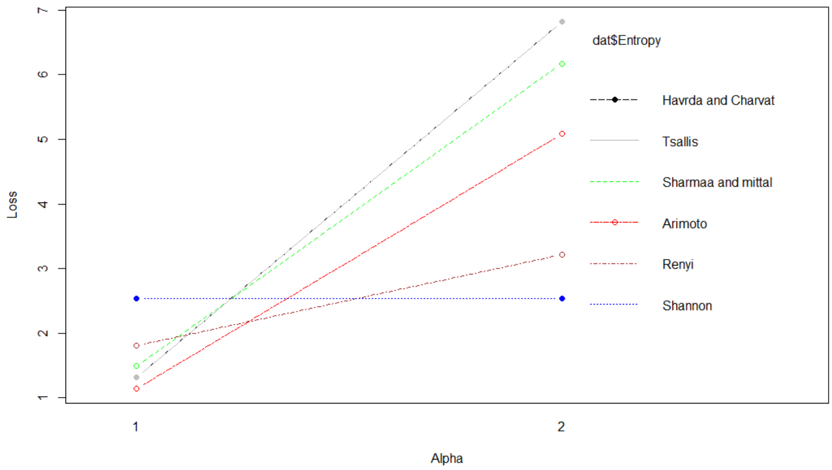

Figure (28): Relative loss of entropies according to α

So according to different values of alpha we see that performance of entropy measures is sorted ascending as follows: Renyi, Arimoto, Sharma and Mittal, Tsallis, Havrda and Charvat, Shannon.

5. Conclusions and future research directions

We have calculated and compared six different measures of entropy and their relative loss for the noncentral Fisher distribution. The comparison was done according to all existed parameters. We conclude that the most suitable entropy measure for noncentral Fisher distribution which has the lowest significant relative loss is Renyi entropy, then comes Shannon, Arimoto, Sharma and Mittal, Havrda and Charvat and at last Tsallis.

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