



# Neutrosophic Pythagorean Fuzzy Shortest Path in a Network

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## Abstract

We began a novel technique to dealing with the Neutrosophic Pythagorean shortest route problem in a network in this paper by representing each edge weight as a triangular fuzzy Pythagorean number with dependent Neutrosophic components and Pythagorean fuzzy graph condition.  $0 \leq \mu_1(v'_i)^2 + \beta_1(v'_i)^2 + \sigma_1(v'_i)^2 \leq 2$ . The main purpose of this article is to show how to use Neutrosophic Pythagorean fuzzy graphs. As a result, we created the proposed method, which also delivers the shortest path length from the source node (SN) to the destination node by using a ranking function for the Neutrosophic Pythagorean fuzzy Triangular number. Finally, an illustrative instance is supplied for validation.

**Keywords:** Neutrosophic Pythagorean fuzzy Triangular number (NPFTN); Score function (SF); Accuracy function (AF); Neutrosophic Pythagorean shortest path problem (NPSPP).

## 1. Introduction

Finding a route among two vertices (or nodes) in a graph that causes the sum of the weights of its individual edges is reduced is known as the "shortest path issue" in graph theory. A special form of the shortest path (SP) issue in graphs, with the vertices denoting crossings and the edges denoting road segments, each weighted by the segment's lengths, may be used to mimic the challenge of determining the shortest path between two crossings on a road map.

In order to deal with erroneous data in SP problems, Zadeh [11], created the fuzzy set theory. As a result, academics have tried a number of alternative approaches addressing specific SP problems in fuzzy environments. Okada [16], suggested an approach for solving the fuzzy SP issue based on possibilities theory to ascertain the level of possibility for each arc. In order to solve the resultant problem, Keshavarz and Khorram [9], reduced the fuzzy SP issue to a bi-level problem is formulated and provided an effective solution based on the parametric SP problem. Dou et al. used hazy multi-criteria decision-making techniques based on similarity measures to tackle the fuzzy SP problem in the multiple restrictions network.

Using the graded mean integrated model of fuzzy integers, Deng et al. [3] enhanced the Dijkstra method to solve fuzzy SP issues. Also, other authors concentrated on finding the shortest path in a network that has various fuzzy arc costs using heuristic techniques. Fuzzy sets, however, can just express membership functions and are not capable of expressing non-membership functions. The degree of non-membership in this case is just the opposite of the degree of membership. The analysis was then expanded to include the non-membership degree thanks to Atanassov's [7], introduction of intuitionistic FS (IFS). In this instance, the total of the membership degree and the

non-membership degree is one or less. Several academics focus more on addressing SP problems using intuitionistic fuzzy arc costs in an IFS setting. Mukherjee took into account the SP problem in a fuzzy intuitionistic setting. In order to solve the IFSP problem, Geetharamani and Jayagowri [5], suggested an innovative algorithm that makes use of intuitionistic fuzzy shortest path length technique and similarity measure. In order to find the shortest path seen between source node and the final node, Biswas et al [18], invented an approach.

Smarandache [4], initially discussed neutrosophy in 1995, and he defined one of the most significant new computational formulas neutrosophic set theory—between 1999 and 2005. Fuzzy sets and intuitionistic fuzzy sets cannot handle problems including imprecise, incomplete, indeterminate, and distorted data. As a generalisation of the concepts of fuzzy sets and intuitionistic fuzzy sets, Smarandache created the concepts of neutrosophic set and neutrosophic logic. Three separate membership degrees—truth membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree—define the notion of a neutrosophic set (F). An approach for resolving the neutrosophic shortest path problem based on score function was proposed by Broumi et al. [23]. The same authors also proposed a study of a network's single valued triangular neutrosophic shortest path. Moreover, Broumi et al. [21], suggested creating an algorithm for identifying the shortest path on a network where the edge weights are expressed by bipolar neutrosophic values.

Finally, Ajay and Chellamani [2], was discussed about Pythagorean Neutrosophic fuzzy graphs. Out of the three possible dependency instances in a neutrosophic set, we select the one exceptional case known as a neutrosophic pythagorean set, in which indeterminacy is independent and truth and falsity are entirely dependents are  $0 \leq \mu_1(v_i')^2 + \beta_1(v_i')^2 + \sigma_1(v_i')^2 \leq 2$ . Then we apply neutrosophic pythagorean sets to fuzzy graphs and investigate neutrosophic pythagorean graph. In this paper, the Neutrosophic Pythagorean Fuzzy Shortest Path Problem is defined in terms of triangular numbers, and the destination nodes are determined using a ranking function.

