Algorithms for Computing Pythagoras Triples and 4-Tiples in Some Neutrosophic Commutative Rings

Hamiyet Merkepci, Ahmed Hatip

Department of Mathematics , Gaziantep University, Gaziantep, Turkey
Emails: hamiyetmerkepci@gmail.com; Kollnaar5@gmail.com

Abstract

This paper is dedicated to study the number theoretical Pythagoras triples\4-tiples problem in several kinds of neutrosophic algebraic systems, where it finds an algorithm to find Pythagoras triples\4-tiples in commutative neutrosophic rings and refined neutrosophic rings too. Besides, the necessary and sufficient condition for a triple\4-tiple to be Pythagoras triple\4-tiple (quadruples) is obtained and proven in term of theorems. In addition, many numerical examples will be illustrated.

Keywords: Neutrosophic ring; refined neutrosophic ring; Pythagoras triple; neutrosophic number theory

1. Introduction

Neutrosophic logic was supposed by Smarandache [1] as a new insight through fuzzy generalizations. This concept has found its way into algebraic structures, where we find many neutrosophic algebraic structures built over the idea of indeterminacy such as neutrosophic rings [2], refined neutrosophic rings [3], and spaces [4,14,17-20].

Neutrosophic number theory is a new branch of study concerning with the properties of neutrosophic rings especially neutrosophic integers [5]. Concepts such as divisibility, congruencies, Diophantine equations in neutrosophic rings were studied and handled [6-7,15].

In classical number theory, a triple (x, y, z) is called a Pythagoras triple in the commutative ring R if and only if $x^2 + y^2 = z^2$. These triples have a deep connection with Fermat's last theorem [8]. Pythagoras triples were studied widely in [9,10,11]. In addition, a Pythagoras 4-tiple (Quadruple) is a 4-tiple (x, y, t, z) such that $x^2 + y^2 + t^2 = z^2$[12]. These special 4-tiples have been studied in many works, see [13].

In neutrosophic rings and refined neutrosophic rings, the problem of finding Pythagoras triples\4-tiples is still open. This motivates us to study this problem and to find the conditions of Pythagoras triples in neutrosophic rings and refined neutrosophic rings. Also, we will present algorithms for transforming a classical Pythagoras triple to its corresponding neutrosophic and refined neutrosophic one.

All rings in theorems are considered commutative.

2. Main Discussion

Definition [2]:

Let R be a ring, then $R(I) = \{a + bI; a, b \in R\}$ is called a neutrosophic ring.

The addition on R(I) is defined as follows:

$(a + bI) + (c + dI) = a + c + (b + d)I.$

The multiplication on R(I) is defined as follows:

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

$(a+bI).(c+dI) = a.c + I(ad+bc+bd)$.

Remark that, the element I is defined with the logical property $I^2 = I$.

Definition [3]:

Let R be a ring, the refined neutrosophic ring is defined as follows:

$R(I_1, I_2) = \{a + bI_1 + cI_2; a, b, c \in R\}$.

The operations on $R(I_1, I_2)$ are defined with the logical property $I_1I_1 = I_1, I_1I_2 = I_2I_1 = I_1, I_2I_2 = I_2$.

For more details about neutrosophic and refined neutrosophic rings, see [3-7].

3. Neutrosophic Pythagoras Triples

Definition:

Let R(I) be a commutative neutrosophic ring, the triple $(x, y, z); x = a + bI, y = c + dI, z = m + nI$

is called a Pythagoras triple if and only if

$x^2 + y^2 = z^2$.

Theorem:

Let R(I) be a commutative neutrosophic ring, and the triple $(x, y, z); x = a + bI, y = c + dI, z = m + nI$ is a Pythagoras triple, then $(a, c, m), (a + b, c + d, m + n)$ are Pythagoras triples in the ring R.

Proof:

From the equation $x^2 + y^2 = z^2$, we get:

$a^2 + c^2 + I(b^2 + d^2 + 2ab + 2cd) = m^2 + I(2mn + n^2)$. This implies that

$\begin{cases} a^2 + c^2 = m^2 \\ b^2 + d^2 + 2ab + 2cd = 2mn + n^2 \end{cases}$, by adding the first equation to the second one, we get:

$(a+b)^2 + (c+d)^2 = (m+n)^2$, thus $(a, c, m), (a + b, c + d, m + n)$ are Pythagoras triples in the ring R.

