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On Intuitionistic Fuzzy Subgroups of (M-N) Type and Their Algebraic Properties

Ahmed Hatip

Gaziantep University, Department of Mathematics, Gaziantep, Turkey

Email: kollnaar5@gmail.com

Abstract

The objective of this paper is to define the intuitionistic fuzzy subgroup of type (M-N) and to study some of its elementary properties and substructures such as normality, direct and inverse images.Also, many related theorems and examples will be presented and illustrated.

Keywords: fuzzy set; intuitionistic fuzzy set; (M-N) subgroup

1. Introduction and Preliminaries

The concept of fuzzy set was suggested by Zadeh in 1965, as a generalization of the classical logic approach, and then it was generalized by many concepts such as intuitionistic fuzzy sets, fuzzy sets, and their structures [1-3].

The algebraic structures generated by fuzzy sets and their extensions were studied widely, for example, we have fuzzy groups, kernels, intuitionistic fuzzy groups, and M-intuitionistic fuzzy groups [4-7].

First of all, we recall some basic concepts.

Definition:

A group G is called (M-N) if there exists two sets M and N such that:

1) M acts on G by
$M \times G \to G$

$(m, g) \to mg.$

2) N acts on G by
$G \times N \to G$

$(\boldsymbol{g}, \boldsymbol{n}) \to \boldsymbol{gn}.$

3) The following condition is true $(\boldsymbol{mg})\boldsymbol{n} = \boldsymbol{m}(\boldsymbol{gn}) \quad \forall \boldsymbol{m} \in \boldsymbol{M}, \boldsymbol{n} \in \boldsymbol{N}, \boldsymbol{g} \in \boldsymbol{G}.$

Definition:

Let G_1, G_2 be two (M-N) groups, and $f: G_1 \to G_2$ is a homomorphism, then f is called (M-N) homomorphism if:

$\boldsymbol{f}(\boldsymbol{mx}) = \boldsymbol{mf}(\boldsymbol{x}), \boldsymbol{f}(\boldsymbol{xn}) = \boldsymbol{f}(\boldsymbol{x})\boldsymbol{n} \ \forall \boldsymbol{m} \in \boldsymbol{M}, \boldsymbol{n} \in \boldsymbol{N}, \boldsymbol{x} \in \boldsymbol{G_1}.$

Definition:

Let X be a non-empty set, an intuitionistic fuzzy subset A is defined as follows: $A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle, \forall x \in X\}$, where $\mu_A: X \to [0,1]$ is a membership function and $\vartheta_A: X \to [0,1]$ is a non-membership function with

$0 \le \mu_A(x) + \vartheta_A(x) \le 1.$

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Definition:

Let X be a non-empty set, and consider

$\boldsymbol{B} = \{\langle x, \mu_B(x), \vartheta_B(x)\rangle, \forall x \in X\}$, $A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle, \forall x \in X\}$, as two intuitionistic fuzzy subsets of X, then

$\overline{\boldsymbol{A}} = \{\langle x, \vartheta_A(x), \mu_A(x)\rangle, \forall x \in X\}$

$\boldsymbol{A} = \boldsymbol{B}$ $\boldsymbol{iff}$ $\forall x \in X : \mu_A(x) = \mu_B(x), \vartheta_A(x) = \vartheta_B(x)$

$\boldsymbol{AB}$ $\boldsymbol{iff}$ $\forall x \in X : \mu_A(x) \le \mu_B(x), \vartheta_A(x) \ge \vartheta_B(x)$.

Definition:

Let X, Y be two non-empty subsets, and the mapping $f: X \to Y$, let A,B be two intuitionistic fuzzy subsets of X,Y respectively, then we define:

$f(A) = \{\langle y, f(\mu_A)(y), f(\vartheta_A)(y)\rangle : y \in Y\}$,

$f(\mu_A)(y) = \begin{cases} Sup_{x \in f^{-1}(y)}\ \mu_A(x) & : f^{-1}(y) \neq 0 \\ 0 & : \quad otherwise \end{cases}$,

$f(\vartheta_A)(y) = \begin{cases} inf_{x \in f^{-1}(y)}\ \vartheta_A(x) & : f^{-1}(y) \neq 0 \\ 0 & : \quad otherwise \end{cases}$,

$f^{-1}(B)(x) = \mu_{f^{-1}(B)}(x), \vartheta_{f^{-1}(B)}(x) = \left(\mu_{(B)}\big(f(x)\big), \vartheta_{(B)}\big(f(x)\big)\right)$,

Definition:

Let G be a group, and $A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle, \forall x \in G\}$ be an intuitionistic fuzzy subset, then A is called an intuitionistic fuzzy subgroup if:

$\mu_A(xy) \ge min\{\mu_A(x), \mu_A(y)\}$,

$\vartheta_A(xy) \le Max\{\vartheta_A(x), \vartheta_A(y)\}$ $\forall x, y \in G$,

$\mu_A(x^{-1}) = \mu_A(x)$, $\vartheta_A(x^{-1}) = \vartheta_A(x)$ $\forall x \in G$,

Main discussion

Definition:

Let G be an (M-N) group and A be an intuitionistic fuzzy subgroup of G, we say that A is an (M-N) intuitionistic fuzzy subgroup if

$$\mu_A(mx) \ge \mu_A(x), \vartheta_A(mx) = \vartheta_A(x)$$

$\mu_A(xn) \ge \mu_A(x), \vartheta_A(xn) = \vartheta_A(x)$ $\forall n \in N, m \in M, x \in G$.

Example:

Consider the group G=U(10)={1,3,7,9}, and M={1,3}, N={9,13}, we define:

$(m, g) \to (m \times g)\ mod2$, $(g, n) \to (g \times n)\ mod4$,

$A = [\langle 1,0.8,0.1\rangle, \langle 3,0.6,0.2\rangle, \langle 7,0.6,0.2\rangle\langle 9,0.5,0,3\rangle]$, it is clear that A is an intuitionistic fuzzy (M-N) subgroup.

Theorem:

The intersection of two intuitionistic fuzzy (M-N) subgroups of G is an (M-N) intuitionistic fuzzy subgroup.

Proof:

Let A, B be two (M-N) intuitionistic fuzzy subgroups, it is known that $A \cap B$ is an intuitionistic fuzzy subgroup.

$$\forall x \in G, m \in M, n \in N :$$

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$$\mu_{A\cap B}(mx) = \min(\mu_A(xn), \mu_B(mx)) \geq \min(\mu_A(x), \mu_B(x)) = \mu_{A\cap B}(x)$$

$$\mu_{A\cap B}(xn) = \min(\mu_A(xn), \mu_B(mx)) \geq \min(\mu_A(x), \mu_B(x)) = \mu_{A\cap B}(x)$$

$$\vartheta_{A\cap B}(mx) = \max(\vartheta_A(xn), \vartheta_B(xn)) \leq \max(\vartheta_A(x), \vartheta_B(x)) = \vartheta_{A\cap B}(x)$$

$$\vartheta_{A\cap B}(xn) = \max(\vartheta_A(xn), \vartheta_B(xn)) \leq \max(\vartheta_A(x), \vartheta_B(x)) = \vartheta_{A\cap B}(x)$$

So that, the proof is complete.

Theorem:

Let $f: G_1 \to G_2$ be an (M-N) homomorphism, then:

1)The direct image of an intuitionistic fuzzy (M-N) subgroup is an intuitionistic (M-N) fuzzy subgroup.

2)The inverse image of an intuitionistic fuzzy (M-N) subgroup is an intuitionistic (M-N) fuzzy subgroup.

Proof:

Let A,B be two intuitionistic fuzzy (M-N) subgroups of G_1, G_2, we have:

$A = \{\langle y, f(\mu_A)(y), f(\vartheta_A)(y)\rangle, y \in Y\}$,

$\mu_{f(A)}(my) = Sup_{z\in f^{-1}(my)}\mu_{(A)}(z) \geq Sup_{mx\in f^{-1}(my)}\mu_{(mx)}$,

$= Sup_{f(mx)=(my)}\mu_{(A)}(mx) = Sup_{mf(x)=(my)}\mu_{(A)}(mx)$,

$\geq Sup_{f(x)=(y)}\mu_{(A)}(x) = \mu_{f(A)}(y)$. Also,

$\mu_{f(A)}(yn) = Sup_{z\in f^{-1}(yn)}\mu_{(A)}(z) \geq Sup_{xn\in f^{-1}(yn)}\mu_{(mx)}$,

$= Sup_{f(xn)=(yn)}\mu_{(A)}(xn) = Sup_{f(x)n=(yn)}\mu_{(A)}(xn)$,

$\geq Sup_{f(x)=(y)}\mu_{(A)}(x) = \mu_{f(A)}(y)$,

Thus $(x \in f^{-1}(y))$.