## 2. PREFACE

### Definition 2.1 [2]

A Neutrosophic Pythagorean set with truth, falsity as dependent Neutrosophic components on non-empty universe

$$E = \{(e, \mu_E(e), \beta_E(e), \sigma_E(e)) : e \in U\}, \text{ where}$$

$\mu_E(e), \beta_E(e), \sigma_E(e) \in [0,1]$ ,  $0 \leq (\mu_E(e))^2, (\beta_E(e))^2, (\sigma_E(e))^2 \leq 2, \forall e \in U$ ,  $\mu_E(e), \beta_E(e), \sigma_E(e)$  are the degrees of membership, indeterminacy and non-membership respectively. Here  $\mu_E(e)$  and  $\sigma_E(e)$  are dependent and  $\beta_E(e)$  is independent.

### Definition 2.2 [2]

A Neutrosophic Pythagorean Fuzzy Graph (NPFPG) is  $G' = (V', E')$ , where  $V' = \{v'_1, v'_2, \dots, v'_n\}$  such that  $\mu'_1, \beta'_1$  and  $\sigma'_1$  from  $V' \rightarrow [0,1]$  with  $0 \leq \mu_1(v'_i)^2 + \beta_1(v'_i)^2 + \sigma_1(v'_i)^2 \leq 2, \forall v'_i \in V'$  signifies membership, indeterminacy and non-membership functions correspondingly and  $E' \subseteq V' \times V'$  where  $\mu_2, \beta_2, \sigma_2$  from  $V' \times V' \rightarrow [0,1]$  such that,

$$\mu_2(v'_i v'_j) \leq \mu_1(v'_i) \wedge \mu_1(v'_j)$$

$$\beta_2(v'_i v'_j) \leq \beta_1(v'_i) \wedge \beta_1(v'_j)$$

$$\sigma_2(v'_i v'_j) \leq \sigma_1(v'_i) \vee \sigma_1(v'_j)$$

$$\text{With } 0 \leq (\mu_2(v'_i v'_j))^2 + (\beta_2(v'_i v'_j))^2 + (\sigma_2(v'_i v'_j))^2 \leq 2, \forall (v'_i v'_j) \in E.$$

## 3. Neutrosophic Pythagorean Triangular Fuzzy Number and Algebraic Operations:

The neutrosophic Pythagorean triangular fuzzy values for provide a convenient notation for

$$\tilde{Z}' = \langle (\tilde{p}, \tilde{q}, \tilde{r}), (\tilde{s}, \tilde{t}, \tilde{u}), (\tilde{w}, \tilde{x}, \tilde{y}) \rangle$$

$$\text{where, } (\mu_{\tilde{p}_1}'(v'))^2, (\mu_{\tilde{p}_2}'(v'))^2, (\mu_{\tilde{p}_3}'(v'))^2 = (\tilde{p}, \tilde{q}, \tilde{r}),$$

$$(\beta_{\tilde{p}_1}'(v'))^2, (\beta_{\tilde{p}_2}'(v'))^2, (\beta_{\tilde{p}_3}'(v'))^2 = (\tilde{s}, \tilde{t}, \tilde{u}),$$

$$(\sigma_{\tilde{p}_1}(v'))^2, (\sigma_{\tilde{p}_2}(v'))^2, (\sigma_{\tilde{p}_3}(v'))^2 = (\tilde{w}, \tilde{x}, \tilde{y}).$$

**Definition 3.1**

A neutrosophic Pythagorean triangular fuzzy number (NPTFN)  $\tilde{Z}' = \langle (\tilde{p}, \tilde{q}, \tilde{r}), (\tilde{s}, \tilde{t}, \tilde{u}), (\tilde{w}, \tilde{x}, \tilde{y}) \rangle$  is said to be zero NPTFNs iff,

$$(\tilde{p}, \tilde{q}, \tilde{r}) = (0,0,0); (\tilde{s}, \tilde{t}, \tilde{u}) = (1,1,1); (\tilde{w}, \tilde{x}, \tilde{y}) = (1,1,1)$$

**Definition 3.2**

Let  $\tilde{Z}' = \langle \tilde{Z}'_1, \tilde{Z}'_2 \rangle$ , then  $\tilde{Z}'_1 = \langle (\tilde{p}_1, \tilde{q}_1, \tilde{r}_1), (\tilde{s}_1, \tilde{t}_1, \tilde{u}_1), (\tilde{w}_1, \tilde{x}_1, \tilde{y}_1) \rangle$  and