Theorem:

Let R be a commutative ring, R(I) be its corresponding neutrosophic ring. Assume that

$(a, c, m), (b, d, n)$ are two Pythagoras triples in the ring R, hence

$(a + (b - a)I, c + (d - c)I, m + (n - m)I)$ is a Pythagoras triple in R(I).

Proof:

Let us compute $(a + (b - a)I)^2 + (c + (d - c)I)^2 = a^2 + c^2 + I(b^2 + a^2 - 2ab + c^2 + d^2 - 2dc + 2ab - 2a^2 + 2cd - 2c^2) = m^2 + I(n^2 - m^2) = (m + (n - m)I)^2$

This means that $(a + (b - a)I, c + (d - c)I, m + (n - m)I)$ is a Pythagoras triple in R(I).

According to the previous two theorems, we are able to present an algorithm to find all neutrosophic Pythagoras triples.

The Algorithm of finding neutrosophic Pythagoras triples:

To find all non-trivial Pythagoras triples in the commutative neutrosophic ring R(I), follow these steps:

Step 1: Find all Pythagoras triples in the classical ring R (if their number is finite), and write them as follows:

$(a_1, b_1, c_1), (a_2, b_2, c_2), \dots (a_n, b_n, c_n)$.

Step 2: Write the previous triples as elements in the direct product of the ring R with itself, i.e. $(a_i, a_j), (b_i, b_j), (c_i, c_j)$.

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

Step 3: Compute the corresponding neutrosophic element as follows:

$(a_i, a_j) \rightarrow (a_i + I(a_j - a_i)), (b_i, b_j) \rightarrow (b_i + I(b_j - b_i)), (c_i, c_j) \rightarrow (c_i + I(c_j - c_i))$, then we get the corresponding neutrosophic Pythagoras triple.

Example:

Consider $Z_3 = \{0,1,2\}$ the ring of integers modulo 4. The Pythagoras triples in Z_3 are:

$(0,0,0), (0,1,1), (0,2,1), (0,1,2), (1,0,1), (2,0,1)$.

First of all, we will write the triples in the ring $Z_3 \times Z_3$

Since has 36 Pythagoras triples, we will put only 5 of them to check the algorithm.

$\{(0,0), (0,1), (0,1)\}, \{(0,0), (1,0), (1,0)\}, \{(0,0), (0,0), (0,0)\},$

$\{(0,0), (0,1), (0,2)\}, \{(0,0), (1,1), (1,2)\}.$

The corresponding neutrosophic Pythagoras triples are:

$(0, I, I), (0,1 - I, 1 - I), (0,0,0), (0, I, 2I), (0,1,1 + I)$. And so on.

Example:

Let Z be the ring of integers, Z(I) be the corresponding neutrosophic ring.

It is known that (3,4,5),(5,12,13) are two Pythagoras triples in Z.

The corresponding Pythagoras triples in the ring $Z \times Z$ are:

$\{(3,5), (4,12), (5,13)\}, \{(5,3), (12,4), (13,5)\}, ,$

$\{(3,3), (4,4), (5,5)\}, \{(5,5), (12,12), (13,13)\}.$

Now, we can get the corresponding neutrosophic Pythagoras triples:

$\{(3 + 2I, 4 + 8I, 5 + 8I), (5 - 2I, 12 - 8I, 13 - 8I)\}, \{(3,4,5)\}, \{(5,12,13)\}.$

Remark: If we exchanged the positions between the first and the second element, we get the same triple, for example $(4,3,5)$ is the same of $(3,4,5)$.

Experimental Results on Some Rings:

Table (1)

Pythagoras Triples' pairs In Z_4	**Corresponding Pythagoras Triple In Z_4(I)**
(0,2,0), (0,2,0)	(0,2,0)
(0,2,2), (0,2,2)	(0,2,2)
(1,0,3), (1,0,3)	(1,0,3)
(3,0,1), (3,0,1)	(3,0,1)
(0,2,0), (0,2,2)	*(0,2,2I)*
(0,2,0), (1,0,3)	*(I,2-2I,3I)*
(0,2,0), (3,0,1)	*(3I,2,I)*
(0,2,2),(1,0,3)	*(I,2-2I,2+I)*
(0,2,2), (3,0,1)	*(3I,2-2I,2-I)*
(1,0,3), (3,0,1)	*(1-2I,0,3-2I)*

Table (2)

Pythagoras Triples' pairs In Z_3	**Corresponding Pythagoras Triple In Z_3(I)**
(0,2,1), (0,2,1)	(0,2,1)
(0,1,2), (0,1,2)	(0,1,2)
(1,2,0), (1,2,0)	(1,2,0)
(0,2,1), (0,1,2)	(0,2-I,1+I)