Now,

$\vartheta_{f\,A}(my) = inf_{z\in f^{-1}(my)}\vartheta_A(z) \leq inf_{mx\in f^{-1}(my)}\vartheta_A(mx)$,

$= inf_{f(mx)=(my)}\,\vartheta_A(mx) = inf_{mf(x)=(my)}\,\vartheta_A(mx)$,

$\leq inf_{f(x)=(y)}\,\vartheta_A(mx) = \vartheta_{fA}(y)$,

thus

$\vartheta_{f\,A}(yn) = inf_{z\in f^{-1}(yn)}\vartheta_A(z) \leq inf_{xn\in f^{-1}(yn)}\vartheta_A(xn)$,

$= inf_{f(xn)=(yn)}\,\vartheta_A(xn) = inf_{f(x)n=(yn)}\,\vartheta_A(xn)$,

$\leq inf_{f(x)=(y)}\,\vartheta_A(x) = \vartheta_{fA}(y)$,

$(x \in f^{-1}(y))$

On the other hand,

$$f^{-1}(B)(x) = \left(\mu_{f^{-1}(B)}(x), \vartheta_{f^{-1}(B)}(x)\right) = (\mu_B(f(x)), \vartheta_B(f(x))$$

$$\mu_{f^{-1}(B)}(mx) = \mu_B(f(mx)) \geq \mu_B(f(x)) = \mu_{f^{-1}(B)}(x)$$

$$\mu_{f^{-1}(B)}(mx) = \mu_B(f(mx)) \geq \mu_B(f(x)) = \mu_{f^{-1}(B)}(x)$$

$$\vartheta_{f^{-1}(B)}(mx) = \vartheta_B(f(mx)) \geq \vartheta_B(f(x)) = \vartheta_{f^{-1}(B)}(x)$$

$$\vartheta_{f^{-1}(B)}(xn) = \vartheta_B(f(xn)) \geq \vartheta_B(f(x)) = \vartheta_{f^{-1}(B)}(x)$$

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$\forall x \in G_1, m \in M, n \in N$

Definition:

Let G be an (M-N) group and B is an intuitionistic (M-N) fuzzy subgroup of G, we equip the set $G/B = \{xB: x \in G\}$ by the following law: $(xB)^\circ(yB) = (xy)B \;\; \forall x, y \in G$.

Theorem:

G/B is an (M-N) group.

Proof:

We define: $M \times G/B \to G/B$, with $(m, gB) \to m(gB) = (mg)B$, and

$G/B \times N \to G/B$, with
$(gB, n) \to (gB)n = (gn)B$, we have:

$\forall aB \in G/B \;, m \in M, n \in N$:

$\big(m(aB)\big)n = \big((ma)B\big)n = \big((ma)n\big)B$

$= \big(m(an)\big)B$

$\big(m(an)\big)B = m\big((aB)n\big)$.

Definition:

Let be an (M-N) group with A as an intuitionistic (M-N) fuzzy subgroup, we define:

$A/B = \left(\mu_{A/B}\,, \vartheta_{A/B}\right) \;\; : \;\; A/B : G/B \to [0,1]$

$\mu_{A/B}(aB) = Sup_{xB=aB}\mu_A(x)$

$\vartheta_{A/B}(aB) = inf_{xB=aB}\vartheta_A(x) \qquad \forall aB \in G/B$

Theorem:

A/B is an intuitionistic (M-N) fuzzy subgroup of G/B.

Proof:

A/B is an intuitionistic fuzzy subgroup of G/B, that is because:

$$\forall xB, yB \in G/B : \mu_{A/B}\big((xB)(yB)\big) = \mu_{A/B}\big((xy)B\big) = Sup_{zB=xyB}\mu_A(z)$$

$$= Sup_{abB=xyB}\mu_A(ab) = Sup_{\substack{aB=xB\\ bB=yB}}\mu_A(ab)$$

$$\geq \min\big\{Sup_{aB=xB}\mu_A(a), Sup_{bB=yB}\mu_A(b)\big\}$$

$$\geq \min\big\{\mu_{A/B}(xB), \mu_{A/B}(yH)\big\}$$

$$\forall\, xB, yB \in G/B: \vartheta_{A/B}\big((xB)(yB)\big) = \vartheta_{A/B} = \big((xy)B\big) = inf_{zB=xyB}\vartheta_A(z)$$

$$= inf_{abB=xyB}\vartheta_A(ab) = inf_{\substack{aB=xB\\ bB=yB}}\vartheta_A(ab)$$

$$\leq Max\,\{inf_{aB=xB}\vartheta_A(a), inf_{bB=yB}\vartheta_A(b)\}$$

$$\leq Max\big\{\vartheta_{A/B}(xB), \vartheta_{A/B}(yB)\big\}.$$

$$\forall (xB)^{-1} \in G/B : \mu_{A/B}((xB)^{-1}) = \mu_{A/B}(x^{-1}B)$$

$$Sup_{a^{-1}B=x^{-1}B}\mu_A(a^{-1}) = Sup_{aB=xB}\mu_A(a) = \mu_{A/B}(xB).$$

$$\forall (xB)^{-1} \in G/B : \vartheta_{A/B}((xB)^{-1}) = \vartheta_{A/B}(x^{-1}B)$$