$\tilde{Z}'_2 = \langle (\tilde{p}_2, \tilde{q}_2, \tilde{r}_2), (\tilde{s}_2, \tilde{t}_2, \tilde{u}_2), (\tilde{w}_2, \tilde{x}_2, \tilde{y}_2) \rangle$  be two neutrosophic Pythagorean triangular fuzzy Values (NPTFV) in set of  $\mathcal{R}$ , and  $\lambda > 0$ . Next, the operational guidelines are,

$$(i) \tilde{Z}'_1 \oplus \tilde{Z}'_2 = \left\langle \sqrt{(\tilde{p}_1^2 + \tilde{p}_2^2 - \tilde{p}_1^2 \tilde{p}_2^2)}, \sqrt{(\tilde{q}_1^2 + \tilde{q}_2^2 - \tilde{q}_1^2 \tilde{q}_2^2)}, \sqrt{(\tilde{r}_1^2 + \tilde{r}_2^2 - \tilde{r}_1^2 \tilde{r}_2^2)}, (\tilde{s}_1 \tilde{s}_2, \tilde{t}_1 \tilde{t}_2, \tilde{u}_1 \tilde{u}_2), (\tilde{w}_1 \tilde{w}_2, \tilde{x}_1 \tilde{x}_2, \tilde{y}_1 \tilde{y}_2) \right\rangle$$

$$(ii) \tilde{Z}'_1 \otimes \tilde{Z}'_2 = \left\langle \sqrt{(\tilde{s}_1^2 + \tilde{s}_2^2 - \tilde{s}_1^2 \tilde{s}_2^2)}, \sqrt{(\tilde{t}_1^2 + \tilde{t}_2^2 - \tilde{t}_1^2 \tilde{t}_2^2)}, \sqrt{(\tilde{u}_1^2 + \tilde{u}_2^2 - \tilde{u}_1^2 \tilde{u}_2^2)}, \sqrt{(\tilde{w}_1^2 + \tilde{w}_2^2 - \tilde{w}_1^2 \tilde{w}_2^2)}, \sqrt{(\tilde{x}_1^2 + \tilde{x}_2^2 - \tilde{x}_1^2 \tilde{x}_2^2)}, \sqrt{(\tilde{y}_1^2 + \tilde{y}_2^2 - \tilde{y}_1^2 \tilde{y}_2^2)}, (\tilde{p}_1 \tilde{p}_2, \tilde{q}_1 \tilde{q}_2, \tilde{r}_1 \tilde{r}_2) \right\rangle$$

where  $\lambda > 0$ .

To compare the NPTFS grades, the concepts of the score function S and accuracy function H are used. It is possible to rank various NPTFS and idea routes.

**Definition 3.3**

Let  $\tilde{Z}'_1 = \langle (\tilde{p}_1, \tilde{q}_1, \tilde{r}_1), (\tilde{s}_1, \tilde{t}_1, \tilde{u}_1), (\tilde{w}_1, \tilde{x}_1, \tilde{y}_1) \rangle$  be a NPTFV then, the score function  $\bar{S}'(\tilde{A}'_1)$  and an accuracy function  $\bar{H}'(\tilde{A}'_1)$  of NPTFV are defined as follows.

$$(i) \bar{S}'(\tilde{A}'_1) = \frac{1}{12} [8 + (\tilde{p}_1^2 + 2\tilde{q}_1^2 + \tilde{r}_1^2) - (\tilde{s}_1^2 + 2\tilde{t}_1^2 + \tilde{u}_1^2) - (\tilde{w}_1^2 + 2\tilde{x}_1^2 + \tilde{y}_1^2)]$$

$$(ii) \bar{H}'(\tilde{A}'_1) = \frac{1}{4} [(\tilde{p}_1^2 + 2\tilde{q}_1^2 + \tilde{r}_1^2) - (\tilde{w}_1^2 + 2\tilde{x}_1^2 + \tilde{y}_1^2)]$$

Following that, the comparisons among two NPTFVs should be made in the order of their relationships.

**Definition 3.4**

Let  $\tilde{Z}'_1 = \langle (\tilde{p}_1, \tilde{q}_1, \tilde{r}_1), (\tilde{s}_1, \tilde{t}_1, \tilde{u}_1), (\tilde{w}_1, \tilde{x}_1, \tilde{y}_1) \rangle$  and  $\tilde{Z}'_2 = \langle (\tilde{p}_2, \tilde{q}_2, \tilde{r}_2), (\tilde{s}_2, \tilde{t}_2, \tilde{u}_2), (\tilde{w}_2, \tilde{x}_2, \tilde{y}_2) \rangle$  be two NPTFVs in the set of real numbers. Then, we define a ranking method as follows.