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

(0,2,1), (1,2,0)	*(I,2,1-I)*
(0,1,2), (1,2,0)	*(I,1+I,2-2I)*

Table (3)

Pythagoras Triples' pairs In *Z*	**Corresponding Pythagoras Triple In *Z*(I)**
(3,4,5),(3,4,5)	(3,4,5)
(6,8,10), (6,8,10)	(6,8,10)
(5,12,13),(5,12,13)	(5,12,13)
(3,4,5),(4,3,5)	(3+I,4-I,5)
(3,4,5), (6,8,10)	*(3+3I,4+4I,5+5I)*
(3,4,5),(8,6,10)	*(3+5I,4+2I,5+5I)*
(4,3,5),(8,6,10)	*(4+4I,3+3I,5+5I)*
(6,8,10),(8,6,10)	*(6+2I,8-2I,10)*
(6,8,10),(5,12,13)	*(6-I,8+4I,10+3I)*
(6,8,10),(12,5,13)	*(6+6I,8-3I,10+3I)*
(8,6,10),(12,5,13)	*(8+4I,6-I,10+3I)*
(3,4,5),(5,12,13)	*(3+2I,4+8I,5+8I)*
(3,4,5),(12,5,13)	*(3+9I,4+I,5+8I)*
(4,3,5),(12,5,13)	*(4+8I,3+2I,5+8I)*
(4,3,5),(5,12,13)	*(4+I,3+9I,5+8I)*

Refined Neutrosophic Pythagoras Triples

Theorem:

Let (x,y,z); $x = x_0 + x_1 I_1 + x_2 I_2, y = y_0 + y_1 I_1 + y_2 I_2, z = z_0 + z_1 I_1 + z_2 I_2$

be a triple in the commutative refined neutrosophic ring $R(I_1, I_2)$, then (x,y,z) is a Pythagoras triple if and only if (x_0, y_0, z_0),

$(x_0 + x_1 + x_2, y_0 + y_1 + y_2, z_0 + z_1 + z_2), (x_0 + x_2, y_0 + y_2, z_0 + z_2)$.

Are Pythagoras triples in the classical ring R.

Proof:

From the equation $x^2 + y^2 = z^2$, we get:

$x_0{}^2 + y_0{}^2 + I_1(x_1{}^2 + y_1{}^2 + 2x_0x_1 + 2y_0y_1 + 2x_1x_2 + 2y_1y_2) + I_2(x_2{}^2 + y_2{}^2 + 2x_0x_2 + 2y_0y_2) = z_0{}^2 + I_1(z_1{}^2 + 2z_0z_1 + 2z_1z_2) + I_2(z_2{}^2 + 2z_0z_2)$, so that, we get the following equations:

$$\begin{cases} x_0{}^2 + y_0{}^2 = z_0{}^2 \ (1) \\ x_1{}^2 + y_1{}^2 + 2x_0x_1 + 2y_0y_1 + 2x_1x_2 + 2y_1y_2 = z_1{}^2 + 2z_0z_1 + 2z_1z_2 (2) \\ x_2{}^2 + y_2{}^2 + 2x_0x_2 + 2y_0y_2 = z_2{}^2 + 2z_0z_2 \ (3) \end{cases}$$

Equation (3) can be written as follows:

$(x_0 + x_2)^2 - (x_0)^2 + (y_0 + y_2)^2 - y_0{}^2 = (z_0 + z_2)^2 - (z_0)^2$, thus

$(x_0 + x_2)^2 + (y_0 + y_2)^2 = (z_0 + z_2)^2$.

On the other hand, from equation (2), by a similar discussion we can get:

$(x_0 + x_1 + x_2)^2 + (y_0 + y_1 + y_2)^2 = (z_0 + z_1 + z_2)^2$.

This means that (x,y,z) is a Pythagoras triple in the refined neutrosophic ring $R(I_1, I_2)$ if and only if

$(x_0, y_0, z_0), (x_0 + x_1 + x_2, y_1 + y_2 + y_3, z_1 + z_2 + z_3), (x_0 + x_2, y_0 + y_2, z_0 + z_2)$ are Pythagoras triples in the corresponding classical ring R.

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

Example:

Let $Z_5 = \{0,1,2,3,4\}$ be the ring of integers modulo $5, Z_5(I_1, I_2)$ be the corresponding refined neutrosophic ring.