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$$inf_{a^{-1}B=x^{-1}B}\vartheta_A(a^{-1}) = inf_{aB=xB}\vartheta_A(a) = \vartheta_A(xB)$$

On the other hand,

$$\forall \text{ aB} \in G/B\,, m \in M, n \in N:$$

$$\mu_{A/B}(\boldsymbol{m}(\boldsymbol{aB})) = \mu_{A/B}((ma)B) = Sup_{mxB=maB}\mu_A(mx)$$

$$\geq Sup_{xB=aB}\mu_A(x) = \mu_{A/B}(aB)$$

$$\mu_{A/B}((\boldsymbol{aB})\boldsymbol{n}) = \mu_{A/B}((an)B) = Sup_{xnB=anB}\mu_A(xn)$$

$$\geq Sup_{xB=aB}\mu_A(x) = \mu_{A/B}(aB)$$

$$\vartheta_{A/B}(\boldsymbol{m}(\boldsymbol{aB})) = \vartheta_{A/B}((ma)B) = inf_{mxB=maB}\vartheta_A(mx)$$

$$\leq inf_{xB=aB}\vartheta_A(x) = \vartheta_{A/B}(aB)$$

$$\vartheta_{A/B}((\boldsymbol{aB})\boldsymbol{n}) = \vartheta_{A/B}((an)B) = inf_{xnB=anB}\vartheta_A(xn)$$

$$\leq inf_{xB=aB}\vartheta_A(x) = \vartheta_{A/B}(aB)$$

Thus, the proof is complete.

Theorem:

Let G/B be an (M-N) group, A,C be two intuitionistic fuzzy (M-N) subgroups of G, then

$C \subseteq A \Rightarrow C/B \subseteq A/B$ .i
$C/B \cap A/B = C \cap A/B$ .ii

Proof:

$\forall xB \in G/B\ \mu_{A/B}(xB) = Sup_{xB=zB}\mu_A(z)$

$\geq Sup_{xB=zB}\mu_c(z) = \mu_{c/B}(xB)$

$\forall xB \in G/B\ \vartheta_{A/B}(xB) = inf_{xB=zB}\vartheta_A(z)$

$C/B \subseteq A/B\,, \leq inf_{xB=zB}\vartheta_c(z) = \vartheta_{C/B}(xB)$

$\boldsymbol{II}$ **.** $\forall xB \in G/B\ \mu_{A\cap C/B}(xB) = Sup_{xB=zB}\mu_{A\cap C}(z)$

$Sup_{xB=zB}\{\boldsymbol{min}\{\mu_A(z), \mu_C(z)\}\}$

$\boldsymbol{min}\{Sup_{xB=zB}\mu_A(z), Sup_{xB=zB}\mu_c(z)\}$

$\boldsymbol{min}\{\mu_{A/B}(z), \mu_{C/B}(z)\} = \mu_{A/B\cap C/B}(xB)$

$\forall xB \in G/B\ \vartheta_{A\cap C/B}(xB) = inf_{z\in xH}\vartheta_{A\cap C}(z)$

$inf_{xB=zB}\{\boldsymbol{max}\{\vartheta_A(z), \vartheta_C(z)\}\}$

$\boldsymbol{max}\{inf_{xB=zB}\vartheta_A(z), inf_{xB=zB}\vartheta_c(z)\}$

$\boldsymbol{max}\{\vartheta_{A/B}(z), \vartheta_{C/B}(z)\} = \vartheta_{A/B\cap C/B}(xB)$

References

[1].Zhan, J. Tan, Z. 2004. Intuitionistic *M*-Fuzzy Groups-Soochow Journal of Mathematics, V 30, pp. 85-90.
[2].Atanassov, K., 1986. *Intuitionistic fuzzy sets*-Fuzzy Sets and Systems, V. 20, PP. 87-96.
[3].Sharma, P., 2011. (α, β)-*Cut* of intuitionistic fuzzy groups - International

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Mathematical Forum, V.6, pp. 2605-2614.
[4].Xiong, Q., 1978. Modern algebra - science and technology publishment in shang hai.
[5].Sharma, P., 2012. On the Direct Product of Intuitionistic Fuzzy Subgroups- International Mathematical Forum, V.7, pp. 523-530.
[6].Mordeson, J. Wierman, M. Clark, T. Pham, A. Redmond, M., 2012. Linear Models in the Mathematics of Uncertainty- Springer- Verlag.
[7].Sharma, P., 2011. Homomorphism of Intuitionistic Fuzzy Groups- International Mathematical Forum, V. 6,PP. 3169 - 3178.

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