- (i) If  $\bar{S}'(\tilde{A}'_1) > \bar{S}'(\tilde{A}'_2)$ , then  $\tilde{A}'_1 > \tilde{A}'_2$ , that is,  $\tilde{A}'_1$  is superior to  $\tilde{A}'_2$ , denoted by  $\tilde{A}'_1 > \tilde{A}'_2$ .
- (ii) If  $\bar{S}'(\tilde{A}'_1) = \bar{S}'(\tilde{A}'_2)$ , and  $\bar{H}'(\tilde{A}'_1) > \bar{H}'(\tilde{A}'_2)$  then  $\tilde{A}'_1 > \tilde{A}'_2$ , that is  $\tilde{A}'_1$  is superior to  $\tilde{A}'_2$ , denoted by  $\tilde{A}'_1 > \tilde{A}'_2$

**Network Terminology: -**

Assume of a directed network  $G(V, E)$ . Consisting of a set of  $m$  directed edges  $E \subseteq V \times V$  and a finite set of nodes  $V = \{1,2,3, \dots, n\}$ . Each edge is identified by an ordered pair  $(i, j)$  with  $i, j \in V$  and  $i$  not equal to  $j$ . We take into consideration two nodes in this network, s and t, which stand for source node and the destination nodes,

respectively. We build a path with consecutive nodes and edges,  $\tilde{P}_{ij} = \{i = i, (i_1, i_2), i_2, \dots, (i_{l-1}, i_l), i_l = j\}$ . Every  $i \in V - \{S\}$  is presumed to have at least one path  $\tilde{P}_{s_i}$  in  $G(V, E)$ .

$\tilde{d}_{ij}$  denotes NPTFN associated with the edge  $(i, j)$ , the Pythagorean distance along the path  $\tilde{P}$  is given as  $\tilde{d}'(\tilde{P})$ , where  $d$  is the length required to travel  $(i, j)$  from  $i$  to  $j$ ,  $\tilde{d}'(\tilde{P}) = \sum_{(i,j \in \tilde{P})} \tilde{d}'_{ij}$ .

**Remark:** - A node  $i$  is said to be predecessor node  $j$  if,

- (i) Node  $i$  is directly connected to node  $j$ .
- (ii) The direction of path connecting node  $i$  and  $j$  from  $i$  to  $j$ .

#### 4. Neutrosophic Pythagorean Triangular Fuzzy Path Problem

The edge length in a network is regarded in this paper as a NPN, specifically a NPTFN. The SP algorithm is composed of six steps.

**Step – I:** Assume  $\tilde{d}'_1 = \langle (0,0,0), (1,1,1), (1,1,1) \rangle$  and label the source nod is nod 1 as

$$[\tilde{d}'_1 = \langle (0,0,0), (1,1,1), (1,1,1) \rangle, \dots]$$

**Step – II:** Find  $\tilde{d}'_j = \min\{\tilde{d}'_i \oplus \tilde{d}'_{ij}\}$ ,  $j = 2,3,4, \dots, n$ .

**Step – III:** Label nod  $j$  as  $[\tilde{d}'_j, r]$  if min occurs equivalent to the singular value of  $i$  i.e.,  $i = r$ . Choose any value of  $I$  since the Neutrosophic Pythagorean Triangular distances along the path is  $\tilde{d}'_j$  if minimum exists according to more than one value of  $i$  which denotes that there are more than one interval-valued Neutrosophic Pythagorean Paths (NPP) between source node and nod  $j$ .

**Step – IV:** Let the destination nod (nod  $n$ ) be labelled as  $[\tilde{d}'_n, l]$ , then the NPTFS distance between source nod is  $\tilde{d}'_n$ .

**Step – V:** Since destination nod is labelled as  $[\tilde{d}'_n, l]$ . So, to find the NPTFSP between source nod and destination nod, check the label of nod 1. Let it be  $[\tilde{d}'_n, \tilde{P}]$  now we check the label of nod  $\tilde{P}$  and so on. Rerun the same procedure until nod 1 is obtained.

**Step – VI:** Now the NPTFSP can be obtained by combining all the nodes obtained by the step – V.

**Remark 4.1**

Let  $\tilde{A}'_i = 1,2, \dots, n$  be a set of NPTFN, if  $\bar{S}'(\tilde{A}'_k) < \bar{S}'(\tilde{A}'_l)$ , for all  $i$ , the TPFN is the min of  $(\tilde{A}'_k)$ .