For example, consider the following triple of refined neutrosophic integers $x = 1 + I_1 + I_2, y = 2 + 2I_2, z = 1 + 3I_1 + I_2$. Now, we will check if it is a Pythagoras triple.

$f(x) = (1,3,2), f(y) = (2,4,4), f(z) = (1,0,2)$. It is clear that:

$1^2 + 2^2 \neq 1^2, 3^2 + 4^2 = 0^2, 2^2 + 4^2 \neq 2^2$, this means that (x, y, z) is a not a refined neutrosophic Pythagoras triple.

Theorem:

Let $R(I_1, I_2)$ be a refined neutrosophic ring, suppose that $(x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2)$ are three Pythagoras triples in R. Hence

$(x, y, z) = (x_0 + I_1(x_1 - x_2) + I_2(x_2 - x_0), y_0 + I_1(y_1 - y_2) + I_2(y_2 - y_1), z_0 + I_1(z_1 - z_2) + I_2(z_2 - z_0)$ is a Pythagoras triple in $R(I_1, I_2)$.

Proof:

By computing $x^2 + y^2$, we get:

$x^2 + y^2 = {x_0}^2 + {y_0}^2 + I_1({x_1}^2 + {y_1}^2 - {x_2}^2 - {y_2}^2) + I_2({x_2}^2 + {y_2}^2 - {x_0}^2 - {y_0}^2)=$

${z_0}^2 + I_1({z_1}^2 - {z_2}^2) + I_2({z_2}^2 - {z_0}^2) = z^2$. Thus (x, y, z) is a Pythagoras triple.

An algorithm for finding Pythagoras triples in $R(I_1, I_2)$

To find Pythagoras triples in the commutative refined neutrosophic ring $R(I_1, I_2)$, follow these steps:

Step 1: Find Pythagoras triples in the ring $\times R \times R$.

Step 2: For every Pythagoras triple $((x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2))$ in the ring $\times R \times R$, there exists a corresponding Pythagoras triple in the refined neutrosophic ring $R(I_1, I_2)$ as follows:

$(x, y, z) = (x_0 + I_1(x_1 - x_2) + I_2(x_2 - x_0), y_0 + I_1(y_1 - y_2) + I_2(y_2 - y_1), z_0 + I_1(z_1 - z_2) + I_2(z_2 - z_0)$.

Example:

Let Z be the ring of integers, it is known that $((3,4,5), (6,8,10), (5,12,13))$ are three Pythagoras triples. Now, we can get a Pythagoras triple in the refined neutrosophic ring of integers $Z(I_1, I_2)$ as follows:

$x = 3 + (6 - 5)I_1 + (5 - 3)I_2 = 3 + I_1 + 2I_2, y = 4 + (8 - 12)I_1 + (12 - 4)I_2 = 4 - 4I_1 + 8I_2, z = 5 + (10 - 13)I_1 + (13 - 5)I_2 = 5 - 3I_1 + 8I_2$.

It is clear that $x^2 + y^2 = z^2$.

Neutrosophic Pythagoras 4-tiples

Theorem:

Let R be any commutative ring, R(I) be its corresponding neutrosophic ring, then a neutrosophic 4-tiple $(x, y, t, z); x = x_0 + x_1 I, y = y_0 + y_1 I, t = t_0 + t_1 I, z = z_0 + z_1 I$ is a Pythagoras 4-tiple if and only if

$(x_0, y_0, t_0, z_0), (x_0 + x_1, y_0 + y_1, t_0 + t_1, z_0 + z_1)$,

are Pythagoras 4-tiples in R.

Proof:

Firstly, suppose that (x, y, t, z) is a Pythagoras 4-tiple in R(I), hence

$x^2 + y^2 + t^2 = z^2$, this implies that

${x_0}^2 + {y_0}^2 + {t_0}^2 + I[(x_0 + x_1)^2 + (y_0 + y_1)^2 + (t_0 + t_1)^2 - {x_0}^2 - {y_0}^2 - {t_0}^2] = {z_0}^2 + I[(z_0 + z_1)^2 - {z_0}^2]$, thus

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

${x_0}^2+{y_0}^2+{t_0}^2={z_0}^2, and\ (x_0+x_1)^2+(y_0+y_1)^2+(t_0+t_1)^2-{x_0}^2-{y_0}^2-{t_0}^2=(z_0+z_1)^2-{z_0}^2$, so that

$(x_0+x_1)^2+(y_0+y_1)^2+(t_0+t_1)^2=(z_0+z_1)^2$, thus, we get

$(x_0,y_0,t_0,z_0),(x_0+x_1,y_0+y_1,t_0+t_1,z_0+z_1)$,

are Pythagoras 4-tiples in R.