**Example 4.2**

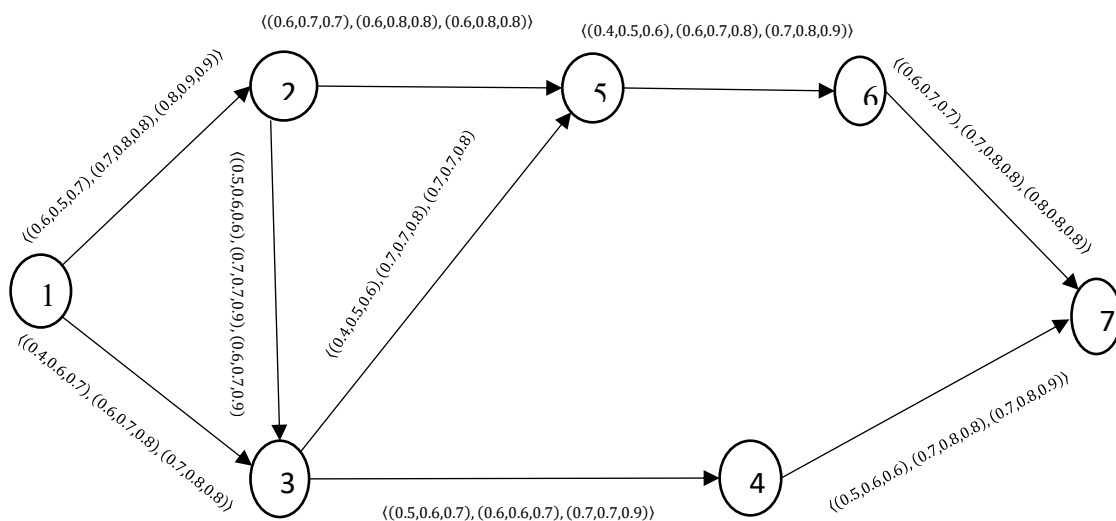


Figure 1: NPFT Directed Graph.

This network each edge have been assigned to the NPTFN as follows:

Table 1: Weights of the PNTFN.

| Node   | NPTFN   |
|--------|---|
| 1 - 2: | $\langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle$ |
| 1 - 3: | $\langle\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle\rangle$ |
| 2 - 3: | $\langle\langle(0.5,0.6,0.6), (0.7,0.7,0.9), (0.6,0.7,0.9)\rangle\rangle$ |
| 2 - 5: | $\langle\langle(0.6,0.7,0.7), (0.6,0.8,0.8), (0.6,0.8,0.8)\rangle\rangle$ |
| 3 - 5: | $\langle\langle(0.4,0.5,0.6), (0.7,0.7,0.8), (0.7,0.7,0.8)\rangle\rangle$ |
| 3 - 4: | $\langle\langle(0.5,0.6,0.7), (0.6,0.6,0.7), (0.7,0.7,0.9)\rangle\rangle$ |
| 5 - 6: | $\langle\langle(0.4,0.8,0.8), (0.7,0.7,0.7), (0.8,0.8,0.8)\rangle\rangle$ |
| 4 - 7: | $\langle\langle(0.5,0.6,0.6), (0.7,0.8,0.8), (0.7,0.8,0.9)\rangle\rangle$ |
| 6 - 7: | $\langle\langle(0.6,0.7,0.7), (0.7,0.8,0.8), (0.8,0.8,0.8)\rangle\rangle$ |

From the characteristics, edges and PNTF distance at the network are established. (Fig.1).

### Numerical Explanation:

Since nod 7 is the destination nod, so  $n = 7$ .

Consider  $\widetilde{d}_1' = \langle\langle(0,0,0), (1,1,1), (1,1,1)\rangle\rangle$  and label the source nod is nod 1 as  $[\langle\langle(0,0,0), (1,1,1), (1,1,1)\rangle\rangle, - - -]$ , then the values of  $\widetilde{d}_j$ ;  $j = 2,3,4,5,6,7$  can be attained:

#### Iteration 1:

When  $i = 1$  and  $j = 2$  are used in step 2 of the proposed algorithm (PA) so only node 1 is the predecessor node of node 2, the  $\widetilde{d}_2'$  value that is,

$$\begin{aligned}\widetilde{d}_2' &= \min \{ \widetilde{d}_1' \oplus \widetilde{d}_{12}' \} = \min \{ \langle\langle(0,0,0), (1,1,1), (1,1,1)\rangle\rangle \oplus \langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle \} \\ &= \min \{ \langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle \} = \langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle\end{aligned}$$

From the above calculation, min occurs comparable to the nod 1 and then label nod 2 as

$$[\langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle, 1]. \widetilde{d}_2' = \langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle$$

**Iteration 2:** The predecessor nod of nod 3 are nod 1 and nod 2, so put  $i = 1,2$  and  $j = 3$  is given by the PA, then the path value of  $\widetilde{d}_3'$  is,

$$\begin{aligned}\widetilde{d}_3' &= \min \{ \widetilde{d}_1' \oplus \widetilde{d}_{13}', \widetilde{d}_2' \oplus \widetilde{d}_{23}' \} \\ &= \min \left\{ \begin{array}{l} \langle\langle(0,0,0), (1,1,1), (1,1,1)\rangle\rangle \oplus \langle\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle\rangle, \\ \langle\langle(0.6,0.5,0.7), (0.7,0.8,0.8), (0.8,0.9,0.9)\rangle\rangle \oplus \langle\langle(0.5,0.6,0.6), (0.7,0.7,0.9), (0.6,0.7,0.9)\rangle\rangle \end{array} \right\}\end{aligned}$$

$$\widetilde{d}_3' = \min \left\{ \begin{array}{l} \langle\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle\rangle, \\ \langle\langle(0.7211,0.7211,0.821), (0.49,0.56,0.64), (0.48,0.63,0.81)\rangle\rangle \end{array} \right\}$$

From the above values using score functions, then the obtained value  $\widetilde{d}_3'$  is

$$\widetilde{d}_3' = [\langle\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle\rangle, 1], \widetilde{d}_3' = \langle\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle\rangle.$$

**Iteration 3:** The predecessor of nod 4 is node 3, so put the values of  $i = 3$  and  $j = 4$  respectively in step – 2 of the given PA, then the path value of  $\widetilde{d}_4'$  is,

$$\begin{aligned}\widetilde{d}_4' &= \min \{ \widetilde{d}_3' \oplus \widetilde{d}_{34}' \} \\ &= \min \{ \langle\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle\rangle \oplus \langle\langle(0.5,0.6,0.7), (0.6,0.6,0.7), (0.7,0.7,0.9)\rangle\rangle \}\end{aligned}$$

$$\widetilde{d}_4' = \langle\langle(0.6083,0.7684,0.8602), (0.36,0.42,0.56), (0.49,0.56,0.72)\rangle\rangle$$

**Iteration 4:** The predecessor nodes for nod 5 are nod 2 and nod 3, so put  $i = 2,3$  and  $j = 5$  in step – 2 of the given PA, then the path value of  $\widetilde{d}_5'$  is,

$$\begin{aligned} \widetilde{d}_5' &= \min \{ \widetilde{d}_2' \oplus \widetilde{d}_{25}', \widetilde{d}_3' \oplus \widetilde{d}_{35}' \} \\ &= \min \{ \langle (0.6, 0.5, 0.7), (0.7, 0.8, 0.8), (0.8, 0.9, 0.9) \rangle \oplus \langle (0.6, 0.7, 0.7), (0.6, 0.8, 0.8), (0.6, 0.8, 0.8) \rangle, \\ &\quad \langle (0.4, 0.6, 0.7), (0.6, 0.7, 0.8), (0.7, 0.8, 0.8) \rangle \oplus \langle (0.4, 0.5, 0.6), (0.7, 0.7, 0.8), (0.7, 0.7, 0.8) \rangle \} \end{aligned}$$

$$\widetilde{d}_5' = \min \{ \langle (0.7684, 0.7858, 0.8602), (0.42, 0.64, 0.64), (0.48, 0.72, 0.72) \rangle, \langle (0.5426, 0.7211, 0.8207), (0.42, 0.49, 0.64), (0.49, 0.56, 0.64) \rangle \}$$

by using score functions, then the obtained value  $\widetilde{d}_5'$  is

$$\widetilde{d}_5' = \langle (0.7684, 0.7858, 0.8602), (0.42, 0.64, 0.64), (0.48, 0.72, 0.72) \rangle$$

The above calculation, min occurs comparable to the nod 2 and then label nod 5 as  $\{ \langle (0.7684, 0.7858, 0.8602), (0.42, 0.64, 0.64), (0.48, 0.72, 0.72) \rangle, 2 \}$ .

**Iteration 5 :** The predecessor nod for nod 6 is nod 5, so put  $i = 5$  and  $j = 6$  in step – 2 of the given PA, then the path value of  $\widetilde{d}_6'$  is,

$$\begin{aligned} \widetilde{d}_6' &= \min \{ \widetilde{d}_5' \oplus \widetilde{d}_{56}' \} \\ &= \min \left\{ \langle (0.7684, 0.7858, 0.8602), (0.42, 0.64, 0.64), (0.48, 0.72, 0.72) \rangle \oplus \right. \\ &\quad \left. \langle (0.4, 0.8, 0.8), (0.7, 0.7, 0.7), (0.8, 0.8, 0.8) \rangle, \right\} \end{aligned}$$

$$\widetilde{d}_6' = \min \{ \langle (0.8099, 0.9286, 0.9520), (0.294, 0.448, 0.448), (0.384, 0.576, 0.576) \rangle \}$$

by using score functions, then the obtained value  $\widetilde{d}_6'$  is,

$$\widetilde{d}_6' = \langle (0.8099, 0.9286, 0.9520), (0.294, 0.448, 0.448), (0.384, 0.576, 0.576) \rangle$$