For the converse, we suppose that $(x_0,y_0,t_0,z_0),(x_0+x_1,y_0+y_1,t_0+t_1,z_0+z_1)$,

are Pythagoras 4-tiples in R, this means that ${x_0}^2+{y_0}^2+{t_0}^2={z_0}^2, and\ (x_0+x_1)^2+(y_0+y_1)^2+(t_0+t_1)^2=(z_0+z_1)^2$.

Now, we can write:

$x^2+y^2+t^2={x_0}^2+{y_0}^2+{t_0}^2+I[(x_0+x_1)^2+(y_0+y_1)^2+(t_0+t_1)^2-{x_0}^2-{y_0}^2-{t_0}^2]={z_0}^2+I[(z_0+z_1)^2-{z_0}^2]=z^2$, this implies that (x,y,t,z) is a Pythagoras 4-tiple in R(I).

Example:

Consider the commutative ring $R=Z[i]=\{a+bi;a,b\in Z\}$, $R(I)=\{x+yI;x,y\in Z[i]\}$ is the corresponding neutrosophic ring.

We have $(i,1,1,1),(1,-i,-1,1)$ are two Pythagoras 4-triples in R, that is because:

$i^2+1^2+1^2=1^2, 1^2+(-i)^2+-1^2=1^2$.

Now, we will find the corresponding Pythagoras 4-tiples in R(I).

The previous two Pythagoras 4-tiples can be written as duplets as follows:

$(i,1),(1,-i),(1,-1),(1,1)$. The neutrosophic elements which are related with those duplets are:

$x=i+I(1-i), y=1+I(-i-1), t=1+I(-1-1)=1-2I, 1+I(1-1)=1$.

Now, let us check $x^2+y^2+t^2=-1+I(-2i+2i+2)+1+I(2i-2i-2)+1+4I-4I=1=z^2$.

Refined neutrosophic Pythagoras 4-tiples

Theorem:

Let (x,y,t,z); $x=x_0+x_1I_1+x_2I_2, y=y_0+y_1I_1+y_2I_2, t=t_0+t_1I_1+t_2I_2, z=z_0+z_1I_1+z_2I_2$

be a 4-triple in the commutative refined neutrosophic ring $R(I_1,I_2)$, then (x,y,t,z) is a Pythagoras 4-triple if and only if (x_0,y_0,t_0,z_0),

$(x_0+x_1+x_2,y_0+y_1+y_2,t_0+t_1+t_2,z_0+z_1+z_2),(x_0+x_2,y_0+y_2,t_0+t_2,z_0+z_2)$.

Are Pythagoras 4-triples in the classical ring R.

Proof:

Firstly, suppose that (x,y,t,z) is a 4-tiple in $R(I_1,I_2)$, hence it is a Pythagoras 4-tiple if and only if

$x^2+y^2+t^2=z^2$, this is equivalent to

${x_0}^2+{y_0}^2+{t_0}^2+I_1[(x_0+x_1+x_2)^2+(y_0+y_1+y_2)^2+(t_0+t_1+t_1)^2-(x_0+x_2)^2-(y_0+y_2)^2-(t_0+t_2)^2]+I_2[(x_0+x_2)^2+(y_0+y_2)^2+(t_0+t_2)^2-{x_0}^2-{y_0}^2-{t_0}^2]=$

${z_0}^2+I_1[(z_0+z_1+z_2)^2-(z_0+z_2)^2]+I_2[(z_0+z_2)^2-{z_0}^2]$,

Thus, we get

$$\begin{cases} {x_0}^2+{y_0}^2+{t_0}^2={z_0}^2 \\ (x_0+x_1+x_2)^2+(y_0+y_1+y_2)^2+(t_0+t_1+t_1)^2=(z_0+z_1+z_2)^2. \\ (x_0+x_2)^2+(y_0+y_2)^2+(t_0+t_2)^2=(z_0+z_2)^2 \end{cases}$$

Thus, the proof is complete.

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

Example:

Consider the commutative ring $R = Z[i] = \{a + bi; a, b \in Z\}$, $R(I_1, I_2) = \{x + yI_1 + zI_2; x, y, z \in Z[i]\}$ is the corresponding refined neutrosophic ring.