**Iteration 6 :** The predecessor nodes for nod 7 are nod 4 and nod 6, so put  $i = 4, 6$  and  $j = 7$  in step – 2 of the given PA, then the path value of  $\widetilde{d}_7'$  is,

$$\begin{aligned} \widetilde{d}_7' &= \min \{ \widetilde{d}_4' \oplus \widetilde{d}_{47}', \widetilde{d}_6' \oplus \widetilde{d}_{67}' \} \\ &= \min \left\{ \begin{array}{l} \langle (0.6083, 0.7684, 0.8602), (0.36, 0.42, 0.56), (0.49, 0.56, 0.72) \rangle \\ \oplus \langle (0.5, 0.6, 0.6), (0.7, 0.8, 0.8), (0.7, 0.8, 0.9) \rangle, \\ \langle (0.8099, 0.9286, 0.9520), (0.294, 0.448, 0.448), (0.384, 0.576, 0.576) \rangle \oplus \\ \langle (0.6, 0.7, 0.7), (0.7, 0.8, 0.8), (0.8, 0.8, 0.8) \rangle \end{array} \right\} \\ \widetilde{d}_7' &= \min \left\{ \langle (0.7263, 0.8589, 0.9129), (0.252, 0.336, 0.448), (0.343, 0.448, 0.648) \rangle, \right. \\ &\quad \left. \langle (0.8831, 0.9642, 0.9758), (0.2058, 0.3584, 0.3584), (0.3072, 0.4608, 0.4608) \rangle \right\} \end{aligned}$$

by using score functions, then the obtained value  $\widetilde{d}_7'$  is,

$$\widetilde{d}_7' = \langle (0.7263, 0.8589, 0.9129), (0.252, 0.336, 0.448), (0.343, 0.448, 0.648) \rangle$$

The above calculation, min occurs comparable to the nod 4 and then label node 7 as  $\{ \langle (0.7263, 0.8589, 0.9129), (0.252, 0.336, 0.448), (0.343, 0.448, 0.648) \rangle, 4 \}$ .

Then the path  $1 \rightarrow 3 \rightarrow 4 \rightarrow 7$  is identified the shortest path and the shortest distance between source nod and destination nod for

$$\langle (0.7263, 0.8589, 0.9129), (0.252, 0.336, 0.448), (0.343, 0.448, 0.648) \rangle$$

By using the given proposed algorithm, shortest path for all nodes from source node and labelling is given:

Table 2: Neutrosophic Pythagorean Triangular Fuzzy distance and Shortest Path.

| Node | $\widetilde{d}_i'$  | NPTFSP between $j^{\text{th}}$ and $1^{\text{st}}$ nod |
|------|---|--|
| 2    | $\langle (0.6, 0.5, 0.7), (0.7, 0.8, 0.8), (0.8, 0.9, 0.9) \rangle$ | $1 \rightarrow 2$                                      |

|   |  |   |
|---|--|---|
| 3 | $\langle(0.4,0.6,0.7), (0.6,0.7,0.8), (0.7,0.8,0.8)\rangle$                      | $1 \rightarrow 3$                             |
| 4 | $\langle(0.6083,0.7684,0.8602), (0.36,0.42,0.56), (0.49,0.56,0.72)\rangle$       | $1 \rightarrow 3 \rightarrow 4$               |
| 5 | $\langle(0.7684,0.7858,0.8602), (0.42,0.64,0.64), (0.48,0.72,0.72)\rangle$       | $1 \rightarrow 2 \rightarrow 3 \rightarrow 5$ |
| 6 | $\langle(0.8099,0.9286,0.9520), (0.294,0.448,0.448), (0.384,0.576,0.576)\rangle$ | $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$ |
| 7 | $\langle(0.7263,0.8589,0.9129), (0.252,0.336,0.448), (0.343,0.448,0.648)\rangle$ | $1 \rightarrow 3 \rightarrow 4 \rightarrow 7$ |

## 5. Conclusion

A method to solve the shortest path problem on a network with neutrosophic Pythagorean triangular fuzzy edges has been devised in this paper. With the aid of fictitious data, we have used an example to demonstrate the methodology. The following algorithm for the neutrosophic Pythagorean triangular fuzzy number shortest path issue in a neutrosophic pythagorean trapezoidal environment will also be extended.

## References

- [1] Nagoor Gani, M. Mohammed Jabarulla, On Searching Intuitionistic Fuzzy Shortest path in a Network, Applied Mathematical Sciences, Vol.4, no.69,3447-3454, 2010.
- [2] D. Ajay, P. Chellamani, International Journal of Neutrosophic Science, Vol 11, No. 2, PP. 108 – 114, 2020.
- [3] Deng Y, Chen Y, Zhang Y, Mahadevan S, Fuzzy Dijkstra algorithm for shortest path problem under uncertain environment. Appl Soft Comput: 12:1231–1237, 2012.
- [4] F. Smarandache, “A geometric interpretation of the neutrosophic set — A generalization of the intuitionistic fuzzy set,” Granular Computing, IEEE International Conference, pp.602 – 606, 2011.
- [5] Geetharamani G, Jayagowri P, Using similarity degree approach for shortest path in Intuitionistic fuzzy network, In: International conference on computing, communication and applications (ICCCA), IEEE, Dindigul, Tamilnadu, India, pp 1–6, 2012.
- [6] H. Garg (ed.), (2021). Pythagorean Fuzzy Sets, Theory and Applications, under exclusive license to Springer Nature Singapore Ptr Ltd.
- [7] K.T. Atanassov (2019). Intuitionistic Fuzzy Sets, Springer.
- [8] Karunambigai MG, Ranagasmay P, Atanassov K, Palaniappan N, an intuitionistic fuzzy graph method for finding the shortest paths in networks. Advances in soft Computing: 42:3 – 10, 2007.
- [9] Keshavarz E, Khorram E, A fuzzy shortest path with the highest reliability. J Comput Appl Math: 230:204–212, 2009.
- [10] Kumar A., Kaur M, A new algorithm for solving network flow with fuzzy arc lengths. An official Journal of Turkish Fuzzy systems Association: 2(1); 1 – 13, 2011.
- [11] L.A. Zadeh, Fuzzy Sets, Information and Control 8(3), 338-353, 1965.
- [12] Lazim Abdullah, Pinxin Goh, Decision making method based on Pythagorean fuzzy sets and its application to solid waste management, Complex and Intelligent System, 5:185-198, 2019.
- [13] L. Sujatha, Hyacinta JD, The Shortest Path Problem on Networks with Intuitionistic Fuzzy Edge Weights, Global Journal of Pure and Applied Mathematics, Vol 13, No.7, PP. 3285-3300, 2017.
- [14] M. Muhammad Akram, Farwa Ilyasa and Arsham Borumand Saeid, Certain Notions of Pythagorean Fuzzy Graphs, Journal of Intelligent and Fuzzy Systems 36, 5857-5874, 2019.
- [15] M. Muhammad Akram, W.A. Dudek, F Ilyas (2019). Group decision-making based on Pythagorean fuzzy TOPSIS method. International Journal of Intelligent system.
- [16] Okada S, Fuzzy Shortest path problems incorporating among paths. Fuzzy sets and Systems: 142:335 – 357, 2004.
- [17] P. Jayagowri and G. Geetha Ramani, (2014). Using Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network, Hindawi Publishing Corporation, Advances in Fuzzy Systems, Article ID 183607.
- [18] P. Biswas, S. Pramanik and B. C. Giri, Aggregation of Triangular Fuzzy Neutrosophic Set Information and its Application to Multiattribute Decision Making, Neutrosophic sets and systems, 12, pp.20-40, 2016.
- [19] R.R. Yager, Pythagorean fuzzy subsets, In: Proc Joint IFSA world congress and NAFIPS Annual Meeting, Edmonton, Canada, PP.57-61, 2013.
- [20] R.R. Yager, Pythagorean membership grades in decision making, IEEE Transactions on Fuzzy Systems 22(4), 958-965, 2014.
- [21] S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu, Shortest path problem under triangular fuzzy neutrosophic information, 10<sup>th</sup> International Conference on software, Knowledge, Information Management & Applications, PP. 169 – 174, 2016.

- [22] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "On Bipolar Single Valued Neutrosophic Graphs," *Journal of New Theory*, N11, pp.84-102, 2016.
- [23] S. Broumi, F. Smarandache, M. Talea and A. Bakali, An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. *Applied Mechanics and Materials*, vol.841, 184-191, 2016.
- [24] S. Broumi, M. Talea, A. Bakali, F. Smarandache, "Single Valued Neutrosophic Graphs," *Journal of New Theory*, N 10, pp. 86-101, 2016.
- [25] S. Broumi, M. Talea, F. Smarandache and A. Bakali, Single Valued Neutrosophic Graphs: Degree, Order and Size. *IEEE World Congress on Computational Intelligence*, 2016, pp.2444-2451, 2016.
- [26] S. Broumi, A. Bakali, M, Talea, and F, Smarandache, Isolated Single Valued Neutrosophic Graphs. *Neutrosophic Sets and Systems*, Vol. 11, pp.74-78, 2016.
- [27] Sujatha L, Hyacinta JD, The SPP on Networks with Intui Fuzzy Edge Weights, *Global Journal of Pure and Applied Mathematics: Vol 13, No.7, PP. 3285-3300, 2017.*
- [28] X. Zhang and Z. Xu, Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets, *International Journal of Intelligent Systems* 29(12), 1061-1078, 2014.
- [29] Xin Zhang, Peide Liu, Method for Aggregating Triangular Fuzzy Intuitionistic Fuzzy Information and its Application to Decision Making, *Baltic Journal on Sustainability*, 16(2); 280-290, 2010.