We have $(i, 1,1,1), (1, -i, -1,1), (1,3, i, -3)$ are three Pythagoras 4-triples in R, that is because:

$i^2 + 1^2 + 1^2 = 1^2, 1^2 + (-i)^2 + -1^2 = 1^2, 1^2 + (3)^2 + i^2 = (-3)^2$.

Now, we will find the corresponding Pythagoras 4-tiples in R(I).

The previous three Pythagoras 4-tiples can be written as classical triples as follows:

$(i, 1,1), (1, -i, 3), (1, -1, i), (1,1, -3)$. The refined neutrosophic elements which are related with those triples are:

$x = i + I_1[1 - 1] + I_2[1 - i] = i + (1 - i)I_2, y = 1 + I_1[-i - 3] + I_2[3 - 1] = 1 + (-3 - i)I_1 + (2)I_2, t = 1 + I_1[-1 - i] + I_2[i - 1] = 1 + (-1 - i)I_1 + (-1 + i)I_2, z = 1 + I_1[1 + 3] + I_2[-3 - 1] = 1 + (4)I_1 + (-4)I_2$.

Now, we check it as follows:

$x^2 + y^2 + t^2 = -1 + I_2[-2i + 2i + 2] + 1 + I_1[8 + 6i - 12 - 4i - 6 - 2i] + I_2[4 + 4] + 1 + I_1[2i - 2 - 2i + 4] + I_2[-2i - 2 + 2i] = 1 + (-8)I_1 + (8)I_2 = z^2$.

4. Conclusion

In this paper, we have studied a famous number theoretical problem in some refined neutrosophic rings, where we have presented novel algorithms to find Pythagoras triples and 4-tiples in commutative neutrosophic rings and refined neutrosophic rings, as well as the necessary and sufficient conditions for these triples\4-tiples. Also, we have illustrated many examples to clarify the validity of our algorithms. As a future research direction, we aim to study the Pythagoras triples\4-tiples problem in n-refined neutrosophic rings, and n-cyclic refined neutrosophic rings.

References

[1] F. Smarandache, "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets," Inter. J. Pure Appl. Math., pp. 287-297, 2005.
[2] Kandasamy, V.W.B., and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phonex, Arizona 2006.
[3] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F.," Refined Neutrosophic Rings II",International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94, 2020.
[4] Agboola, A.A.A,.andAkinleye, S.A,. "Neutrosophic Vector Spaces", Neutrosophic Sets and Systems, Vol. 4, pp. 9-17, 2014.
[5] Ceven, Y., and Tekin, S., " Some Properties of Neutrosophic Integers", Kırklareli University Journal of Engineering and Science, Vol. 6, 2020.
[6] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, 2021.
[7] Abobala, M., "Partial Foundation of Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 39, 2021.
[8] R. A. Trivedi, S. A. Bhanotar, Pythagorean Triplets - Views, Analysis and Classification, IIOSR J. Math., 11 (2015).
[9] J. Rukavicka, Dickson's method for generating Pythagorean triples revisited, Eur. J. Pure Appl.Math., 6 (2013).
[10] J. H. Barnett, Generating Pythagorean triples: the methods of Pythagoras and of Plato via Gnomons, Number Theory, 7 (2017).
[11] E. Maor, The Pythagorean Theorem: A 4,000-Year History, Princeton, New Jersey: Princeton University Press, (2007).
[12] T.Roy and F J. Sonia, A Direct Method To Generate Pythagorean Triples And Its Generalization To Pythagorean Quadruples And n-tuples, , Jadavpur University, India (2010)
[13] Sarkis, M., " On the Solutions Of Fermat's Diophantine Equation In 3-refined Neutrosophic Ring of Integers", Neoma Journal of Mathematics and Computer Science, 2023.
[14] Celik, M., and Olgun, N., " On the Classification Of Neutrosophic Complex Inner Product Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023

[15] Ahmad, K., Bal, M., and Aswad, M., " A Short Note on The Solutions Of Fermat's Diophantine Equation In Some Neutrosophic Rings", Journal Of Neutrosophic and Fuzzy Systems, 2022.
[16] Celik, M., and Olgun, N., " An Introduction to Neutrosophic Real Banach And Hillbert Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.
[17] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.
[18] Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.
[19] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol 5, pp59-66, 2020.
[20] Abobala, M,. "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol 5, pp72-75, 2020.

Doi: https://doi.org/10.54216/IJNS.200310
Received: October 08, 2022 Accepted: March 04, 2023